

Black Holes and Quantum Gravity

Xi Dong



Snowmass Community Summer Study Workshop

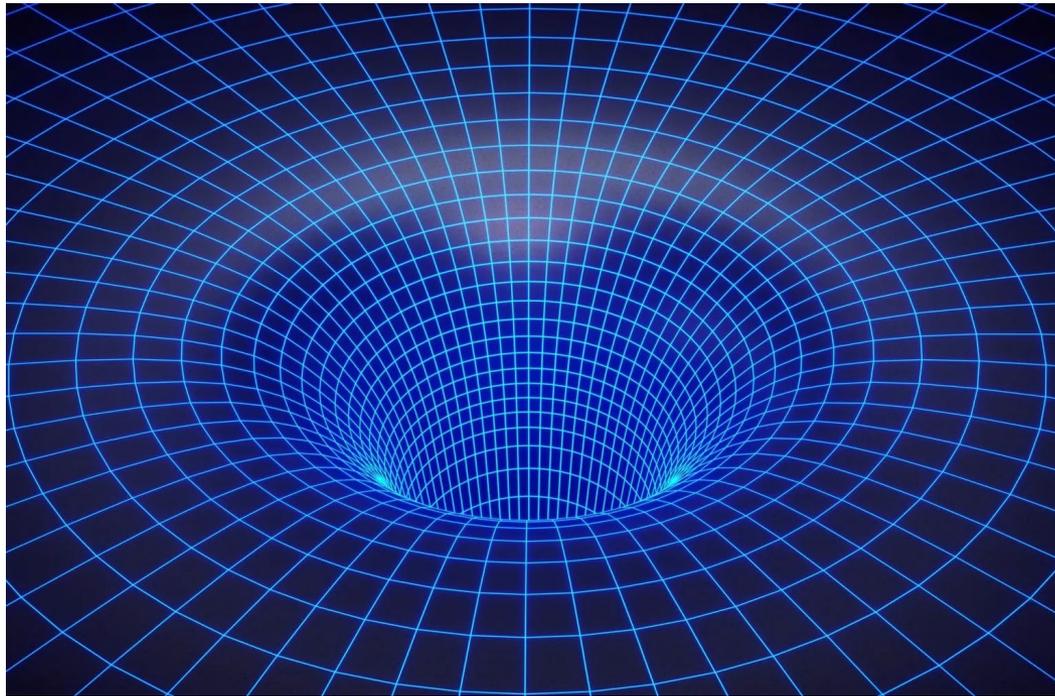
University of Washington, Seattle

July 23, 2022

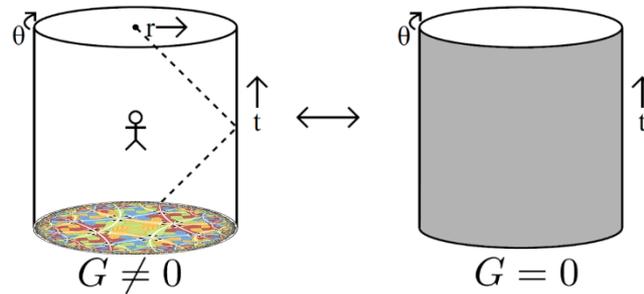
Plan

1. Emergence of spacetime
2. Black hole information paradox
3. Wormholes

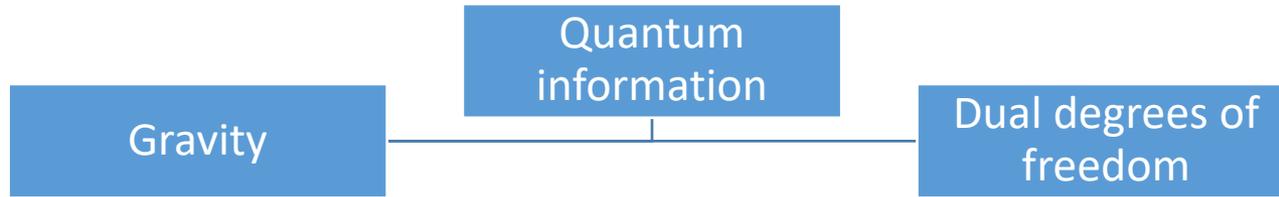
1. Emergence of spacetime



- The last 25 years have seen the discovery of **non-perturbatively** precise theories of quantum gravity: **AdS/CFT** and **matrix models** .
- In these theories, the gravitational spacetime **emerges holographically** from the collective behavior of dual, non-gravitational degrees of freedom.
- This emergence is sharpest in AdS/CFT:



- But even there, the basic mechanism has only been clarified recently.
- Insights from a quantum information perspective have been central to these recent developments.



- These connections grew out of a better understanding of the **fine-grained gravitational entropy**:

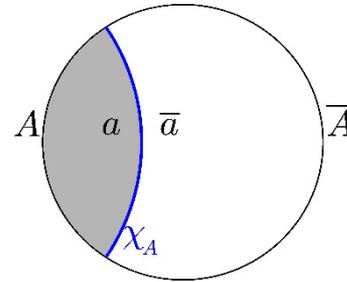
$$S = -\text{Tr } \rho \log \rho$$

- Also known as the von Neumann entropy or entanglement entropy.
- ρ : density matrix of a subsystem.
- I will focus on AdS/CFT, where the subsystems are chosen to be boundary subregions.

Gravitational entropy formula

At leading order, the entropy of a boundary subregion A is given by the **area of a bulk extremal surface χ_A** :

$$S = \text{ext} \frac{\text{Area}}{4G}$$



[Ryu & Takayanagi '06]

- Similar to the entropy of a black hole.
- Works in static as well as dynamical spacetimes.
- Generalized to higher-derivative gravity and Renyi entropy.

[Hubeny, Rangamani & Takayanagi '07]

[XD '13; Camps '13; Miao & Guo '15;
XD '16; XD & Marolf '20; ...]

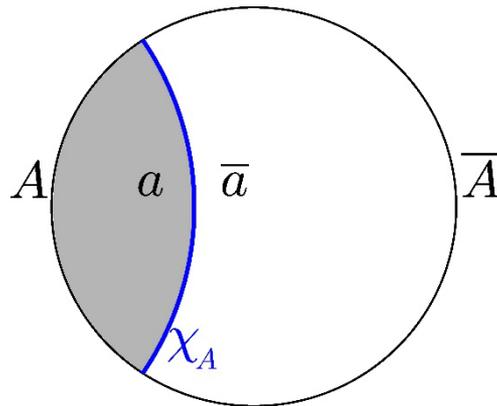
- Quantum corrections now understood.
- Has been derived by semiclassical gravitational path integrals.

[Faulkner, Lewkowycz & Maldacena '13; Engelhardt & Wall '14; XD & Lewkowycz '18; ...]

[Lewkowycz & Maldacena '13; XD, Lewkowycz & Rangamani '16; ...]

Entanglement wedge

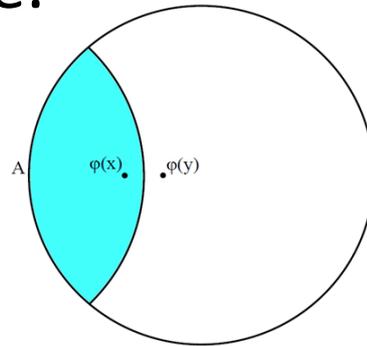
- A bulk region bounded by the extremal surface.
- On a time slice, it is between the boundary subregion A and the extremal surface χ_A :



- A key notion in describing the central concept of **subregion-subregion duality**.

Subregion-subregion duality

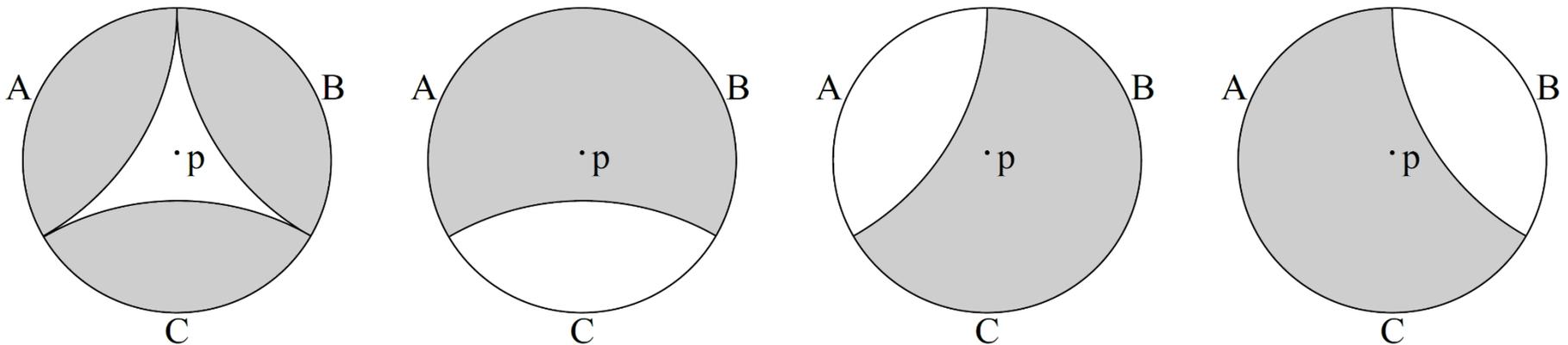
“The quantum information in a boundary subregion is exactly the information needed to describe its entanglement wedge.”



- In particular, bulk operators in the entanglement wedge can be **reconstructed** as boundary operators on that subregion.
- This “**entanglement wedge reconstruction**” refines how spacetime emerges from the boundary.

Holographic code

- A striking aspect of subregion-subregion duality:
It functions as a **quantum error-correcting code**!
- Information about the bulk is stored redundantly on the boundary.



- This is another example showing the importance of the quantum information perspective.

- So why does subregion-subregion duality work?
- The answer lies in the **quantum corrections** to the gravitational entropy formula:

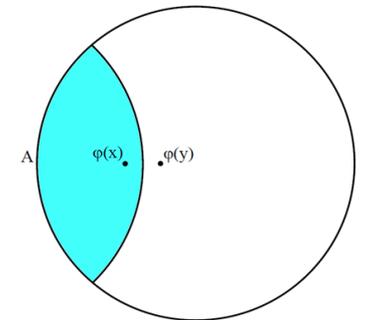
$$S = \text{ext} \frac{\text{Area}}{4G_N}$$

Quantum corrections

These corrections come from bulk matter fields and gravitons.

- The answer is surprisingly simple:

$$S = \text{ext} \frac{\text{Area}}{4G_N} \quad \longrightarrow \quad S = \text{ext} \left(\frac{\text{Area}}{4G_N} + S_{\text{bulk}} \right)$$



[Engelhardt & Wall '14]

- Replaces the extremal surface with a **quantum extremal surface (QES)**.
- Called the QES formula.
- Matches one-loop FLM result. [Faulkner, Lewkowycz & Maldacena '13]
- Has been derived by semiclassical gravitational path integrals.

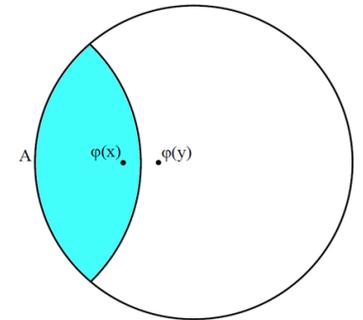
[XD & Lewkowycz '18; Penington, Shenker, Stanford & Yang;
Almheiri, Hartman, Maldacena, Shaghoulian & Tajdini; ...]

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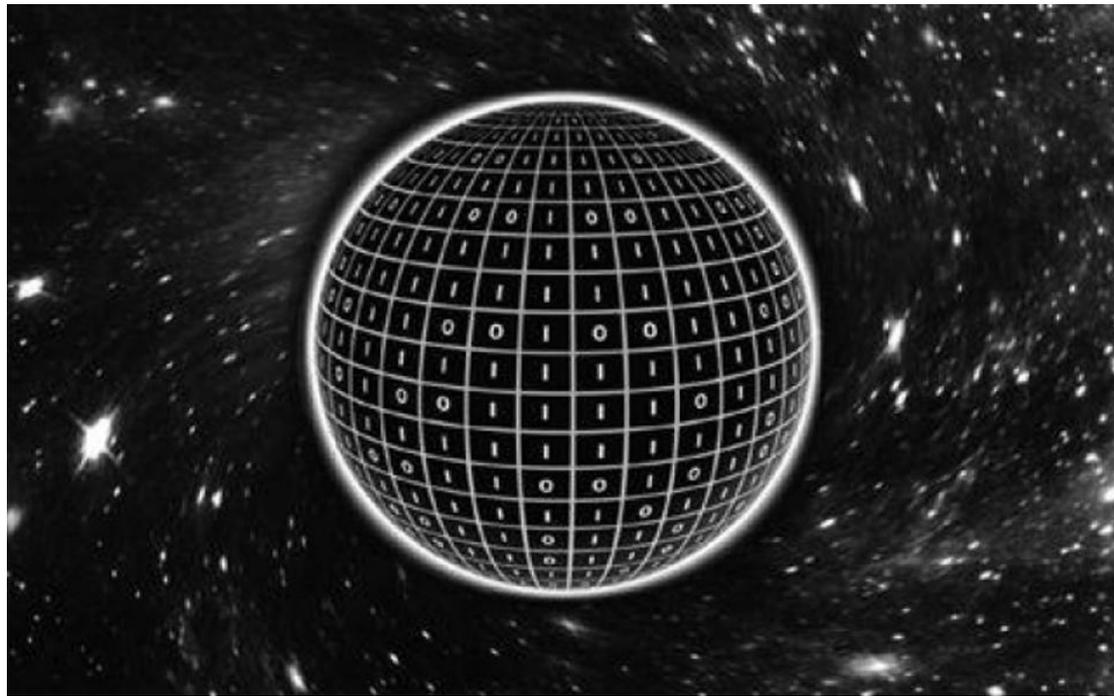
[Engelhardt & Wall '14]

- S_{bulk} defined in the entanglement wedge.
- A change to the state in the entanglement wedge would show up on the boundary subregion, at least entropy-wise.
- This strongly suggests subregion-subregion duality holds. (Can be promoted to a proof.)

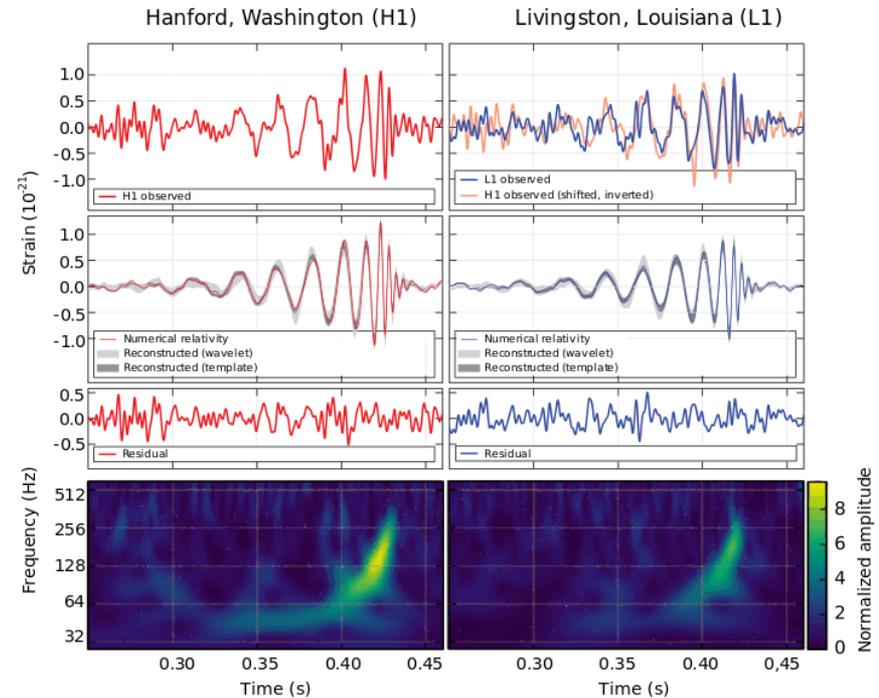
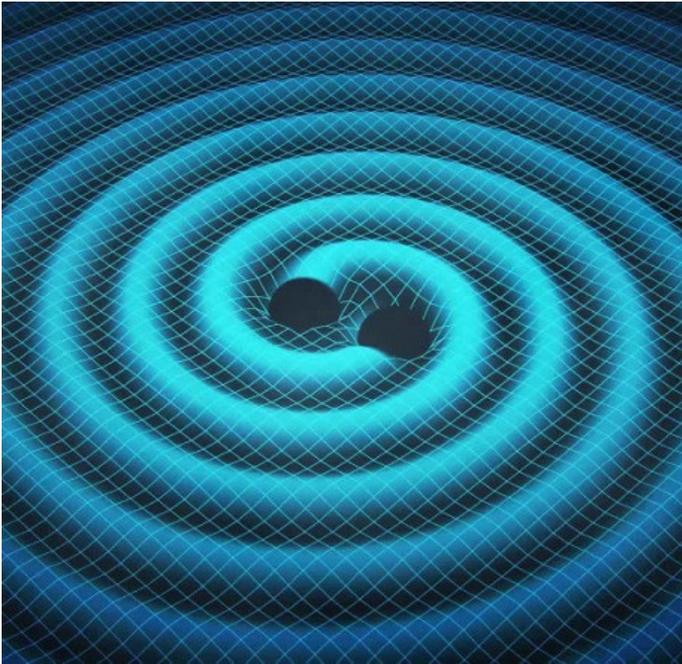
$$S = \text{ext} \left(\frac{\text{Area}}{4G_N} + S_{\text{bulk}} \right)$$

- Often, quantum correction S_{bulk} has a small effect compared to the area term.
- But sometimes, it has a dramatic effect.
- An important example: [old black holes](#).

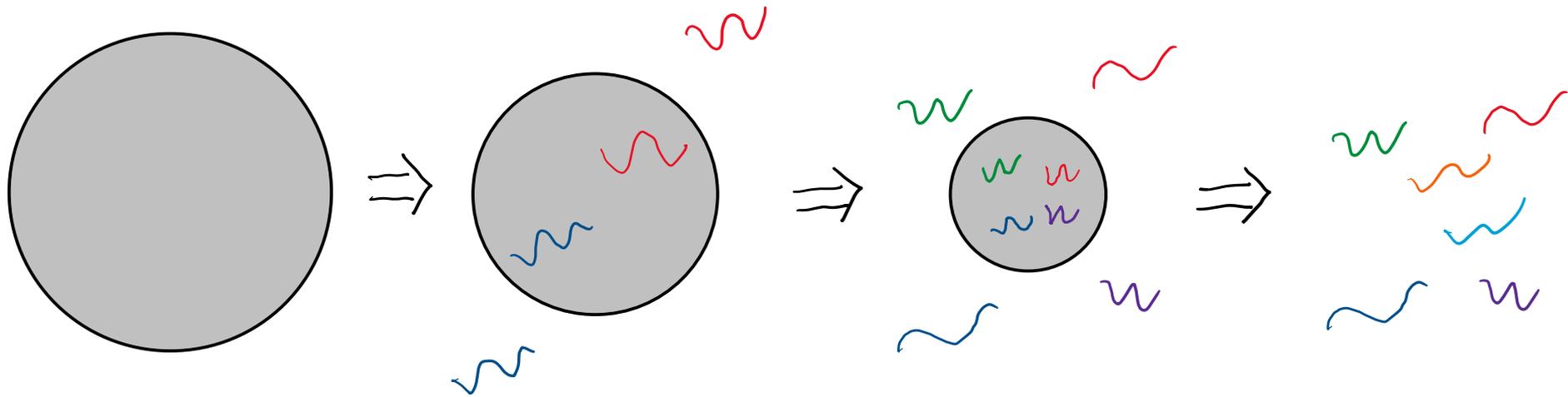
2. Black hole information paradox



- Classically, black holes are well understood conceptually in terms of Einstein's general relativity.

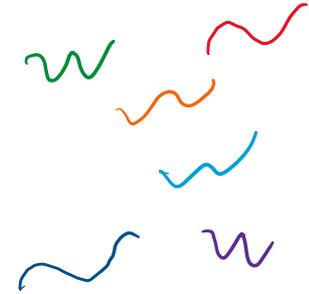
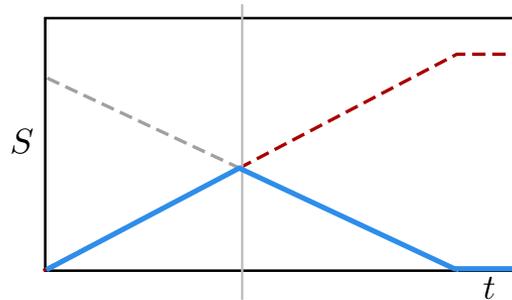
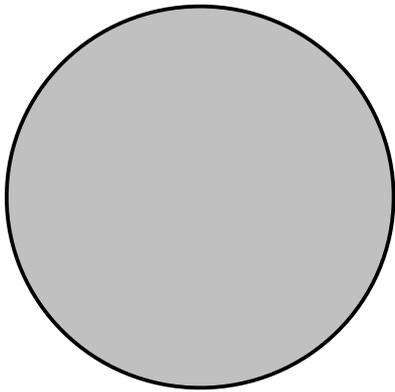


- But quantum mechanically, they are the main obstacle in formulating a satisfactory theory of gravity.



- Quantum black holes evaporate.
- Hawking radiation produces entanglement between the black hole interior and the radiation.
- The ‘paradox’:
Once the black hole completely evaporates, there is nothing for the radiation to be entangled with.

- A sharp way of thinking about the paradox is the **entropy of Hawking radiation**.
- Consider a black hole formed in a pure state.

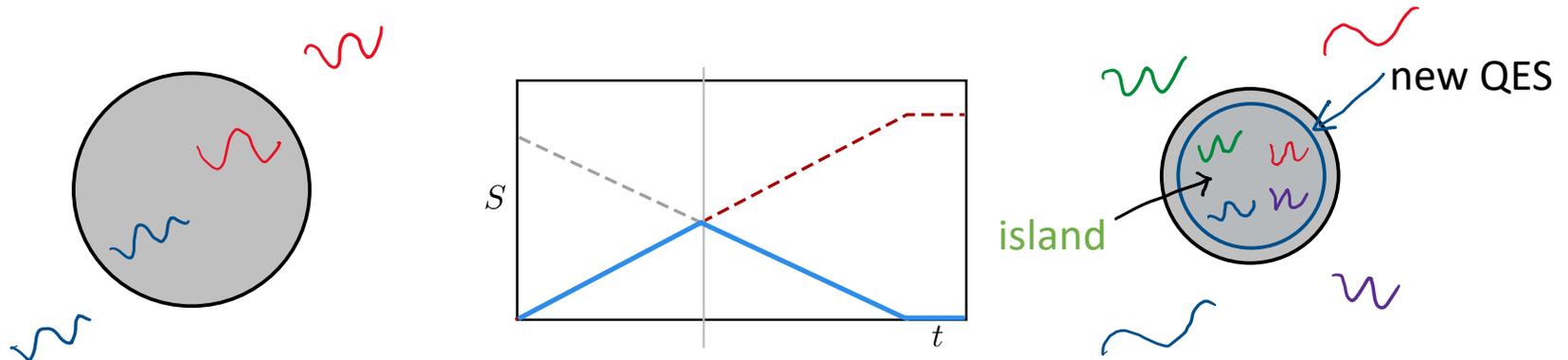


- Hawking's calculation \Rightarrow S **grows monotonically**.
- But unitary evaporation \Rightarrow the **Page curve**.

Entropy of Hawking radiation

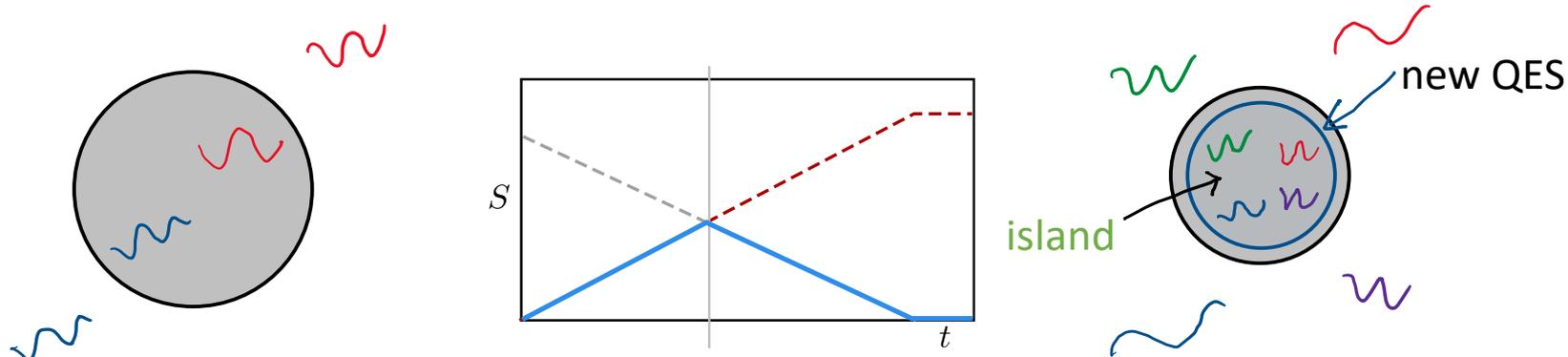
- Remarkably, the QES formula agrees with unitarity!

$$S = \text{ext} \left(\frac{\text{Area}}{4G_N} + S_{\text{bulk}} \right)$$



- A **new QES** in the black hole interior becomes dominant at late stages of evaporation.
- This new QES encloses an **'island'** in the old black hole.

[Penington; Almheiri, Engelhardt, Marolf & Maxfield; Almheiri, Mahajan, Maldacena & Zhao; Penington, Shenker, Stanford & Yang; Almheiri, Hartman, Maldacena, Shaghoulian & Tajdini; ...]



- The entanglement wedge of the Hawking radiation now contains the island in the black hole interior.
- According to subregion-subregion duality, the island is actually encoded in the Hawking radiation!

- But how do we know the QES formula predicting the island is correct?

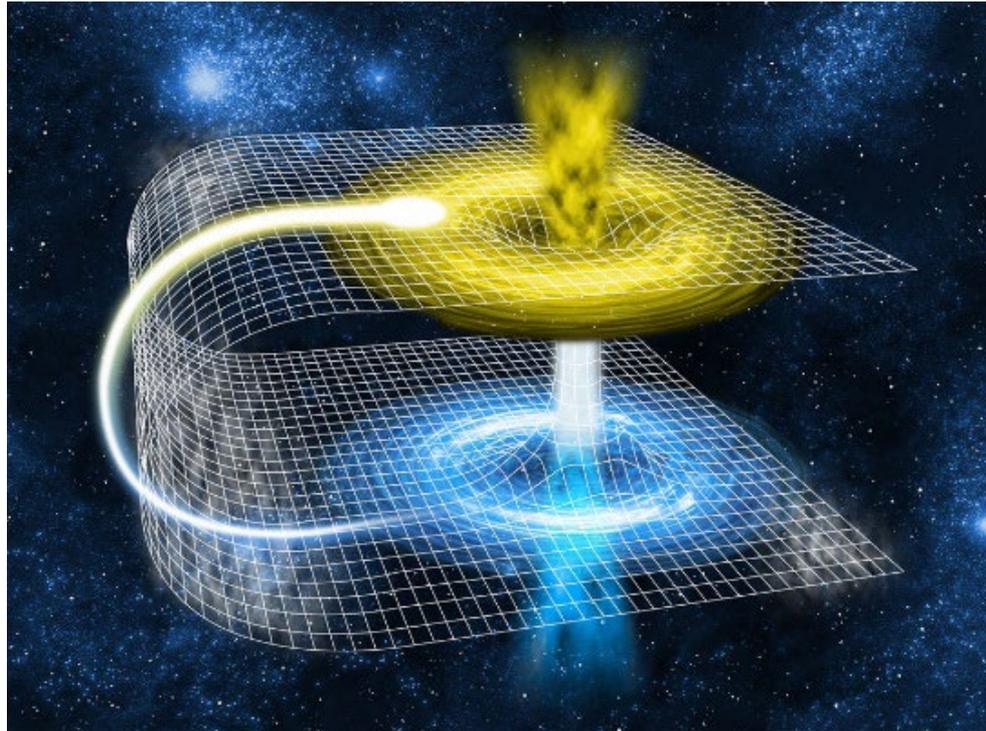
1. It has been derived by semiclassical gravitational path integrals.

[Lewkowycz & Maldacena '13; XD & Lewkowycz '18; ...]

2. Digging into how this works for old black holes reveals new gravitational instantons – ‘**replica wormholes**’.

[Penington, Shenker, Stanford & Yang; Almheiri, Hartman, Maldacena, Shaghoulian & Tajdini; ...]

3. Wormholes



Replica wormholes

- Are gravitational instantons with nontrivial topology.

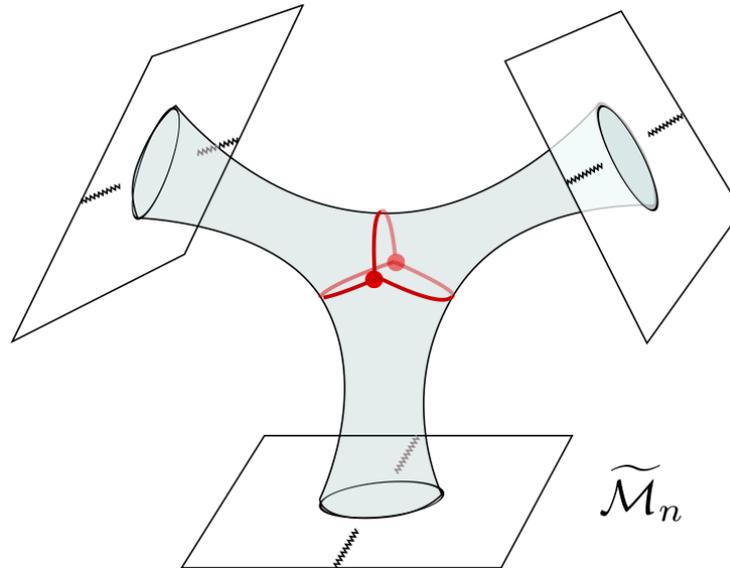


Figure from [Almheiri, Hartman, Maldacena, Shaghoulian & Tajdini]

- Produce the island.

Example: 'purity' (second Renyi)

$$\text{tr}(\rho^2) \quad \begin{cases} = 1, & \text{pure} \\ < 1, & \text{mixed} \end{cases}$$

Is the expectation value of 'swap operator' on a double copy of ρ .

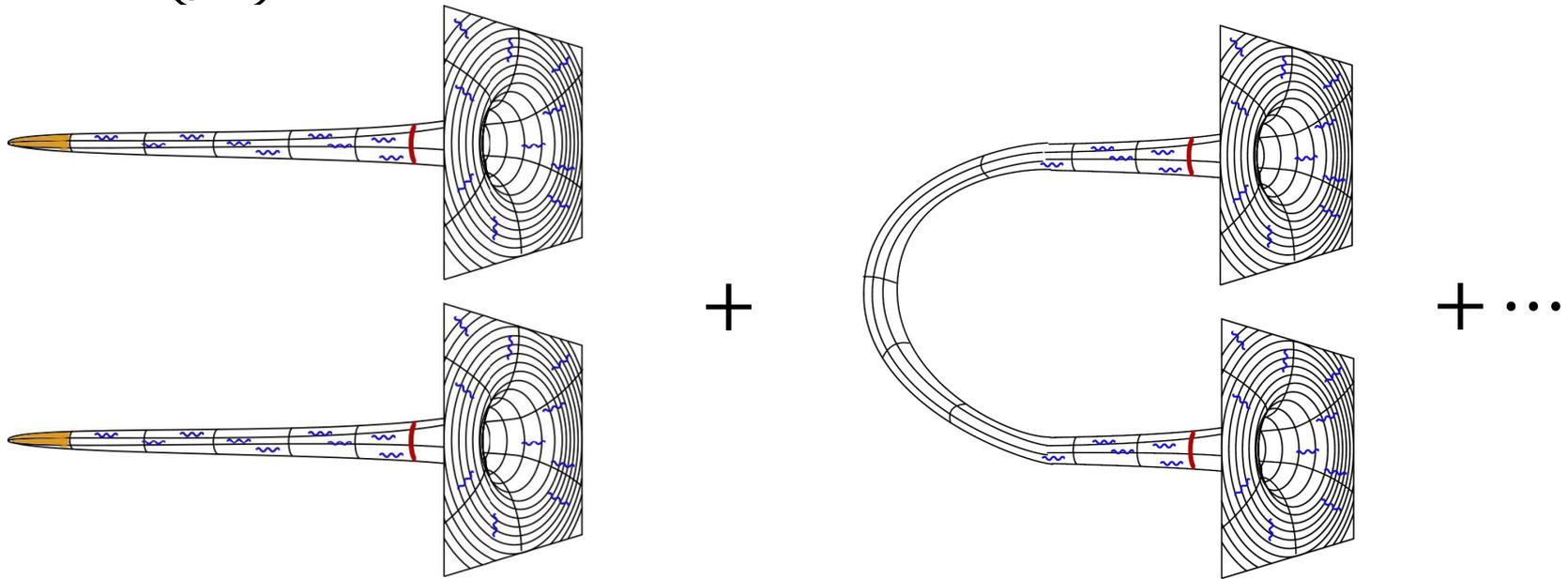


Purity of Hawking radiation

- Calculate it using gravitational path integral:

$$\text{tr}(\rho^2) =$$

[Penington, Shenker, Stanford & Yang; Almheiri, Hartman, Maldacena, Shaghoulian & Tajdini; ...]



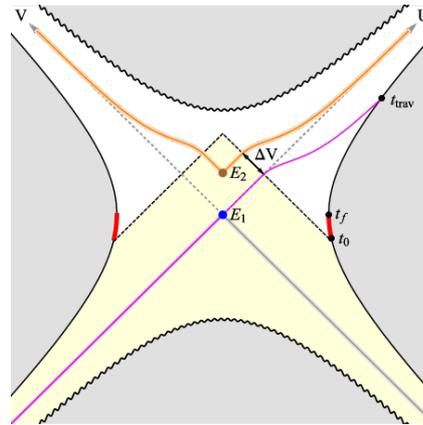
Hawking saddle
Dominates at early times
Highly mixed Hawking radiation

Replica wormhole
Dominates at late times
Produces island and purifies Hawking radiation

Figure from Hartman's
Snowmass talk at KITP

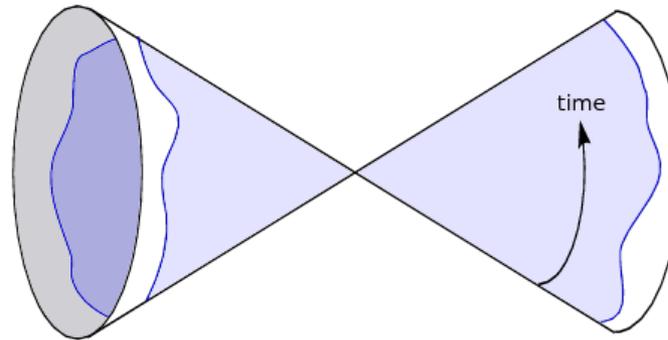
Other wormholes

- Traversable wormholes



[Gao, Jafferis & Wall;
Maldacena, Stanford & Yang;
Maldacena & Qi;
Horowitz, Marolf, Santos & Wang]

- ‘Double cone’ explaining the ramp in the spectral form factor.



[Cotler et al;
Saad, Shenker & Stanford;
Cotler & Jensen]

- Bra-ket wormholes [Page; Anous, Kruthoff & Mahajan; Chen, Gorbenko & Maldacena]
- Wormholes and baby universes

[Giddings, Strominger & Polchinski; Marolf & Maxfield; Giddings & Turiaci]

- It is satisfying to see the QES formula, island, and replica wormholes come together to produce the unitary Page curve.
- But many mysteries remain.
- In particular, we are not done yet with the black hole information paradox.
- For example, how do we see the detailed unitarity of the black hole S-matrix?

- Looking forward, it is important to try a wide range of ideas and approaches in solving future problems.
- There are many interesting ideas that I have not time to mention.
- I will end with a few open questions.
- See white paper for more. [Bousso, XD, Engelhardt, Faulkner, Hartman, Shenker & Stanford]

Open questions

- It seems almost a miracle for the semiclassical gravitational path integral to be able to determine the fine-grained entropy of Hawking radiation.

Why does low-energy gravity know so much about the UV?

What else does it know?

Open questions

- Relatedly, we might have expected string theory – as a well-established UV completion of gravity – to play a larger role in the interplay between quantum information and spacetime.

So can we study gravitational entropy and bulk reconstruction in string theory?

To what extent can we understand the gravitational entropy formula as arising from stringy edge modes?

Open questions

- Low dimensional toy models of gravity, such as the SYK model and its low energy limit – JT gravity, involve some sort of averaging over an ensemble of quantum systems.
- Necessary to solve the factorization problem caused by wormholes.

What role does averaging play in higher dimensional, more realistic gravity?

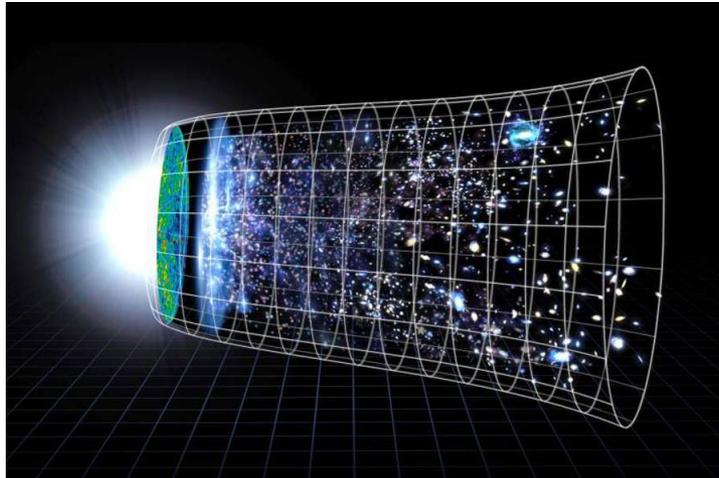
Open questions

- How are basic features of the bulk gravitational theory represented by properties of the holographic code?

Does this help clarify long-standing puzzles such as the origin of local physics on sub-AdS scales?

Open questions

- How do we use all these ideas to further decode physics behind a horizon, or in cosmologies resembling our world?



- To what extent will ‘quantum gravity in the lab’ allow us to gain experimental insight into some of these issues?

Thank you!