Theory Perspectives on the W Mass

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Seattle Snowmass Summer Meeting 2022

22 July 2022

Outline

1. Current Status
2. Theory Cross Checks of CDF
   - Resummation Studies
   - PDF Studies
   - Width Studies
3. Future Studies
   - Non-Perturbative Functions
   - Mixed QCD-EW Corrections
   - Corrections to Factorization

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Standard Model: W Mass

Standard Model EW Fit

$$M_W^2\left(1-\frac{M_W^2}{M_Z^2}\right)=\frac{\pi\alpha}{\sqrt{2}G_F}\left(1+\Delta r\right)$$

$$\Delta r=\Delta\alpha-\frac{c_W^2}{s_W^2}\Delta\rho+\Delta r_{\text{rem}}\left(M_H\right)\ ,$$

where s_W^2 is the Weinberg angle, $\Delta\alpha$ is the correction to α from the light fermions, $\Delta\rho$ is the correction to the ρ parameter, and Δr_{rem} contains all corrections containing the Higgs mass.

Parameter	Fit Result
G_μ [GeV^{-2}]	1.1663787 $\times 10^{-5}$
$\alpha(0)^{-1}$	137.035999139
$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$	0.027627 $\pm$ 0.000096
M_Z [GeV]	91.1883 $\pm$ 0.0021
M_H [GeV]	125.21 $\pm$ 0.12
m_t [GeV]	172.75 $\pm$ 0.44
M_W [GeV]	80.3591 $\pm$ 0.0052

Table reproduced from: `HEPFit` Group (2112.07274).

Experimental Measurements

- CDF Run II results most precise
- 7σ tension with SM
- 3σ tension between CDF-II and ATLAS result
- Missing LHCb result: 80,354 $\pm$ 32 MeV

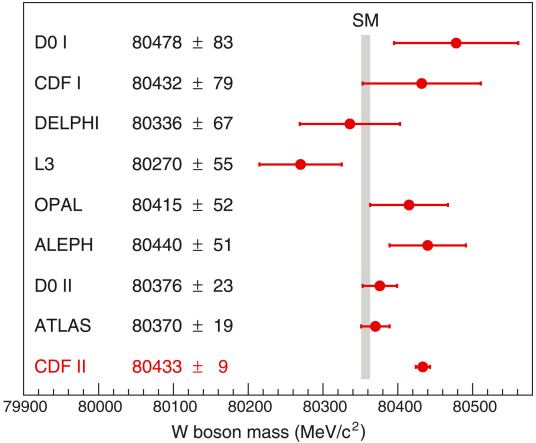

Figure reproduced from CDF-II measurement (Science 376, 170).

Extracting W Mass from Data

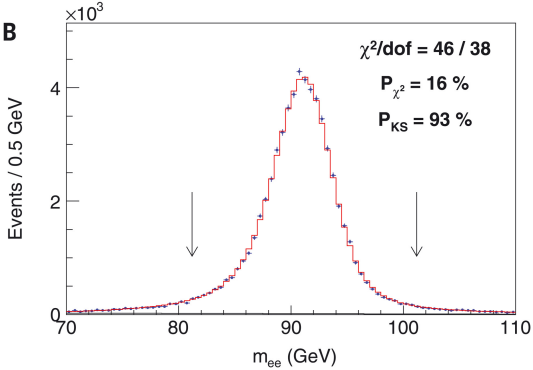

Figure reproduced from CDF-II measurement (Science 376, 170).

- Can't measure invariant mass directly due to neutrino
- Look at sensitive observables
  - $M_T = \sqrt{2\left(p_T^\ell p_T^\nu - \vec{p}_T^{\,\ell} \cdot \vec{p}_T^{\,\nu}\right)}$
  - p_T^ℓ
  - p_T^ν with $(\vec{p}_T^{\,\nu} = -\vec{p}_T^{\,\ell} - \vec{u}_T)$
- Requires precise theory calculation
- Fit theory templates with varying M_W

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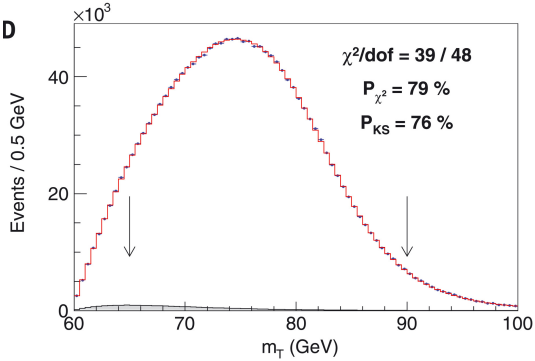

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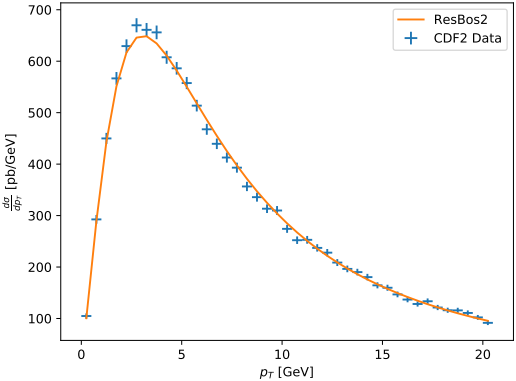
Breakdown of Fixed Order

- Perturbative series has terms proportional to $\alpha_s^n \log^m \left(\frac{p_T^2}{M_W^2} \right)$, $m \leq 2n$
- As $p_T^W \to 0$ the series no longer converges
- Need to include corrections to all orders by resumming the series



Analytic vs. Numeric Resummation

Analytic:

- Formal resummation (focus here on b-space CSS resummation)
- Pros:
  - High precision and accuracy
- Cons:
  - Inclusive only
  - Numerically expensive
- Used by CDF to obtain M_W

Numerical

- Parton Showers (Pythia, Sherpa, Herwig, Dire, Vincia)
- Pros:
  - Exclusive final states
  - Quick
- Cons:
  - Currently only LL with some subleading effects included
- Used by ATLAS to obtain M_W

Parton Showers

Evolution Equation

$$\frac{df_a(x,t)}{d\ln t} = \sum_{b=q,g} \int_0^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} \left[P_{ab}(z)\right]_+ f_b\left(\frac{x}{z}, t\right)$$

- $f_a(x,t)$ is the obervable being evolved
- $P_{ab}(z)$ is the evolution (splitting) kernel
- Solve using Markovian Monte-Carlo algorithms
- Treat P_{ab} as a probability
- Virtual corrections defined at kinematic endpoints by $+$ prescription

Collins-Soper-Sterman Formalism

Resummation

$$\frac{d\sigma_{\text{res}}}{dQ^2 d^2\vec{q}_T dy d\Omega} = \sigma \int \frac{d^2 b}{(2\pi)^2} e^{i\vec{q}_T\cdot\vec{b}} \tilde{W},$$

$$\tilde{W} = e^{-S(b)} \;\; C \otimes f(x_A, C_3/b) C \otimes f(x_B, C_3/b)$$

$$S(b) = \int_{\frac{C_1^2}{b^2}}^{C_2^2 Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \left[\ln\left(\frac{C_2^2 Q^2}{\bar{\mu}^2}\right) \; A(\bar{\mu}) \; + \; B(\bar{\mu}) \right]$$

[Collins, Soper, Sterman, '85] [...]

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- Electroweak cross section

[Collins, Soper, Sterman, '85] [...]

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- Sudakov factor

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- Collinear factors

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- Electroweak cross section
- Sudakov factor
- Collinear factors
- Perturbative Coefficients (A, B, C)

[Collins, Soper, Sterman, '85] [...]

Order Definitions

Order	Boundary Condition	Anomalous Dimension γ_i (non-cusp)	Anomalous Dimension Γ_{cusp}, β	Fixed Order Matching
LL	1	-	1-loop	-
NLL	1	1-loop	2-loop	-
NLL' (+ NLO)	α_s	1-loop	2-loop	α_s
NNLL (+ NLO)	α_s	2-loop	3-loop	α_s
NNLL' (+ NNLO)	α_s^2	2-loop	3-loop	α_s^2
N^3LL (+ NNLO)	α_s^2	3-loop	4-loop	α_s^2
N^3LL' (+ N^3LO)	α_s^3	3-loop	4-loop	α_s^3
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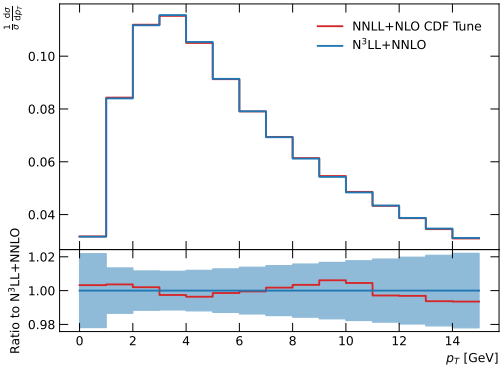
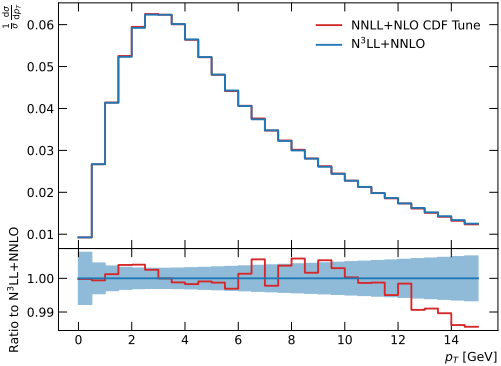
- All terms known to this accuracy [2104.07509]

Higher Order Correction effects

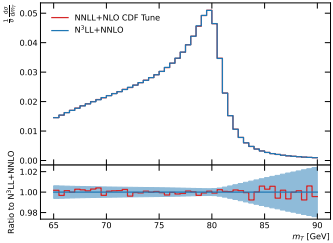




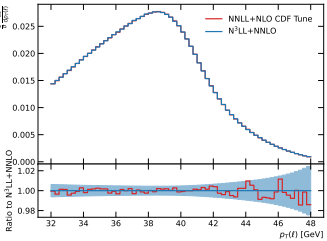
Tuned result: [2205.02788]

- Fit to $p_T(Z) < 15$ GeV
- $g_2 = 0.662$ GeV^2
- $\alpha_S(M_Z) = 0.120$
- Tuned PDF set: `CT18NNLO_as_120`

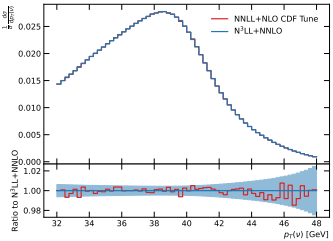
Higher Order Correction effects



Best Fit: $M_W = 80,386$ MeV



Best Fit: $M_W = 80,388$ MeV



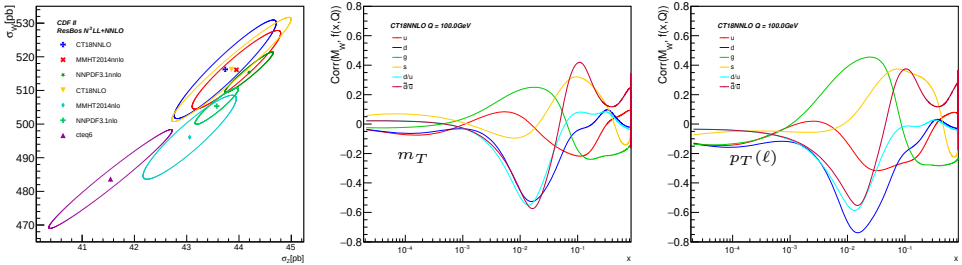
Best Fit: $M_W = 80,389$ MeV

	Mass Shift [MeV]	
Observable	ResBos2	+Detector Effect+FSR
m_T	1.5 ± 0.5	$0.2 \pm 1.8 \pm 1.0$
$p_T(\ell)$	3.1 ± 2.1	$4.3 \pm 2.7 \pm 1.3$
$p_T(\nu)$	4.5 ± 2.1	$3.0 \pm 3.4 \pm 2.2$

[2205.02788]

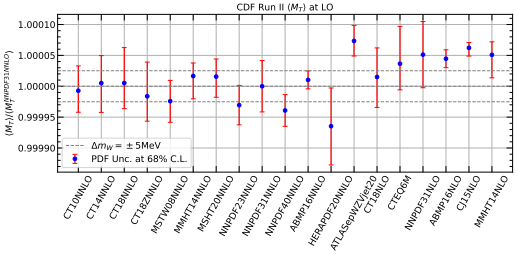
PDF Studies

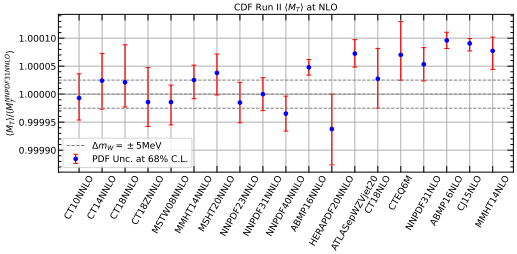
	m_T		$p_T(\ell)$		$p_T(\nu)$	
PDF Set	NNLO	NLO	NNLO	NLO	NNLO	NLO
CT18	0.0 ± 1.3	1.8 ± 1.2	0.0 ± 15.9	2.0 ± 14.3	0.0 ± 15.5	2.9 ± 14.2
MMHT2014	1.0 ± 0.6	2.6 ± 0.6	6.2 ± 7.8	36.7 ± 7.0	3.9 ± 7.5	36.0 ± 6.7
NNPDF3.1	1.1 ± 0.3	2.1 ± 0.4	2.1 ± 3.8	13.5 ± 4.9	5.4 ± 3.7	10.0 ± 4.9
CTEQ6M	N/A	2.8 ± 0.9	N/A	19.0 ± 10.4	N/A	20.9 ± 10.2



PDF Studies

δM_W in MeV	sta.	NNPDF3.1	CT18	MMHT14	NNPDF4.0	MSHT20	CTEQ6M
$\langle M_T \rangle$(LO)	–	$0^{+8.3}_{-8.3}$	$-1.0^{+8.3}_{-11.4}$	$-3.3^{+7.4}_{-4.2}$	$+7.8^{+5.1}_{-5.1}$	$-3.1^{+6.7}_{-5.7}$	$-7.3^{+8.4}_{-12.0}$
χ^2 fit (LO)	8.0	$0^{+7.6}_{-7.6}$	$-1.0^{+5.4}_{-8.6}$	$-3.3^{+6.1}_{-3.0}$	$+8.0^{+3.7}_{-3.7}$	$-3.0^{+5.0}_{-4.0}$	$-7.3^{+5.6}_{-9.3}$
$\langle M_T \rangle$(NLO)	–	$0^{+5.9}_{-5.9}$	$-4.2^{+8.8}_{-13.3}$	$-5.0^{+6.7}_{-5.3}$	$+6.9^{+6.2}_{-6.2}$	$-7.6^{+7.9}_{-6.7}$	$-14.0^{+9.0}_{-11.9}$
χ^2 fit (NLO)	8.0	$0^{+4.2}_{-4.2}$	$-4.3^{+5.4}_{-10.1}$	$-5.1^{+4.8}_{-3.4}$	$+7.1^{+4.5}_{-4.5}$	$-7.8^{+5.7}_{-4.5}$	$-14.6^{+5.8}_{-5.4}$
CDF	9.2	$0^{+3.9}_{-3.9}$	–	–	–	–	-3.3





[2205.03942]

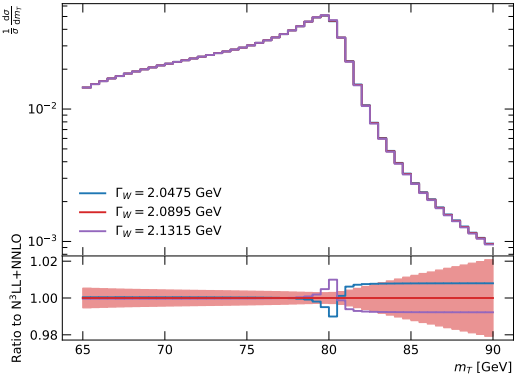
Width Studies

Results from: 2205.02788 (at $N^3LL + NNLO$)

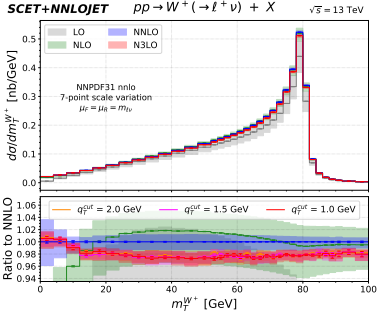
Width	Mass Shift [MeV]
2.0475 GeV	2.0 ± 0.5
2.1315 GeV	0.3 ± 0.5
NLO	1.2 ± 0.5

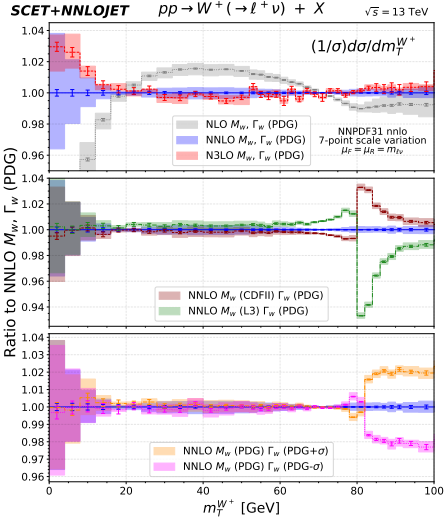
Results from: 2205.03942 (at NLO)

Variation	$\Gamma_W = 2,085 \pm 42$
χ^2 fit (NLO)	$0^{+7.1}_{-6.8}$



N^{3}LO Corrections to Width





Results from SCET+NNLOJet [2205.11426]

- Calculation using q_T subtraction method
- Need to propagate through analysis to determine impact on M_W, but should be minor

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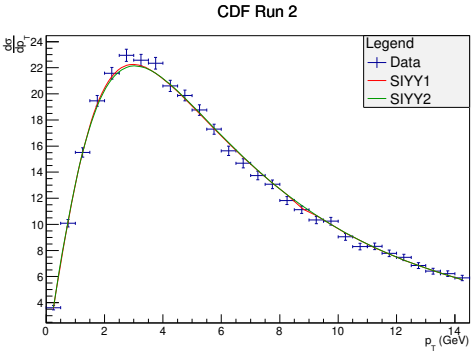
- Lower limit goes to zero as b goes to infinity
- Requires evaluation of $\alpha_s(C_1/b)$ which is non-perturbative
- Need to introduce a non-perturbative cutoff (b^*-prescription):

$$b^* = \frac{b}{\sqrt{1 + \frac{b^2}{b_{\text{max}}^2}}}$$

BLNY Form

$$S_{NP}(b) = -b^2 \left(g_1 + g_2 \log\left(\frac{Q}{2Q_0}\right) + g_1 g_3 \log(100 x_1 x_2) \right)$$

- g_1 and g_3 extracted from global fit
- g_2 tuned to reproduce CDF-II p_T^Z
- M_W vs. M_Z captured in Q dependence
- No flavor dependence included
- Ongoing LQCD studies [1011.1213, 1111.4249, 1506.07826, 1706.03406, 1911.00800, 2003.06063, 2005.14572, 2103.16991, 2106.13027, 2107.11930, 2204.00200, 2205.02788]



NOTE: SIYY2 is the same functional form as BLNY, but with $b_{\text{max}} = 1.5$ GeV^{-1} [1406.3073]

Flavor Dependence

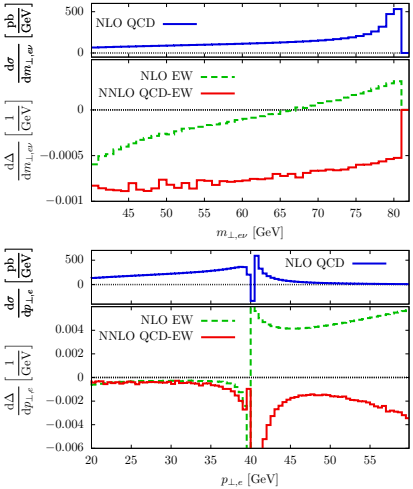
- Study on flavor dependence for $\sqrt{s} = 7$ TeV LHC
- $S_{NP}(b) = -b^2(g_a + g_{evo}\log(Q^2/Q_0^2)$, where g_a is the flavor dependent piece
- Found shift could be up to 10 MeV
- Additional studies are required to validate
- Unclear what the global shift would be
- Investigate flavor dependence on the lattice

Set	u_v	d_v	u_s	d_s	others
1	0.34	0.26	0.46	0.59	0.32
2	0.34	0.46	0.56	0.32	0.51
3	0.55	0.34	0.33	0.55	0.30
4	0.53	0.49	0.37	0.22	0.52
5	0.42	0.38	0.29	0.57	0.27

Set	ΔM_W^+		ΔM_W^-	
	M_T	p_T^ℓ	M_T	p_T^ℓ
1	0	-1	-2	3
2	0	-6	-2	0
3	-1	9	-2	-4
4	0	0	-2	-4
5	0	4	-1	-3

Table reproduced from: Phys. Letters B 788 (2019) 542-545

Mixed QCD-EW Corrections



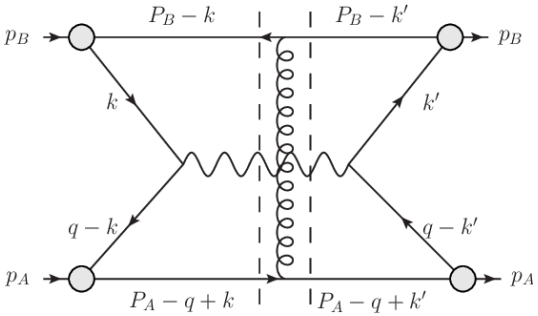
[2009.10386]

- Mixed QCD-EW corrections calculated for Z and W boson at LHC energies

  [1909.08428, 2005.10221, 2009.10386, 2103.02671]
- Rough estimated impact at LHC of 17 ± 2 MeV
- Detailed studies required for CDF
- Need to understand how to combine these mixed corrections with resummation calculations

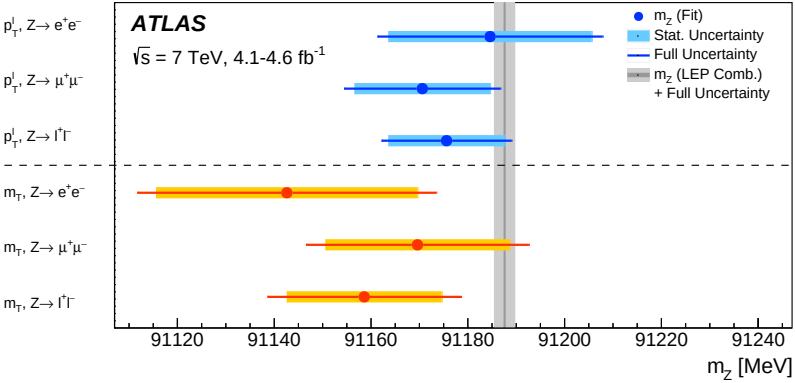
Corrections to Factorization

- Super hard to even start evaluating
- Corrections of order $\Lambda_{QCD}^2/M_W^2 \sim 10$ MeV for M_W [Phys. Lett. B 134 (1984) 263]



[1405.2080]

Question for ATLAS



- Treat one lepton as neutrino and perform M_W measurement procedure
- ATLAS uses this for consistency check, why not calibration?

[1701.07240]

Conclusions

- Tension between SM prediction and CDF
- Not a result of higher order corrections
- More work on PDFs and widths are needed
- Non-perturbative flavor dependence needs to be studied, cannot disentangle with Z data
- Mixed QCD-EW effects at the Tevatron need to be investigated
- Work on understanding power corrections to factorization may be important at current precision