Precision Measurements and Field Theory: A Virtuous Cycle

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This talk

Highlight the connection between precision measurements, (the necessary) precision calculations, and advances on our understanding of field theory

based on …

"not-exact-but-nevertheless-effective"

Magic Zeroes and Hidden Symmetries

N. Craig, **IGG**, A. Vainshtein, Z. Zhang [2112.05770]

which was inspired by…

Naturalness and the muon magnetic moment

N. Arkani-Hamed, K. Harigaya [2106.01373]

which was prompted by…

Muon g-2 @ FNAL [2104.03281]
Muon g-2 @ BNL [hep-ex/0602035]

+

Muon g-2 in the SM [2006.04822]

The Muon Magnetic Dipole

Kannike, *et al* [1111.2551]; Freitas, *et al* [1402.7065]
Arkani-Hamed, Harigaya [2106.01373]

Extension of the SM with two vector-like $SU(2)_L$ lepton doublets L and L^c, and two singlets S and S^c

for concreteness $v \ll m_S, m_L$

$$\Delta\mathcal{L} = -\{m_L L^c L + m_S S^c S\} + \text{h.c.}$$

Yukawas among heavy fermions

$$-\{Y'_V H L S^c + Y_V H^\dagger L^c S\} + \text{h.c.}$$

$$-\{Y_L H l S^c + Y_R H^\dagger L e^c\} + \text{h.c.}$$

mixing between heavy and SM leptons

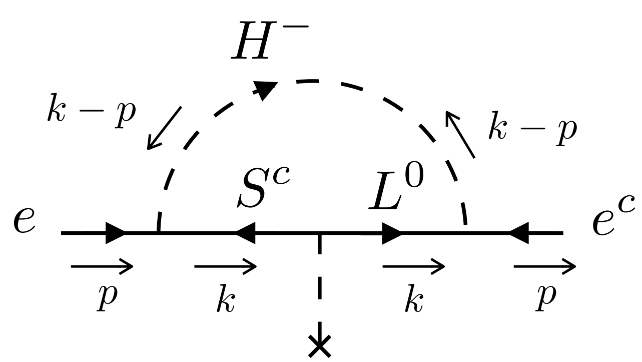
- Two additional mass eigenstates (mostly doublet and mostly singlet)
- All $U(1)_A$ and $U(1)_V$ symmetries broken, except for $U(1)_{V,\text{diag}}$
- Breaking SM $U(1)_A$ requires $Y_L, Y_R \neq 0$ and either Y_V or $Y'_V \neq 0$

WLOG, I'll focus on effects proportional to Y'_V only

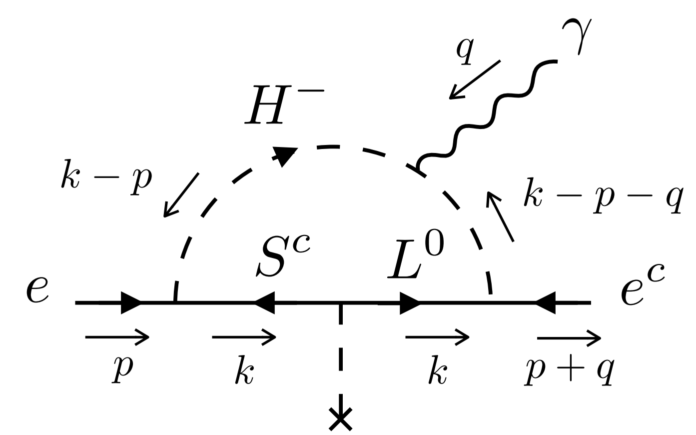
A Magic Zero

$$Y_L, Y_R, Y'_V \neq 0 \;\Rightarrow\; \text{muon } U(1)_A \text{ is broken}$$

Mass



Dipole



Expect: $\Delta m \sim \dfrac{Y_L Y_R Y'^*_V}{16\pi^2} v$

$$\Delta\tau_\mu \sim \frac{e\,\Delta m}{m^2}$$

$$\sim \frac{e\,Y_L Y_R Y'^*_V}{16\pi^2}\frac{v}{m^2} \times \left\{1 + \mathcal{O}\left(\frac{v^2}{m^2}\right)\right\}$$

Find: $\Delta m \sim \dfrac{Y_L Y_R Y'^*_V}{16\pi^2} v$

$$\Delta\tau_\mu \sim 0 + \frac{e\,Y_L Y_R Y'^*_V}{16\pi^2}\frac{v^3}{m^4}$$

Arkani-Hamed, Harigaya (2106.01373)

Too Fast...

$$\mathcal{L}_{\text{free}} = \bar{\Psi}\, i\, \gamma^\mu\, \partial_\mu \Psi = e^\dagger\, i\, \bar{\sigma}^\mu\, \partial_\mu e + e^{c\dagger}\, i\, \bar{\sigma}^\mu\, \partial_\mu e^c = \chi_i^\dagger\, i\, \bar{\sigma}^\mu\, \partial_\mu \chi^i$$

$$\Psi = \begin{pmatrix} e \\ e^{c\dagger} \end{pmatrix} \qquad \chi^1 \equiv e\,, \;\; \chi^2 \equiv e^c$$

Full symmetry of free Dirac lagrangian: $\boxed{U(2) = U(1)_A \times SU(2)_V}$

Mass: $-m\epsilon^{\alpha\beta} e_\beta e^c_\alpha + \text{h.c.} = -\frac{1}{2} m_{ij} \epsilon^{\alpha\beta} \chi^i_\beta \chi^j_\alpha + \text{h.c.}$ $\quad U(2) \xrightarrow{m \neq 0} U(1)_V$

$$m_{12} = m_{21} = m$$

Dipole: $\tau F^{\alpha\beta} e_\beta e^c_\alpha + \text{h.c.} = -\frac{1}{2}(\tau \epsilon_{ij}) F^{\alpha\beta} \chi^i_\beta \chi^j_\alpha + \text{h.c.}$ $\quad U(2) \xrightarrow{\tau \neq 0} SU(2)_V$

$\Rightarrow$ Possible to have $\Delta m = 0$ but $\Delta\tau \neq 0$ if additional d.o.f. respect $SU(2)_V$

Voloshin [1988]

Too Fast...

Voloshin [1988]

$\Rightarrow$ Possible to have $\Delta m = 0$ but $\Delta \tau \neq 0$ if additional d.o.f. respect $SU(2)_V$

Used by Voloshin to build models with large $\Delta\tau_\nu$ but tiny m_ν

On compatibility of small mass with large magnetic moment of neutrino

M. B. Voloshin [1988]

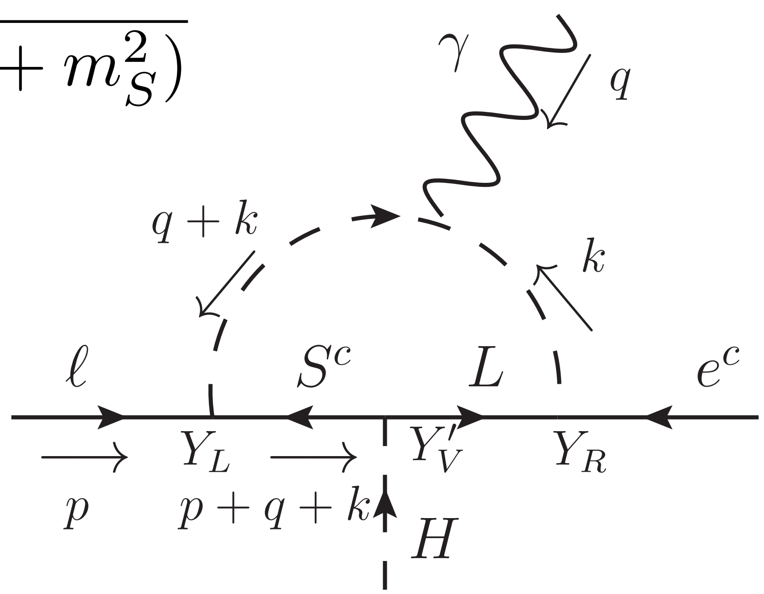
Inspired by experimental anomaly suggesting that solar neutrino flux undergoes time-variations anti-correlated with solar magnetic activity

However… $\Delta m \neq 0$ and $\Delta \tau = 0$ cannot be explained by unbroken subgroup

A total derivative?

Arkani-Hamed, Harigaya [2106.01373]: No symmetry reason, but rather magic zero arises because loop integral is a "total derivative"

$$\mathcal{M} \propto \int \frac{d^4k}{(2\pi)^4} \frac{\epsilon \cdot k}{(k^2)^2} f\left[(k+p)^2\right] \qquad f[u] = \frac{u}{(u+m_L^2)(u+m_S^2)}$$



Expanding to linear order in external momenta:

$$\mathcal{M} \propto \int \frac{d^4k}{(2\pi)^4} \frac{(\epsilon \cdot k)(k \cdot p)}{(k^2)^2} f'[k^2] \propto \int_0^\infty \frac{du}{u} u f'[u] = f(\infty) - f(0)$$

with $f(\infty) = f(0) = 0$

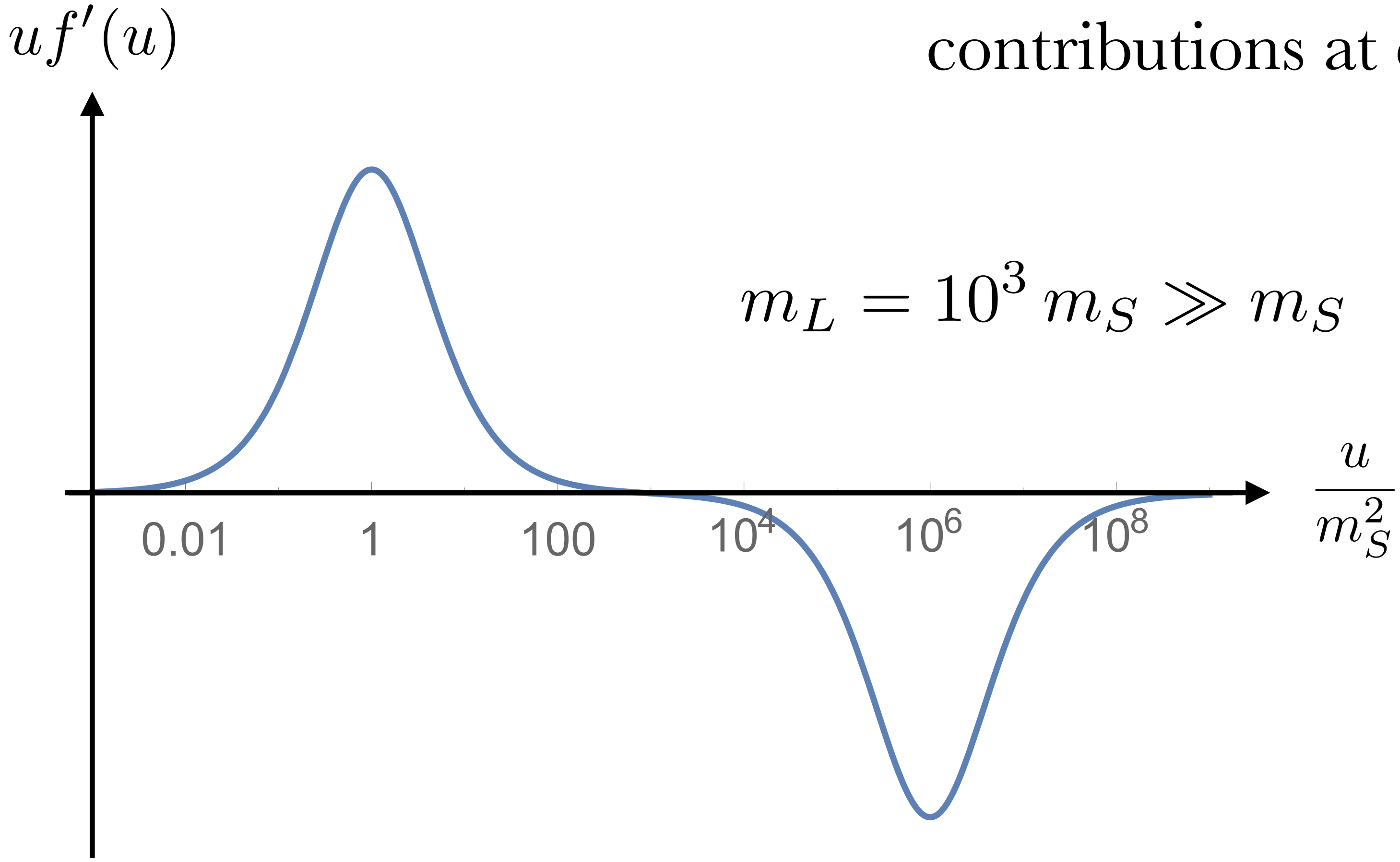
$$\begin{cases} f(\infty) = 0 & \text{because } g\text{-}2 \text{ is calculable} \\ f(0) = 0 & \text{because operator } \left(lH^\dagger\right) HHe^c \text{ is identically zero} \end{cases}$$

cute, but not enough

Cancellation across scales

Arkani-Hamed, Harigaya [2106.01373]

Integral vanishes due to equal and opposite contributions at different scales, m_s and m_L



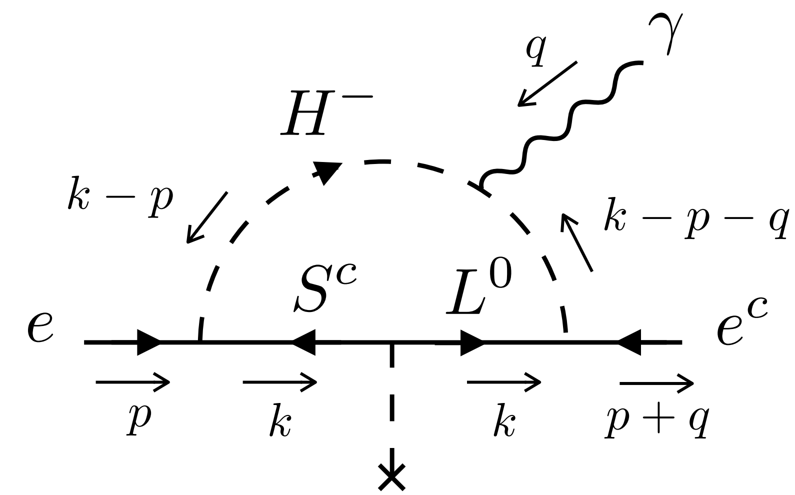
$$m_L = 10^3\, m_S \gg m_S$$

Weird! Not the usual scale-by-scale cancellation that would be enforced by an unbroken symmetry…

Gauge to Mass Basis

In the gauge/flavor basis:

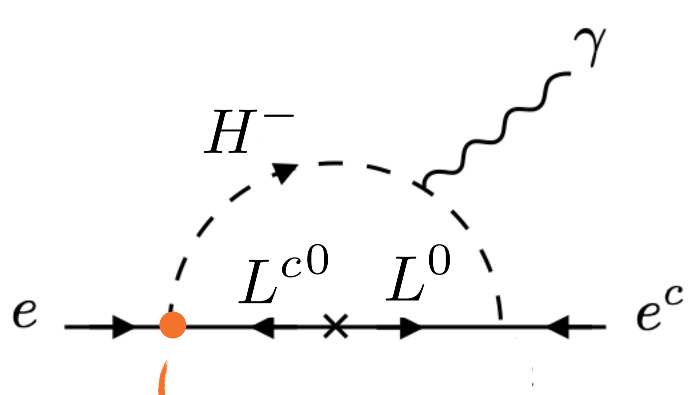
Arkani-Hamed, Harigaya [2106.01373]



flavor $\longrightarrow$ mass basis

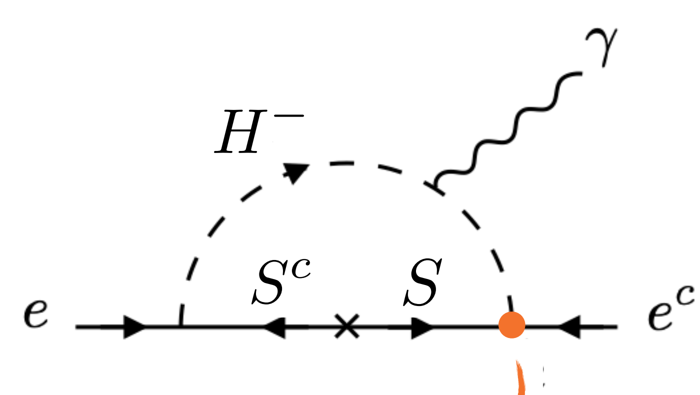
$$\begin{pmatrix} L^0 \\ S \end{pmatrix} \to \begin{pmatrix} 1 & -\theta^* \\ \theta & 1 \end{pmatrix} \begin{pmatrix} L^0 \\ S \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} L^{c0} \\ S^c \end{pmatrix} \to \begin{pmatrix} 1 & -\theta^c \\ \theta^{c*} & 1 \end{pmatrix} \begin{pmatrix} L^{c0} \\ S^c \end{pmatrix}$$

In the mass basis:



$\propto \theta^{c*}$

$+$



$\propto \theta^*$

A Toy Model

$$\Psi_e = \begin{pmatrix} e \\ e^{c\dagger} \end{pmatrix}$$

(later: muon)

ϕ with $Q_{\rm EM}(\phi) = 1$

(later: H^+)

$$\Psi = \begin{pmatrix} \psi \\ \psi^{c\dagger} \end{pmatrix} \qquad \hat{\Psi} = \begin{pmatrix} \hat{\psi} \\ \hat{\psi}^{c\dagger} \end{pmatrix}$$

(later: doublet- and singlet-like fermions)

$$\Delta\mathcal{L} = \Big\{ - m\, \psi^c \psi + y_L\, \phi(\psi^c e) + y_R^*\, \phi^* (e^c \psi) \Big\} + \text{h.c.}$$

$$+ \Big\{ - \hat{m}\, \hat{\psi}^c \hat{\psi} + \hat{y}_L\, \phi(\hat{\psi}^c e) + \hat{y}_R^*\, \phi^* (e^c \hat{\psi}) \Big\} + \text{h.c.}$$

symmetry of free fermion sector

$$U(6) \xrightarrow{\Delta\mathcal{L}} U(1)_{V,\text{diag}}$$

only "fermion number" left unbroken

Spurion analysis using the subgroup: $U(1)^6 \times P_\Psi \subset U(6)$

$$P_\Psi :\ \psi \leftrightarrow \psi^c,\ \hat{\psi} \leftrightarrow \hat{\psi}^c \qquad \underline{\text{and}} \qquad m \leftrightarrow \hat{m},\ y_L \leftrightarrow \hat{y}_L,\ y_R \leftrightarrow \hat{y}_R$$

(Toy) Spurion Analysis

Mass and dipole: must carry two units of $U(1)_A$ SM charge,

and be neutral under all other symmetries, including P_Ψ

Mass: $$\Delta m = y_L y_R^* \, m^* \left(\alpha \log \frac{\mu^2}{|m|^2} + \beta \right) + \hat{y}_L \hat{y}_R^* \, \hat{m}^* \left(\alpha \log \frac{\mu^2}{|\hat{m}|^2} + \beta \right)$$

in general, logs and no-log pieces

(Explicit calculation: $\alpha = 1/16\pi^2, \ \beta = 0$)

Dipole: $$\Delta \tau = \gamma e \boxed{\left(\frac{y_L y_R^*}{m} + \frac{\hat{y}_L \hat{y}_R^*}{\hat{m}} \right)}$$

no logs!

(there cannot be divergences because we cannot write counterterms)

$$\Delta \tau = 0 \ \ \text{provided} \ \ \left(\frac{y_L y_R^*}{m} + \frac{\hat{y}_L \hat{y}_R^*}{\hat{m}} \right) = 0$$

Spurion Analysis

This toy model accounts for the model in Arkani-Hamed, Harigaya [2106.01373], after making the identifications:

$$\phi \to H^+ \qquad \{\psi, \psi^c\} \to \{L^0, L^{c0}\} \qquad \{\hat{\psi}, \hat{\psi}^c\} \to \{S, S^c\}$$

$$m = m_L \qquad y_L = Y_L\,\theta^{c*} \qquad y_R^* = -Y_R$$

$$\hat{m} = m_S \qquad \hat{y}_L = Y_L \qquad \hat{y}_R^* = Y_R\,\theta^*$$

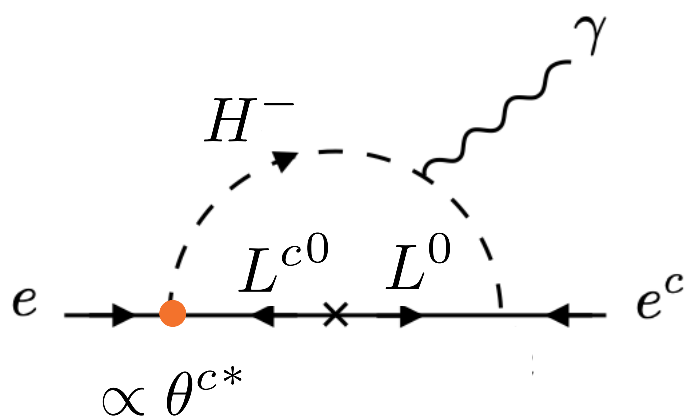
At leading order in the Higgs vev:

$$\theta = \frac{v}{\sqrt{2}} \frac{m_S^* Y_V'}{|m_L|^2 - |m_S|^2} \qquad \text{and} \qquad \theta^c = \frac{v}{\sqrt{2}} \frac{m_L^* Y_V'}{|m_L|^2 - |m_S|^2}$$

$$\Rightarrow \qquad \frac{y_L y_R^*}{m} = -\frac{v}{\sqrt{2}} \frac{Y_L Y_R Y_V'^*}{|m_L| - |m_S|^2} \qquad \text{and} \qquad \frac{\hat{y}_L \hat{y}_R^*}{\hat{m}} = +\frac{v}{\sqrt{2}} \frac{Y_L Y_R Y_V'^*}{|m_L| - |m_S|^2}$$

$$\text{Thus} \quad \left(\frac{y_L y_R^*}{m} + \frac{\hat{y}_L \hat{y}_R^*}{\hat{m}} \right) = 0$$

(More) Spurion Analysis



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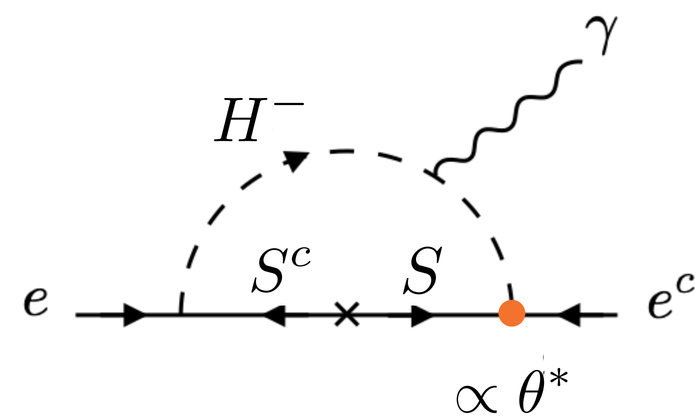


Vanishing dipole due to $\dfrac{\theta^c}{\theta} = \dfrac{m_L^*}{m_S^*}$

Is there a symmetry behind this relationship?

Before rotating to the mass basis, the mass lagrangian for the neutral fermions:

$$\mathcal{L} \supset \left\{ -m_L L^0 L^{c0} - m_S S S^c - \frac{Y_V' v}{\sqrt{2}} L^0 S^c \right\} + \text{h.c.}$$

$$C' :\ L^0 \leftrightarrow S^c,\ L^{c0} \leftrightarrow S \quad \text{and} \quad m_L \leftrightarrow m_S \quad \Rightarrow \quad \theta^c = -\theta\Big|_{m_S \leftrightarrow m_L}$$

$U(1)^6$ spurion analysis: $\theta = N \dfrac{m_S^* Y_V' v/\sqrt{2}}{|m_L|^2 - |m_S|^2} \xrightarrow[C']{} \theta^c = N \dfrac{m_L^* Y_V' v/\sqrt{2}}{|m_L|^2 - |m_S|^2}$ ✔

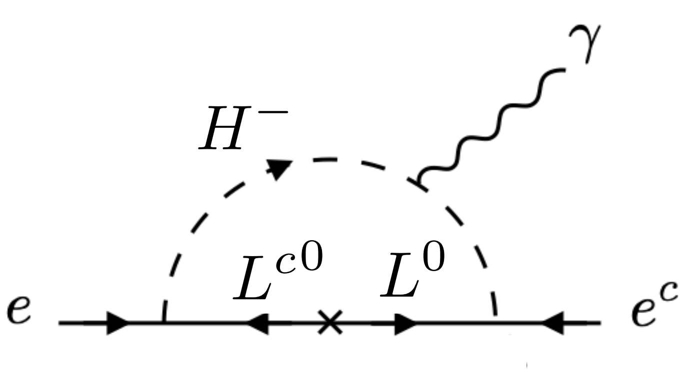
Surprise... or expectation?

We can go back to the flavor basis, before setting the Higgs to its vev:

$$\mathcal{L} \supset \left\{ -m_L L^0 L^{c0} - m_S S S^c - Y_V' H^0 L^0 S^c + Y_L H^+ e S^c - Y_R H^- L^0 e^c \right\} + \text{h.c.}$$

Extend C' to act on the SM fields as "usual" charge conjugation:

$$C' : e \leftrightarrow e^c,\ H^+ \leftrightarrow H^-,\ A_\mu \leftrightarrow -A_\mu \qquad \text{and} \qquad Y_L \leftrightarrow -Y_R$$



$$\tau_{(L)} = \frac{e}{32\pi^2} \frac{v}{\sqrt{2}} \frac{Y_L Y_R Y_V'^*}{|m_L|^2 - |m_S|^2} \xrightarrow{C'} -\tau_{(L)}$$

but

SM dipoles must be C' even $\Rightarrow$ 2nd loop such that $\tau_{(S)} = -\tau_{(L)}$

Gauge basis

can we avoid going to the mass basis??

Discussion in mass basis cumbersome (partly) because discrete symmetry doesn't respect the $SU(2)_L$ gauge structure of the SM…

Yes, but…

$$
P_{LR}\quad
\begin{array}{cccccc}
l^i = \begin{pmatrix} \nu \\ e \end{pmatrix} & & L^i = \begin{pmatrix} L^0 \\ L^- \end{pmatrix} & & L_i^c = \begin{pmatrix} -L^{c0} \\ L^{c+} \end{pmatrix} & \Big\} \; SU(2)_L \\
\updownarrow & & \updownarrow & & \updownarrow & \\
r_{i'}^c = \begin{pmatrix} \nu^c \\ e^c \end{pmatrix} & & R_{i'}^c = \begin{pmatrix} S^c \\ E^c \end{pmatrix} & & R^{i'} = \begin{pmatrix} -S \\ E \end{pmatrix} & \Big\} \; SU(2)_R
\end{array}
$$

Requires introducing full $SU(2)_L \times SU(2)_R$ and $P_{LR} : SU(2)_L \leftrightarrow SU(2)_R$

$$\Sigma^i{}_{j'} = (\Sigma^i{}_1,\ \Sigma^i{}_2) = (\widetilde{H}^i,\ H^i) = \begin{pmatrix} H^{0*} & H^+ \\ -H^- & H^0 \end{pmatrix}$$

Gauge basis

Repeat spurion analysis…

$$\textbf{Dipole} \;\propto\; \left[\mathsf{m}_L \Delta\mathsf{m}^{-2}\,\mathsf{m}_L^{-1} - \mathsf{m}_R^{-1}\Delta\bar{\mathsf{m}}^{-2}\,\mathsf{m}_R\right]_{u}{}^{v}{}_{r'}{}^{s'}$$

$$\text{with}\quad \begin{cases} \left[\Delta\mathsf{m}^2\right]_i{}^j{}_{k'}{}^{l'} \equiv \delta_i^j \left[\mathsf{m}_R^\dagger \mathsf{m}_R\right]_{k'}{}^{l'} - \left[\mathsf{m}_L^\dagger \mathsf{m}_L\right]_i{}^j \delta_{k'}^{l'} \\ \left[\Delta\bar{\mathsf{m}}^2\right]_i{}^j{}_{k'}{}^{l'} \equiv \delta_i^j \left[\mathsf{m}_R \mathsf{m}_R^\dagger\right]_{k'}{}^{l'} - \left[\mathsf{m}_L \mathsf{m}_L^\dagger\right]_i{}^j \delta_{k'}^{l'} \end{cases}$$

Choosing $\left[\mathsf{m}_L\right]_i{}^j = \delta_i^j\, m_L\,, \qquad \left[\mathsf{m}_R\right]_1{}^1 = m_S\,, \quad \left[\mathsf{m}_R\right]_2{}^2 = m_E\ (\to \infty)$

$$\Rightarrow\ \textbf{Dipole} \equiv 0$$

✔

(Requires some intermediate steps, including field redefinitions to eliminate Yukawas in favor of derivative operators:

e.g. $S \to S - \dfrac{Y}{m_S} HL \quad \Rightarrow \quad m_S S^c S + Y HL S^c \to m_S S^c S + \text{derivative}$)

Conclusions

Contributions not forbidden by symmetries are compulsory still true,
but requires a broader understanding of "symmetry"

symmetry breaking structure can play key role in forbidding
certain contributions, more in the spirit of "selection rules"

Not just a "curiosity", but can have important pheno implications,
e.g. lower scale of vector-like masses makes model testable in future

Arkani-Hamed, Harigaya [2106.01373]

… although hard to see how analogous structure
could be built around the Higgs mass-squared

Also see Delle Rose, von Harling, Pomarol [2201.10572]: on-shell methods
to see the vanishing of dipole contribution from tree-level amplitudes

Thank you!