# Naturalness in EFT

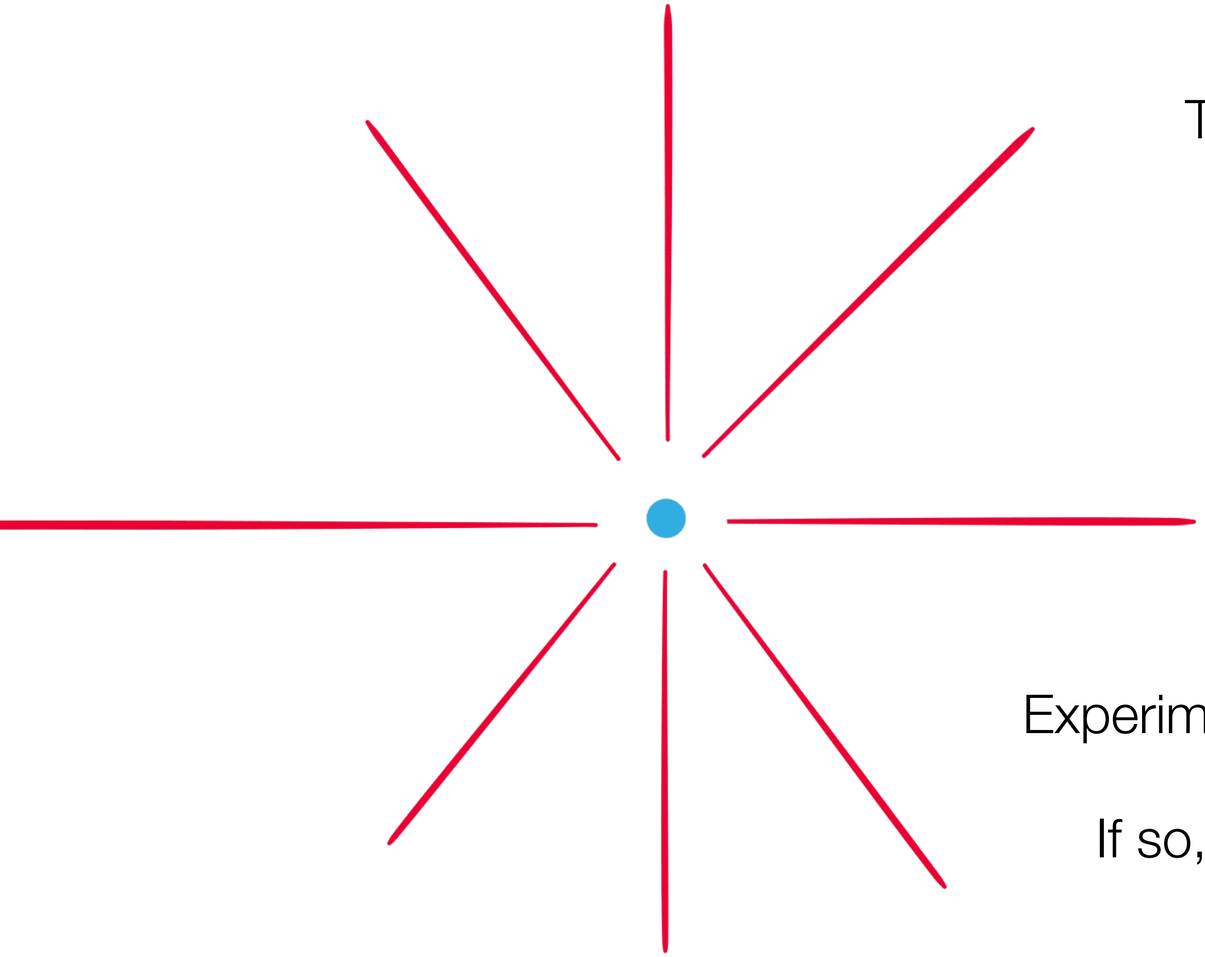
**Nathaniel Craig**
University of California, Santa Barbara
UCSB

# Naturalness in Snowmass

- N. Blinov, NC, M. Dolan, J de Vries, P. Draper, I. Garcia Garcia, B. Lillard, J. Shelton, *Snowmass White Paper: Strong CP Beyond Axion Direct Detection*, 2203.07218
- P. Agrawal, K.V. Berghaus, J. Fan, A. Hook, G. Marques-Tavares and T. Rudelius, *Some open questions in axion theory*, 2203.08026
- P. Berglund, L. Freidel, T. Hubsch, J. Kowalski-Glikman, R.G. Leigh, D. Mattingly et al., *Infrared Properties of Quantum Gravity: UV/IR Mixing, Gravitizing the Quantum – Theory and Observation*, 2202.06890
- P. Draper, I.G. Garcia and M. Reece, *Snowmass White Paper: Implications of Quantum Gravity for Particle Physics*, 2203.07624
- B. Batell, M. Low, E.T. Neil and C.B. Verhaaren, *Review of Neutral Naturalness*, 2203.05531
- C. Dvorkin et al., *The Physics of Light Relics,* 2203.07943
- P. Asadi et al., *Early-Universe Model Building,* 2203.06680
- NC, *Naturalness: A Snowmass White Paper*, 2205.05708

- strong CP problem
- cosmological constant problem
- **electroweak hierarchy problem**

# The Naturalness Strategy



The naturalness strategy: an **analogy** from E&M

$$\Delta E_C = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_e}$$

$$(m_e c^2)_{\rm obs} = (m_e c^2)_{\rm bare} + \Delta E_C$$

Experimentally $r_e \lesssim 10^{-18}\,\mathrm{cm} \Rightarrow \Delta E_C \gtrsim 100\,\mathrm{GeV}$

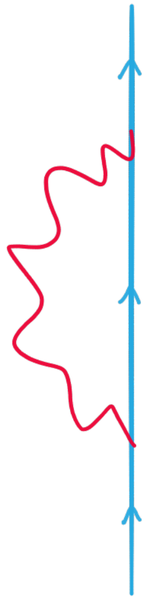
If so, $0.511 = -99999.489 + 100000.000\,\mathrm{MeV}$

To avoid fine-tuning, i.e. for the theory to be “natural”, need picture to change on scales below 2.8 × $10^{-13}$ cm

# The Naturalness Strategy

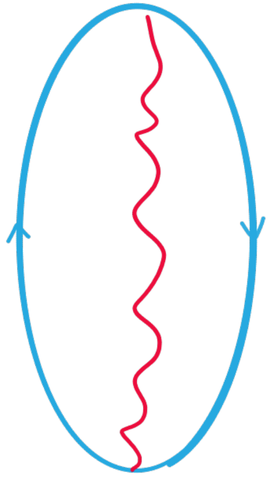
**Dirac (1928-29):** There is a new state in the relativistic quantum theory

**Weisskopf (1934-39):** Compute the self-energy including the positron

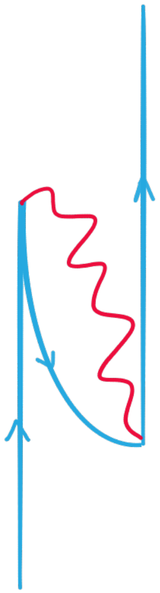


$$\Delta E = \Delta E_C + \ldots$$

$$\Delta t \sim \hbar/\Delta E \sim \hbar/(2m_e c^2)$$



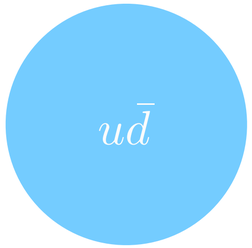
$$d \sim c\Delta t \sim 200 \times 10^{-13}\,\mathrm{cm}$$



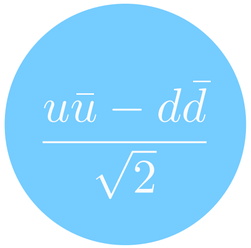
$$\Delta E = -\Delta E_C + \ldots$$

$$\Delta E = \Delta E_C - \Delta E_C + \frac{3\alpha}{4\pi} m_e c^2 \log \frac{\hbar}{m_e c r_e}$$

# The Naturalness Strategy




$\pi^+$



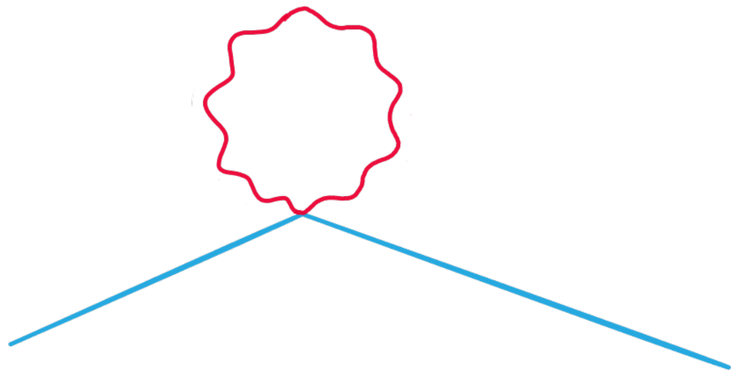

$\pi^0$

*What about scalars?*

Consider the charged & neutral pions…

Another divergence!
In terms of scale $\Lambda \sim \hbar c/r$:

$$m^2_{\pi^\pm} - m^2_{\pi^0} = \frac{3\alpha}{4\pi}\Lambda^2$$



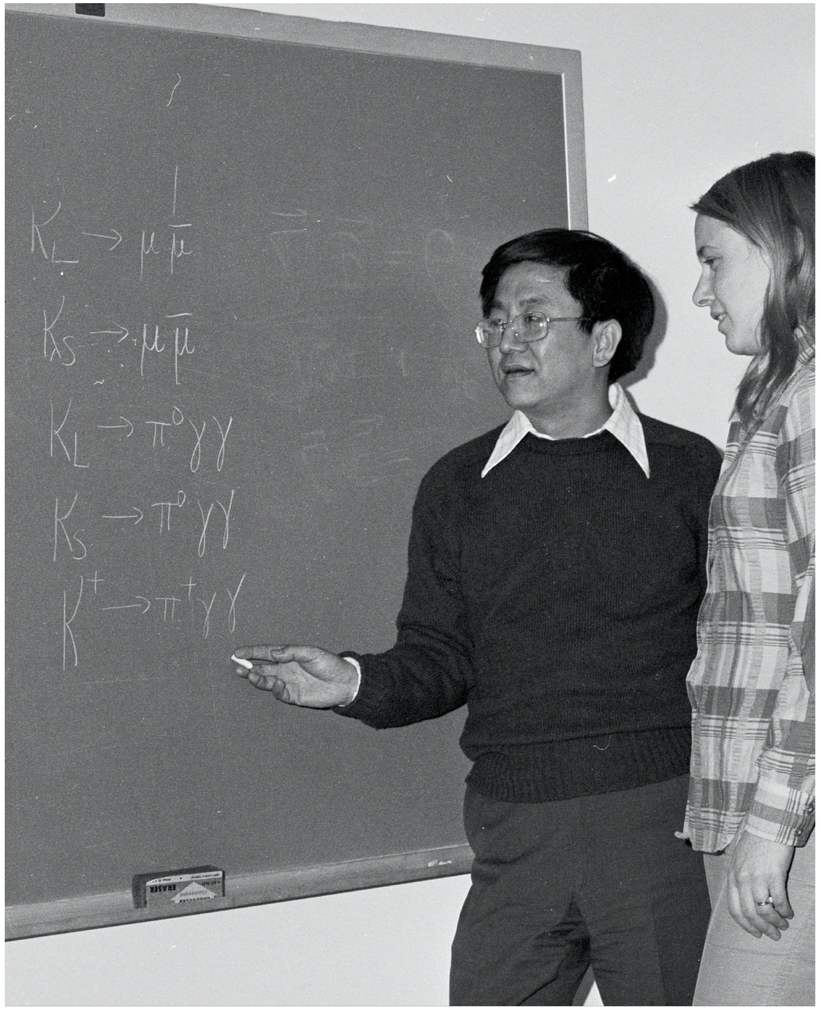
Given observed splitting, *predict* scale of new physics:

$$m^2_{\pi^\pm} - m^2_{\pi^0} = (35.5\,\mathrm{MeV})^2 \Rightarrow \Lambda \lesssim 850\,\mathrm{MeV}$$ ✓

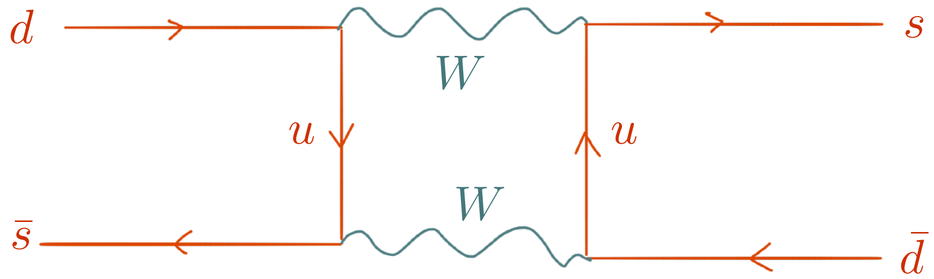
[Das, Guralnik, Mathur, Low, Young '67]

$$m^2_{\pi^\pm} - m^2_{\pi^0} \approx \frac{3\alpha}{4\pi}\frac{m_\rho^2 m_{a_1}^2}{m_\rho^2 + m_{a_1}^2}\log\left(\frac{m_{a_1}^2}{m_\rho^2}\right)$$

# The Naturalness Strategy




$\frac{d\bar{s} + s\bar{d}}{\sqrt{2}}$ $K_L$ $\frac{d\bar{s} - s\bar{d}}{\sqrt{2}}$ $K_S$

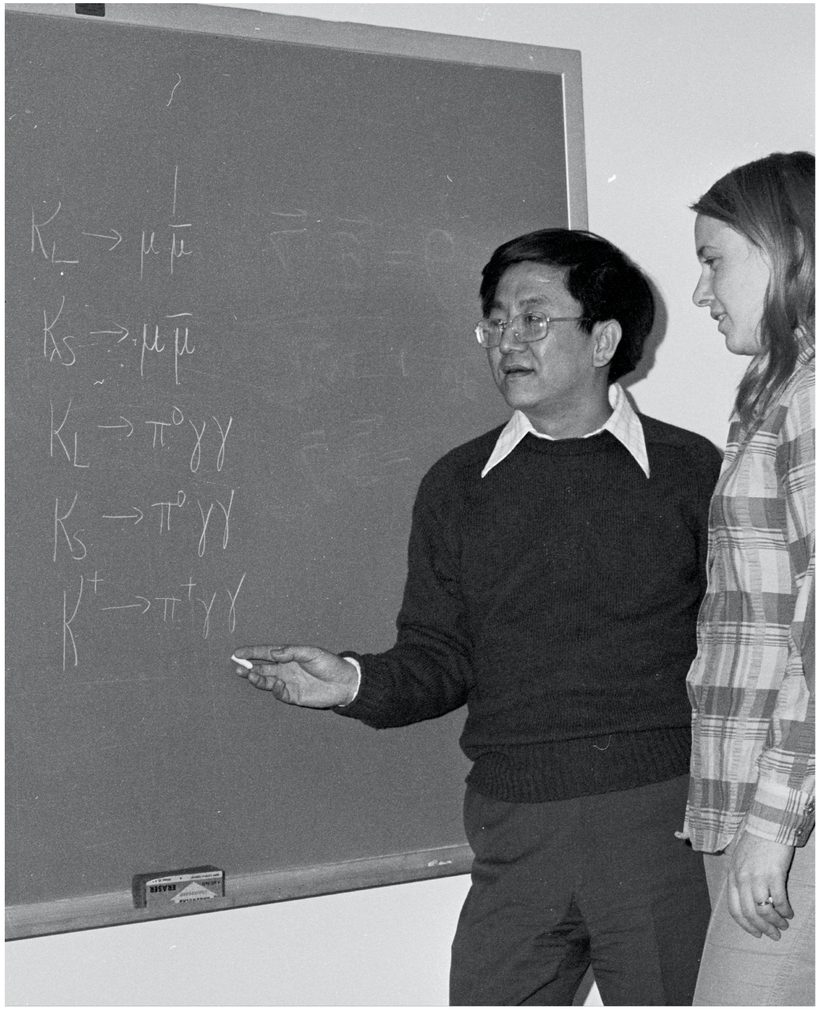



$$m_{K_L} - m_{K_S} \simeq \frac{1}{16\pi^2} m_K f_K^2 G_F^2 \sin^2\theta_C \cos^2\theta_C \times \Lambda^2$$

[Ioffe & Shabalin, 1968]

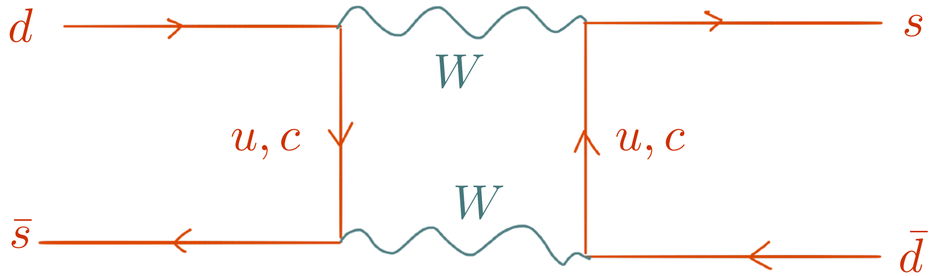
Given observed splitting, expect new physics around $\Lambda \sim 3$ GeV

# The Naturalness Strategy




[Glashow, Iliopoulos, Maiani, 1970]

Propose a 4th quark, charm




$$m_{K_L} - m_{K_S} \simeq \frac{1}{4\pi^2} m_K f_K^2 G_F^2 \sin^2\theta_C \cos^2\theta_C \times m_c^2$$

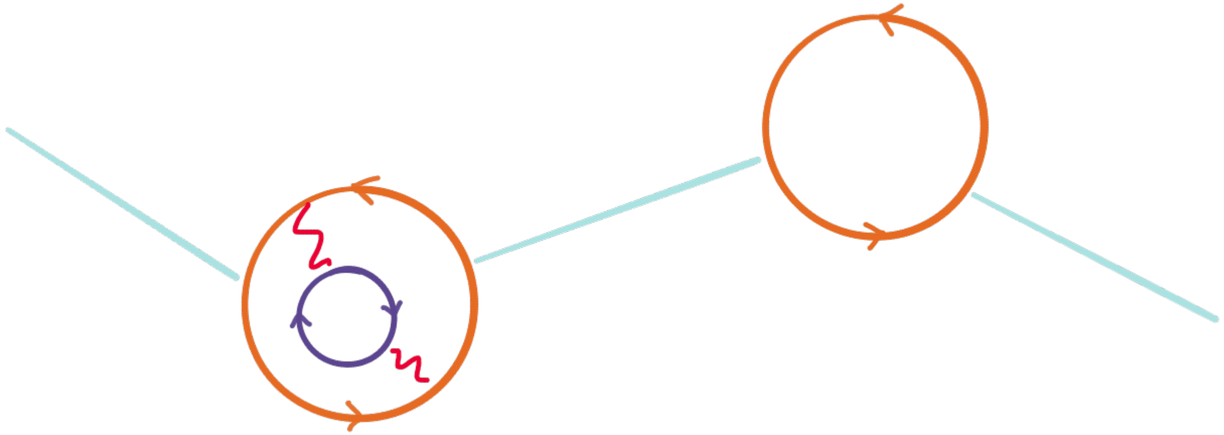
[Gaillard & Lee, 1974]

*Predict* $m_c \simeq 1.5\,\mathrm{GeV}$

Charm discovered in 1974, $m_c = 1.2\,\mathrm{GeV}$

# The “Hierarchy Problem”

The Higgs is an apparently elementary scalar w/ self-energy due to its couplings to Standard Model particles



Assuming SM is valid down to some length scale $r_{\rm new} \equiv \dfrac{\hbar c}{\Lambda}$ then we have

$$\Delta m_H^2 = \frac{\Lambda^2}{16\pi^2}\left[-6y_t^2 + \frac{9}{4}g_2^2 + \frac{3}{4}g_Y^2 + 6\lambda + \ldots\right]$$

Expecting new physics at Λ such that $\Delta m_H^2 \sim m_H^2$ is a *strategy*.

# The Naturalness Strategy

| Param | UV sensitivity | Natural if | NP | Scale | Natural? |
|---|---|---|---|---|---|
| **“$m_e$”** | $e^2\Lambda$ | $\Lambda \lesssim 5$ MeV | Positron | 511 keV | ✓ |
| **$m_{\pi^\pm}^2 - m_{\pi^0}^2$** | $\frac{3\alpha}{4\pi}\Lambda^2$ | $\Lambda \lesssim 850$ MeV | Rho | 770 MeV | ✓ |
| **$m_{K_L} - m_{K_S}$** | $\frac{s_c^2 f_K^2 m_{K_L^0}}{24\pi^2 v^4}\Lambda^2$ | $\Lambda \lesssim 3$ GeV | Charm | 1.2 GeV | ✓ |
| **$m_H^2$** | $-\frac{6y_t^2}{16\pi^2}\Lambda^2$ | $\Lambda \lesssim 500$ GeV | ? | ? | ? |

# A Reminder

Often said that the quadratic divergence "is" the hierarchy problem:

$$\Delta m_H^2 = \frac{\Lambda^2}{16\pi^2}\left[-6y_t^2 + \frac{9}{4}g_2^2 + \frac{3}{4}g_Y^2 + 6\lambda + \dots\right]$$

But this is misleading, as it depends on regularization scheme (try dim reg!).

Only meaningful if $\Lambda$ is a **physical scale**. From the bottom-up, $\Lambda$ a proxy for the scale of new physics. Given a UV completion, "quadratic divergence" replaced by finite, calculable, regularization scheme-independent contributions.

*Tempting to "solve" hierarchy problem by positing no physical scale above weak scale. Still need to explain why scale of quantum gravity, hypercharge Landau pole do not contribute.*

# From the “naturalness strategy” to new physics

**At this level, we expect**

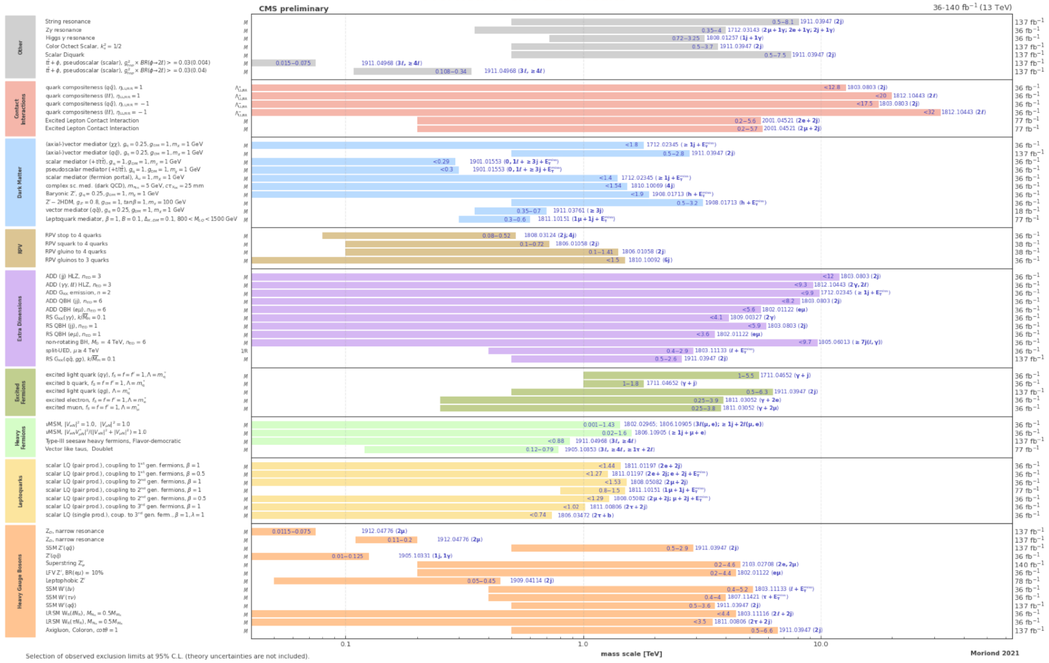
- New physics around the TeV scale…
- …coupling to the Higgs

Strong motivation for beyond-the-Standard Model physics connected to Higgs! But maybe too broad to be useful guidance to experiment.

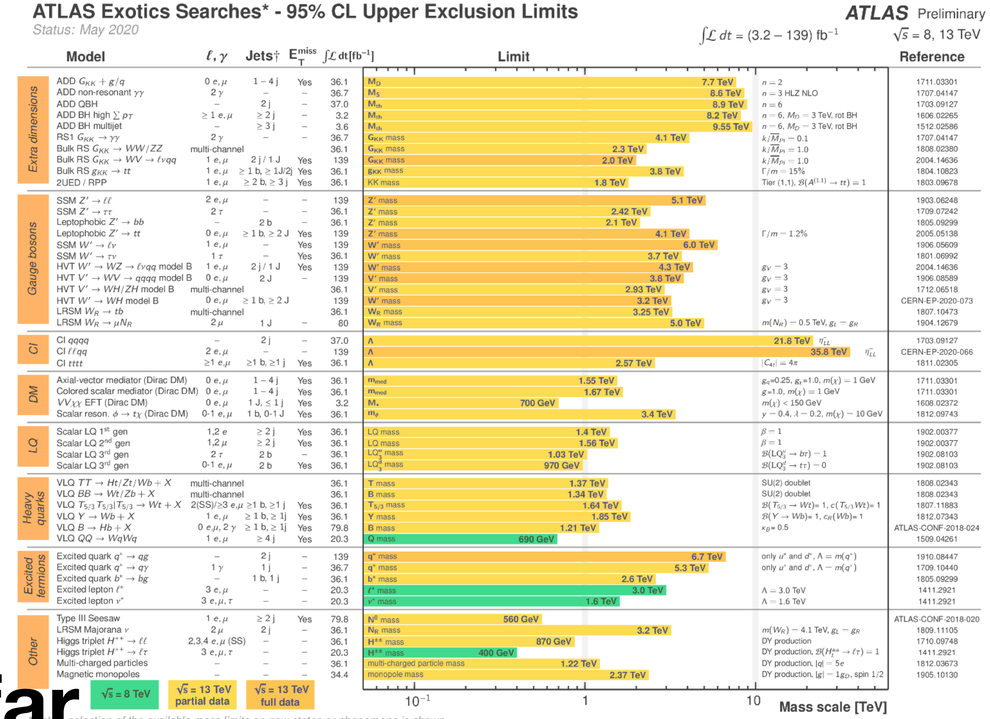
To make further progress, we can come up with models.
**Easiest path:** work by analogy with known examples.

**Supersymmetry:** Relate Higgs to a fermion, *a la* electron.

**Compositeness:** Higgs is a composite particle, *a la* the pion.

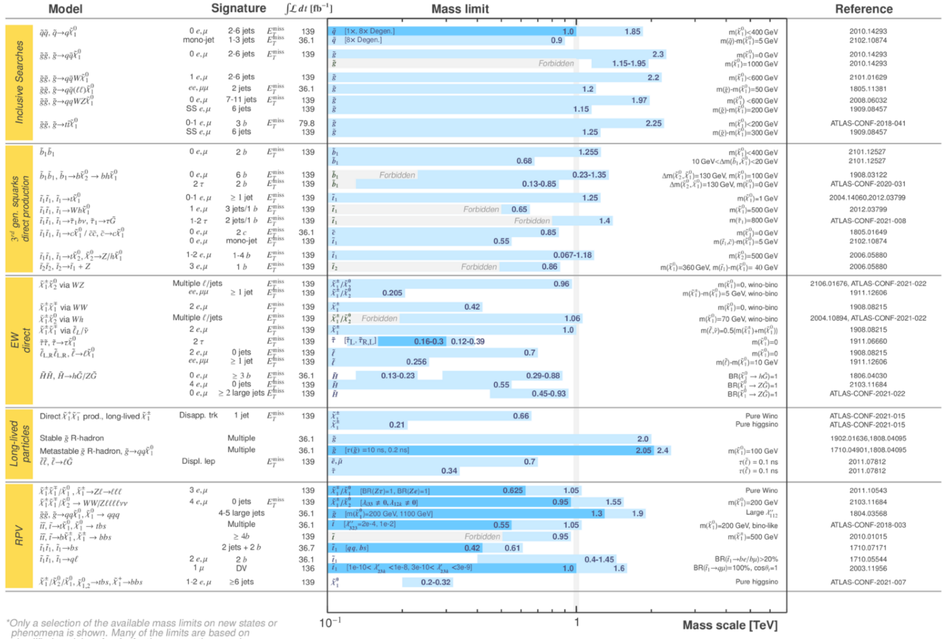



Selection of observed exclusion limits at 95% C.L. (theory uncertainties are not included).

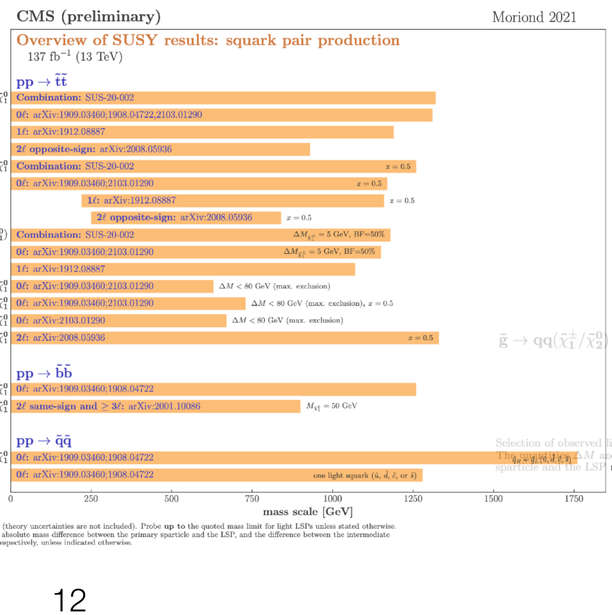



Only a selection of the available mass limits on new states or phenomena is shown.
†Small-radius (large-radius) jets are denoted by the letter j (J).

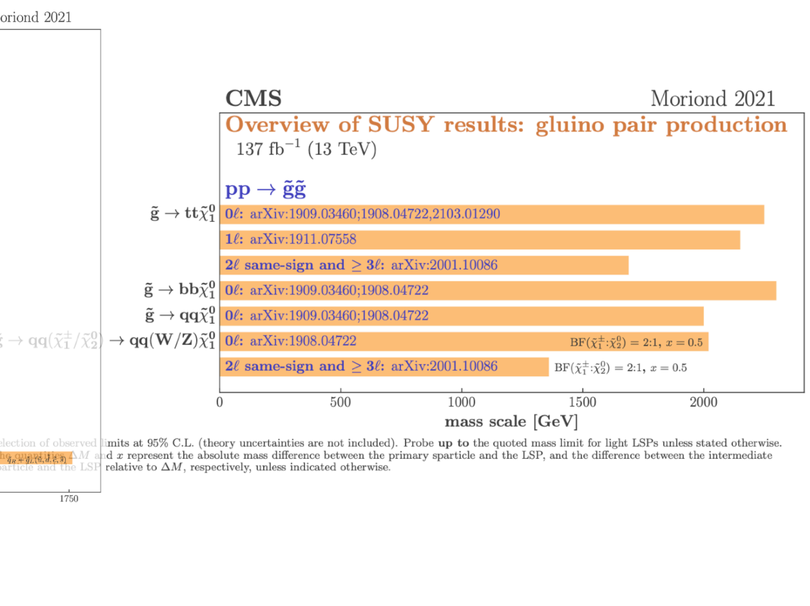
# But thus far ...




*Only a selection of the available mass limits on new states or phenomena is shown. Many of the limits are based on simplified models, c.f. refs. for the assumptions made.




Selection of observed limits at 95% C.L. (theory uncertainties are not included). Probe **up to** the quoted mass limit for light LSPs unless stated otherwise. The quantities $\Delta M$ and $x$ represent the absolute mass difference between the primary sparticle and the LSP, and the difference between the intermediate sparticle and the LSP relative to $\Delta M$, respectively, unless indicated otherwise.

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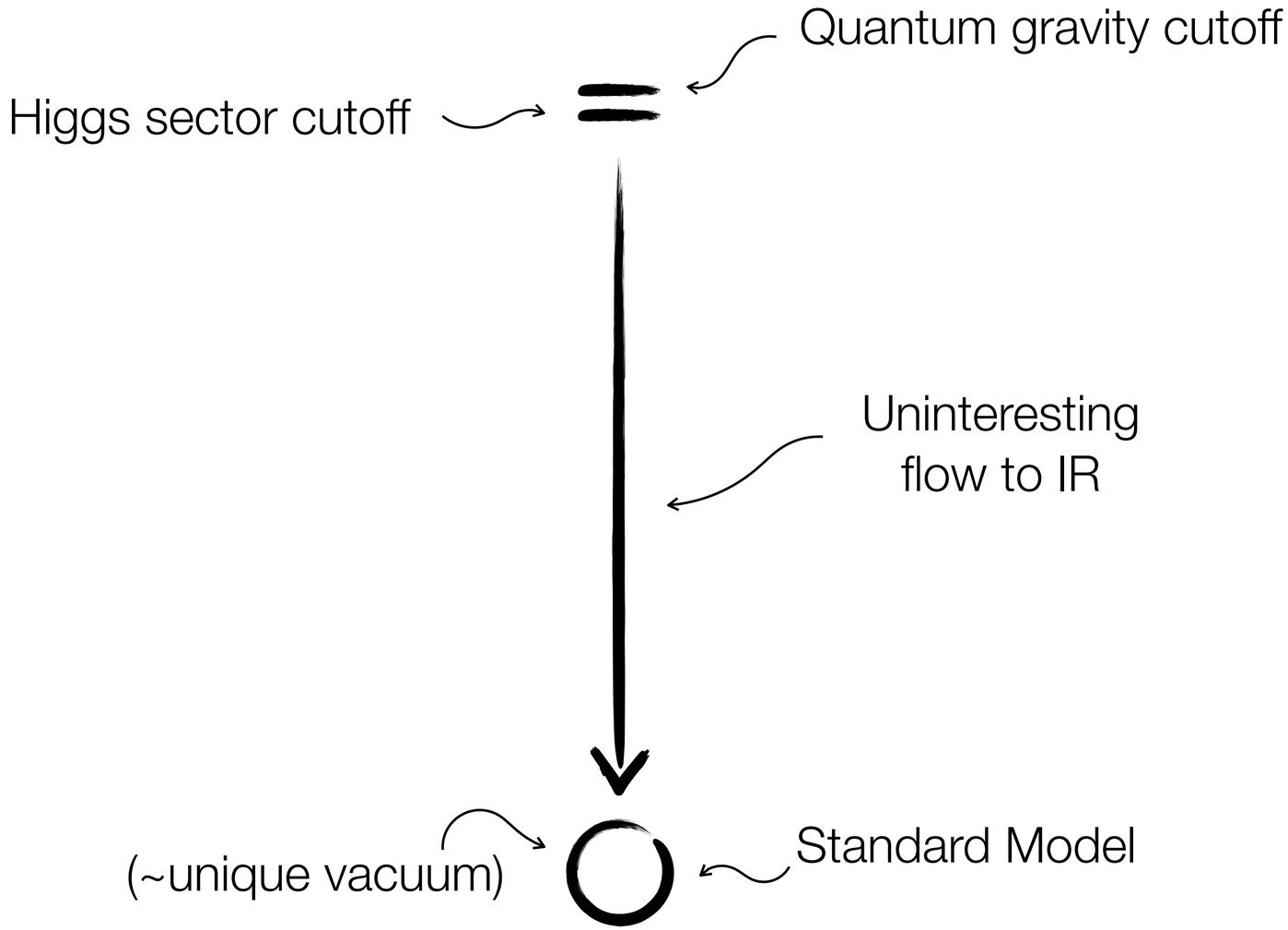
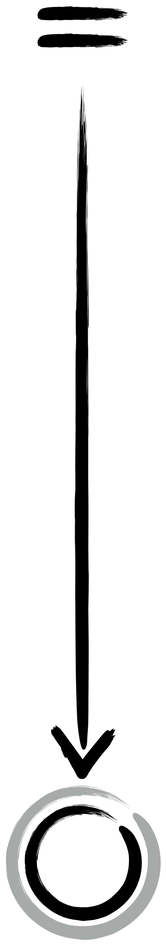
# The Hierarchy Problem Cartoon




$m_H$ is not technically natural ⇒ *hierarchy problem*

# Adding a symmetry

…and (sometimes) breaking it softly



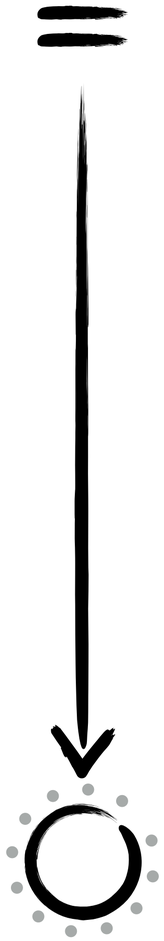
1. **Supersymmetry** (*a la* the electron)
2. **Global symmetry** (*a la* the pion)
3. **Discrete symmetry**
4. **Modular invariance**
   [Dienes et al. ’94-’01, …]

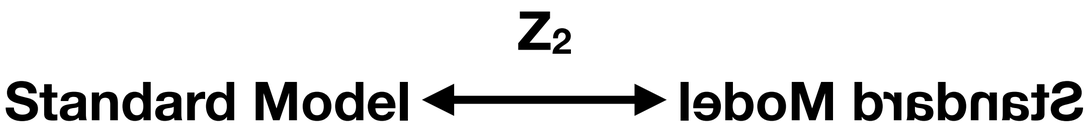
Experimental signals: partner particles

- The familiar host of prompt signals (with or without missing energy)
- Rich variety of displaced decays (RPV, twin higgs, folded SUSY, …)

# Neutral Naturalness

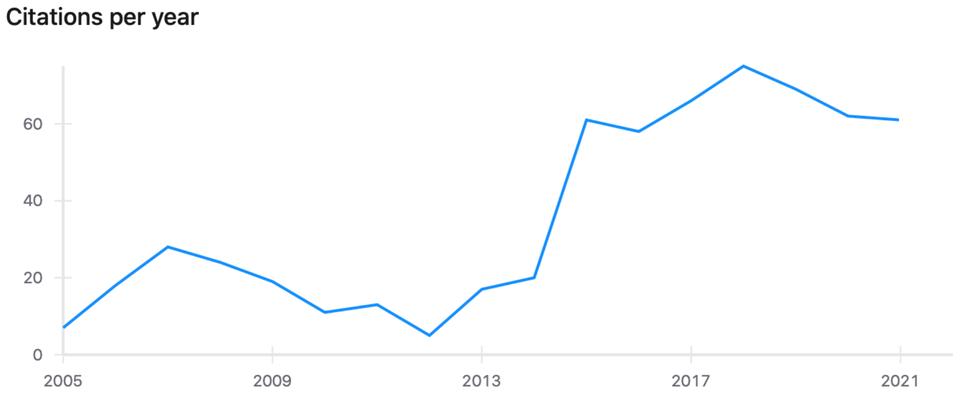
Twin Higgs [Chacko, Goh, Harnik '05]





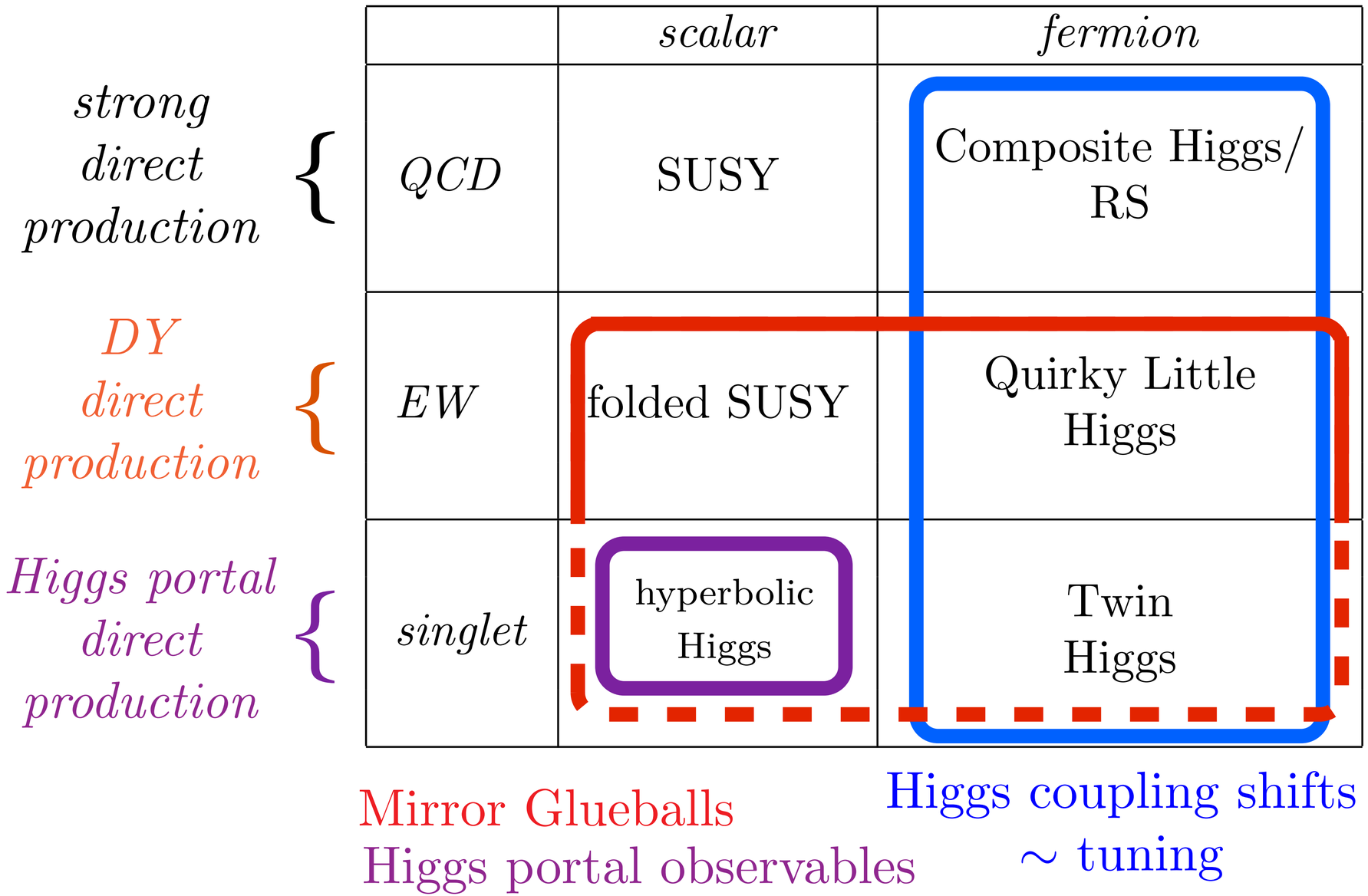

Radiative corrections to mass-squared are SU(4) symmetric thanks to $Z_2$:

$$V(H) \supset \frac{\Lambda^2}{16\pi^2}\left(-6y_t^2 + \frac{9}{4}g^2 + \dots\right)\left(|H_A|^2 + |H_B|^2\right)$$




See [B. Batell, M. Low, E.T. Neil and C.B. Verhaaren, *Review of Neutral Naturalness*, 2203.05531]

From [Curtin, Verhaaren '15]



# Lowering the cutoff

…in diverse dimensions

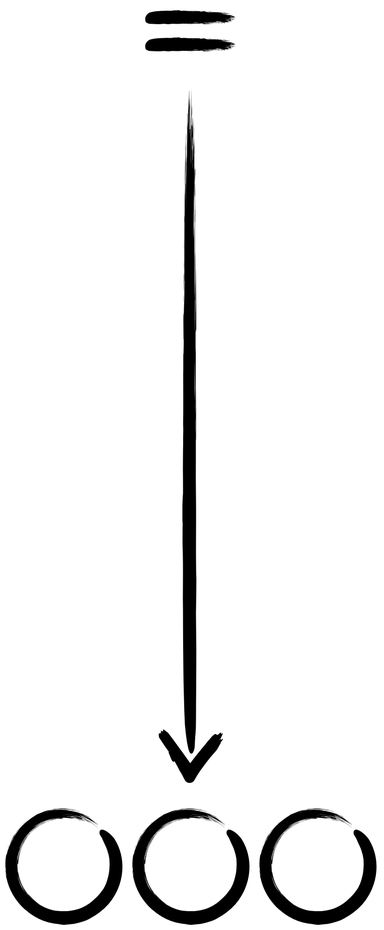
5. **RS / Technicolor**
   [Randall, Sundrum ’99; Weinberg ’79; Susskind ‘79]
6. **LED / $10^{32}$ x SM**
   [Arkani-Hamed, Dimopoulos, Dvali ’98; Antoniadis + ibid. ’98; Dvali, Redi ‘09]
7. **LST / Clockwork**
   [Antoniadis, Dimopoulos, Giveon ’01; Kaplan, Rattazzi ’15; Giudice, McCullough ‘16]
8. **Classicalization**
   [Dvali, Giudice, Gomez, Kehagias ‘10]
9. **Disorder**
   [Rothstein ‘12]

Experimental signals: resonances, …

- Primary distinctions are in spacing & coupling of resonances
- Potential goldmine of relatively unexplored signals for LST — e.g. perturbative string excitations

# Selecting a vacuum

Vacuum is one of many; end up in observed vacuum through dynamical process or anthropic constraint.



10. **Anthropics (pressure)**
11. **Relaxation (rolling)** [Graham, Kaplan, Rajendran ‘15]
12. **NNaturalness (reheating)** [Arkani-Hamed et al ‘16]
13. **Crunching away (collapse)** [Csaki et al ’20, *see also* Geller, Hochberg, Kuflik ’18, Cheung & Saraswat ’18, …]

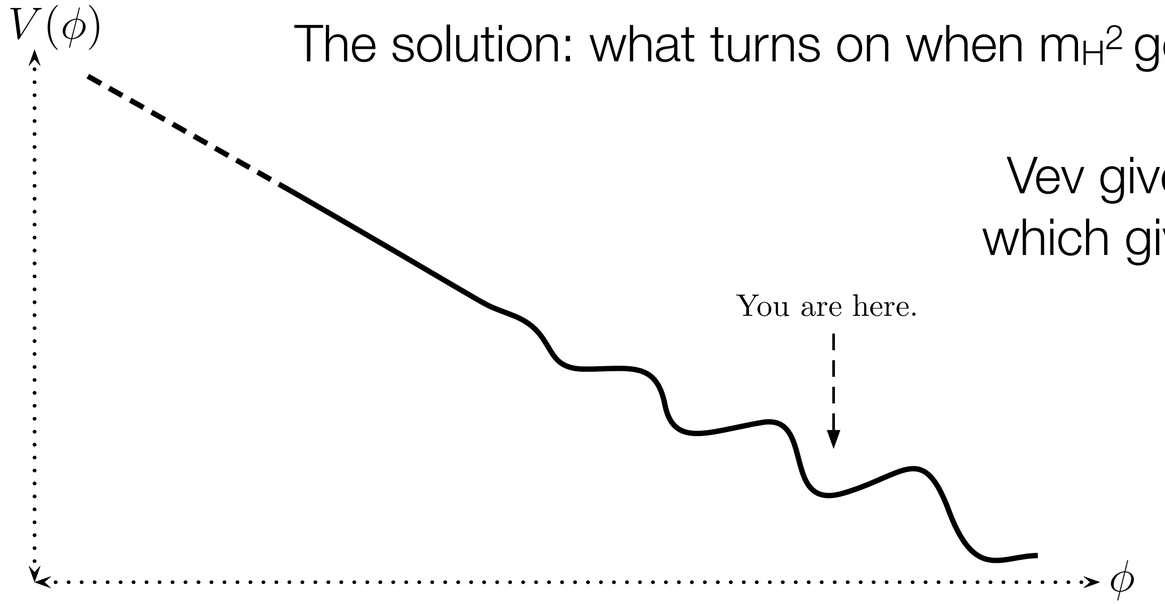
Experimental signals: Diverse, but typically

- Cosmology (Bubble collisions; axions; contributions to $N_{eff}$ and $\sum m_\nu$)
- Exotic lab signals (displaced decays, hidden sector confinement, rare/precision frontier, …)

# Relaxion

**What if the weak scale is selected by dynamics, not symmetries?**

The idea: couple Higgs to field whose minimum sets $m_H=0$
The problem: How to make $m_H=0$ a special point of potential?




The solution: what turns on when $m_H^2$ goes negative?

Vev gives quark masses
which give axion potential.

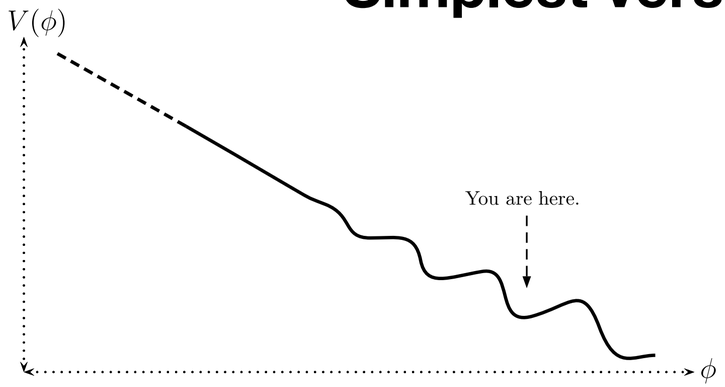
**“Relaxion”**

[Graham, Kaplan,
Rajendran ‘15]

*But: immense energy stored in evolving field, need dissipation.*

# Relaxion

**Simplest version: an axion coupled to QCD during inflation.**




$$\Lambda^4(H)\cos(\phi/f) + F(g\phi)$$
$$+\left(-M^2 + g\phi\right)|H|^2$$

Axion-like potential

Scanning term

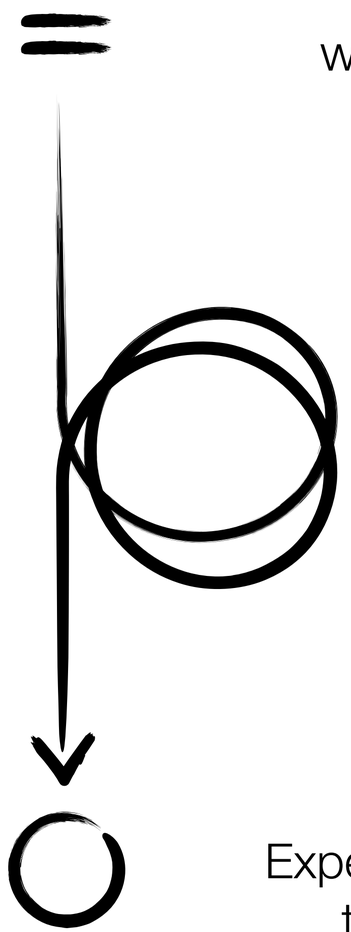
Large bare Higgs mass

Various other subtleties regarding technical naturalness, trans-Planckian field excursions, fine-tuning to inflationary sector; need to solve strong CP problem. *New UV considerations.*

Extensive development, e.g. [Espinosa et al. ’15; Hardy ’15; Gupta et al ’15; Batell, Giudice, McCullough ’15; Choi, Im ’15; Kaplan, Rattazzi ’15; Di Chiara et al. ’15; Ibanez et al. ’15; Hook, Marques-Tavares ’16; Nelson, Prescod-Weinstein ’17; …]

See [P. Asadi et al., *Early-Universe Model Building,* 2203.06680]

# Complicating the flow



SM is reached from some intermediate fixed point where, say, a generalized Veltman condition is satisfied

$$\delta m_H^2 = \sum_i c_i \frac{g_{i,\star}^2}{16\pi^2} \Lambda_i^2 = 0$$

*This is a sense in which*

**14. Conformal symmetry**

*could address the hierarchy problem*

**Top-down: Embed SM in orbifold of N=4 SYM**
[Frampton, Vafa '99; Csaki, Skiba, Terning '99]

**Bottom-up: "Little conformal symmetry"**
[Houtz, Colwell, Terning '16]

Experimental signals: Not fully explored, but expect new particles w/ SM quantum numbers around the TeV scale. Novelty is that statistics, irreps & couplings differ from more familiar solutions.

# Exploding the cutoff

Gravity doesn't provide a UV scale & the SM takes care of itself

**15. Asymptotic fragility**
[Dubovsky, Gorbenko, Mirbabayi '13]

**16. Agravity** [Salvio, Strumia '14]

Scale $M_{Pl}$ not associated with relevant operator becoming strong, not "felt" by non-grav physics.

In IR, looks like CFT perturbed by irrelevant operators; in UV, no UV fixed point; cannot define local observables.

Example in 2d, no proposal for 4d.

Gravity has no intrinsic length scale and is "renormalizable"

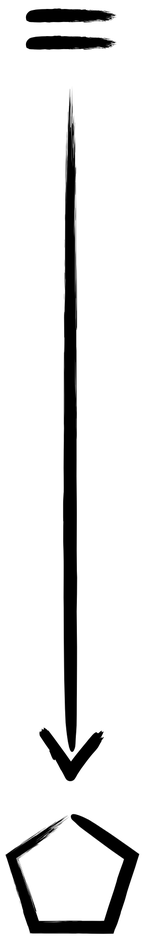
$$S \sim \int d^4x\sqrt{g}\left(\frac{R^2}{f_1^2} + \frac{\frac{1}{3}R^2 - R_{\mu\nu}^2}{f_2^2} + \dots\right)$$

(E-H term via vev of some field)

Can be re-written in terms of 2-derivative fields w/ ghosts.

Experimental signals: Crucially, must render SM couplings asymptotically free. Not a property of the SM itself, so entails low-scale unification.

# Not actually the SM



## 17. Lee-Wick (higher derivative scalar)

[Grinstein, O'Connell, Wise '06]

**Lee-Wick:** higher-derivative theory

$$\sim \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2M^2}(\partial^2\phi)^2 + \dots$$

Write as normal field + new field w/ wrong-sign quadratic action

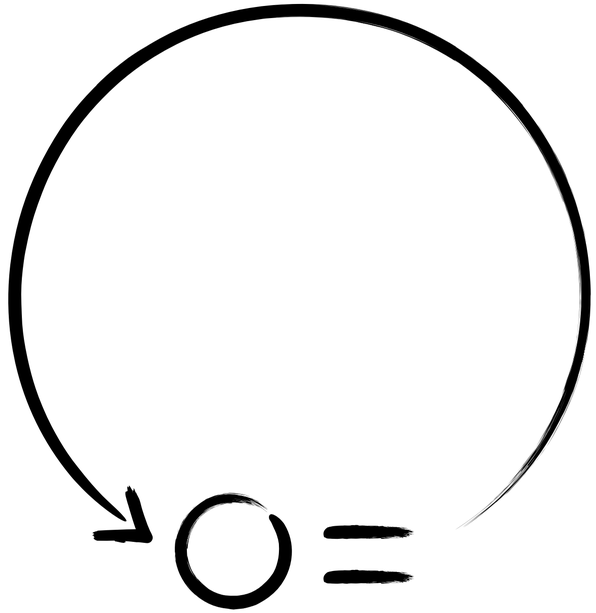
$$-\frac{1}{2}\partial_\mu\tilde{\phi}\partial^\mu\tilde{\phi} + \frac{1}{2}M^2\tilde{\phi}^2 + \dots$$

Improves UV convergence of diagrams, introduce for every SM field

$$\frac{1}{p^2-m^2} - \frac{1}{p^2-M^2} = \frac{m^2-M^2}{(p^2-m^2)(p^2-M^2)}$$

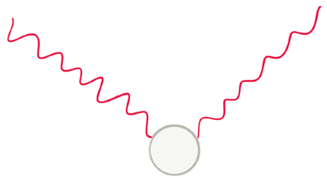
Can be defined in a unitary, Lorentz-invariant manner with only microscopic acausality. But who ordered that?

# Connecting UV & IR

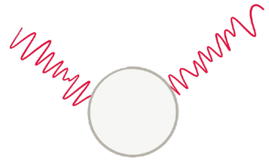


Essential feature of the hierarchy problem: the UV doesn't know about the IR…unless it does?

Two frameworks exhibiting UV/IR mixing: quantum gravity & non-commutative QFT



QG (cartoon version): collide sufficiently energetic particles, make a black hole. More energetic particles → bigger black hole.



NCQFT (cartoon version): non-commutativity of the form $[x^\mu,x^\nu]=i\theta^{\mu\nu}$, qualitatively a position-position uncertainty principle $\Delta x^\mu \Delta x^\nu \geq \theta/2$ [Filk '96, Minwalla, Seiberg, Van Raamsdonk '99, NC, Koren '19]

*Two ways to put this to work for hierarchy problem:*

**18. Indirect UV/IR mixing**

**19. Direct UV/IR mixing**

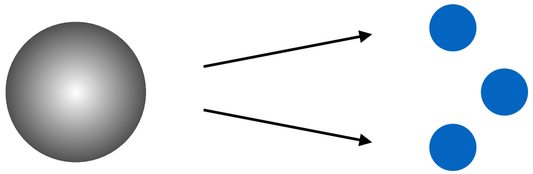
# Indirect UV/IR: WGC

**(Electric) weak gravity conjecture:** an abelian gauge theory must contain a state of charge $q$ and mass $m$ satisfying

$$gq \geq \frac{m}{M_{\rm Pl}}$$

[Arkani-Hamed, Motl, Nicolis, Vafa '07]

**"Justification": consider BH of charge *Q*, mass *M* decaying to this particle**



# particles produced = $Q/q$

Energy conservation: $mQ/q < M$

**Then BH satisfies** $Z = Q\,M_{Pl}/M < z = q\,M_{Pl}/m$

Extremal BH (Z=1) stable unless there exists a state with z > 1

**⇒ $q > m/M_{Pl}$ to avoid stable black holes, remnants, in conflict w/ holography**

# A Family of Conjectures

Electric WGC: $m \leq (gq) M_{\rm Pl}$
[Arkani-Hamed, Motl, Nicolis, Vafa '07]

Magnetic WGC: $\Lambda \lesssim g M_{\rm Pl}$
[Arkani-Hamed, Motl, Nicolis, Vafa '07]

+Scalar WGC: $m \leq \sqrt{g^2 q^2 - \mu^2}\, M_{\rm Pl}$
[Palti '17]

dS WGC: $m^2 \gtrsim g q M_{\rm Pl} H$
[Montero, Van Riet, Venken '19]

Axion WGC: $f \leq (1/S) M_{\rm Pl}$
[Arkani-Hamed, Motl, Nicolis, Vafa '07]

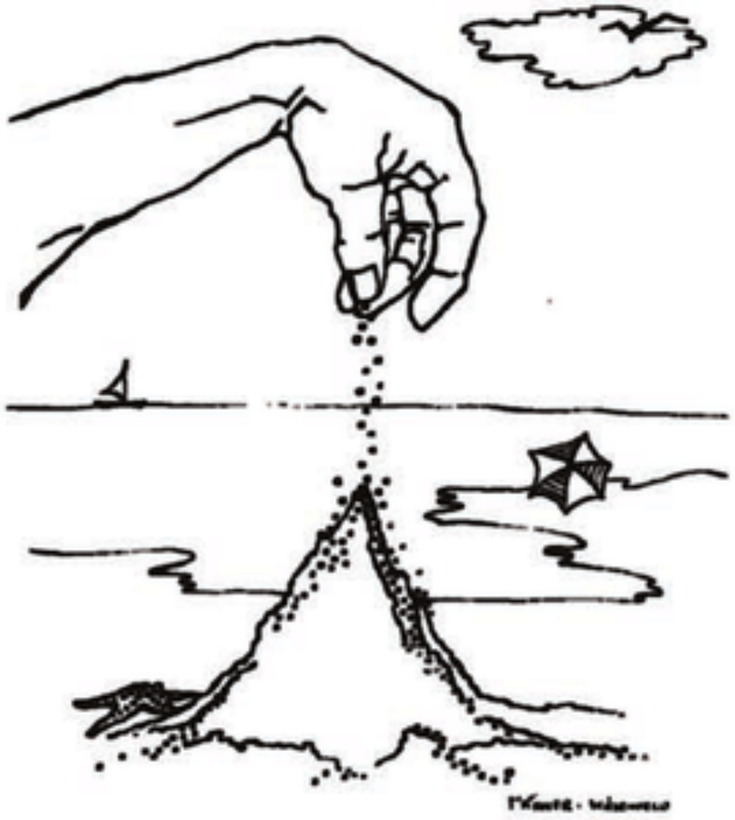
See [P. Draper, I.G. Garcia and M. Reece, *Snowmass White Paper: Implications of Quantum Gravity for Particle Physics*, 2203.07624]

***New hierarchies from EFT + gravity.***

Relevant for the hierarchy problem [Cheung & Remmen '14; Ibañez, Martin-Lozano, Valenzuela '17; NC, Garcia Garcia, Koren '19; March-Russell, Petrossian-Byrne '20]

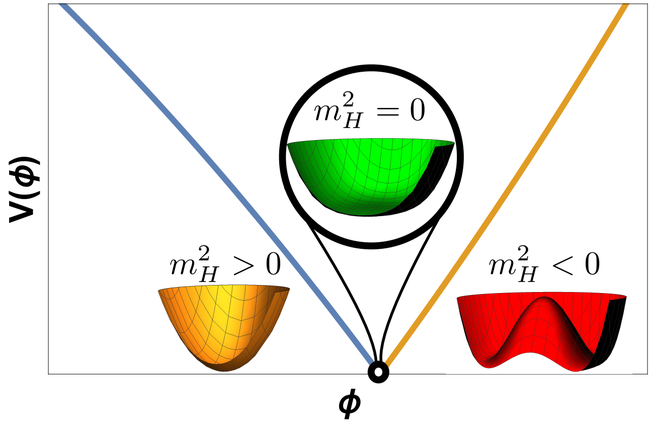
# Self-Organized Criticality

Some systems evolve into critical states on their own (sandpiles, a la [Bak, Tang, Wiesenfeld ’84]).
Wouldn’t that be nice? [Giudice ’08, Kaplan ‘97]

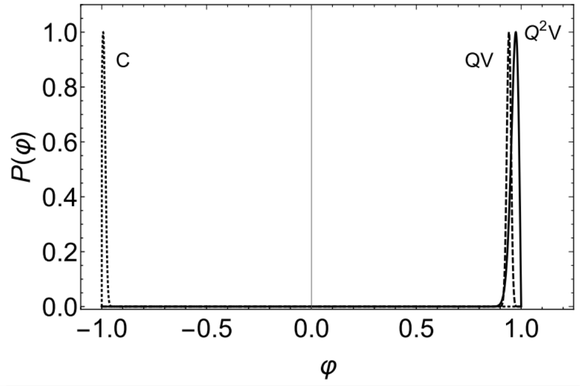


**20. Self-organized Criticality**

Vanishing Higgs mass coinciding with potential minimum for an extra-dimensional modulus field
[Eroencel, Hubisz, Rigo ’18]




Localization of scalar fields exponentially close to critical points during eternal inflation
[Giudice, McCullough, You ’21]
[Khoury et al. ’19-’20]



1. Supersymmetry
2. Global symmetry
3. Discrete symmetry
4. Modular invariance
5. RS/Technicolor
6. LED/$10^{32}$xSM
7. LST/Clockwork
8. Classicalization
9. Disorder
10. Anthropics
11. Relaxation
12. NNaturalness
13. Crunching away
14. Conformal symmetry
15. Asymptotic fragility
16. Agravity
17. Lee-Wick Theory
18. Weak gravity conjecture
19. Non-commutative QFT
20. Self-organized criticality
21. …

*With apologies for the many omissions…*

# What next?

See [C.Cordova, T. Dumitrescu, K. Intriligator, S-H, Shao, *Snowmass White Paper: Generalized Symmetries in QFT and Beyond*, 2205.09545]

**Generalized Global Symmetry**

| Properties of symmetry op. | Ordinary symmetry | Higher-form symmetry | Non-invertible symmetry |
|---|---|---|---|
| Codimension in spacetime | 1 | $> 1$ | $\geq 1$ |
| Topological | yes | yes | yes |
| Fusion rule | group $g_1 \times g_2 = g_3$ | group $g_1 \times g_2 = g_3$ | fusion ring $a \times b = \sum_c N_{ab}^c c$ |

From [Shu-Heng Shao, KITP Snowmass Theory Frontier Conference]

Thank you!