

Lattice + Heavy Flavor Physics

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Heavy Flavors on the Lattice

- ▶ Quark masses

up ~ 0.002 GeV

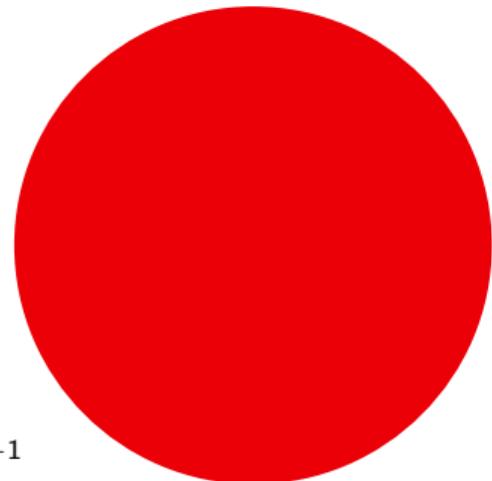


charm ~ 1.25 GeV

down ~ 0.005 GeV

• strange ~ 0.095 GeV

top ~ 175 GeV



bottom
 ~ 4.2 GeV

- ▶ Lattice simulations have a cutoff a^{-1}

→ Fully relativistic quarks require $am \ll 1$ i.e. $m \ll a^{-1}$

→ Typically $a^{-1} \gtrsim 2$ GeV $\Rightarrow m_{\text{charm}} \lesssim a^{-1} \lesssim m_{\text{bottom}}$

→ Charm but in particular bottom quarks require special considerations

Simulating heavy flavors

- ▶ Traditionally: simulate charm and bottom using **effective actions**
 - Heavy quark effective Theory (HQET), Non-Relativistic QCD, Relativistic Heavy Quark (RHQ, Fermilab, Tsukuba)
 - Allow to simulate charm and bottom quarks on coarser lattices
 - Additional systematic uncertainties, partly perturbative renormalization, ...
 - Few percent total errors
- ▶ State-of-the-art: **fully relativistic** simulations at $a^{-1} > 2 \text{ GeV}$
 - Heavy Highly Improved Staggered Quarks (HISQ), Heavy Domain-Wall Fermions (DWF), ...
 - Same action for light (up/down/strange) as for heavy (charm/bottom) quarks
 - ~~ Simulate heavier than charm and extrapolate
 - Fully nonperturbative renormalization straight-forward, reduced systematic uncertainties
 - Sub-percent precision feasible ~~ **QED effects** become relevant

White Paper: arXiv:2205.15373

CERN-TH-2022-036 FERMILAB-CONF-22-433-SCD-T JLAB-THY-22-3582
MITP-22-020 MIT-CTP/5413 MS-TP-22-07 SI-HEP-2022-11

A lattice QCD perspective on weak decays of b and c quarks Snowmass 2022 White Paper

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Leptonic Decays

Exclusive Semi-Leptonic Decays

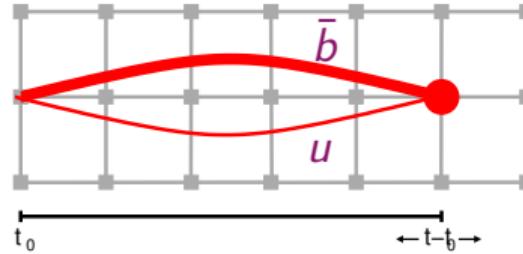
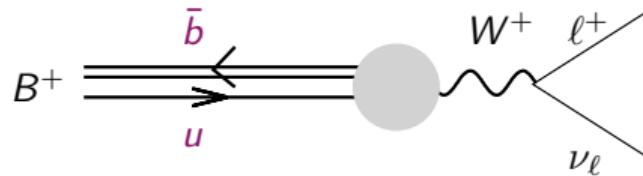
Meson Mixing and Lifetimes

b and c Quark Masses

Radiative Decays and Corrections

Inclusive Decays on the Lattice

Leptonic Decays



- ▶ Conventional parametrization

$$\Gamma(B \rightarrow \ell \nu_\ell) = \frac{m_B}{8\pi} G_F^2 f_B^2 |V_{ub}|^2 m_\ell^2 \left(1 - \frac{m_\ell^2}{m_B^2}\right)^2$$

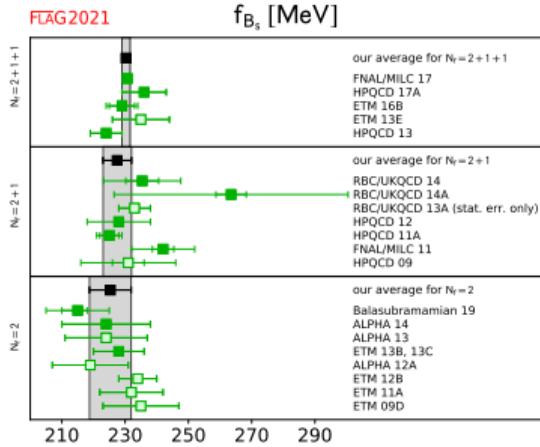
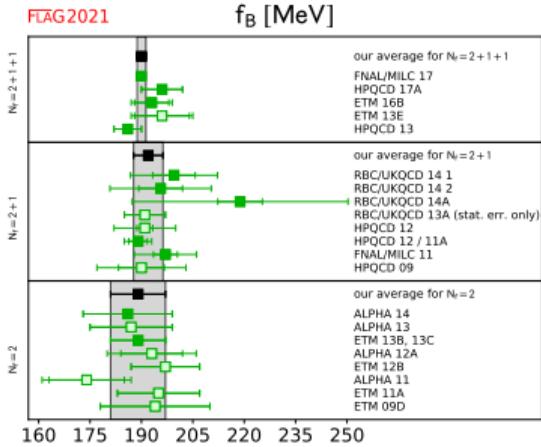
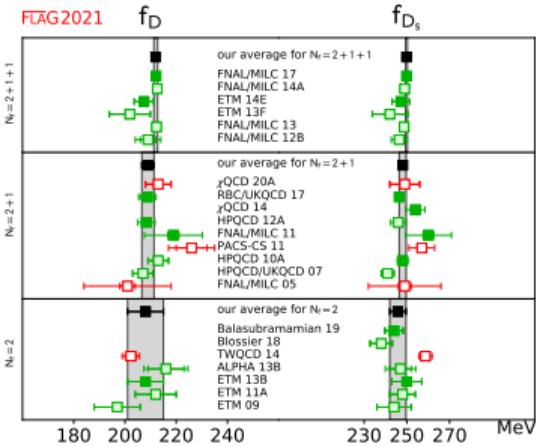
experiment nonperturbative input CKM known

- ▶ Determine $|V_{ub}|$ by combining f_B with $B \rightarrow \tau \nu$ experimental measurement
- ▶ Measure matrix element with axial current

$$\langle 0 | A_\mu | B(p) \rangle = i f_B p_\mu$$

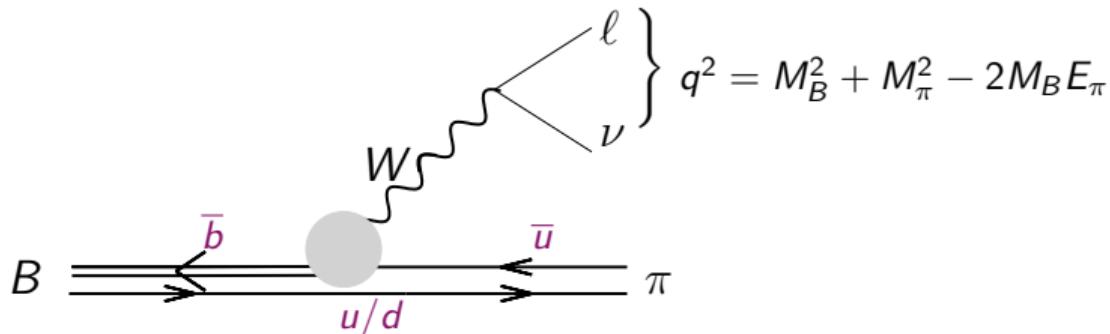
Leptonic Decays

[FLAG 2021]



- ▶ Results have reached sub-percent precision, good agreement between different collaborations
- ▶ Further progress requires including QED and strong-isospin breaking effects
 - ~~ Radiative decays

Exclusive Semi-Leptonic Decays: Example $B \rightarrow \pi \ell \nu$

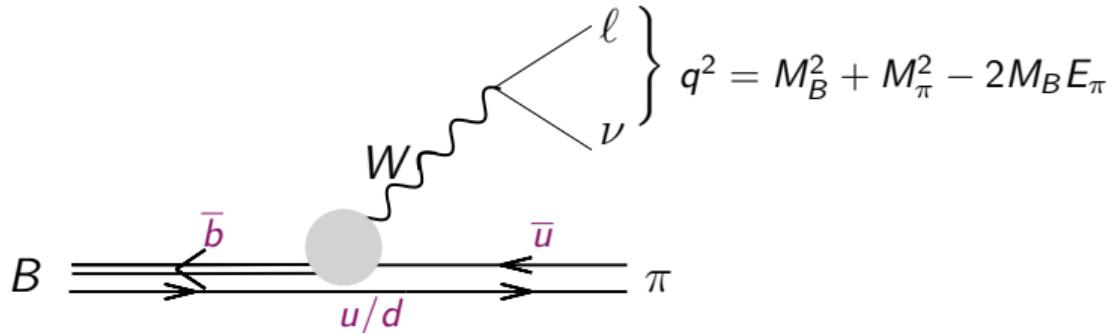


- ▶ Conventionally parametrized placing the B meson at rest

$$\frac{d\Gamma(B \rightarrow \pi \ell \nu)}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} \frac{(q^2 - m_\ell^2)^2 \sqrt{E_\pi^2 - M_\pi^2}}{q^4 M_B^2}$$

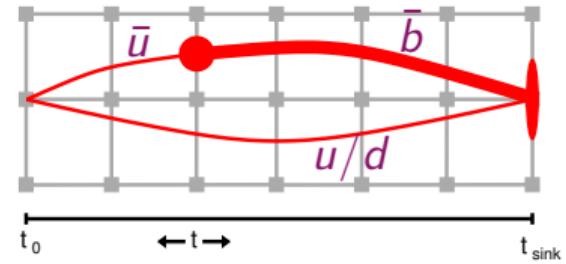
experiment	CKM $\times \left[\left(1 + \frac{m_\ell^2}{2q^2}\right) M_B^2 (E_\pi^2 - M_\pi^2) f_+(q^2) ^2 + \frac{3m_\ell^2}{8q^2} (M_B^2 - M_\pi^2)^2 f_0(q^2) ^2 \right]$	known nonperturbative input
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Exclusive Semi-Leptonic Decays: Example $B \rightarrow \pi \ell \nu$

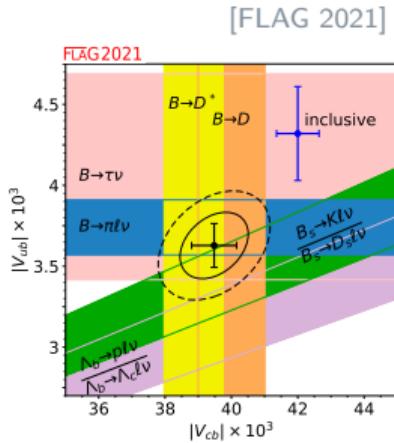
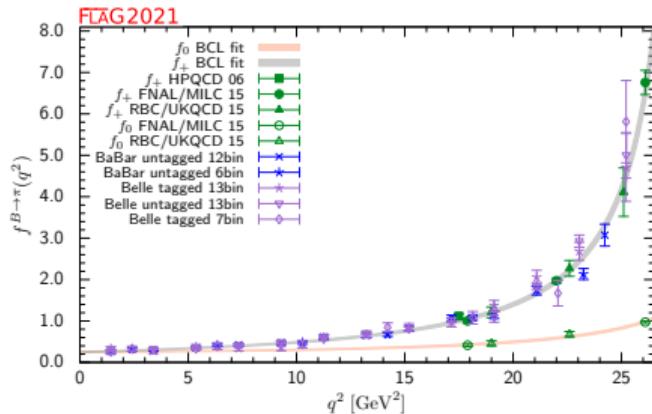
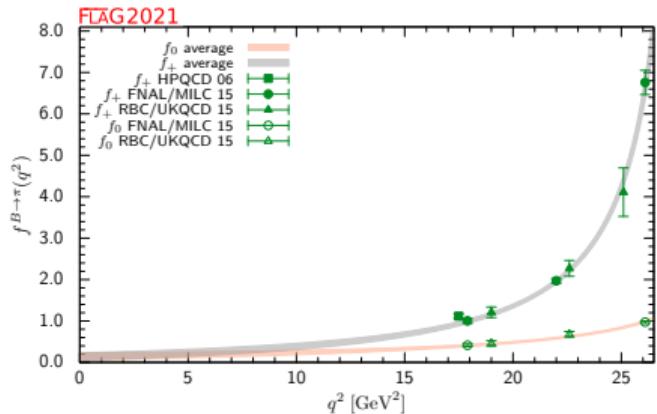


- ▶ Calculate hadronic matrix element for the flavor changing vector current V^μ in terms of the form factors $f_+(q^2)$ and $f_0(q^2)$

$$\langle \pi | V^\mu | B \rangle = f_+(q^2) \left(p_B^\mu + p_\pi^\mu - \frac{M_B^2 - M_\pi^2}{q^2} q^\mu \right) + f_0(q^2) \frac{M_B^2 - M_\pi^2}{q^2} q^\mu$$



Exclusive Semi-Leptonic Decays: Example $B \rightarrow \pi \ell \nu$



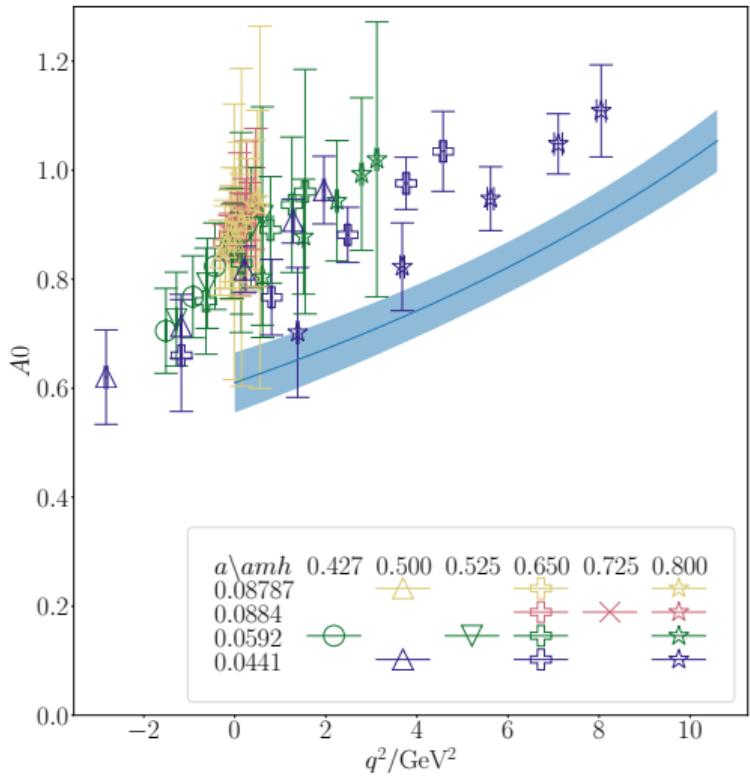
- Used effective actions only allowed determinations of form factors at large q^2
- Combined fit with experimental data from BaBar and Belle gives $|V_{ub}^{\text{excl}}|$
- Shape of lattice data consistent with experimental data
- New calculation by JLQCD [Colquhoun et al. arXiv:2203.04938]
- Updates from other collaborations expected relatively soon

Exclusive Semi-Leptonic Decays: Example $B_s \rightarrow D_s^* \ell \nu$

$$\begin{aligned}\langle D_s^*(k, \varepsilon_\nu) | \mathcal{V}^\mu | B_s(p) \rangle &= V(q^2) \frac{2i\varepsilon^{\mu\nu\rho\sigma} \varepsilon_\nu^* k_\rho p_\sigma}{M_{B_s} + M_{D_s^*}} \\ \langle D_s^*(k, \varepsilon_\nu) | \mathcal{A}^\mu | B_s(p) \rangle &= A_0(q^2) \frac{2M_{D_s^*} \varepsilon^* \cdot q}{q^2} q^\mu \\ &\quad + A_1(q^2) (M_{B_s} + M_{D_s^*}) \left[\varepsilon^{*\mu} - \frac{\varepsilon^* \cdot q}{q^2} q^\mu \right] \\ &\quad - A_2(q^2) \frac{\varepsilon^* \cdot q}{M_{B_s} + M_{D_s^*}} \left[k^\mu + p^\mu - \frac{M_{B_s}^2 - M_{D_s^*}^2}{q^2} q^\mu \right]\end{aligned}$$

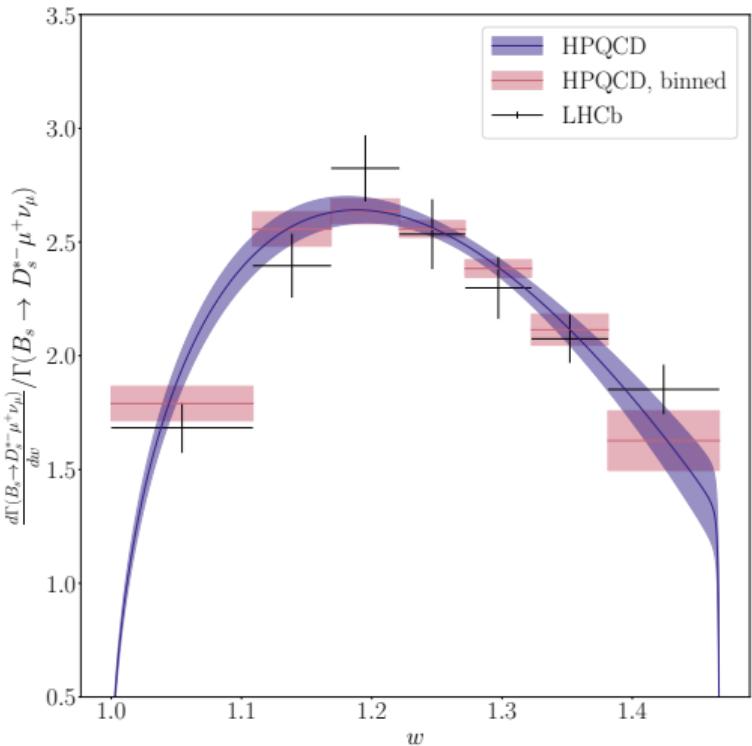
- ▶ Determine the four form factors $V(q^2)$, $A_0(q^2)$, $A_1(q^2)$, $A_2(q^2)$
or in HQE convention $h_V(w)$, $h_{A_0}(w)$, $h_{A_1}(w)$, $h_{A_2}(w)$
- ▶ Narrow-width approximation i.e. D_s^* is treated as a QCD-stable particle

Exclusive Semi-Leptonic Decays: Example $B_s \rightarrow D_s^* \ell \nu$



- ▶ HPQCD [Judd, Davies PRD105(2022).094506]
 - All-HISQ setup
 - Fully non-perturbative renormalization
 - Simulate heavier-than-charm \rightarrow close-to-bottom
 - Directly cover most of the allowed q^2 range at the finest lattice spacing
- ~ Parametrize pole mass for different charm masses

Exclusive Semi-Leptonic Decays: Example $B_s \rightarrow D_s^* \ell \nu$



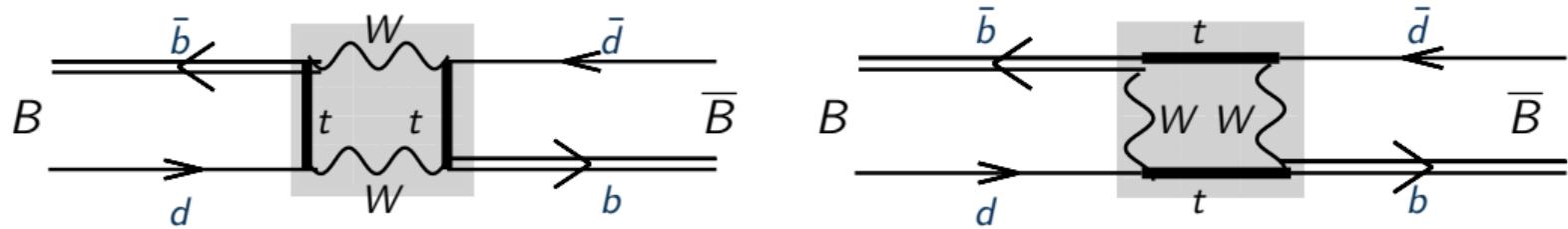
- ▶ HPQCD [Judd, Davies PRD105(2022).094506]
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 - Simulate heavier-than-charm \rightarrow close-to-bottom
 - Directly cover most of the allowed q^2 range at the finest lattice spacing
- ~ Parametrize pole mass for different charm masses
- ▶ Combination with LHCb [LHCb JHEP12(2020)144]
 - $|V_{cb}|_{BGL} = 42.7(1.5)_{\text{lat}}(1.7)_{\text{exp}}(0.4)_{\text{em}} \times 10^{-3}$
 - $|V_{cb}|_{CLN} = 41.6(1.5)_{\text{lat}}(1.6)_{\text{exp}}(0.4)_{\text{em}} \times 10^{-3}$

Beyond the narrow width approximation

General $1 \rightarrow 2$ formalism has been developed: $B \rightarrow (K^* \rightarrow K\pi)\ell^+\ell^-$ or $B \rightarrow (\rho \rightarrow \pi\pi)\ell\nu$
[Lüscher CMP105(1986)153][NPB364(1991)237] [Lellouch, Lüscher CMP219(2001)219] [Hansen, Sharpe PRD86(2012)016007]
[Briceño et al. PRD91(2015)034501] [Briceño et al. PRD91(2021)054509] ...

- ▶ Rigorous non-perturbative relation between finite-volume Euclidean quantities and physical, infinite-volume $1 \rightarrow 2$ decay amplitude
- ▶ Conceptually different calculations and substantially larger computational effort
- ▶ Pioneered for $K \rightarrow \pi\pi$ by RBC-UKQCD [Blum et al. PRL108(2012)141601] [Bai et al. PRL115(2015)212001]
- ▶ $B_{(s)}$, $D_{(s)}$ decays typically more challenging because the large decaying meson mass allows kinematically additional final states
- ▶ Difficulty mainly determined by energy of the two-hadron final state
 - ⇒ Semi-leptonic calculations where leptons carry away much initial energy are more accessible
- ▶ Honest calculations of physical processes requires a formalism treating rigorously all important open channels including e.g. four-particle states
 - Ongoing work to extend the general $1 \rightarrow 2$ formalism to more particles [Müller, Rusetsky JHEP03(2021)152] [Hansen et al. JHEP04(2021)113]

Meson Mixing and Lifetimes



- ▶ B -meson mixing dominated by top-loops \Rightarrow short distance

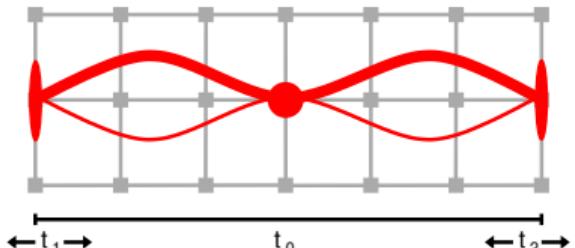
$$\Delta m_q = \frac{G_F^2 m_W^2}{6\pi^2} \eta_B S_0 M_{B_q} f_{B_q}^2 B_{B_q} |V_{tq}^* V_{tb}|^2, \quad q = d, s$$

- ▶ Constants and lattice uncertainties cancel in ratio [Bernard, Blum, Soni PRD58(1998)014501]

$$\frac{\Delta m_s}{\Delta m_d} = \frac{M_{B_s}}{M_{B_d}} \xi^2 \frac{|V_{ts}|^2}{|V_{td}|^2}, \quad \xi^2 = \frac{f_{B_s}^2 B_{B_s}}{f_{B_d}^2 B_{B_d}}$$

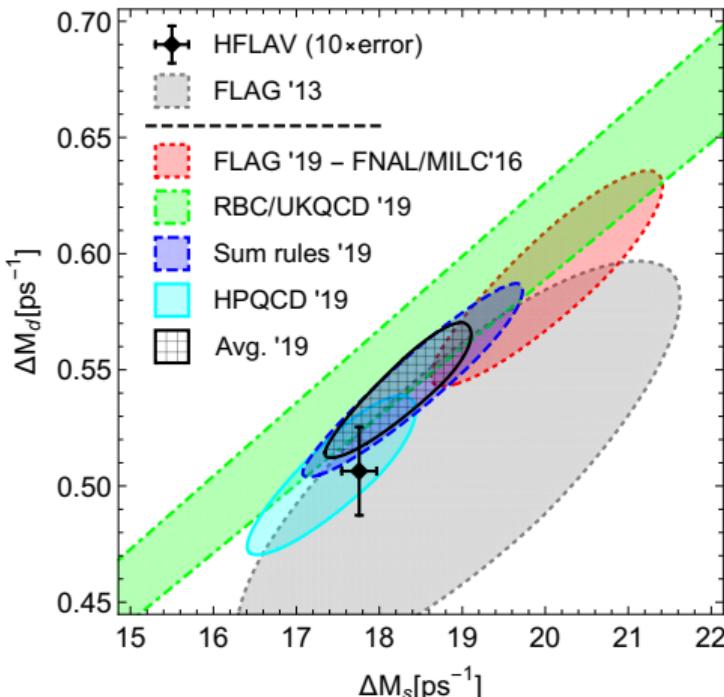
Meson Mixing and Lifetimes

[Di Luzio et al. JHEP12(2019)009]



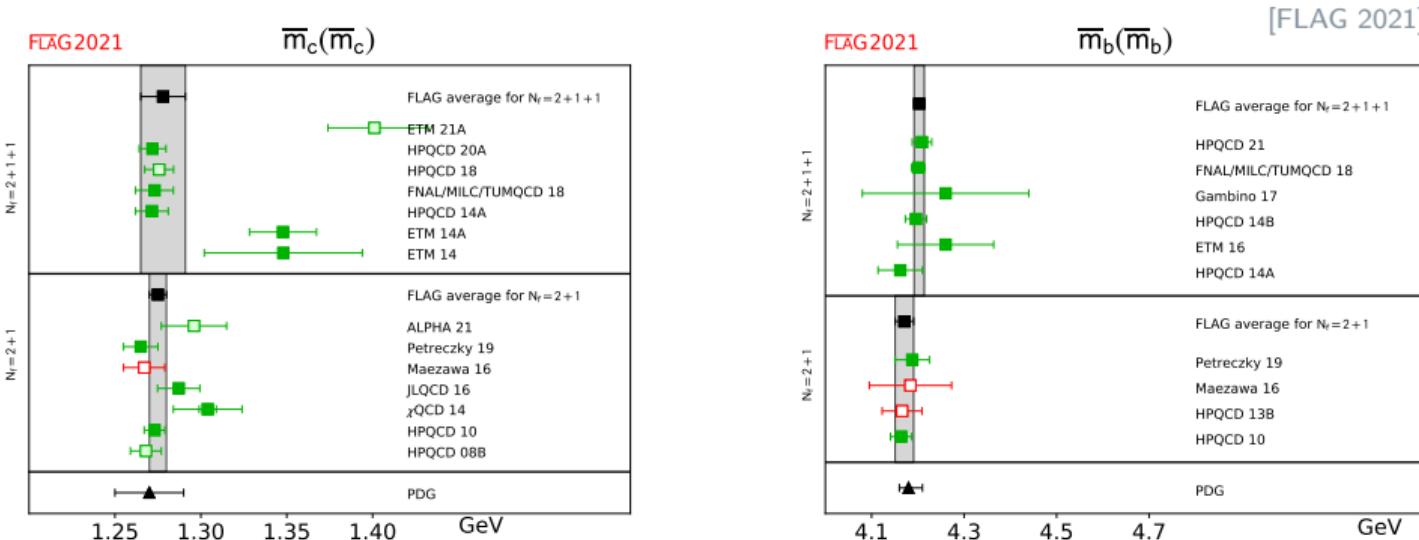
- ▶ Determine full five operator basis
→ SM Δm_q

$$\begin{aligned} \langle \bar{B}_q^0 | [\bar{b} \gamma^\mu (1 - \gamma_5) q] [\bar{b} \gamma_\mu (1 - \gamma_5) q] | B_q^0 \rangle \\ = \frac{3}{8} f_{B_q}^2 M_{B_q}^2 B_{B_q} \end{aligned}$$



- ▶ Add dim-7 operators [Davies et al. PRL124(2020)082001]
- ▶ Lifetimes much harder: operators mix, disconnected contributions

b and c Quark Masses



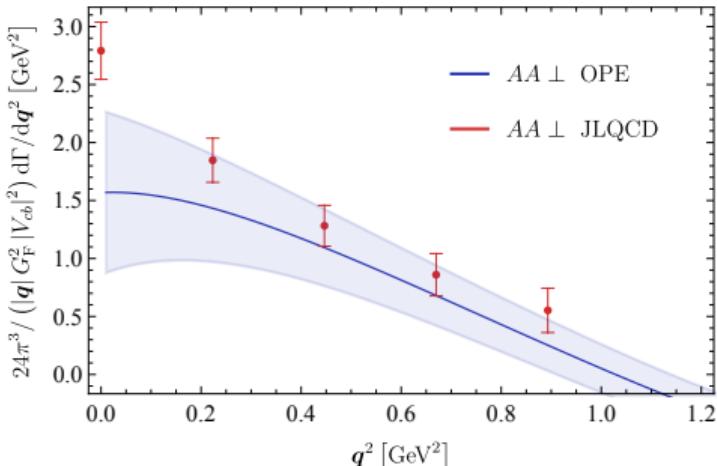
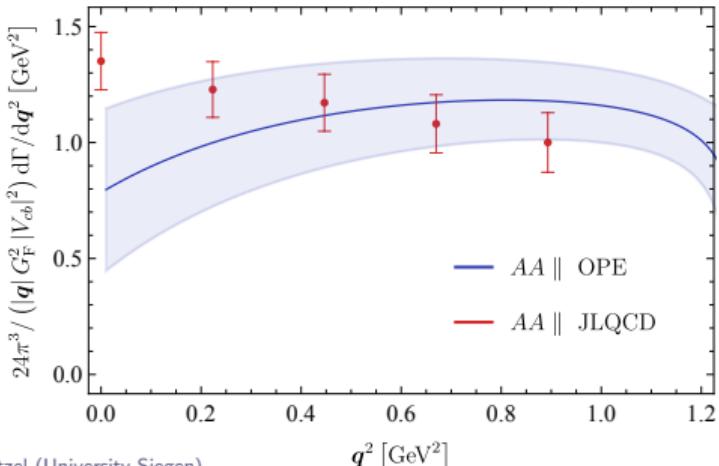
- ▶ Determinations based on a variety of different methods, with different systematics
- ▶ Sub-percent level precision, QED effects are small

Radiative Decays and Corrections

- ▶ Hadronic processes including QED effects
 - Real photon amplitudes required to determine CKM matrix elements at $\mathcal{O}(\alpha_{\text{em}})$
[Carrasco et al. PRD91(2015)074506] [DiCarlo et al. PRD100(2019)034514]
 - Radiative decays lift helicity suppression [Atwood, Eilam, Soni MPL A11(1996)1061]
 - For large photon energy the process $B \rightarrow \ell\nu\gamma$ cleanest probe of $1/\lambda_B$
first inverse moment of the B -meson light-cone distribution amplitude
- ▶ First principles calculation for structure dependent form factors are planned e.g. for
 $B \rightarrow \ell\nu\ell\gamma$, $D_{(s)} \rightarrow \ell\nu\ell\gamma$, $B_{(s)} \rightarrow \ell^+\ell^-\gamma$, $D_{(s)} \rightarrow \ell^+\ell^-\gamma$
[Kane et al. PoS Lattice2019 134] [Kane et al. PoS Lattice2021 162]

Inclusive Decays on the Lattice

- ▶ Longstanding $2 - 3\sigma$ tensions between inclusive and exclusive $|V_{ub}|$ and $|V_{cb}|$ determinations
- ▶ Extract Minkowski-space spectral densities from finite-volume Euclidean correlation functions
- ▶ Several promising new ideas: [Hashimoto PTEP(2017)053B03] [Hansen, Meyer, Robaina PRD96(2017)094513]
[Gambino, Hashimoto PRL125(2020)032001]
 - Evaluation of 4-point functions on the lattice
- ▶ First numerical comparisons to OPE [Gambino et al. JHEP07(2022)083]



Leptonic decays, c & b Quark Masses

- Sub-percent level precision achieved, need QED effects
- Good agreement between collaborations/methods → More results with sub-percent level errors

Semi-leptonic Decays

- Push for (sub-)percent level precision
- Increase range of directly simulated q^2
- Overall good agreement of lattice data
- ⇒ Overcome narrow width approximation
- ⇒ Shape comparisons
- More processes, more collaborations,
more baryonic decays

Neutral Meson Mixing

- Push B_{b_q} to percent-level precision (5 operators)
- Resolve small tension between lattice results
- ⇒ Determine dim-7 operators
- More results, different actions

Explore: radiative decays, lifetimes, inclusive decays, pdf, ...
... non-leptonic decays