Motivation

- There are many motivations for studying models of new physics Beyond the Standard Model (BSM). Dark matter, neutrino mass, Higgs sector, flavor puzzles...

- Perturbative extensions of the SM are nice for the theorist, but sometimes physics is strongly coupled, e.g. QCD!

- If BSM physics is strongly coupled, lattice can give physics information that is otherwise inaccessible.
Approaches to lattice BSM*

• 1) “**Top-down BSM**”: pick a specific strongly-coupled model, put it on the lattice, calculate things of pheno relevance.

• 2) “**Bottom-up BSM**”: choose a class of theories with common properties or shared description: large-Nc expansion, near-conformal (dilaton EFT), etc. Try to study the broad

• 3) “**Pure exploration**”: studies of non-perturbative phenomena, strongly-coupled QFT purely for theory interest, with no immediate pheno connection.

*not a unique way to organize; many lattice calculations have motivation/application in more than one area. Never a “waste” even if we rule out a specific theory…
1) (Example) top-down models

**Composite Higgs:** new strongly-coupled sector at the electroweak scale; Higgs is a composite bound state. (W/Z, top often have some composite part too.)

**Composite dark matter:** dark “hidden sector” which is strongly coupled. Rich spectrum of possible DM candidates: dark baryons, dark mesons (e.g. SIMPs), dark glueballs...

**Neutral naturalness:** mirror copy of SM components in hidden sector. Rich structure for e.g. dark matter. Lattice SU(3) results (particularly w/ heavy quarks) can be relevant.

2) Understanding the larger space

Large-N limit(s) provide analytic structure for predictions; lattice can **test** these analytic expansion and **compute** numerical values for expansion coefficients.

Conformal phase transition cuts across the parameter space; low-energy theory near the edge (**dilaton EFT**) can be tested and probed by lattice.
• Emergence of light 0++ state near conformal transition - “pseudo-dilaton” - in multiple theories. Above: SU(3) Nf=2 sextet (left), SU(3) Nf=8 fundamental (right). Dynamical surprise (although speculation in literature earlier) - first indications from lattice!

• Model-building work is ongoing to understand the effective theory of pions + light scalar (“dilaton EFT”). Some work on pheno-viable composite Higgs models based on such theories, e.g. arXiv:2205.03320.
3) Pure exploration has the most variety. One example is lattice calculation in N=4 SYM. (Building SUSY into lattice is quite difficult - current projects are culmination of many years of work on theory and algorithms!)

Points above: scaling of Wilson loops vs. ’t Hooft coupling λ, from lattice; lines show holography-predicted, non-perturbative scaling behavior.
Another example: quantum finite elements (QFE) approach generalizes ordinary finite element method for use in lattice QFT.

Enables simulation of QFT on curved manifolds, allowing radial quantization to study CFT w/ exponential scale separations.

Results for $\phi^4$ theory in 3d show excellent convergence to known results from conformal bootstrap (left), particularly w/irrelevant Ricci term included.

Conclusion

• Lattice is often used as a source of high-precision results for QCD

• But lattice is also very useful as a “numerical laboratory” to gain qualitative insights about other new theories!

• Study specific strongly-coupled theories; look at general properties of classes of theories; learn new things about QFT.
Backup
Aside: the infrared-conformal phase

Many theories in the space are “cousins” of QCD: color charges confine into a spectrum of “hadron” bound states.

In the infrared-conformal phase, gauge coupling $g$ will approach an “infrared fixed point”, freezing at some $g = g^* > 0$.

This freezing restores scale invariance - we recover a conformal field theory (CFT). CFTs have a unique “spectrum” of operator anomalous dimensions; no confined bound states.

These CFTs can appear in many models of new physics, but only with broken scale invariance, since our world isn’t conformal! Still, learning about the symmetric limit is useful and important. (Analogous to supersymmetric BSM models: SUSY must be broken, but it’s still useful to describe the physics.)
What about effective theories?

- We can take a more bottom-up approach and say: just identify the right effective field theory (EFT) for collider physics, dark matter detection, etc.

- Nothing wrong with this approach, but using only the EFT has limited predictive power: need to fix many (infinite!) low-energy constants from experiment.

- Plus, EFT comes with an energy cutoff: fine for working in the low-energy limit at the threshold of discovery, but many details of the full theory are out of reach.

- **EFT + lattice** allows analytic calculation but many LECs are determined from a handful of underlying UV parameters - best of both worlds!

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\mathcal{L}_{\text{EFT}} = c_1 + c_2 + c_3 + c_4 + \ldots
\]

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\mathcal{L}_{\text{UV}} = a + b
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