Neutrino Theory and Particle Physics

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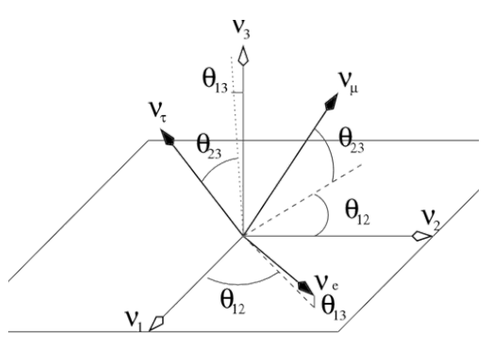
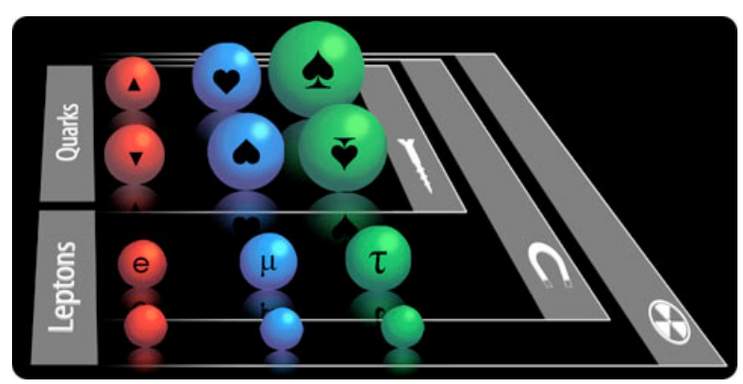
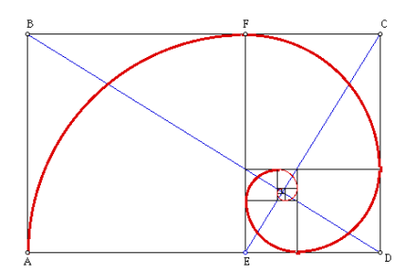
Seattle Snowmass Summer Meeting, 2022

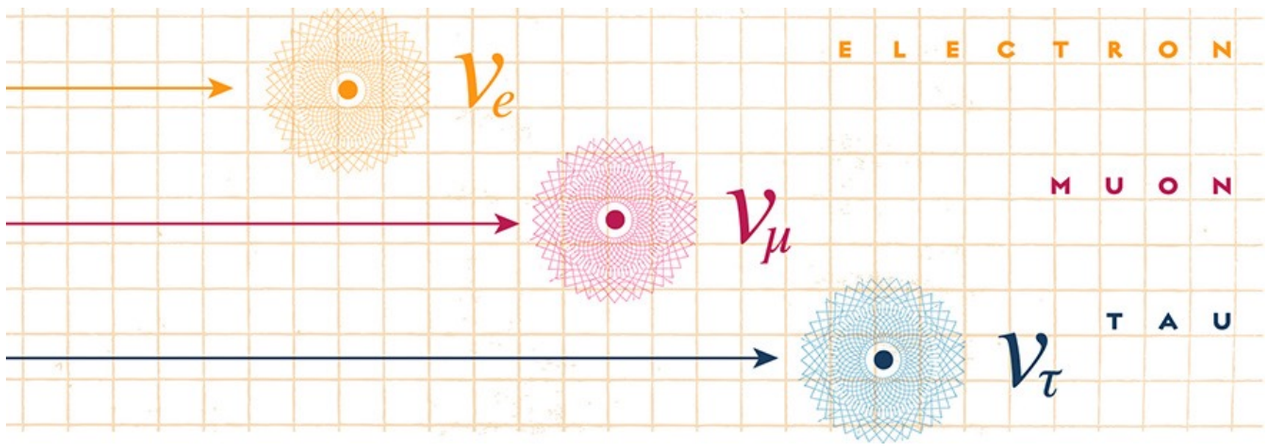






(image credits: Wikipedia, E. Palti)

A wealth of neutrino data…



(image credit: C. Wiens)

neutrino oscillations discovery!

SuperK '98

2 decades+ of results…

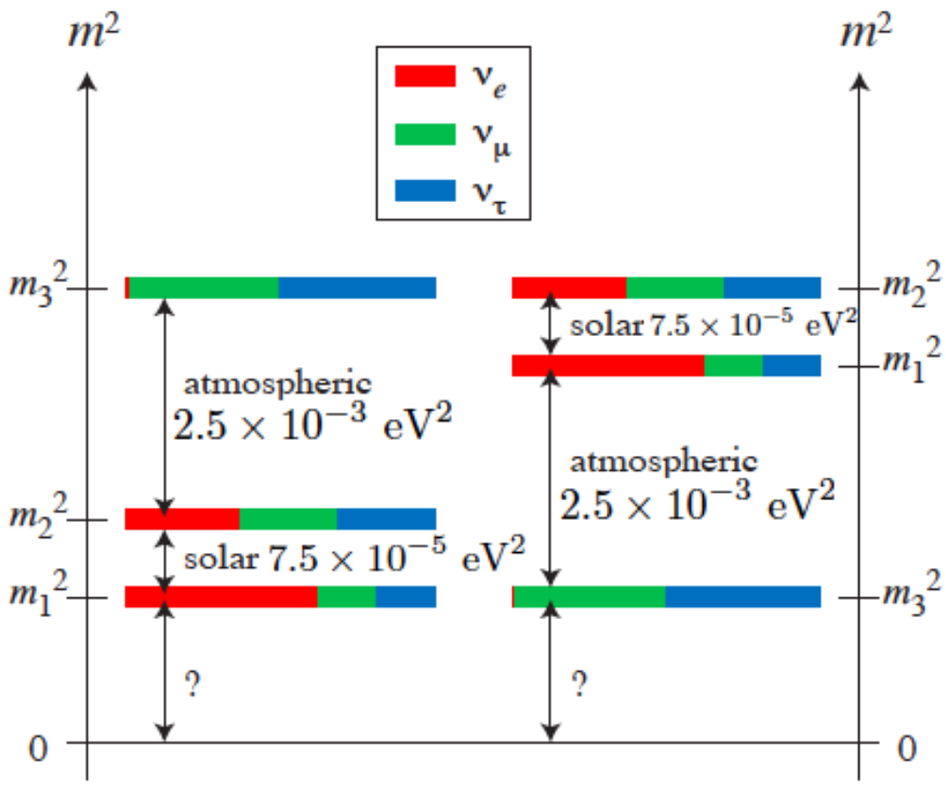
Some highlights:

- 1998: atmospheric ν_μ disappearance **(SK)**
- 2002: solar ν_e disappearance **(SK)**
- 2002: solar ν_e appear as ν_μ, ν_τ **(SNO)**
- 2004: reactor $\overline{\nu}_e$ oscillations **(KamLAND)**
- 2004: accelerator ν_μ disappearance **(K2K)**
- 2006: accelerator ν_μ disappearance **(MINOS)**
- 2011: accel. ν_μ appear as ν_e **(T2K,MINOS)**
- 2012: reactor $\overline{\nu}_e$ disappear **(Daya Bay, RENO,…)**
  reactor angle measured!
- 2014: CP violation hint? **(T2K)**
- 2015: normal hierarchy hint? **(SK, T2K, NOvA)**
- 2016: non-maximal atm hint? **(NOvA)**
- 2018: CP cons disfavored at 2σ **(T2K)**
- 2021: improved direct mass limit **(KATRIN)**

The emergent picture…

a (seemingly) robust 3-neutrino mixing scheme

(image credit: King, Luhn)



(normal ordering) (inverted ordering)

$$\sum m_i < 0.12 \text{ eV}$$ **(cosmology)**

$$m_\nu < 0.8 \text{ eV}$$ **KATRIN (2021)**

Global Fits: **Esteban et al. '20, (www.nu-fit.org)**

Capozzi et al. '21, deSalas et al. '20

NuFIT 5.1 (2021)

		Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 2.6$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
without SK atmospheric data	$\sin^2\theta_{12}$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$	$0.304^{+0.012}_{-0.012}$	$0.269 \to 0.343$
	$\theta_{12}/^\circ$	$33.44^{+0.77}_{-0.74}$	$31.27 \to 35.86$	$33.45^{+0.77}_{-0.74}$	$31.27 \to 35.87$
	$\sin^2\theta_{23}$	$0.573^{+0.018}_{-0.023}$	$0.405 \to 0.620$	$0.578^{+0.017}_{-0.021}$	$0.410 \to 0.623$
	$\theta_{23}/^\circ$	$49.2^{+1.0}_{-1.3}$	$39.5 \to 52.0$	$49.5^{+1.0}_{-1.2}$	$39.8 \to 52.1$
	$\sin^2\theta_{13}$	$0.02220^{+0.00068}_{-0.00062}$	$0.02034 \to 0.02430$	$0.02238^{+0.00064}_{-0.00062}$	$0.02053 \to 0.02434$
	$\theta_{13}/^\circ$	$8.57^{+0.13}_{-0.12}$	$8.20 \to 8.97$	$8.60^{+0.12}_{-0.12}$	$8.24 \to 8.98$
	$\delta_{CP}/^\circ$	194^{+52}_{-25}	$105 \to 405$	287^{+27}_{-32}	$192 \to 361$
	$\frac{\Delta m^2_{21}}{10^{-5}\text{ eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$
	$\frac{\Delta m^2_{3\ell}}{10^{-3}\text{ eV}^2}$	$+2.515^{+0.028}_{-0.028}$	$+2.431 \to +2.599$	$-2.498^{+0.028}_{-0.029}$	$-2.584 \to -2.413$

		Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 7.0$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
with SK atmospheric data	$\sin^2\theta_{12}$	$0.304^{+0.012}_{-0.012}$	$0.269 \to 0.343$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$
	$\theta_{12}/^\circ$	$33.45^{+0.77}_{-0.75}$	$31.27 \to 35.87$	$33.45^{+0.78}_{-0.75}$	$31.27 \to 35.87$
	$\sin^2\theta_{23}$	$0.450^{+0.019}_{-0.016}$	$0.408 \to 0.603$	$0.570^{+0.016}_{-0.022}$	$0.410 \to 0.613$
	$\theta_{23}/^\circ$	$42.1^{+1.1}_{-0.9}$	$39.7 \to 50.9$	$49.0^{+0.9}_{-1.3}$	$39.8 \to 51.6$
	$\sin^2\theta_{13}$	$0.02246^{+0.00062}_{-0.00062}$	$0.02060 \to 0.02435$	$0.02241^{+0.00074}_{-0.00062}$	$0.02055 \to 0.02457$
	$\theta_{13}/^\circ$	$8.62^{+0.12}_{-0.12}$	$8.25 \to 8.98$	$8.61^{+0.14}_{-0.12}$	$8.24 \to 9.02$
	$\delta_{CP}/^\circ$	230^{+36}_{-25}	$144 \to 350$	278^{+22}_{-30}	$194 \to 345$
	$\frac{\Delta m^2_{21}}{10^{-5}\text{ eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$
	$\frac{\Delta m^2_{3\ell}}{10^{-3}\text{ eV}^2}$	$+2.510^{+0.027}_{-0.027}$	$+2.430 \to +2.593$	$-2.490^{+0.026}_{-0.028}$	$-2.574 \to -2.410$

Caveat: light sterile neutrino(s)?

(image credit: ParticleBites)

Anomalies:

1995: $\overline{\nu}_e$ appearance **(LSND)**

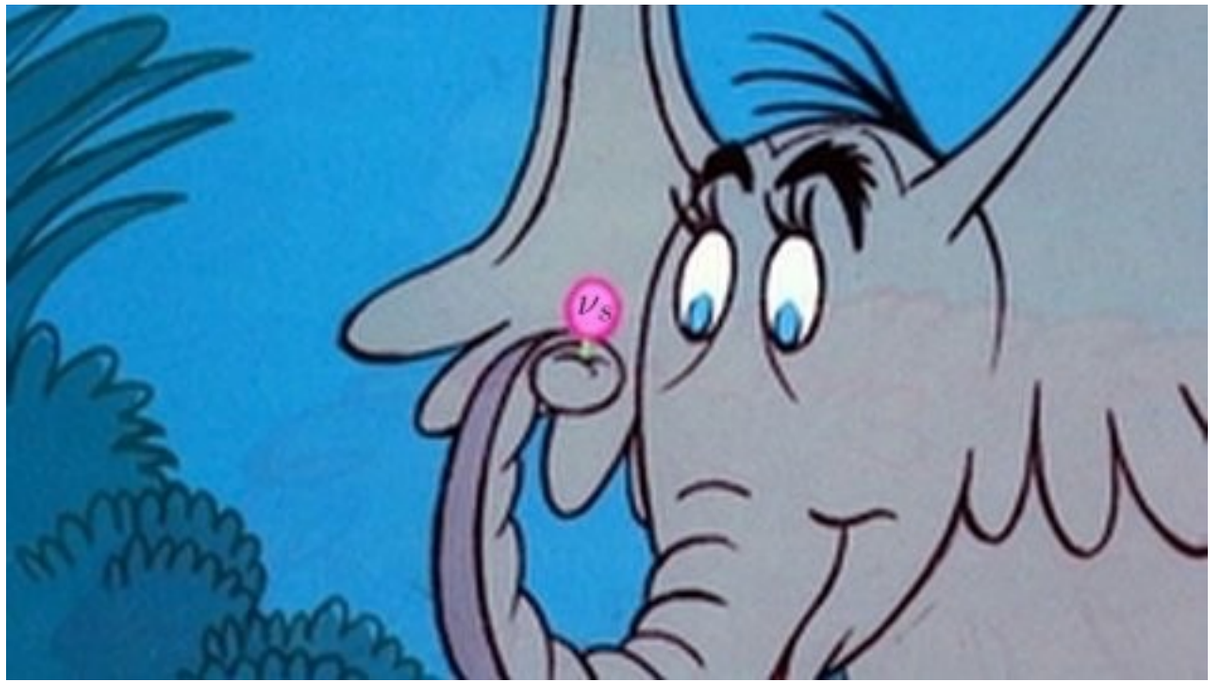
2007: $\overline{\nu}_e$ appearance **(MiniBooNE)**

2012: ν_e appearance **(MiniBooNE)**

1995: ν_e disappearance **(Gallium)**

2011: ν_e disappearance **(Reactor)**

[well-documented tension between appearance and disappearance data]



Recent update: Kopp's talk at Neutrino 2022

(see also: Arguelles' talk at Neutrino 2022)

status remains unclear!

Restrict focus here to 3 active light families only

Physics Beyond the Standard Model!

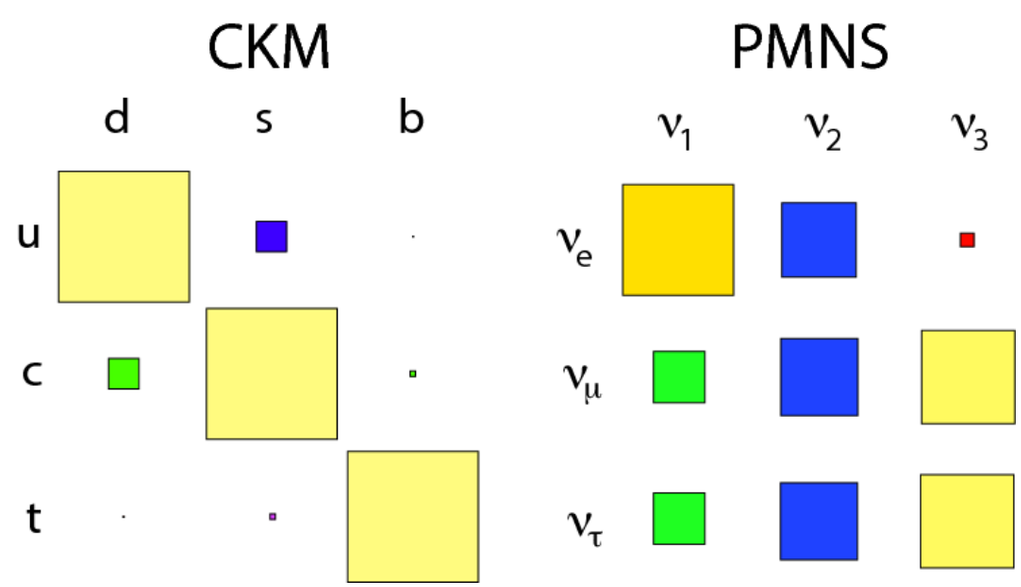
- **suppression of overall neutrino mass scale**

  window to new physics, possibly at very high energy scales!

- **implications for the flavor puzzle of the Standard Model**

  ✓ **two large lepton mixing angles (or more**)**

  ✓ **mass pattern: nature of mass hierarchy, neutrino mass ordering**



(image credit: S.Stone)

- **connections to cosmology, astrophysics (matter-antimatter asymmetry,…)**

(image credits: FB, Wikipedia)

Neutrino Masses

- **Minimal extensions of SM:**

  ✓ **include NR terms** $\mathcal{L} = \mathcal{L}_{\rm SM} + \mathcal{L}_5 + \ldots$

  ✓ **include RH neutrinos** $\mathcal{L} = \mathcal{L}_{\rm SM} + \mathcal{L}_4 + \ldots$

  which (if any) global symmetries are preserved in the truncation…

main question: are neutrinos Dirac or Majorana?

Dirac

$\Delta L = 0$

Majorana

$\Delta L = 2$

Critically important question, to be settled by experiment!

Majorana neutrinos

- **minimal scenario: SM → "νSM"**

$$\mathcal{L}_5 \simeq \frac{\lambda_{ij}}{\Lambda} L_i L_j H H$$

Weinberg '79

✓ **violates lepton number** $\Delta L = 2$

✓ **Majorana neutrino masses upon EWSB**

$$\boxed{\mathcal{M}_\nu^{\rm Maj} \simeq \frac{\lambda_{ij}\langle H\rangle^2}{\Lambda}}$$

if $\lambda \sim O(1)$

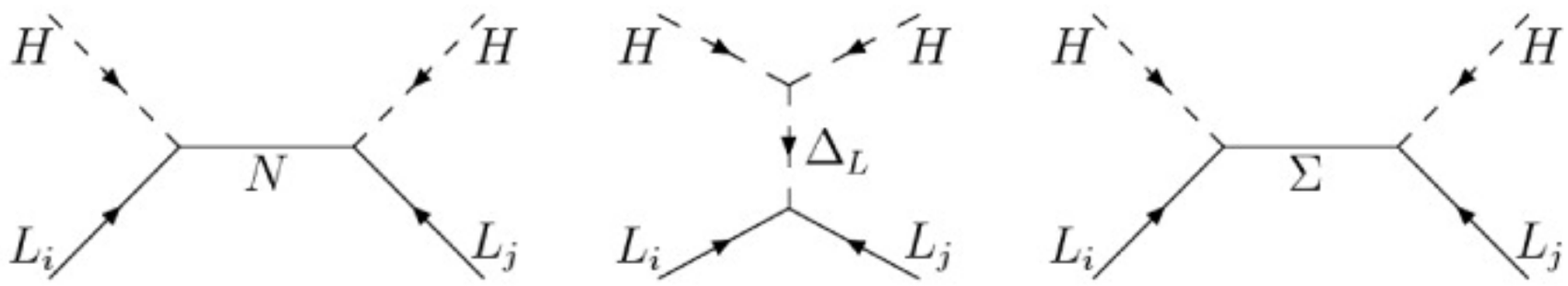
$\Lambda \gg \langle H\rangle \sim O(100\ \text{GeV})$

"seesaw"

Many ways to UV complete the νSM →

At tree level:

see Ma '98



Type I **Type II** **Type III**

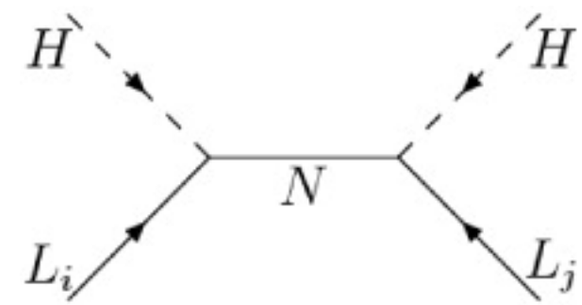
Type I seesaw

Minkowski '77, Gell-Mann, Ramond, Slansky '79, Yanagida '79
Mohapatra, Senjanovic '80, Schecter, Valle '80…

✓ **introduce right-handed neutrinos**

$$\mathcal{L} \simeq Y_{ij} L_i N_j H + M_{R\,ij} N_i N_j$$

$$\boxed{\mathcal{M}_\nu \sim \langle H \rangle^2 Y M_R^{-1} Y^T}$$



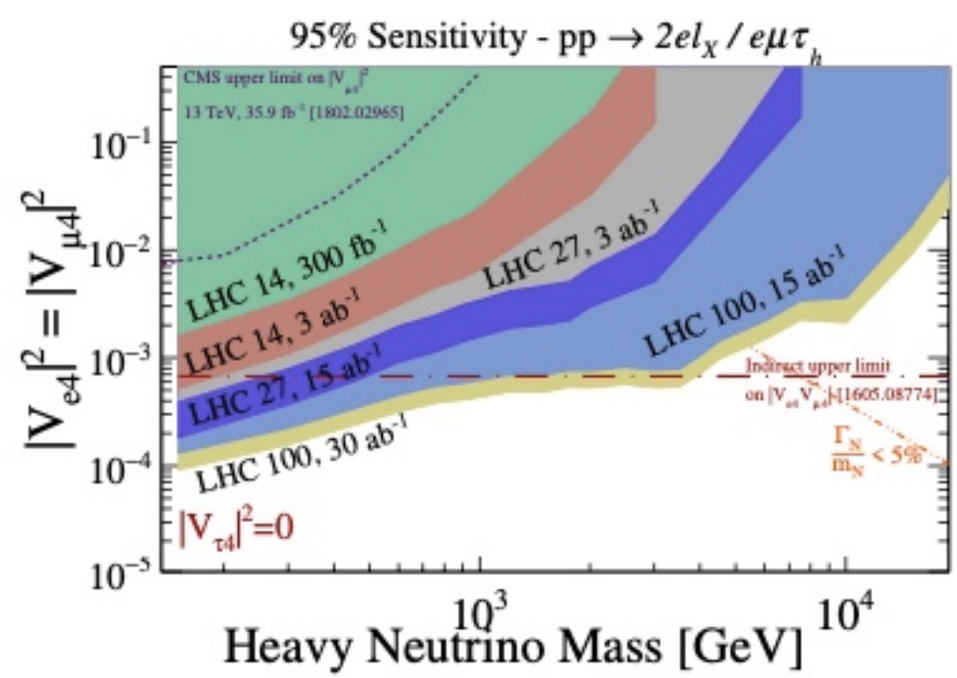
$$\mathcal{M}_\nu = \begin{pmatrix} 0 & m \\ m & M \end{pmatrix} \qquad m_1 \sim \frac{m^2}{M} \qquad m_2 \sim M \gg m_1$$ **(canonical version)**

"phenomenological" Type I seesaw: active — (heavy) sterile mixing

$$\nu_\ell = \sum_{m=1}^{3} U^*_{\ell m} \nu_m + \sum_{m'=4}^{3+n_R} V^*_{\ell m'} N_{m'}$$

Han et al., 2203.06131
Snowmass TF11

see also: del Aguila et al. '08, Atre et al. '09
Pascoli et al. '18,…

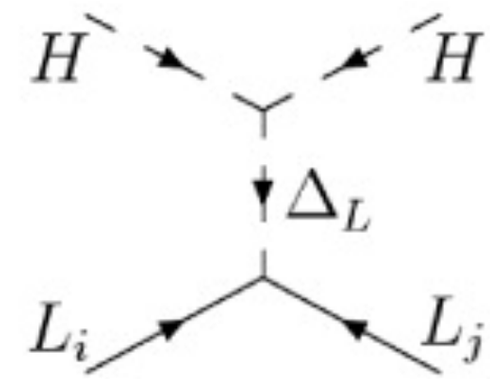


Type II seesaw

Magg, Wetterich '80, Lazarides, Shafi, Wetterich '81,
Cheng, Li '80, Schechter, Valle '80, Mohapatra, Senjanovic '81

✓ **introduce EW triplet Higgs scalar** $\Delta \sim (\mathbf{3}, 2)$

$$\mathcal{L} \simeq Y_{\Delta\, ij} L_i L_j \Delta + \mu_\Delta H H \Delta$$



$$\boxed{\mathcal{M}_\nu \sim \langle H \rangle^2 Y_\Delta \mu_\Delta / M_\Delta^2}$$

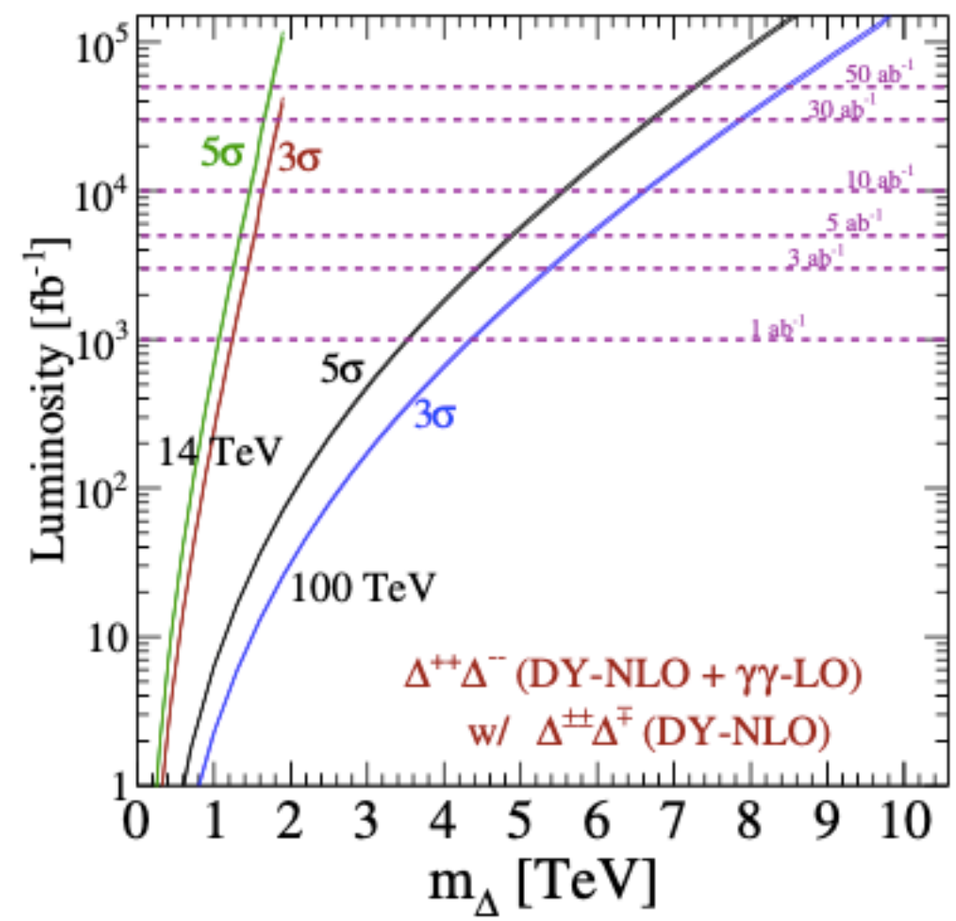
$$\sqrt{2}\Delta = \begin{pmatrix} \Delta^+ & \sqrt{2}\Delta^{++} \\ \sqrt{2}\Delta^0 & -\Delta^+ \end{pmatrix}$$

couplings to SM leptons directly connected to PMNS parameters!

Han et al., 2203.06131
Snowmass TF11

Fileviez Perez et al. '08, Gavela et al. '09,…

Nemeviesk et al. '16, Fuks et al. '19 ,…



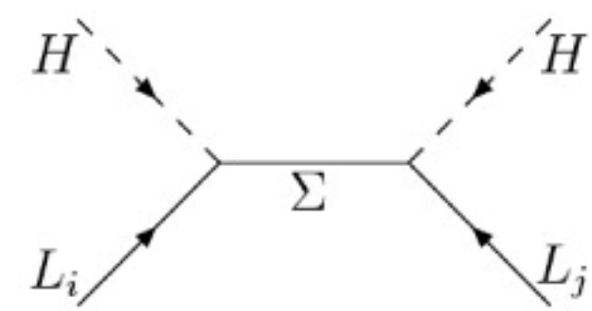
Type III seesaw

Foot, He, Joshi '89, Ma '98,…

✓ **introduce electroweak triplet fermions** $\Sigma \sim (\mathbf{3}, 0)$

$$\mathcal{L} \simeq Y_{\Sigma\, ij} L_i \Sigma_j H + M_{\Sigma\, ij} \Sigma_i \Sigma_j$$

$$\boxed{\mathcal{M}_\nu \sim \langle H \rangle^2 Y_\Sigma M_\Sigma^{-1} Y_\Sigma^T}$$



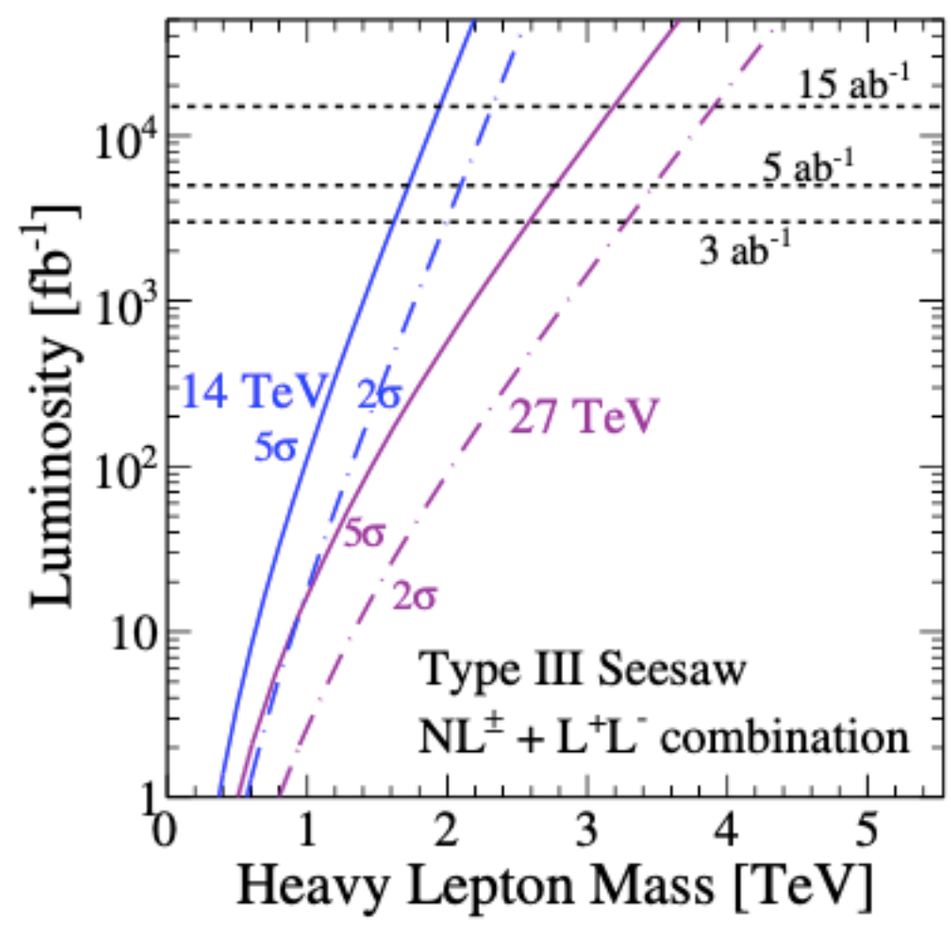
triplet mass can be at high or low scales

"phenomenological" Type III seesaw:

mixing w/charged leptons

$$M_\Sigma \sim O(\mathrm{TeV})$$

Han et al., 2203.06131
Snowmass TF11



del Aguila et al. '08, Franchesini et al. '08,…

Abada et al. '07, Gavela et al. '09,…

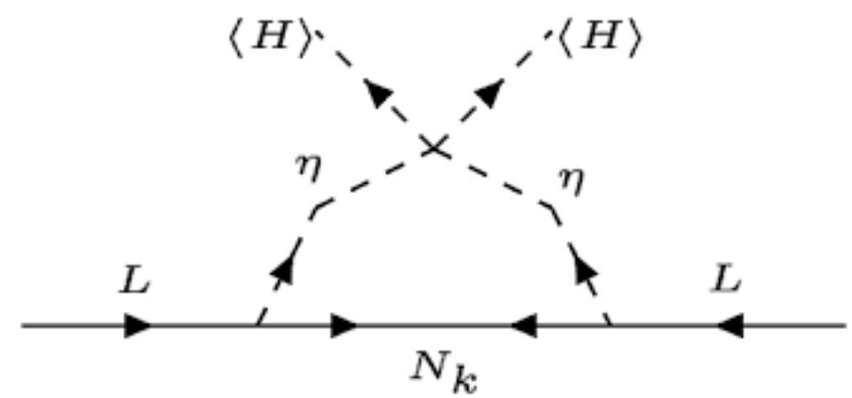
Radiative possibilities: ν**SM**

Zee '86, Babu '88, Ma '98,…

Canonical example: "scotogenic" model Ma '06,…

"radiative Type I seesaw"

$$\mathcal{M}_\nu \sim \lambda \frac{\langle H \rangle^2}{16\pi^2} Y M_R^{-1} Y^T$$



can also have radiative Type III, Type II

Ma, Suematsu '09,… Fraser et al. '15,…

hybrid versions: "scoto-seesaw,"…

Rojas et al. '18, Valle et al., '20,…

Radiative models: **beyond** ν**SM** $\Delta L = 2$

Zee '86, Babu '88, Ma '98,…

Babu et al. '01, de Gouvea et al. '07, Bonnet et al. '12 …

d=7

$LLLe^cH$ $LLQd^cH$

$LL\overline{Q}\overline{u}^cH$ $L\overline{e}^c\overline{u}^cd^cH$

d=9

$LLLe^cLe^c$ (Zee-Babu)

$LLQd^cQd^c$

…

An excellent review:

Cai et al. 1706.08524

✓ **possible connections to** **dark matter** **(discrete symmetries)**

✓ **possible connections to** **flavor physics anomalies** **muon g-2**

many authors!

Many other ideas for Majorana masses

✓ **inverse seesaw:** **3 RH neutrinos and 3 new singlet fermions**

Mohapatra, Valle '86,…

$$\mathcal{L} \simeq Y_{ij} L_i N_j H + M_{ij} S_i N_j + \mu_{ij} S_i S_j$$

$$M_\nu \sim \langle H \rangle^2 (Y^T (M^T \mu^{-1} M)^{-1} Y)$$

$$\mathcal{M}_\nu \sim \begin{pmatrix} 0 & m & 0 \\ m & 0 & M \\ 0 & M & \mu \end{pmatrix} \qquad \mu, m \ll M$$

Recent examples:

Pati-Salam + conn. to B anomalies, muon g-2 — Fileviez Perez et al. '21

flavor+CP model with $\Delta(3n^2),\ \Delta(6n^2)$ — Hagedorn et al. '21

✓ **SUSY with R-parity violation**

Aulakh, Mohapatra '82, Hall, Suzuki '84,…
Borzumati et al. '96, Grossman et al. '03,…

✓ **…**

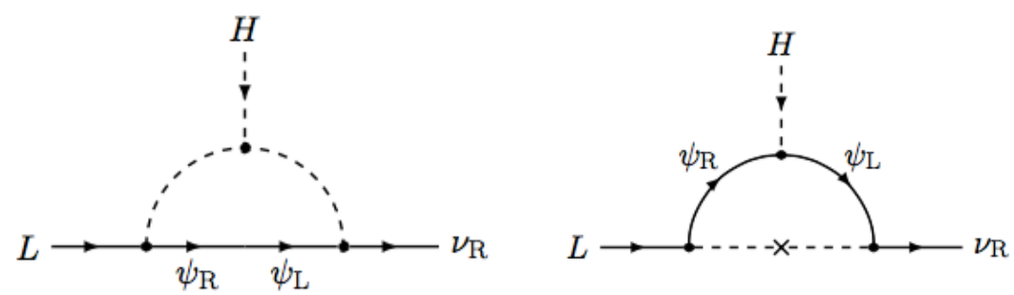
Dirac neutrinos

$$\Delta L = 0$$

- **introduce right-handed neutrinos** $\mathcal{L} = \mathcal{L}_{\rm SM} + \mathcal{L}_4 + \ldots$

  like charged fermions: but stronger suppression $|Y_\nu| \leq 10^{-12}$

  ✓ **radiative**



Cheng, Li '78,

Mohapatra '87, '88,…

Ma, Popov '16,…

✓ **extended gauge sectors**

many examples! see e.g. Langacker '11 for review,…

e.g. *B-L* with Stückelberg mech Fileviez Perez et al. '20,…

✓ **discrete symmetry realization of *L*** **"lepton quarticity"** Chulia et al. '16,…

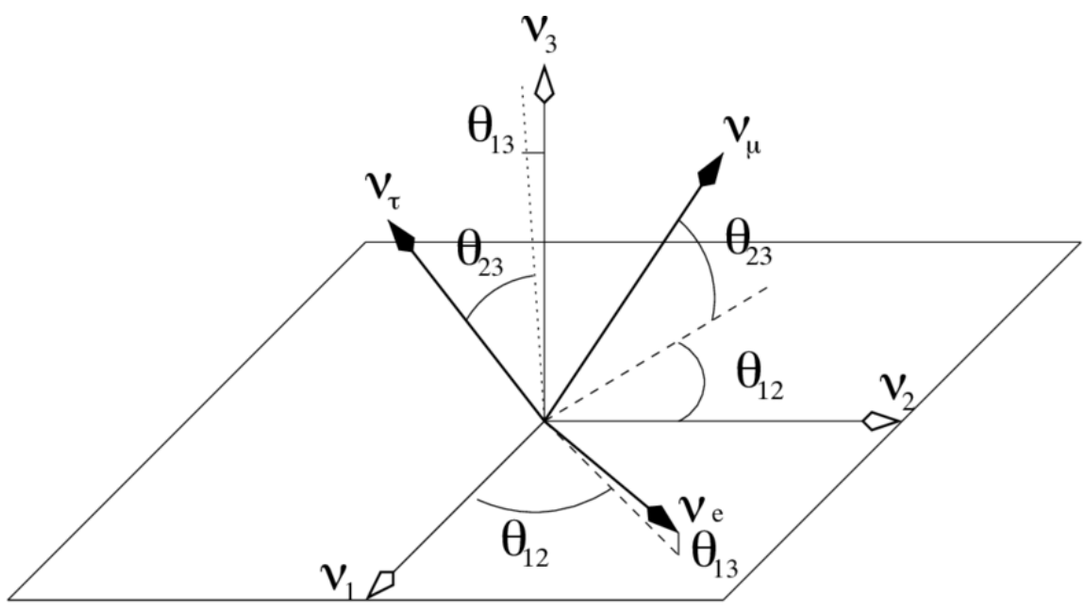
✓ **SUSY breaking (Kähler potential)** Arkani-Hamed et al. '00,…

✓ **warped extra dimensions** Grossman et al. '99, Huber et al. '00,…

… many authors!

Lepton mixing

$$U_{\rm PMNS} = \mathcal{R}_1(\theta_{23})\mathcal{R}_2(\theta_{13},\delta)\mathcal{R}_3(\theta_{12})\mathcal{P}$$



NuFIT 5.1 (2021)

$$|U|_{3\sigma}^{\text{w/o SK-atm}} = \begin{pmatrix} 0.801 \to 0.845 & 0.513 \to 0.579 & 0.143 \to 0.156 \\ 0.232 \to 0.507 & 0.459 \to 0.694 & 0.629 \to 0.779 \\ 0.260 \to 0.526 & 0.470 \to 0.702 & 0.609 \to 0.763 \end{pmatrix}$$

$$|U|_{3\sigma}^{\text{with SK-atm}} = \begin{pmatrix} 0.801 \to 0.845 & 0.513 \to 0.579 & 0.144 \to 0.156 \\ 0.244 \to 0.499 & 0.505 \to 0.693 & 0.631 \to 0.768 \\ 0.272 \to 0.518 & 0.471 \to 0.669 & 0.623 \to 0.761 \end{pmatrix}$$

Two large mixing angles: $\theta_{23},\ \theta_{12}$

Is θ_{13} "large" or "small"?

large reactor angle **vs.** **small reactor angle**

↓ ↓

anarchy **structure**

Anarchy

Hall, Murayama, Weiner '99

(character: Watterson)

$\mathcal{U}_\nu$: random draw of unbiased distribution of 3x3 unitary matrices

statistical tests: lower bound on $|\mathcal{U}_{e3}|^2$

basis independence:

distribution invariant upon unitary transformations

Haba, Murayama '00

Post-reactor angle measurement: renewed focus

de Gouvea, Murayama '12

Altarelli et al. '12,…

Some recent highlights: RG analysis — Brdar, Konig, Kopp '15

model-building + quark sector — Babu et al. '16,…

CP violation — Fortin et al. '17,…

Anarchy hypothesis alone does not provide information on Δm^2

Structure

Achieve special structures via symmetries:

- **Postulate family symmetry** G_f

  spontaneously broken by "flavon" fields $\{\varphi_a\}$

Recall often-used quark paradigm:

$$\boxed{Y_{ij} H \cdot \bar{\psi}_{Li} \psi_{Rj} \longrightarrow \left(\frac{\langle\varphi\rangle}{\Lambda}\right)^{n_{ij}} H \cdot \bar{\psi}_{Li} \psi_{Rj}}$$

$$\epsilon = \left(\frac{\langle\varphi\rangle}{\Lambda}\right) \qquad \Lambda > \langle\varphi\rangle > \langle H\rangle \qquad \epsilon \ll 1$$

Froggatt, Nielsen '79

natural identification: $\boxed{\epsilon \sim O(\lambda)}$ $\qquad \lambda = \sin\theta_c$

(typically related to ratios of quark masses)

Unique theoretical starting point: $\mathcal{U}_{\rm CKM} \sim 1 + O(\lambda)$

Now, for leptons:

✓ ***spontaneously broken** G_f

"traditional" choice → **discrete non-Abelian group**

✓ $U_{\rm PMNS} = \mathcal{W} + O(\lambda')$ $\qquad \lambda' \ll 1$

↑ **"bare" mixing angles** **(diagonal charged lepton basis)**

$(\theta^\nu_{12}, \theta^\nu_{23}, \theta^\nu_{13})$

A priori, **expansions in quark and lepton sectors unrelated**

unification paradigm (broad sense): $\lambda' = \lambda$

Ideas of quark-lepton complementarity and "Cabibbo haze"

↑

$\theta_{23} = \theta_{12} + \theta_c$ **(empirical)**

Raidal '04, Minakata+Smirnov '04,...

("haze" terminology from Datta et al. '05)

pre-measurement, idea that θ_{13} **might be a Cabibbo effect:**

$\theta^\nu_{13} = 0 \qquad \theta_{13} = \lambda/\sqrt{2}$

Vissiani '98, '01

Ramond '04

Family symmetry models (traditional)

Almumin et al., 2204.08668
Snowmass TF11

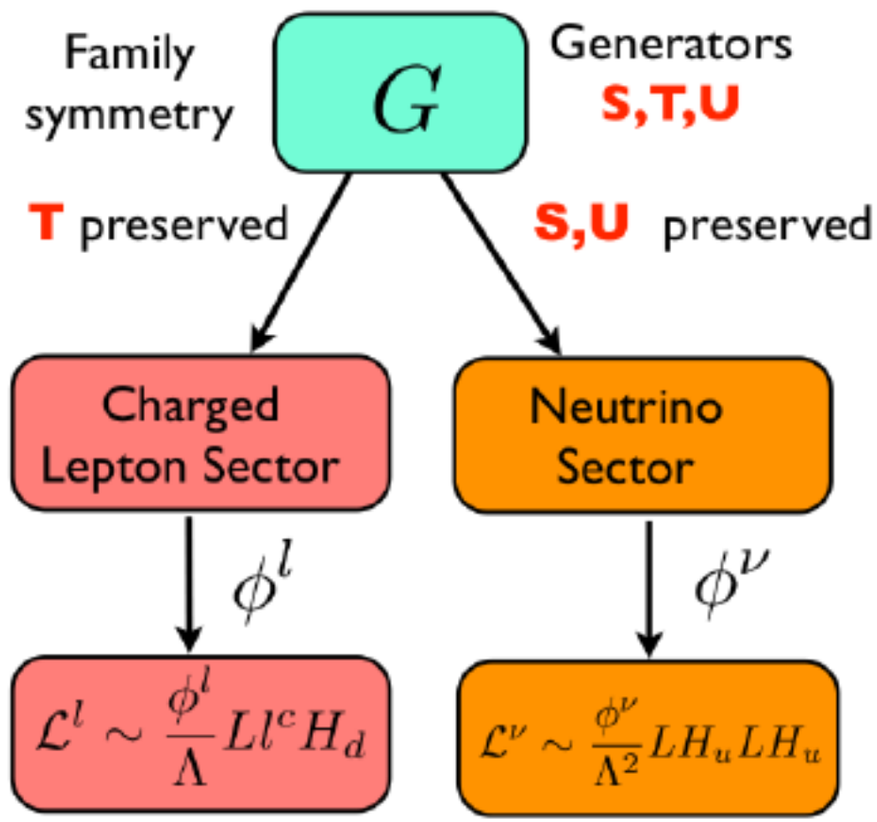
Gehrlein et al., 2203.06219
Snowmass TF11

✓ **usual choices:** $SU(3),\ SO(3)$ **subgroups:**

"Platonic solid" groups $\mathcal{A}_4 \quad \mathcal{S}_4 \quad \mathcal{A}_5 \quad \Delta(3n^2) \quad \Delta(6n^2) \quad \mathcal{D}_n \quad \mathcal{T}' \quad \mathcal{I}' \ldots$

example (Majorana):

(image credit: King, Luhn)



see reviews by King, Luhn '13, King '17

Flavons:

$$\phi^l, \phi^\nu$$

Residual symmetries:

$$T\langle\phi^l\rangle \approx \langle\phi^l\rangle$$
$$S, U\langle\phi^\nu\rangle \approx \langle\phi^\nu\rangle$$

(or broken further, e.g. only *S* or *U* unbroken)

often in SUSY context

also often needed:

"driving fields"

Many papers and authors! Some authors (not comprehensive):

Babu, Chen, Ding, L.E., Feruglio, Grimus, Hagedorn, King, Lam, Luhn, Ma, Merle, Ohlsson, Rodejohann, Stuart,…

see also the very nice recent review of Feruglio and Romanino '19

✓ **Most studied: maximal atm, zero reactor** $\sin^2\theta^\nu_{23} = 1/2 \quad \sin^2\theta^\nu_{13} = 0$

→ **classify scenarios by** **bare solar angle**

tri-bimaximal mixing: $\sin^2\theta^\nu_{12} = 1/3$ — Harrison, Perkins, Scott '02; Xing '02, He, Zee '02, Ma '03…

bimaximal mixing: $\sin^2\theta^\nu_{12} = 1/2$ — Petcov '82, Vissiani '97, Barger et al. '98, Baltz et al. '98

golden ratio (A) mixing: $\sin^2\theta^\nu_{12} = 1/(2+r) \sim 0.276$, $r = (1+\sqrt{5})/2$ — Datta, Ling, Ramond '03; Kajiyama, Raidal, Strumia '08;…

golden ratio (B) mixing: $\sin^2\theta^\nu_{12} = (3-r)/4 \sim 0.345$ — Rodejohann '09,…

hexagonal mixing: $\sin^2\theta^\nu_{12} = 1/4$ — Albright, Duecht, Rodejohann '10, Kimand, Seo '11,…

✓ **can also study nonzero reactor:** $\sin^2\theta^\nu_{13} \neq 0$ — Lam '13, Holthausen et al. '12,… …

All can be obtained via SSB of discrete non-Abelian family symmetries

Residual Symmetries

- **model-independent approach:**

Lam '08, '09, Grimus et al. '09, …
many authors!

determine rows and columns in $\mathcal{U}_{\rm MNSP}$

as pure numbers, independent of masses,

depending on preserved subgroups of finite group G_f

$$T^\dagger \mathcal{M}_e \mathcal{M}_e^\dagger T = \mathcal{M}_e \mathcal{M}_e^\dagger \qquad S^\dagger \mathcal{M}_\nu S = \mathcal{M}_\nu$$

$$G_f \to G_e,\ T \in G_e \qquad G_f \to G_\nu,\ S \in G_\nu$$

$$\longrightarrow \quad \mathcal{U}_{eL}^\dagger T \mathcal{U}_{eL} = T^{\rm diag} \qquad \mathcal{U}_\nu^\dagger S \mathcal{U}_\nu = S^{\rm diag}$$

Majorana: $G_\nu \supseteq Z_2 \times Z_2$ **(Klein group)**

systematic classification of possible mixing matrices

Fonseca, Grimus '14

Very different from quark paradigm (CKM mixing angles as ratios of quark masses)!

Corrections

✓ **charged lepton corrections**

source the reactor angle: $\mathcal{U}_{eL} \sim 1 + O(\lambda)$

correlations among observables → **"sum rules"**

example: $\mathcal{U}_{eL} \sim \mathcal{R}_1(\theta^e_{23}, \delta^e_{23})\mathcal{R}_3(\theta^e_{12}, \delta^e_{12})$

Ge, Dicus, Repko '11, Hanlon et al. '12,

Marzocca et al. '13, Petcov '14, Girardi et al. '14-16,

Ballet et al. '14

Prediction for Dirac phase δ !

$$\cos\delta = \frac{\tan\theta_{23}}{\sin 2\theta_{12}\sin\theta_{13}}\left[\cos 2\theta^\nu_{12} + \left(\sin^2\theta_{12} - \cos^2\theta^\nu_{12}\right)\left(1 - \cot^2\theta_{23}\sin^2\theta_{13}\right)\right]$$

1 model parameter!

Gehrlein et al., 2203.06219
Snowmass TF11

Many authors! Marzocca et al. '13, Petcov '14, Girardi et al. '14-16,…

LE, Ramos, Rock, Stuart '19

✓ **canonical normalization** (Kahler potential corrections)

Many authors.
King '17 (review)

✓ **RG effects** more significant for IO,
heavy neutrino masses
(can be significant for sum rule analysis)

Antusch et al. '03
…
Gehrlein et al. '16

CP Violation

Almumin et al., 2204.08668
Snowmass TF11

- **spontaneous CP violation**

Generalized CP

$$X^T \mathcal{M}_\nu X = \mathcal{M}_\nu^* \qquad Y^\dagger \mathcal{M}_e \mathcal{M}_e^\dagger Y = (\mathcal{M}_e \mathcal{M}_e^\dagger)^*$$

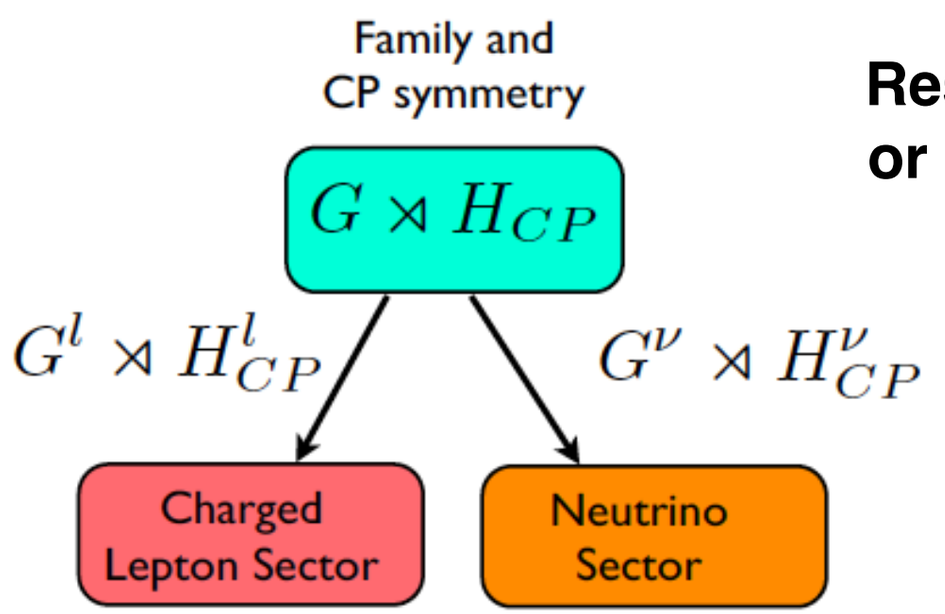
"ordinary" CP has $X = Y = 1$

Branco, Lavoura, Rebelo '86…
Grimus, Rebelo '95

automorphisms of discrete family symmetry

$$X \rho(g)^* X^{-1} = \rho(g')$$

family symmetry



Residual/generalized CP or "CP-like" symmetries

Gehrlein et al., 2203.06219
Snowmass TF11

Holthausen et al. '12,
Feruglio et al. '12,
Chen et al. '14,
…

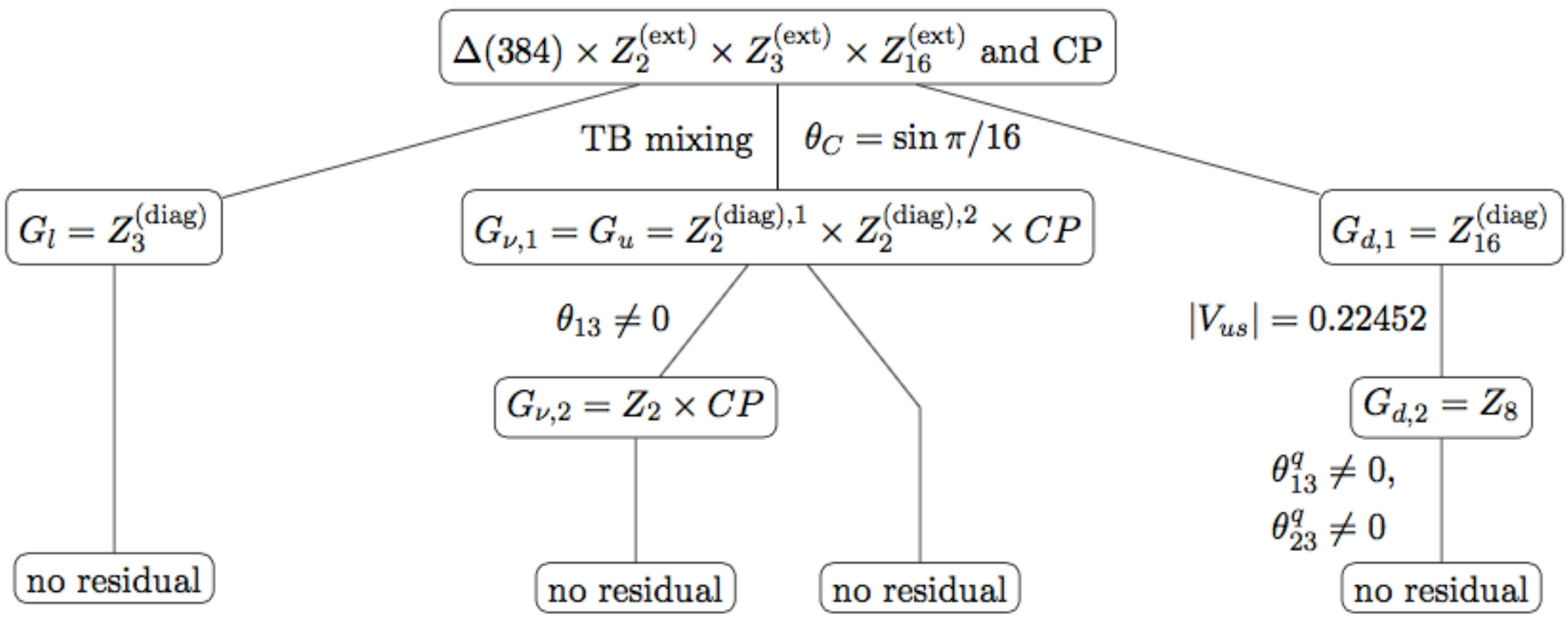
many recent papers!

see recent review of Feruglio and Romanino '19

Example: full quark, lepton flavor model with $\Delta(384)$

Hagedorn and Konig, '18

MSSM framework, Type I seesaw $Q_i, L_i \sim \mathbf{3}$



leptons: **first break to TBM** → **reactor angle generated**

quarks: **first step breaking,** $\lambda \sim \pi/16$ → **Cabibbo angle corrected**

requires intricate sector of flavons+ driving fields (characteristic)

Modular Symmetries

Almumin et al., 2204.08668
Snowmass TF11

Gehrlein et al., 2203.06219
Snowmass TF11

✓ **recent proposal:**

flavor model-building based on modular groups

Feruglio '17,...

many authors!!

modular symmetry: **modulus** $\tau \longrightarrow \dfrac{a\tau + b}{c\tau + d}$ $\quad SL(2, Z)$

$S\colon\ \tau \to -1/\tau \qquad T\colon\ \tau \to \tau + 1 \qquad S^2 = (ST)^3 = 1$ **generates discrete group**

$\Gamma_N \qquad S^2 = (ST)^3 = T^N = 1$ → **finite discrete group, level *N***

(holomorphic) Yukawa couplings as modular forms:

modular weight

$$Y(\tau) \to Y(\gamma(\tau)) = (c\tau + d)^k \rho(\gamma) Y(\tau)$$

group rep matrix

Allows for simplification of flavon sector:

e.g. triplet flavon (A_4) $\quad \phi^\nu = (\phi_1^\nu, \phi_2^\nu, \phi_3^\nu)^T \longrightarrow (Y_1(\tau), Y_2(\tau), Y_3(\tau))^T$ **known functions!**

see also recent talks of Feruglio, Tanimoto at FLASY 2022

Sum Rules

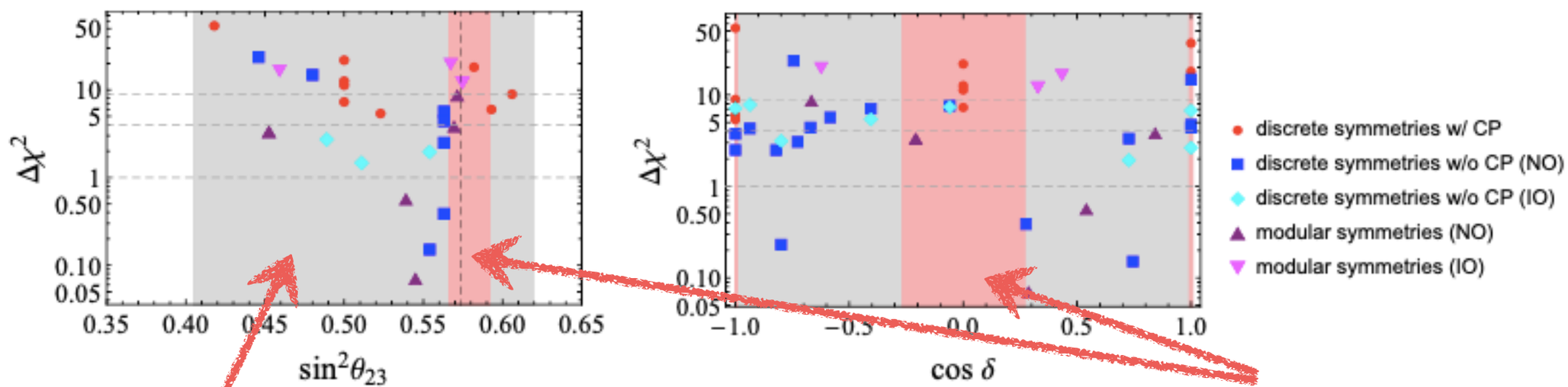
Gehrlein et al., 2203.06219
Snowmass TF11

✓ **modular family symmetries** → **new sum rules**

e.g. modular A_4 **2 dof:** θ, ϕ

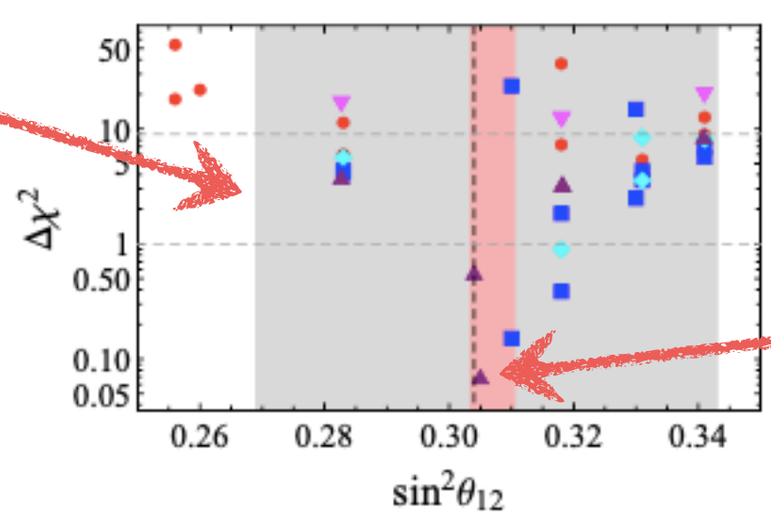
$$\sin^2\theta_{12} = \frac{1}{3 - 2\sin^2\theta}, \quad \sin^2\theta_{13} = (2/3)\sin^2\theta$$

$$\sin^2\theta_{23} = \frac{1}{2} + \frac{\sqrt{2 - 3\sin^2\theta_{13}}}{2(1 - \sin^2\theta_{13})}\sin\theta_{13}\cos\phi, \quad \sin\delta = -\frac{\sin\phi}{\sin 2\theta_{23}}$$



projected, 3σ **DUNE 15 yr**

current 3σ **range, NO**



projected, 3σ **JUNO 6 yr**

Beyond Modular Symmetries

Almumin et al., 2204.08668
Snowmass TF11

Modular symmetries: perhaps a hint at top-down physics

String constructions:

possibilities for mass suppression

✓ **higher-dimensional operators (field theoretic)** — see e.g. Cremades et al. '03, '04; Blumenhagen et al. '06,…; Langacker '11 (review)

geometric suppression (braneworlds)

worldsheet instantons (nonperturbative) — Buchmüller et al. '07

✓ **many fields: many possible RH neutrino candidates**

can have anarchy or family symmetry

discrete symmetries: flavor sector — see e.g. Kobayashi et al. '06; Abe et al. '09,…; Cvetič et al. '18

Bottom-up approach:

modular symmetry and further "stringy" ingredients →

further reduction of *ad hoc* parameters (modular weights,…)

connection to explicit top-down constructions

Recent examples:

✓ **metaplectic** (half-integer modular weights)

connection to magnetized tori

Liu et al. '20, Ding et al. '20, Yao et al. '20,…

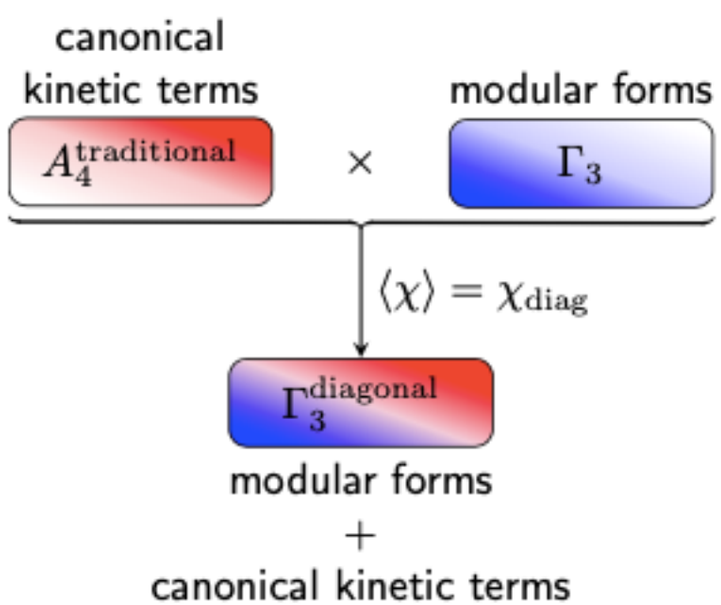
Almumin et al. '21,…

✓ **eclectic** combination of "traditional" G_f and G_{modular}

Nilles et al. '20, '21

Chen et al. '21,…

(including R symmetries, CP or CP-like tmns)



✓ large class of bottom-up constructions

✓ top-down: heterotic orbifolds

(nontrivial issue: moduli/ flavon vev stabilization)

an exciting new direction!

see also recent talks of Chen, Ratz, Trautner at FLASY 2022

Conclusions

Neutrino data has led to a renaissance for beyond SM model-building!

Many mechanisms for suppressing the neutrino mass scale

many TeV-scale scenarios, testable at LHC

For 3 active neutrinos only:

mixings: anarchy or symmetry

(spontaneously broken) symmetries:

discrete non-Abelian groups

modular symmetries and generalizations

More data (atmospheric angle, Dirac CP phase,…) will help enormously!

If sterile neutrinos confirmed: **paradigm shifts again!**

Stay tuned!