Hadron spectroscopy with lattice QCD

M. Padmanath

Mainz, Germany

@ Seattle Snowmass Summer study

20^{th} July, 2022

References and disclaimers

1 USQCD Snowmass proceedings
arXiv:2207.07641

2 White paper on topical group Hadron Spectroscopy (RF7)
arXiv:2203.03230

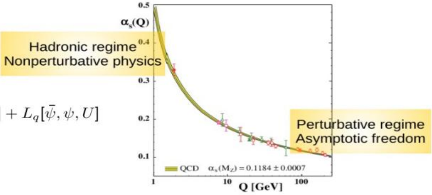
3 Towards a theory of hadron resonances [Review]
arXiv:2206.01477

4 New perspectives on precision Nuclear Physics
arXiv:2202.01105

✿ Disclaimers for my talk:
No review of the subject intended.
For extended list of references, see [3].
Current status will only be skimmed through.

QCD and lattice methodology

Q C D *: the theory of strong interactions*



$$L_{QCD} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \sum_{\alpha=1}^{N_f} \bar{\psi}_\alpha (i\gamma^\mu D_\mu - m_\alpha)\psi_\alpha = L_g[U] + L_q[\bar{\psi}, \psi, U]$$

where $D_\mu \; = \; \partial_\mu - ig\sum_{i=1}^{8} \lambda^a A^a_\mu,$ and

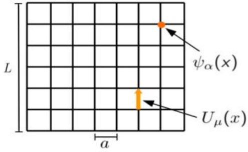
$F_{\mu\nu} \; = \; \frac{i}{g}[D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$

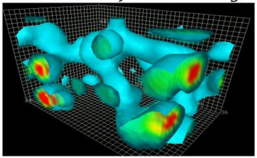
Asymptotic freedom: Perturbation theory at high energies.

Quark confinement and hadronic d.o.f. at low energies.
Need for non-perturbative approaches.

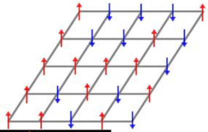
Lattice Q C D *: Only first-principles non-perturbative formulation where calculations can be made in manner to achieve complete control over the systematic and statistical uncertainties.*

Numerial/Computational framework of the theory on a discretized spacetime.



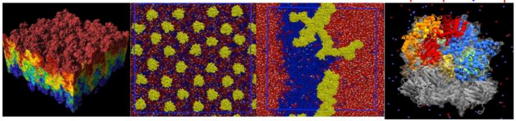


Monte Carlo Techniques



Monte Carlo Simulations:

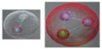
Technology ↔ Statistical physics,
Material science, Chemistry, Biology, ...



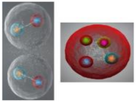
Statistical data analysis: *Common demands in all of the sciences*

High Performance Computing*: Prospects in research and teaching.*

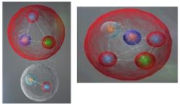
QCD @ hadronic scale:
Effective d.o.f. different from the fundamental d.o.f.
Strongly correlated systems.



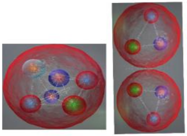
Mesons, Baryons, glueballs, hybrids, ...



Tetraquarks, Molecules, ...



Pentaquarks, Hadroquarkonium, molecules, ...



Hexaquarks, molecules, ...

Hadron spectroscopy and interactions from lattice QCD

- Euclidean two point current-current correlation functions

$$C(t) = \langle 0|\mathcal{O}_i(t)\mathcal{O}_j^\dagger(0)|0\rangle = \sum_n |Z_n|^2 e^{-E_n t}$$

at large times, $C(t) \to |Z_0|^2 e^{-E_0 t}$. Note $e^{-E_n t} = e^{-aE_n . t/a}$

- MonteCarlo Simulations on a finite volume and a spacetime grid:
  Discrete energy spectrum: E_{lat}
  No continuum of energies, no branch cuts, no pole information.
  Maiani and Testa 1990

- Analytic connection $E_{lat} \Leftrightarrow T(E)$ Scattering matrix along $Re(E)$.
  M. Lüscher 1991. Many generalizations, *c.f.* Briceno/Hansen 201x

- Analytic continuation of $T(E)$ to complex E, $T(E^c)$
  and search for pole singularities.

- Accounts only for strong decay channels. Ignores QED and weak interactions.

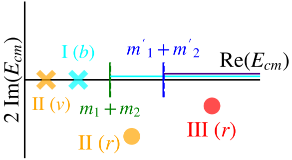
Prospects and challenges

Prospects:

- Resolving the general pattern and microscopic structure of hadron spectrum.
- HVP contribution to $(g-2)_\mu$: Dominant two pion contributions.
- Nucleon structure: understanding the excited state contaminations.
- Important implications on heavy flavor physics: Electroweak transitions

The difficulty level:

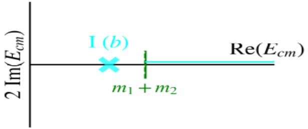
- Straightforward: Strongly stable hadrons and those with OZI suppressed contributions.
- Easy and doable: Near threshold pole singularities, resonances with single decay channel.
- Difficult: Above two or three decay channels, Interactions between hadrons with nonzero spin.
- Challenging: Several open decay channels, three hadron decay channels.

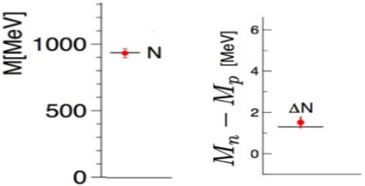


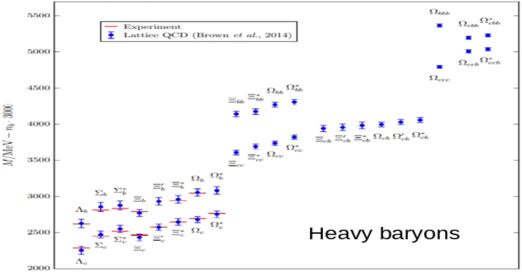
$$T(E) = \begin{bmatrix} T_{aa} & T_{ab} & \dots \\ T_{ba} & T_{bb} & \dots \\ \dots & & \end{bmatrix}$$

The straightforward cases

- D, $B^{(*)}$, $B_c^{(*)}$, p, n, Λ, Λ_c, Ξ_{cc}, ... Masses extracted directly from E_{lat}.

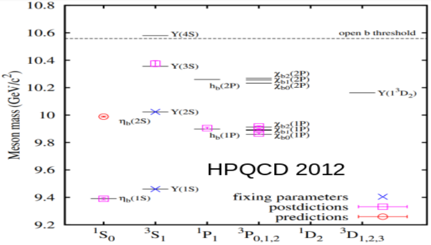






BMW, Science 322, 2008 [M]
BMW, Science 347, 2015 [M_n-M_p]

OZI suppressed contributions: very challenging.
Isospin breaking effects and decays to address.

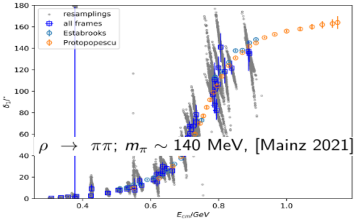


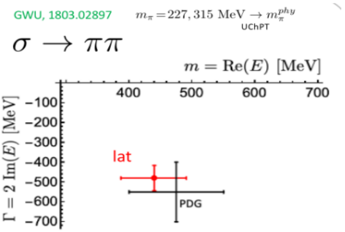
- Heavy quarkonium (η_c, J/ψ, χ_c, η_b, Υ, ...)

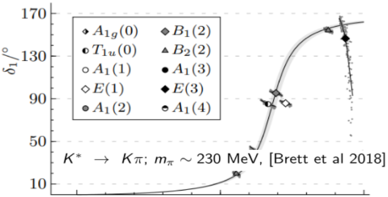
Reviews: 2206.01477, 1905.09651

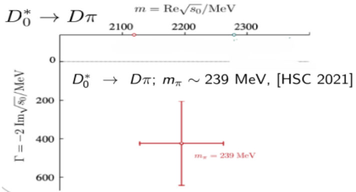
Easy: the ‘vanilla’ resonances

- ρ, σ $\to$ $\pi\pi$; K^*, κ $\to$ $K\pi$; D_0^* $\to$ $D\pi$; $\psi(3770)$ $\to$ $D\bar{D}$, $\Delta \to N\pi$. Widely investigated.









- One to one relation between E_{lat} and the scattering amplitude $T(E)$. More efforts on the $m_{u,d}$ dependence.

See reviews: 2206.01477, 1905.09651

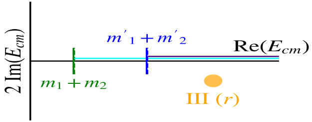
Easy: Near threshold hadrons

- Scalar D_{s0}^* pole below DK: Several studies involving pole determination.
- Scalar B_{s0}^* pole below BK: One study. Lang et al, 2016. More studies anticipated. Discovery awaited.
- X(3872) and its partners in $D\bar{D}^*$: Only two calculations. Prelovsek et al. 2013/15 Need to relax the approximations made.
- Doubly heavy tetraquarks: Several studies. Clear evidence for deeply bound T_{bb}. Only few involving rigorous pole determination. See 2202.10110 related to T_{cc}.
- $bb\bar{b}\bar{b}$ and $cc\bar{c}\bar{c}$: One study. HPQCD 2017. No indications for a bound state.
- light dibaryons (deuteron/dineutron): Several studies, mostly at unphysical $m_{u,d}$.
- H-dibaryon and $\Lambda\Lambda$: Several investigations. Green et al. 2021 addressing continuum limit. Discovery awaited.
- Heavy dibaryons: Several studies addressing different systems. Poles observed in some cases. More lattice studies are desired. Discovery awaited.

Difficult: $R \to H_1 H_2$ and $R \to H_3 H_4$

- A scattering matrix $T(E)$ from finite volume spectrum.
  Energy dependence of each element to be extracted.

$$T(E) = \begin{bmatrix} T_{aa} & T_{ab} & \dots \\ T_{ba} & T_{bb} & \dots \\ \dots & & \end{bmatrix}$$



- Scattering analysis / The finite volume quantization condition is a determinant condition. An underdetermined problem.

- Need to reliably extract low lying excited state finite volume spectra.
  Inverse problem at hand.

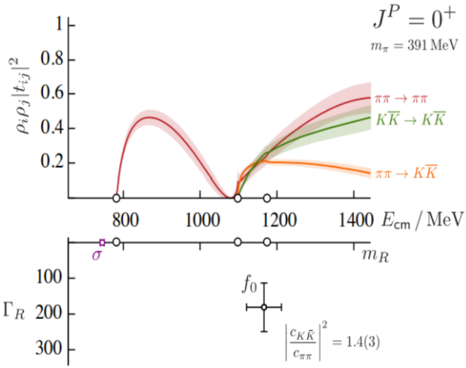
- Most studies performed by the HadSpec collaboration.
  a_0, a_1, b_1, K^*, etc.

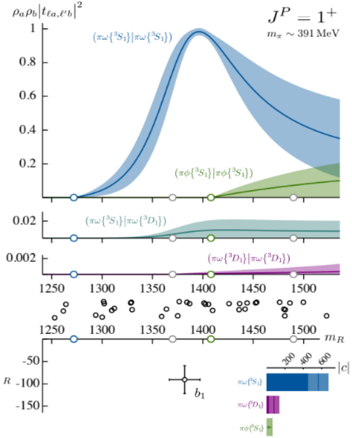
- Heavy sector: Only two studies. More efforts are being made.
  $I = 1$, $D\pi - D\eta - D_s\bar{K}$ [HSC 2016]
  $I = 0$, $D\bar{D} - D_s\bar{D}_s$ [Prelovsek et al. 2020].

- Z_b, Z_c and P_c sectors: Several investigations assuming various approximations. Only partial conclusions available.

Difficult: Some highlights

- σ, f_0 and f_2 resonances from isoscalar $\pi\pi$-$K\bar{K}$-$\eta\eta$ scattering. [left, HSC 2017]





- b_1 resonance in $\pi\omega$-$\pi\phi$ scattering. Scattering particles with non-zero spin. [right, HSC 2019]

Challenging: $R \to H_1 H_2 H_3$

- Many of the hadronic resonances also have allowed 3-particle decay channels.
  $a_1 \to \pi\pi\pi$, $N(1440) \to N\pi\pi$, $X(3872) \to J/\psi\pi\pi$, $T_{cc} \to DD\pi$, ...

- Steady progress in the theoretical formalism to extract the 3-particle force.
  (Sharpe, Hansen, Briceno, Lopez), (Doring, Mai), (Rusetsky, Hammer)

- Impressive mumerical evaluations in the recent times:
  ETMC, GWU, HadSpec, Blanton et al., NPLQCD, ... 2018-2022

- A physical case: $a_1 \to \pi\pi\pi$ GWU 2107.03973 Phys.Rev.Lett.

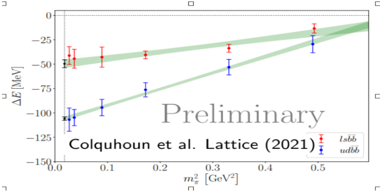
- Future directions:
  - A most general formulation of 3-body systems is still to be developed.
    coupled channels, involving different partial waves, moving frames,
    2- and 3-body channels and scattering particles with nonzero spin.
  - Improving practical strategies to extract the 3-body force.
  - Address increasing complex reactions, and develop systematic procedure
    for extracting the solutions.
  - Four body dynamics, $\rho(700)$, $f_0(500)$.

Challenging: Multiple decay channels and internal structure

- Most heavy quark exotic candidates come with several decay channels. $X(3872)$, $Z_c(3900)$, P_c, $\bar{Q}Q\bar{Q}Q$, $T_{cs}(2900)$, $T_{c\bar{s}}(2900)$, ...

- Necessity of a large #eigenvalues to constrain the scattering amplitude in all channels and partial waves. Expensive contraction programs.

- Scanning the quark mass dependence of the poles. Probing their exotic nature. Cannot be explored in the experiments. However, using lattice QCD one can.



- Search for QCD exotic using the generalized FHT. 1706.09015

- Theoretical formulation for hadron structure using external currents.
  $\langle H|J|H\rangle$ Briceno, Walker-Loud,...

Electroweak transitions

- Stable hadrons, Heavy flavor physics, RF1: significantly explored.
- Theory formalisms on electroweak transitions between stable hadrons to resonances
  Briceno et al., Walker-Loud, Rusetsky.
- For resonances, only $\pi\pi \to \pi\gamma^*$ radiative transitions has been studied.
  This involved the resonant $\rho \to \pi\gamma^*$ transition as well.
  HadSpec 2015, Alexandrou et al. 2018.
- $N\pi \to \Delta \to N\gamma^*$ and $K\pi \to K^* \to K\gamma^*$ amplitudes are also valuable.
- No weak transistions between a stable hadron and a resonance studied yet.
  Except for a study addressing nonleptonic decay $K \to \pi\pi$. RBC/UKQCD 202x
- Weak decays: test cases $D \to \rho l\nu$ and $B \to \rho l\nu$, with $\rho \to \pi\pi$.
- Weak decays that might carry hints to BSM:
  $B \to K^* l\nu$ and $B \to D^* l\nu$, with resonant K^* and D^*, respectively.

The timeline

- Impressive progress in QCD spectroscopy calculations using lattice QCD. Addressing a variety of reactions with increasing complexity.

- Apporach to the physical limit, relaxing several simplifying approximations and addressing several interesting exotic resonances. Major efforts in these directions.

- Great motivation from continuing experimental programs and discoveries.

- Code and algorithmic development: a lot of interdisciplinary prospects.

- Impact of lattice QCD on hadron spectroscopy:
  - Would not have been possible without the extensive support
    - from DOE and NSF to the USQCD lattice-QCD collaboration.
  - Recommend continued support to the collaboration
    - to have a sustained investement in human capital and
    - continued access to computer resources.