Prospects on theory inputs for the first-row CKM unitarity tests

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Anomalies in beta decays

Beta decays place **one of the most stringent tests of SM** through precision measurements of the **first-row CKM matrix elements V_{ud} and V_{us}**

V_{ud}

	$\lvert V_{ud}\rvert$
Superallowed nuclear decays ($0^+ \to 0^+$)	$\boxed{0.97373(31)}$
Free n decay	0.97377(90)
Mirror nuclei decays	0.9739(10)
Pion semileptonic decay (π_{e3})	0.9740(28)

V_{us}

	$\lvert V_{us}\rvert$
Kaon semileptonic decays ($K_{\ell 3}$)	$\boxed{0.22308(55)}$
Tau decays	0.2207(14)
Hyperon decays	0.2250(27)

HFLAV, 2206.07501

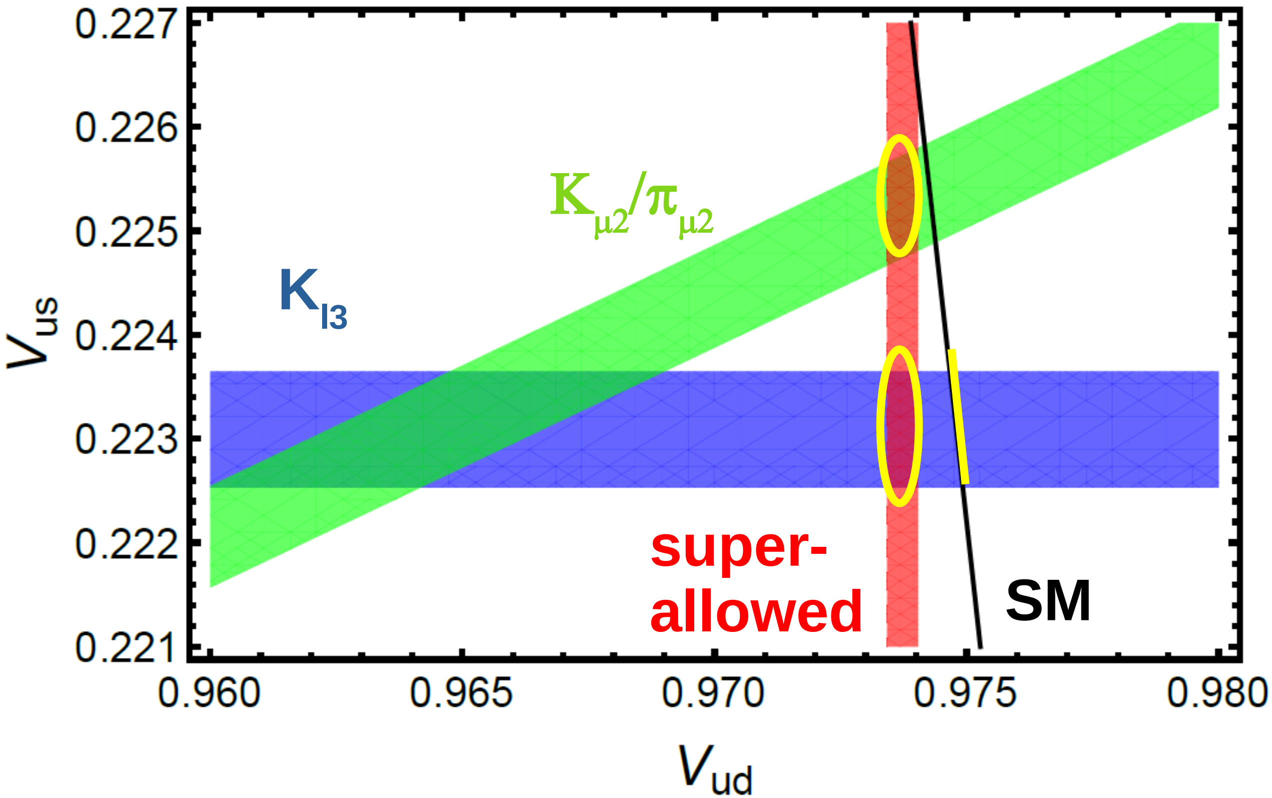
V_{us}/V_{ud}

	$\lvert V_{us}/V_{ud}\rvert$
K/π leptonic decays ($K_{\mu 2}/\pi_{\mu 2}$)	$\boxed{0.23131(51)}$
K/π semileptonic decays ($K_{\ell 3}/\pi_{e3}$)	0.22908(87)

Anomalies in beta decays

Several **anomalies** are recently observed in the **first-row CKM matrix elements**!

SM prediction: $|V_{ud}|^2 + |V_{us}|^2 + \cancel{|V_{ub}|^2} = 1$



"Cabibbo Angle Anomaly (CAA)" ~ 3σ

A glance at the error analysis

An example: First-row CKM unitarity with $|V_{ud}|$ from 0^+ beta decay and $|V_{us}|$ from K_{l3} decay

$$\boxed{|V_{ud}|^2_{0^+} + |V_{us}|^2_{K_{\ell 3}} + \cancel{|V_{ub}|^2} - 1 = -0.0021(7)}$$

SOURCES OF UNCERTAINTY:

$\lvert V_{ud}\rvert^2_{0^+} + \lvert V_{us}\rvert^2_{K_{\ell 3}} - 1$	-2.1×10^{-3}
$\delta\lvert V_{ud}\rvert^2_{0^+}$, **exp**	2.1×10^{-4}
$\delta\lvert V_{ud}\rvert^2_{0^+}$, **RC**	1.8×10^{-4}
$\delta\lvert V_{ud}\rvert^2_{0^+}$, **NS**	5.3×10^{-4}
$\delta\lvert V_{us}\rvert^2_{K_{\ell 3}}$, **exp+th**	1.8×10^{-4}
$\delta\lvert V_{us}\rvert^2_{K_{\ell 3}}$, **lat**	1.7×10^{-4}
Total uncertainty	6.5×10^{-4}
Significance level	3.2σ

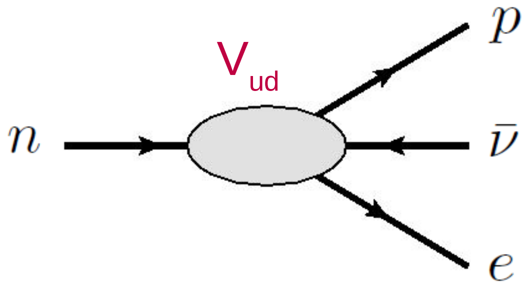
CYS, Galviz, Marciano and Meißner, 2022 PRD

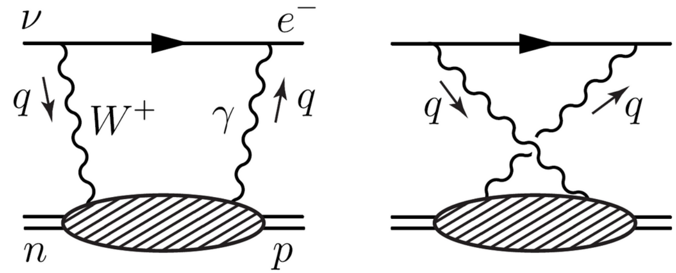
V_{ud}: Theory errors dominate

V_{us}: Theory/experimental errors comparable

Single-nucleon radiative corrections (RC)

Primary source of uncertainty: the **"single-nucleon axial γW-box diagram"**



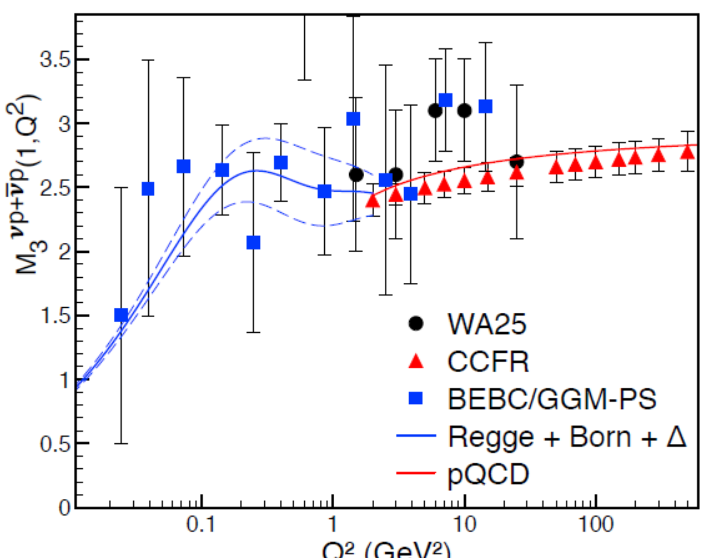


$Q^2=-q^2$

Dispersive representation:

CYS, Gorchtein, Patel and Ramsey-Musolf, 2018 PRL

$$\boxed{\Box^V_{\gamma W} = \frac{\alpha_{em}}{\pi \mathring{g}_V}\int_0^\infty \frac{dQ^2}{Q^2}\frac{M_W^2}{M_W^2+Q^2}\int_0^1 dx\frac{1+2r}{(1+r)^2}F_3^{(0)}(x,Q^2)}$$



Data input: **Parity-odd structure function F_3** from **neutrino-nucleus scattering**

|Vud|: $0.97420(21) \rightarrow 0.97370(14)$

Pre-2018 *2018*

Confirmation by independent studies:

Czarnecki, Marciano and Sirlin, 2019 PRD
CYS, Feng, Gorchtein and Jin, 2020 PRD
Hayen, 2021 PRD
Shiells, Blunden and Melnitchouk, 2021 PRD

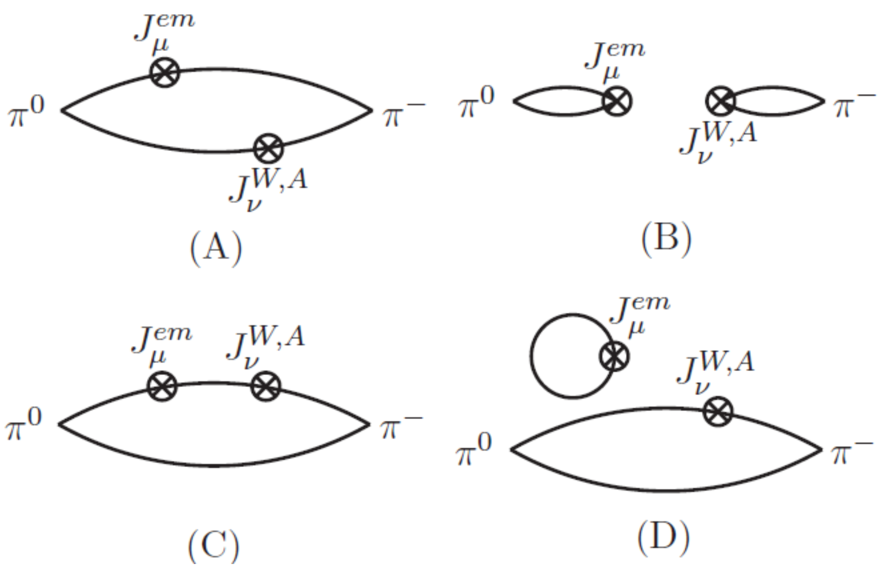
Single-nucleon radiative corrections (RC)

Major limiting factor of the DR treatment: **low quality of the neutrino data** in the most interesting region: **$Q^2 \sim 1\text{GeV}^2$**

Ongoing program: Calculate the box diagram directly with **lattice QCD**

Year 2020: First realistic lattice QCD calculation of the simpler **pion** axial γW-box diagram

Feng, Gorchtein, Jin, Ma and CYS, 2020 PRL



Consequences:

- Significant reduction of the theory uncertainty in **pion semileptonic decay (π_{e3})**
- Indirect implications on the **free-neutron** axial γW-box diagram

CYS, Feng, Gorchtein and Jin, 2020 PRD

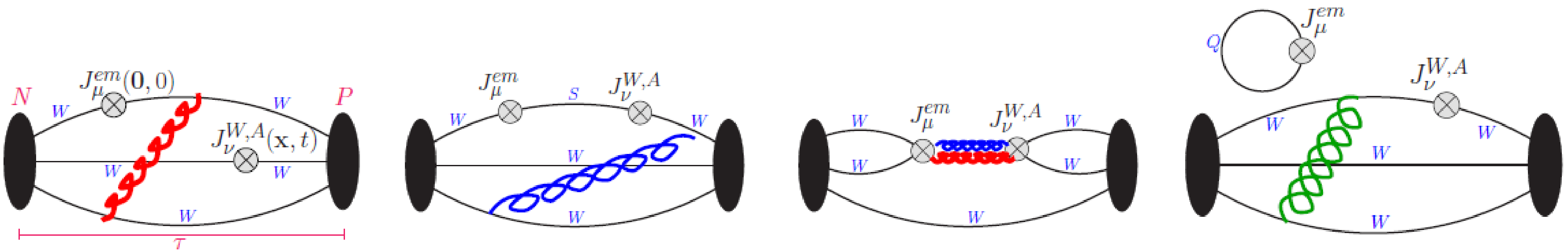
More in Luchang's talk!

Single-nucleon radiative corrections (RC)

Major limiting factor of the DR treatment: **low quality of the neutrino data** in the most interesting region: **$Q^2 \sim 1\mathrm{GeV}^2$**

Ongoing program: Calculate the box diagram directly with **lattice QCD**

Neutron axial γW-box diagram is more complicated, but on the way.



(R. Gupta, Rare Processes and Precision Frontier Townhall Meeting, 2020)

Possible alternative approach using **Feynman-Hellmann theorem (FHT)**

CYS and Meißner, 2019 PRL

Nuclear Structure corrections

Superallowed $0^+ \rightarrow 0^+$ nuclear beta decays provides the best measurement of V_{ud}

Master formula:

$$\boxed{|V_{ud}|^2 = \frac{2984.43\ s}{\mathcal{F}t\left(1+\Delta_R^V\right)}.}$$

Corrected ft (half-life*statistical function)-value:

Measured ft-value

$$\mathcal{F}t = ft\left(1+\delta_{\mathrm{R}}'\right)\left(1+\delta_{\mathrm{NS}}-\delta_{\mathrm{C}}\right)$$

Nucleus-dependent "outer corrections" (under control)

Nuclear structure effects in inner RC

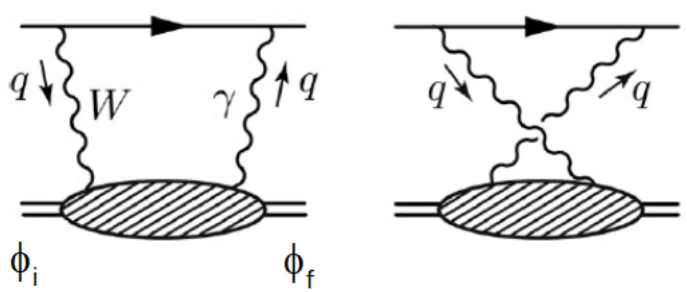
Isospin-breaking corrections

Best-measured decays:

$T_Z = -1$
$^{10}\mathrm{C}\rightarrow{}^{10}\mathrm{B}$
$^{14}\mathrm{O}\rightarrow{}^{14}\mathrm{N}$
$^{22}\mathrm{Mg}\rightarrow{}^{22}\mathrm{Na}$
$^{26}\mathrm{Si}\rightarrow{}^{26}\mathrm{Al}$
$^{34}\mathrm{Ar}\rightarrow{}^{34}\mathrm{Cl}$
$^{38}\mathrm{Ca}\rightarrow{}^{38}\mathrm{K}$
$T_Z = 0$
$^{26m}\mathrm{Al}\rightarrow{}^{26}\mathrm{Mg}$
$^{34}\mathrm{Cl}\rightarrow{}^{34}\mathrm{S}$
$^{38m}\mathrm{K}\rightarrow{}^{38}\mathrm{Ar}$
$^{42}\mathrm{Sc}\rightarrow{}^{42}\mathrm{Ca}$
$^{46}\mathrm{V}\rightarrow{}^{46}\mathrm{Ti}$
$^{50}\mathrm{Mn}\rightarrow{}^{50}\mathrm{Cr}$
$^{54}\mathrm{Co}\rightarrow{}^{54}\mathrm{Fe}$
$^{62}\mathrm{Ga}\rightarrow{}^{62}\mathrm{Zn}$
$^{74}\mathrm{Rb}\rightarrow{}^{74}\mathrm{Kr}$

Nuclear Structure corrections

δ_{NS}: **nuclear modifications of the free-nucleon γW box diagram**



LARGEST source of uncertainty in V_{ud}!

$$\Box^{\text{nucl.}}_{\gamma W} = \Box^{n}_{\gamma W} + \underline{\left[\Box^{\text{nucl.}}_{\gamma W} - \Box^{n}_{\gamma W}\right]}$$

Ongoing program: Nuclear structure functions at low Q^2 with **ab-initio methods**

$$\frac{1}{8\pi}\int d^4x e^{iq\cdot x}\langle\phi_f(p)|[J^{\mu}_{\text{em}}(x), J^{\dagger\nu}_{W}(0)]|\phi_i(p)\rangle$$

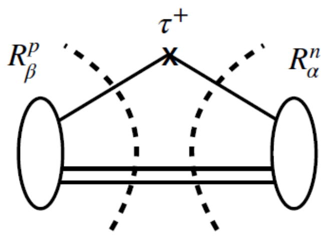
$$= \left(-g^{\mu\nu} + \frac{q^{\mu}q^{\nu}}{q^2}\right)\underline{F_1} + \frac{\hat{p}^{\mu}\hat{p}^{\nu}}{p\cdot q}\underline{F_2} - \frac{i\varepsilon^{\mu\nu\alpha\beta}q_{\alpha}p_{\beta}}{2p\cdot q}\underline{F_3}$$

Inputs to the dispersion integral

Light nuclei: Quantum Monte Carlo, No-Core Shell Model, …
Medium-size nuclei: Coupled-Cluster method, ….

Nuclear Structure corrections

δ_c: isospin-breaking (ISB) corrections to nuclear wavefunctions



Mainly due to Coulomb interaction between protons

$$V_C = V_C^{(0)} + V_C^{(1)}$$

Computed systematically within **shell model** (Hardy-Towner), but were questioned in several aspects:

1. Theoretical inconsistencies *Miller and Schwenk, 2008 PRC, 2009 PRC; Condren and Miller, 2201.10651*

2. Cannot be reproduced by other nuclear theory calculations, which generically predict **smaller δ_c** *Hartree-Fock, DFT, RPA, isovector monopole resonance...*

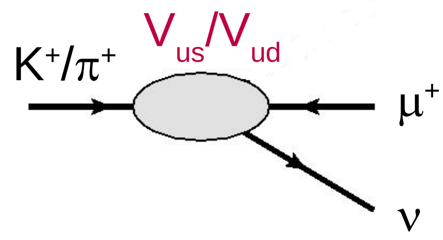
New Thoughts: Possible relation to the **neutron skin** of the stable nuclei?

$$\delta_C \propto \frac{1}{3}\sum_a \frac{|\langle a;0||V_C^{(1)}||g;1\rangle|^2}{(E_{a,0}-E_g)^2} + \frac{1}{2}\sum_{a\neq g} \frac{|\langle a;1||V_C^{(1)}||g;1\rangle|^2}{(E_{a,1}-E_g)^2} - \frac{5}{6}\sum_a \frac{|\langle a;2||V_C^{(1)}||g;1\rangle|^2}{(E_{a,2}-E_g)^2}$$

$$(R_n - R_p)_{V_C^{(1)}} \propto \sum_{a\neq g} \frac{|\langle a;1||V_C^{(1)}||g;1\rangle|^2}{E_{a,1}-E_g} + \sum_a \frac{|\langle a;2||V_C^{(1)}||g;1\rangle|^2}{E_{a,2}-E_g}$$

Kaon/pion leptonic decay ($K_{\mu 2}/\pi_{\mu 2}$)

$$\frac{|V_{us}|f_{K^+}}{|V_{ud}|f_{\pi^+}} = \underbrace{\left[\frac{\Gamma_{K_{\mu 2}} M_{\pi^+}}{\Gamma_{\pi_{\mu 2}} M_{K^+}}\right]}_{\text{“axial ratio” } R_A}{}^{1/2} \; \frac{1 - m_\mu^2/M_{\pi^+}^2}{1 - m_\mu^2/M_{K^+}^2} \,(1 - \delta_{\rm EM}/2)$$



Marciano, 2004 PRL; Cirigliano and Neufeld, 2011 PLB

Lattice QCD inputs: K^+/π^+ decay constants

$$\begin{aligned} N_f = 2+1+1 \quad &: \quad f_{K^+}/f_{\pi^+} = 1.1932(21) \\ N_f = 2+1 \quad &: \quad f_{K^+}/f_{\pi^+} = 1.1917(37) \\ N_f = 2 \quad &: \quad f_{K^+}/f_{\pi^+} = 1.205(18) \end{aligned}$$

FLAG 2021

Electromagnetic RC in ChPT: $\delta_{\rm EM} = \delta_{\rm EM}^K - \delta_{\rm EM}^\pi = -0.0069(17)$

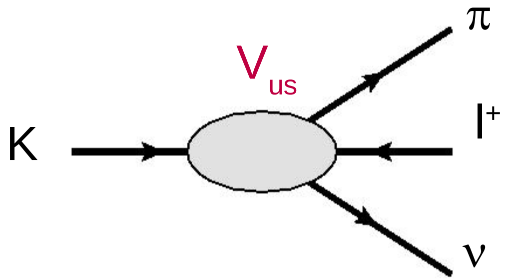
Knecht et al., 2000 EPJC
Cirigliano and Neufeld, 2011 PLB

Advantage: **LECs cancel in the ratio**

Direct lattice QCD calculation of the EMRC+isospin breaking correction (contained in the physical K^+/π^+ decay constants) consistent with ChPT result, with slightly lower uncertainty *Giusti et al, 2018 PRL*

Total: $|V_{us}/V_{ud}| = 0.23131(41)_{\rm lat}(24)_{\rm exp}(19)_{\rm RC}$

Kaon semileptonic decays (K_{l3})



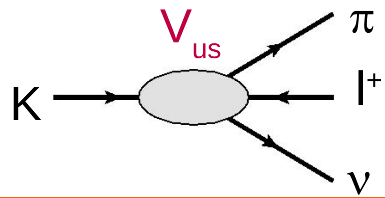
$$\Gamma_{K_{\ell 3}} = \frac{G_F^2 |V_{us}|^2 M_K^5 C_K^2}{192\pi^3} S_{\rm EW} |f_+^{K^0\pi^-}(0)|^2 I_{K\ell}^{(0)} \left(1 + \delta_{\rm EM}^{K\ell} + \delta_{\rm SU(2)}^{K\pi}\right)$$

Measurements of **branching ratio** exist in all channels (most recent: $K^S_{\mu 3}$)

Theory Inputs:

Kπ form factor at t=0
Phase-space factor
Long-distance electromagnetic RC
ISB correction

Kaon semileptonic decays (K_{l3})



Master formula:

$$\Gamma_{K_{\ell 3}} = \frac{G_F^2 |V_{us}|^2 M_K^5 C_K^2}{192\pi^3} S_{\mathrm{EW}} |f_+^{K^0\pi^-}(0)|^2 I_{K\ell}^{(0)} \left(1 + \delta_{\mathrm{EM}}^{K\ell} + \delta_{\mathrm{SU(2)}}^{K\pi}\right)$$

Kπ form factor at t=0: $\langle \pi^-(p') | J_W^\mu | K^0(p) \rangle = f_+^{K^0\pi^-}(t)(p+p')^\mu + f_-^{K^0\pi^-}(t)(p-p')^\mu$

Lattice QCD inputs:

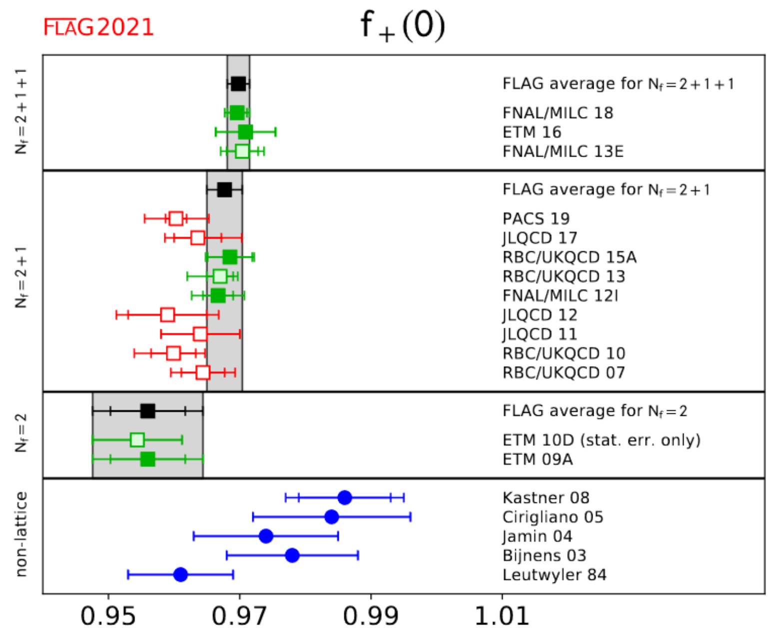
$$N_f = 2+1+1 \quad : \quad f_+(0) = 0.9698(17)$$
$$N_f = 2+1 \quad : \quad f_+(0) = 0.9677(27)$$
$$N_f = 2 \quad : \quad f_+(0) = 0.9560(57)(62)$$

FLAG 2021

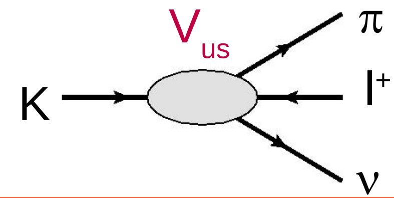
New result from PACS (N_f=2+1):

$$f_+(0) = 0.9615(10)(^{+47}_{-2})(5)$$

Ishikawa et al, 2206.08654



Kaon semileptonic decays (K_{l3})



Master formula:

$$\Gamma_{K_{\ell 3}} = \frac{G_F^2 |V_{us}|^2 M_K^5 C_K^2}{192\pi^3} S_{\rm EW} |f_+^{K^0\pi^-}(0)|^2 I_{K\ell}^{(0)} \left(1 + \delta_{\rm EM}^{K\ell} + \delta_{\rm SU(2)}^{K\pi}\right)$$

Phase-space factor: $$I_{K\ell}^{(0)} = \int_{m_\ell^2}^{(M_K-M_\pi)^2} \frac{dt}{M_K^8} \bar{\lambda}^{3/2} \left(1 + \frac{m_\ell^2}{2t}\right)\left(1 - \frac{m_\ell^2}{t}\right)^2 \left[\bar{f}_+^2(t) + \frac{3m_\ell^2 \Delta_{K\pi}^2}{(2t+m_\ell^2)\bar{\lambda}} \bar{f}_0^2(t)\right]$$

Rescaled
Kπ form factors

probes the **t-dependence** of the Kπ form factors.

Obtained by fitting to the **K_{l3} Dalitz** plot with **specific parameterizations of f(t)** (Taylor expansion, z-expansion, dispersive parameterization, pole parameterization ...)

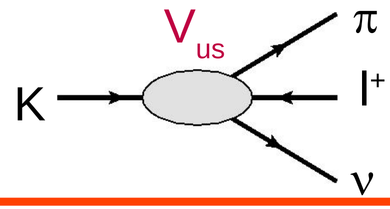
The **dispersive parameterization** currently quotes the smallest uncertainty:

Mode	Update
$K^0{}_{e3}$	**0.15470(15)**
$K^+{}_{e3}$	**0.15915(15)**
$K^0{}_{\mu 3}$	**0.10247(15)**
$K^+{}_{\mu 3}$	**0.10553(16)**

M. Moulson, in the 11th International Workshop on the CKM Unitarity Triangle, 2021

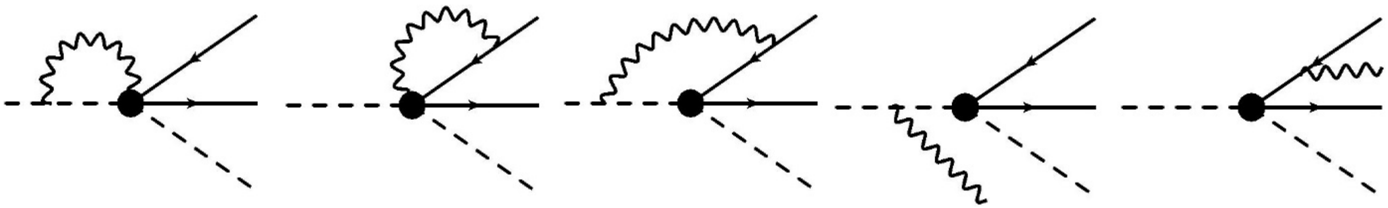
Future: Direct lattice calculation of the t-dependence?

Kaon semileptonic decays (K_{l3})



Master formula:

$$\Gamma_{K_{\ell 3}} = \frac{G_F^2 |V_{us}|^2 M_K^5 C_K^2}{192\pi^3} S_{\rm EW} |f_+^{K^0\pi^-}(0)|^2 I_{K\ell}^{(0)} \left(1 + \delta_{\rm EM}^{K\ell} + \delta_{\rm SU(2)}^{K\pi}\right)$$



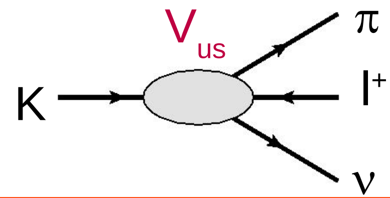
Long-distance electromagnetic RC (10^{-3})

	$\delta_{\rm EM}^{K\ell}$ "Sirlin's representation"	ChPT
$K^0 e$	$11.6(2)_{\rm inel}(1)_{\rm lat}(1)_{\rm NF}(2)_{e^2p^4}$	$9.9(1.9)_{e^2p^4}(1.1)_{\rm LEC}$
$K^+ e$	$2.1(2)_{\rm inel}(1)_{\rm lat}(4)_{\rm NF}(1)_{e^2p^4}$	$1.0(1.9)_{e^2p^4}(1.6)_{\rm LEC}$
$K^0 \mu$	$15.4(2)_{\rm inel}(1)_{\rm lat}(1)_{\rm NF}(2)_{\rm LEC}(2)_{e^2p^4}$	$14.0(1.9)_{e^2p^4}(1.1)_{\rm LEC}$
$K^+ \mu$	$0.5(2)_{\rm inel}(1)_{\rm lat}(4)_{\rm NF}(2)_{\rm LEC}(2)_{e^2p^4}$	$0.2(1.9)_{e^2p^4}(1.6)_{\rm LEC}$

Lattice inputs:
see Luchang's talk!

CYS, Galviz, Gorchtein and Meißner, 2022 JHEP *Cirigliano et al., 2008 JHEP*

Kaon semileptonic decays (K_{l3})



Master formula:

$$\Gamma_{K_{\ell 3}} = \frac{G_F^2 |V_{us}|^2 M_K^5 C_K^2}{192\pi^3} S_{\mathrm{EW}} |f_+^{K^0\pi^-}(0)|^2 I_{K\ell}^{(0)} \left(1 + \delta_{\mathrm{EM}}^{K\ell} + \delta_{\mathrm{SU}(2)}^{K\pi}\right)$$

ISB correction: presents only in the **K⁺** channel by construction.

$$\delta_{\mathrm{SU}(2)}^{K^+\pi^0} \equiv \left(\frac{f_+^{K^+\pi^0}(0)}{f_+^{K^0\pi^-}(0)}\right)^2 - 1 = \frac{3}{2}\frac{1}{Q^2}\left[\frac{\hat{M}_K^2}{\hat{M}_\pi^2} + \frac{\chi_{p^4}}{2}\left(1 + \frac{m_s}{\hat{m}}\right)\right]$$

(neglecting small EM contributions)

$$Q^2 = (m_s^2 - \hat{m}^2)/(m_d^2 - m_u^2)$$

Most recent lattice QCD inputs: *FLAG 2021*

$$Q = 23.3(5)\ , \quad m_s/\hat{m} = 27.42(12) \qquad N_f = 2+1$$

returns: $\delta_{\mathrm{SU}(2)}^{K^+\pi^0} = 0.0457(20)$

Phenomenological inputs from $\eta \to 3\pi$ returns a somewhat larger value:

$$\delta_{\mathrm{SU}(2)}^{K^+\pi^0} = 0.0522(34)$$

Colangelo, Lanz, Leutwyler and Passemar, 2018 EPJC

Summary

- Several **anomalies** at the level ~3σ have been observed in the measurements of the **first-row CKM matrix elements V_{ud} and V_{us}** in **beta decay processes**.
- **SM theory inputs** that require further improvements are:
  - **V_{ud} sector**: RC in single-nucleon and nuclear systems, ISB corrections in nuclear wavefunctions
  - **V_{us} sector**: Lattice inputs of Kaon/pion decay constants and $K\pi$ form factor, RC in leptonic and semileptonic kaon decays, K_{l3} phase-space factor, ISB corrections in $K^{\pm}$ semileptonic decays
- Successful reduction of theory uncertainties above could increase the significance of the anomalies to more than 5σ
- Desirable future **experimental improvements**: K_{l3} and π_{e3} branching ratios, neutron lifetime and g_A, ...

Thanks for your attention!