Lattice QCD input for the first row CKM unitarity tests

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Outline

- **Introduction**
- Pion semi-leptonic decay - γW-box diagram

  PRL 124, 192002 (2020) X. Feng, M. Gorchtein, **LJ**, P.X. Ma, C.Y. Seng
- Kaon semi-leptonic decay - χPT low energy constants

  JHEP 10, 179 (2020) C.Y. Seng, X. Feng, M. Gorchtein, **LJ**, Meißner, Ulf-G.

  PRD 103, 114503 (2021) P.X. Ma, X. Feng, M. Gorchtein, **LJ**, C.Y. Seng
- Conclusion and outlook

CKM matrix elements, $|V_{ud}|$ and $|V_{us}|$

The unitarity relation for the first-row of the CKM matrix is:

$$\Delta_{\text{CKM}} = |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = 0.$$

According to PDG 2020, $\Delta_{\text{CKM}} = -0.0015(3)(4)$, where the first error is the uncertainty from $|V_{ud}|^2$ and the second error is the uncertainty from $|V_{us}|^2$.

- $|V_{ud}| = 0.97370(10)_{\text{exp.,nucl.}}(10)_{\text{RC}}$ [0.14 per mil].
  From superallowed $0^+ \to 0^+$ nuclear β decays, which are pure vector transitions at leading order. PDG 2022 include additional large (27) uncertainty for nuclear structure. Neutron and pion β decay can determine as well, but currently have larger uncertainties.

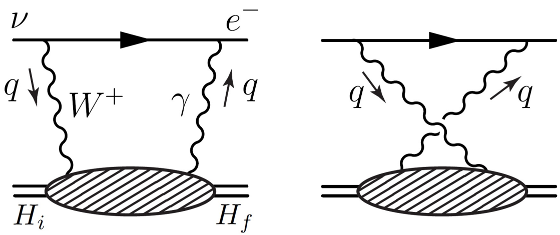
- $|V_{us}| = 0.2245(4)$ [1.8 per mil]
  From combined analysis of $K \to l\nu$ and $K \to \pi l\nu$ decay processes. Lattice QCD inputs are essential and the current limiting factor.

- $|V_{ub}| = 0.00394(36)$. Very small, does not affect Δ_{CKM} much.

"That deviation could be due a problem with $|V_{ud}|$ theory (RC or NP), the lattice determination of $f_+(0)$ or new physics."

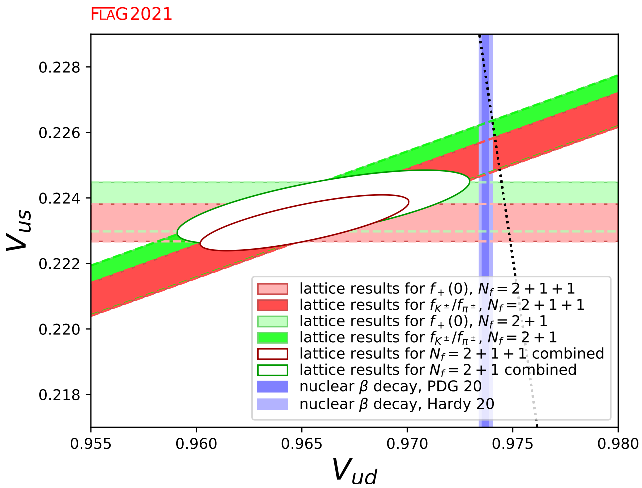
– E. Blucher and W.J. Marciano (PDG 2020)

- In superallowed nuclear β decay rates, one of the dominant uncertainties arises from the nucleus-independent electroweak radiative correction, Δ_R^V, which is universal for both nuclear and free neutron beta decay. J.C. Hardy, I.S. Towner (2015)
- Sirlin established that only the axial γW-box contribution is sensitive to hadronic scales. A. Sirlin (1978)
- Situation is similar for the charged pion β decay.



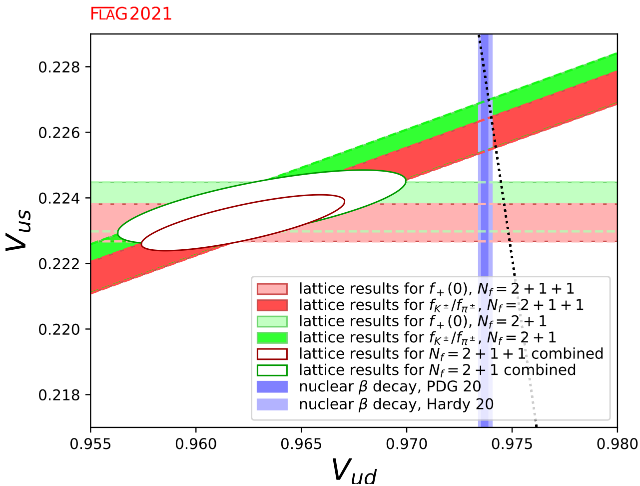
$$H_{\mu\nu}^{VA}(t,\vec{x}) = \langle H_f | T\left[J_\mu^{em}(t,\vec{x}) J_\nu^{W,A}(0)\right] |H_i\rangle$$

$K \to \ell\nu$, $K \to \pi\ell\nu$ and $|V_{us}|$

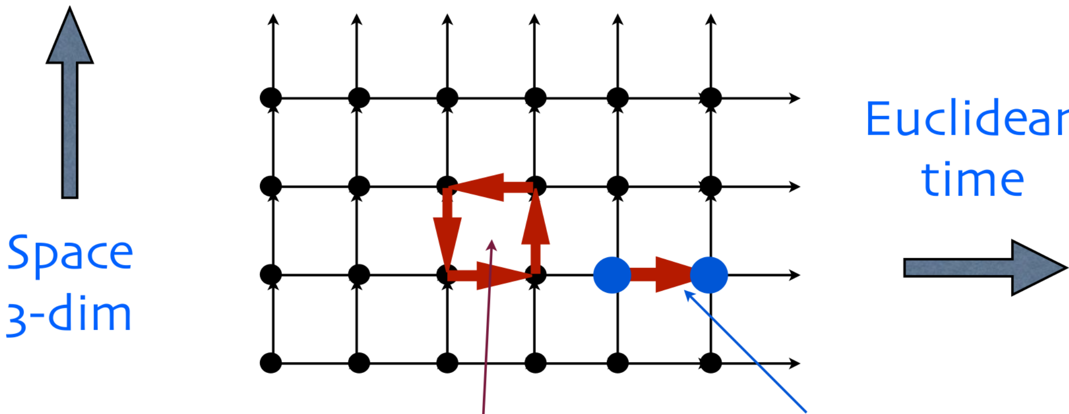


- Dashed line is the CKM matrix first row unitary constraint.
- All the bands and line should cross the same point. There are visible tensions in the plot.
- QED and strong isospin breaking corrections from ChPT calculations.

$K \to \ell\nu$, $K \to \pi\ell\nu$ and $|V_{us}|$



- Dashed line is the CKM matrix first row unitary constraint.
- All the bands and line should cross the same point. There are visible tensions in the plot.
- QED and strong isospin breaking corrections for $f_{K^\pm}/f_{\pi^\pm}$ from lattice QCD. [PRD 100 (2019) 034514] M. Di Carlo, D. Giusti, V. Lubicz, G. Martinelli, C.T. Sachrajda, F. Sanfilippo, S. Simula, N. Tantalo



$$S_E^{\mathrm{latt}} = -\sum_{\Box} \frac{6}{g^2}\,\mathrm{Re}\;\mathrm{tr}_N(U_{\Box,\mu\nu}) - \sum_q \overline{q}(D_\mu^{\mathrm{lat}}\gamma_\mu + am_q)q$$

Wilson gauge action Lattice fermion action

Figure credit: Stephen R. Sharpe.

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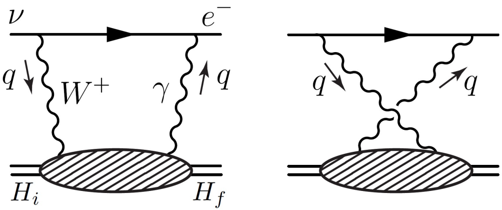
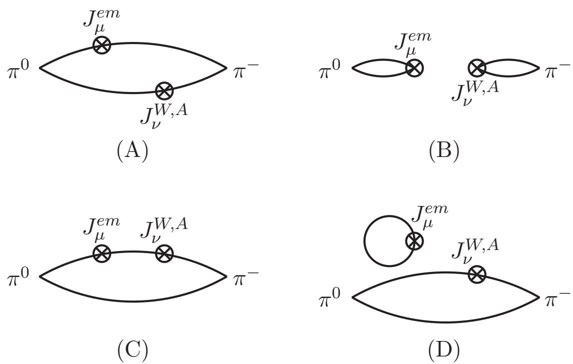
The γW-box contribution for pion



$$H^{VA}_{\mu\nu}(t,\vec{x}) = \langle \pi^0 | T\left[J^{em}_{\mu}(t,\vec{x}) J^{W,A}_{\nu}(0)\right] | \pi^- \rangle$$

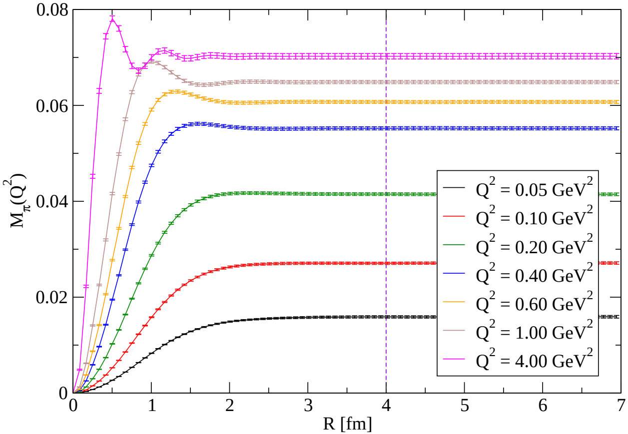
Hadronic function in lattice QCD calculations:



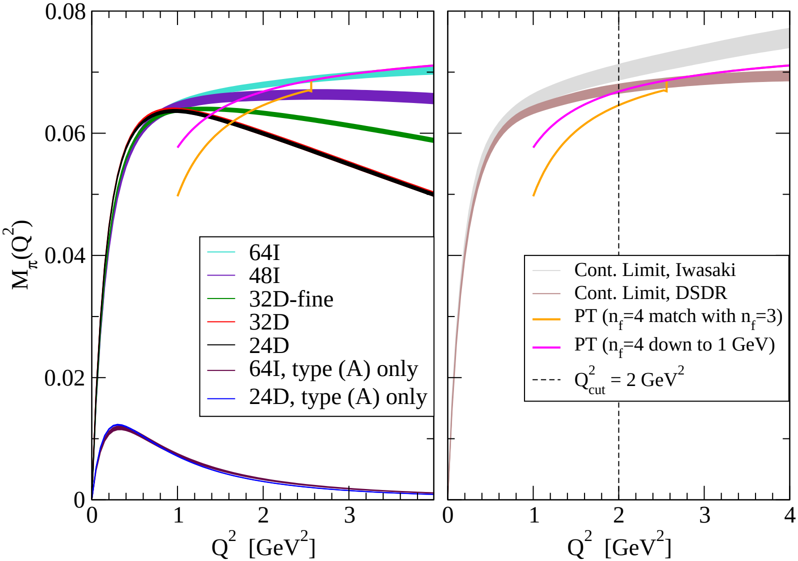
Gluons and quark loops not directly connected to the operators are automatically included in the numerical Monte Carlo integration over all possible QCD gauge field configurations.

$$M_\pi(Q^2) = -\frac{1}{6}\frac{1}{F_+^\pi}\frac{\sqrt{Q^2}}{m_\pi}\int d^4x\,\omega(t,\vec{x})\epsilon_{\mu\nu\alpha 0}x_\alpha H_{\mu\nu}^{VA}(t,\vec{x}),$$

$$\omega(t,\vec{x}) = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{\cos^3\theta\,d\theta}{\pi}\frac{j_1\left(\sqrt{Q^2}|\vec{x}|\cos\theta\right)}{|\vec{x}|}\cos\left(\sqrt{Q^2}t\sin\theta\right)$$



Results from the 64I ensemble (R is the upper limit of the Euclidean space-time range). Note that the weighting function $\omega(t,\vec{x})$ is calculated in the infinite volume. The momentum exchange Q can vary continuously instead of being constrained by $2\pi/L$.



$$\square^{VA}_{\gamma W}\Big|_\pi = \frac{3\alpha_e}{2\pi}\int \frac{dQ^2}{Q^2}\frac{m_W^2}{m_W^2+Q^2}M_\pi(Q^2)$$
$$= 2.830(11)_{\text{stat}}(9)_{\text{PT}}(24)_{\text{a}}(3)_{\text{FV}}\times 10^{-3}$$
$$= 2.830(11)_{\text{stat}}(26)_{\text{sys}}\times 10^{-3}$$

$$\Gamma_{\pi\ell 3} = \frac{G_F^2 |V_{ud}|^2 m_\pi^5 |f_+^\pi(0)|^2}{64\pi^3} (1+\delta)\, I_\pi$$

- Here, m_π is the charged pion mass, I_π is the kinematic factor.
- ChiPT A. Czarnecki, W .Marciano, A. Sirlin (2019)

$$\delta = 0.0334(10)_{\text{LEC}}(3)_{\text{HO}}$$

- Sirlin's parametrization A. Sirlin (1978)

$$\begin{aligned}\delta &= \frac{\alpha_e}{2\pi}\left[\bar{g} + 3\ln\frac{m_Z}{m_p} + \ln\frac{m_Z}{m_W} + \tilde{a}_g\right] + \delta_{\text{HO}}^{\text{QED}} + 2\Box_{\gamma W}^{VA}\Big|_\pi \\ &= 0.0332(1)_{\gamma W}(3)_{\text{HO}}\end{aligned}$$

where $\frac{\alpha_e}{2\pi}\bar{g} = 1.051 \times 10^{-2}$, $\frac{\alpha_e}{2\pi}\tilde{a}_g \approx -9.6 \times 10^{-5}$, $\delta_{\text{HO}}^{\text{QED}} = 0.0010(3)$.

- Remaining theoretical uncertainty mainly comes from corrections of higher order in α_e.

- Further theoretical improvement requires a complete electroweak two-loop analysis.
- At present, with the PIBETA experiment, we have $|V_{ud}| = 0.9740(28)_{\text{exp}}(1)_{\text{th}}$ from pion β decay. Recall PDG 2020 value is $|V_{ud}| = 0.97370(10)_{\text{exp.,nucl.}}(10)_{\text{RC}}$.
- Estimating the difference between pion and nucleon γW-box by hadronic models, combined with the precise lattice results for pion, we obtain a new value for nucleon γW-box. The tension persists.

  PRD 101, 11, 111301 (2020) C.Y. Seng, X. Feng, M. Gorchtein, **LJ**
- This pion calculation provide experiences for the nucleon γW-box lattice calculation. In particular, we expect the discretization error being similar.
- Lattice calculation of the nucleon γW-box diagram in progress.

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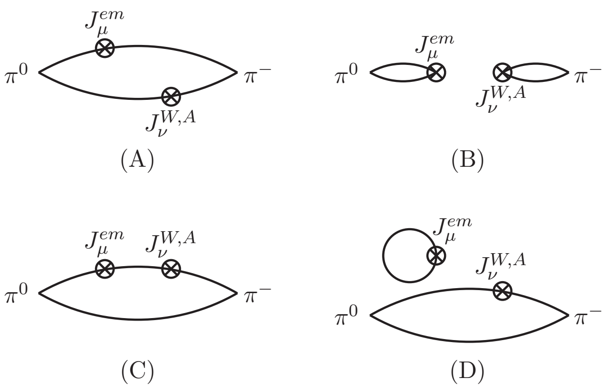
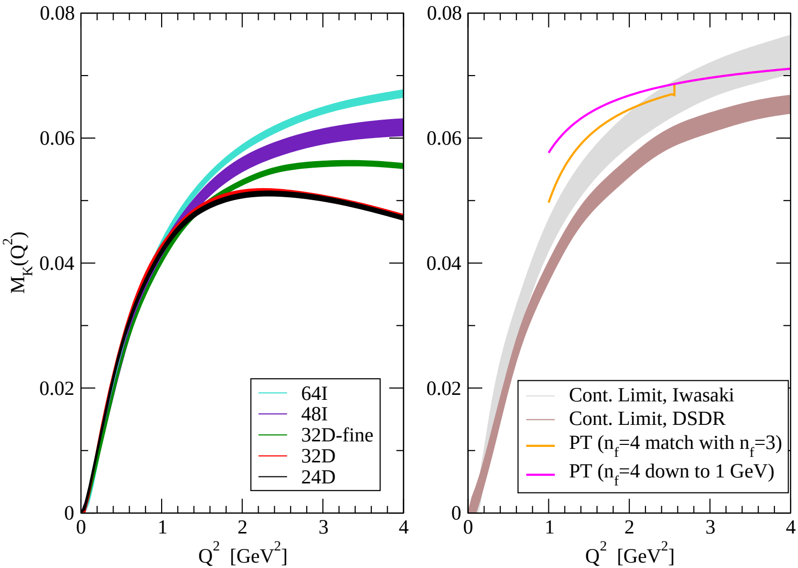
- Goal is to obtain the χPT low energy constants relevant to the QED radiative corrections in kaon semi-leptonic decay, which is largely independent of quark mass.
- Calculate the kaon semi-leptonic decay process in the flavor SU(3) limit with the light meson masses, m_π, m_K, m_η, equal to the physical pion mass.
- Use the same set of contractions and propagators as the pion β decay calculation, but adjust the charge factors appropriately.





$$\Box^{VA}_{\gamma W}\Big|_{K^0,\mathrm{SU(3)}} = \frac{3\alpha_e}{2\pi}\int \frac{dQ^2}{Q^2}\frac{m_W^2}{m_W^2+Q^2}M_K(Q^2)$$

$$= 2.437(20)_{\mathrm{stat}}(15)_{\mathrm{PT}}(36)_{\mathrm{a}}(1)_{\mathrm{FV}} \times 10^{-3}$$

$$= 2.437(20)_{\mathrm{stat}}(39)_{\mathrm{sys}} \times 10^{-3}$$

Using the lattice results of $\Box^{VA}_{\gamma W}\big|_{K^0,\mathrm{SU(3)}}$ evaluated in the flavor SU(3) limit, we can obtain the χPT low energy constants (LECs).

Previously, the LECs are obtained via the minimal resonance model

S. Descotes-Genon, B. Moussallam (2005) B. Ananthanarayan, B. Moussallam (2004)

$$X_1 = -3.7(3.7)\times 10^{-3} \quad \to \quad -2.2(4)\times 10^{-3}$$
$$\tilde{X}_6^{\mathrm{phys}} = 10.4(10.4)\times 10^{-3} \quad \to \quad 13.9(7)\times 10^{-3}.$$

In χPT, the radiative corrections (RCs) have two major sources of theoretical uncertainties:

- The input of the LECs at $\mathcal{O}(e^2p^2)$.
- The unknown $\mathcal{O}(e^2p^4)$ terms in the ChPT expansion.

In unit of percent:

$$\delta^e_{K^0} = 0.99(19)_{e^2p^4}(11)_{\mathrm{LEC}} \quad \to \quad 1.00(19)$$
$$\delta^\mu_{K^0} = 1.40(19)_{e^2p^4}(11)_{\mathrm{LEC}} \quad \to \quad 1.41(19)$$
$$\delta^e_{K^\pm} = 0.10(19)_{e^2p^4}(16)_{\mathrm{LEC}} \quad \to \quad -0.01(19)$$
$$\delta^\mu_{K^\pm} = 0.02(19)_{e^2p^4}(16)_{\mathrm{LEC}} \quad \to \quad -0.09(19).$$

More theory work done in this direction, as mentioned in the previous talk by C.Y. Seng.

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Conclusion and outlook

- Pion β decay (the pion semi-leptonic decay $\pi\ell3$)

$$\Box_{\gamma W}^{VA}\Big|_{\pi} = 2.830(11)_{\text{stat}}(26)_{\text{sys}} \times 10^{-3}$$

$$\delta = 0.0334(10)_{\text{LEC}}(3)_{\text{HO}} \quad \to \quad \delta = 0.0332(1)_{\gamma W}(3)_{\text{HO}}$$

- Kaon semi-leptonic decay in the flavor SU(3) limit (unphysical kaon mass):

$$\Box_{\gamma W}^{VA}\Big|_{K^0,\text{SU(3)}} = 2.437(20)_{\text{stat}}(39)_{\text{sys}} \times 10^{-3}$$

- χPT low energy constants relevant to kaon semi-leptonic decay:

$$X_1 = -3.7(3.7)\times 10^{-3} \to -2.2(4)\times 10^{-3}, \quad \tilde{X}_6^{\text{phys}} = 10.4(10.4)\times 10^{-3} \to 13.9(7)\times 10^{-3}.$$

Conclusion and outlook

- For better $|V_{ud}|$ from nucleus / neutron decay, nucleon $\square_{\gamma W}^{VA}$ lattice calculation underway.
- For better $|V_{ud}|$ from pion β decay (also called pion semi-leptonic decay), more accurate experimental results is needed. The current theoretical determination is very accurate and clean.
- For better $|V_{us}|$ from meson leptonic decay: We are working towards a complete lattice calculation of QED correction to meson leptonic decay with infinite volume QED using the infinite volume reconstruction method. The theoretical framework for this calculation is in preparation: N. Christ, X. Feng, **LJ**, C.T. Sachrajda, T. Wang in preparation.

  First step, pion mass mass splitting calculation finished:

  $$M_{\pi^\pm} - M_{\pi^0} \quad = \quad 4.534(42)_{\mathrm{stat}}(43)_{\mathrm{sys}}\ \mathrm{MeV} \qquad [\text{PDG 2020: } 4.5936(5)\ \mathrm{MeV}]$$

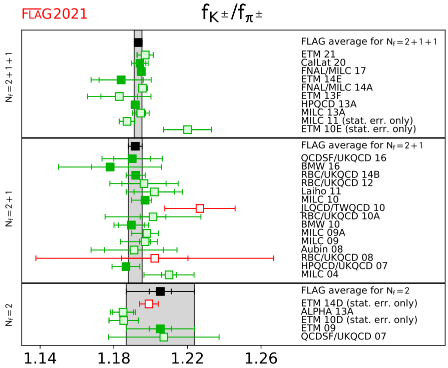
  PRL 128, 052003 (2022) X. Feng, **LJ**, M.J. Riberdy

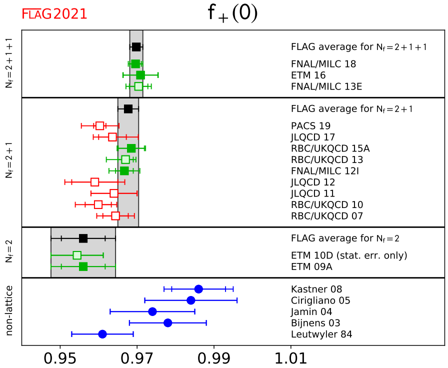
  Note that there are two other groups also working on this:

  - Already finished calculation in QED_L scheme by RM123 group:
    PRD 100 (2019) 034514, M. Di Carlo, D. Giusti, V. Lubicz, G. Martinelli, C.T. Sachrajda, F. Sanfilippo, S. Simula, N. Tantalo
  - Another calculation from RBC-UKQCD also uses QED_L scheme @ physical m_π:
    PoS LATTICE2021 (2022) 110, A. Z. N. Yong, P. Boyle, M. Di Carlo, F. Erben, V. Gülpers, M. T. Hansen, T. Harris, N. Hermansson-Truedsson, R. Hodgson and A. Jüttner, *et al.*

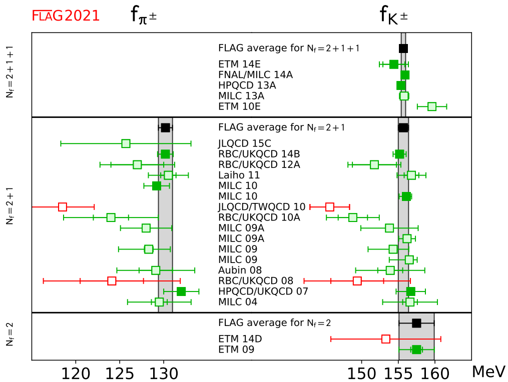
- For better $|V_{us}|$ from meson semi-leptonic decay: a complete lattice calculation of QED correction to meson semi-leptonic decay is more difficult than leptonic decay, but should be feasible as well. Will be the next goal.

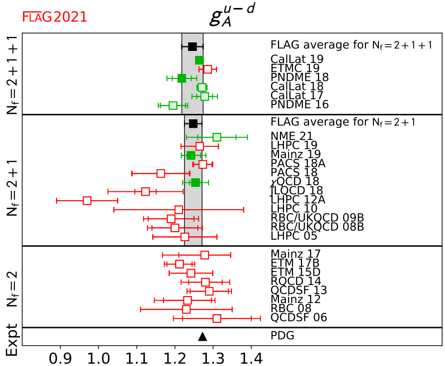
- For better $|V_{us}|$ from meson (semi-)leptonic decay: improving the precision leading order decay constant and form factors are also important.





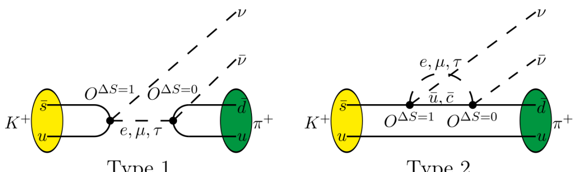
- For better $|V_{ud}|$ from pion leptonic decay: more accurate lattice determination of f_{π^+} and the QED correction to the process are needed.
- For better $|V_{ud}|$ from neutron decay: more accurate g_A is needed. Currently experimental determination is more accurate. But the precision from lattice QCD is improving as well.





- [PRD 102, 054509 (2020)] by the RBC-UKQCD collaborations. Chris Kelly (BNL).
- Final result $\mathrm{Re}(\epsilon'/\epsilon) = 2.17(26)_{\mathrm{stat}}(62)_{\mathrm{sys}}(50)_{\mathrm{isospin}} \times 10^{-3}$
  Or, include the ChPT evaluation of the QED and strong isospin breaking effects: 1.67×10^{-3}. Recall the experimental value is $1.66(23) \times 10^{-3}$.
- Good agreement at this precision. RBC-UKQCD efforts to reduce the error:
  - Repeat the calculation with finer lattices.
  - Non-perturbative 3- to 4-flavor operator matching. Masaaki Tomii.
    M. Tomii, Proc. Sci., LATTICE2018 (2019) 216.
  - Periodic boundary condition $K \to \pi\pi$. Masaaki Tomii and Daniel Hoying.
- Developing method to study the QED and strong isospin breaking effects on the lattice
  - N. Christ and X. Feng, EPJ Web Conf. 175, 13016 (2018)
  - Y. Cai and Z. Davoudi, Proc. Sci., LATTICE2018 (2018) 280
- [PRD 98 (2018) 11, 114512] N. Ishizuka, K. I. Ishikawa, A. Ukawa and T. Yoshié
  Independent calculation with $m_\pi = 260$ MeV.

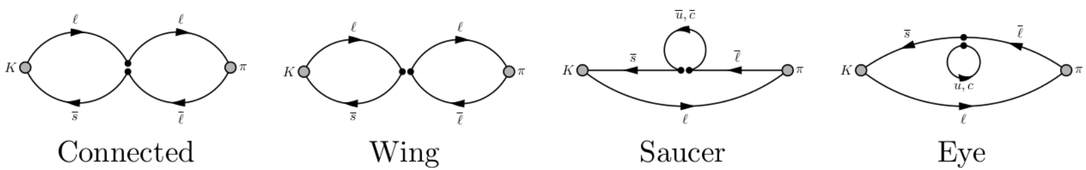
- [PRD 100 (2019) 11, 114506] by the RBC-UKQCD collaborations Xu Feng (Peking University).
- The golden modes: an ideal process in which to search for signs of new physics.
  - NA62 at CERN: $K^+ \to \pi^+\nu\bar{\nu}$ (aim at 10% accuracy)
  - KOTO at J-PARC: $K_L \to \pi^0\nu\bar{\nu}$ (long distance contributions negligible)
- The long distance ($\mathcal{O}(1/m_c)$) contribution is estimated to be about 5% to 10% in $K^+ \to \pi^+\nu\bar{\nu}$.
- Two pioneer lattice calculations @ $m_\pi = 430$ MeV and $m_\pi = 170$ MeV and lighter charm quark mass due to coarse lattice spacing.



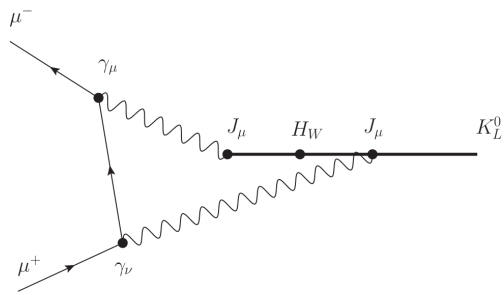
W-W diagram

Rare kaon decays: $K \to \pi\ell^+\ell^-$

- arXiv:2202.08795 [hep-lat], P. A. Boyle, F. Erben, J. M. Flynn, V. Gülpers, R. C. Hill, R. Hodgson, A. Jüttner, F. Ó. hÓgáin, A. Portelli and C. T. Sachrajda
- Similar to $K \to \pi\nu\bar{\nu}$:
  - NA62 at CERN: $K^+ \to \pi^+\ell^+\ell^-$
  - Prospective experiments planned at LHCb to study the: $K_S \to \pi^0\ell^+\ell^-$
  - Lattice calculations @ $m_\pi = 139$ MeV, $a^{-1} = 1.73$ GeV!
  - Statistical error is currently pretty large: $V(z = 0.013(2)) = -0.87(4.44)$. Mostly come from the stochastically estimated quark loops.

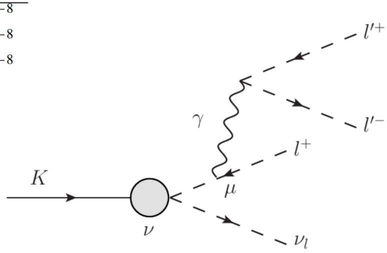


- [PoS LATTICE2019 (2020) 128, PoS LATTICE2019 (2020) 097] by the RBC-UKQCD collaborations
  Yidi Zhao (Columbia University), Norman Christ's graduate student)
- Branching fraction is accurately measured: $\mathrm{BR}(K_L \to \mu^+\mu^-) = 6.84 \pm 0.11) \times 10^{-9}$.
- Two mechanism of comparable sizes for the decay:
  - One-loop, second-order weak process, involving exchange of two weak bosons.
  - $\mathcal{O}(\alpha_{\mathrm{EM}}^2 G_F)$ process shown below.
- First step calculation $\pi \to e^+e^-$ successful.
- Second step calculation $K_L \to \gamma\gamma$ in progress.
- Final goal: lattice calculation of $K_L \to \gamma^*\gamma^* \to \mu^+\mu^-$



- Phys.Rev.D 105 (2022) 5, 054518, X. Y. Tuo, X. Feng, **LJ**, and T. Wang RBC-UKQCD
- Phys.Rev.D 105 (2022) 11, 114507, G. Gagliardi, V. Lubicz, G. Martinelli, F. Mazzetti, C. T. Sachrajda, F. Sanfilippo, S. Simula and N. Tantalo RM123
- Good test of the lattice calculation for the kaon form factors also needed for the QED corrections to the kaon leptonic decay (photon can be emitted from kaon).
- Techniques are developed to treat the four (non-interacting) particle final state.
- Unphysical pion mass ~ 300 MeV is used in both calculations.
- Experimental results from the E865 experiment at BNL. hep-ex/0204006, hep-ex/0505001

Channel	RM123	Tuo et al.	ChPT	Experiment
$\mathrm{Br}[K \to \mu\nu_\mu\, e^+e^-]$	$8.26(13)\, 10^{-8}$	$10.59(33)\, 10^{-8}$	$8.25\, 10^{-8}$	$7.93(33)\, 10^{-8}$
$\mathrm{Br}[K \to e\nu_e\, \mu^+\mu^-]$	$0.762(49)\, 10^{-8}$	$0.72(5)\, 10^{-8}$	$0.62\, 10^{-8}$	$1.72(45)\, 10^{-8}$
$\mathrm{Br}[K \to e\nu_e\, e^+e^-]$	$1.95(11)\, 10^{-8}$	$1.77(16)\, 10^{-8}$	$1.75\, 10^{-8}$	$2.91(23)\, 10^{-8}$
$\mathrm{Br}[K \to \mu\nu_\mu\, \mu^+\mu^-]$	$1.178(35)\, 10^{-8}$	$1.45(6)\, 10^{-8}$	$1.10\, 10^{-8}$	–



Thank You!