Prospects for precise predictions of a_μ in the SM

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Snowmass 2021

RPF3 Session
Snowmass CSS
University of Washington
17-26 July 2022

Outline

- Introduction
- Hadronic corrections in comparison
- HVP
- HLbL
- Timeline
- Summary and Outlook

Based on: ``Prospects for precise predictions or a_μ in the SM": arXiv:2203.15810

also: Belle II: arXiv:2207.06307 (Snowmass WP)

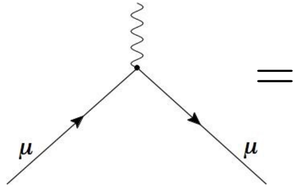
STFC: arXiv:2203.06961,

Chiral Belle arXiv:2205.12847

Anomalous magnetic moment

The magnetic moment of charged leptons (e, μ, τ): $\vec{\mu} = g\frac{e}{2m}\vec{S}$

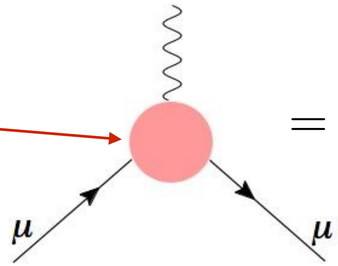
Dirac (leading order): $g = 2$



$$= (-ie)\,\bar{u}(p')\gamma^\mu u(p)$$

Quantum effects (loops):

All SM particles contribute



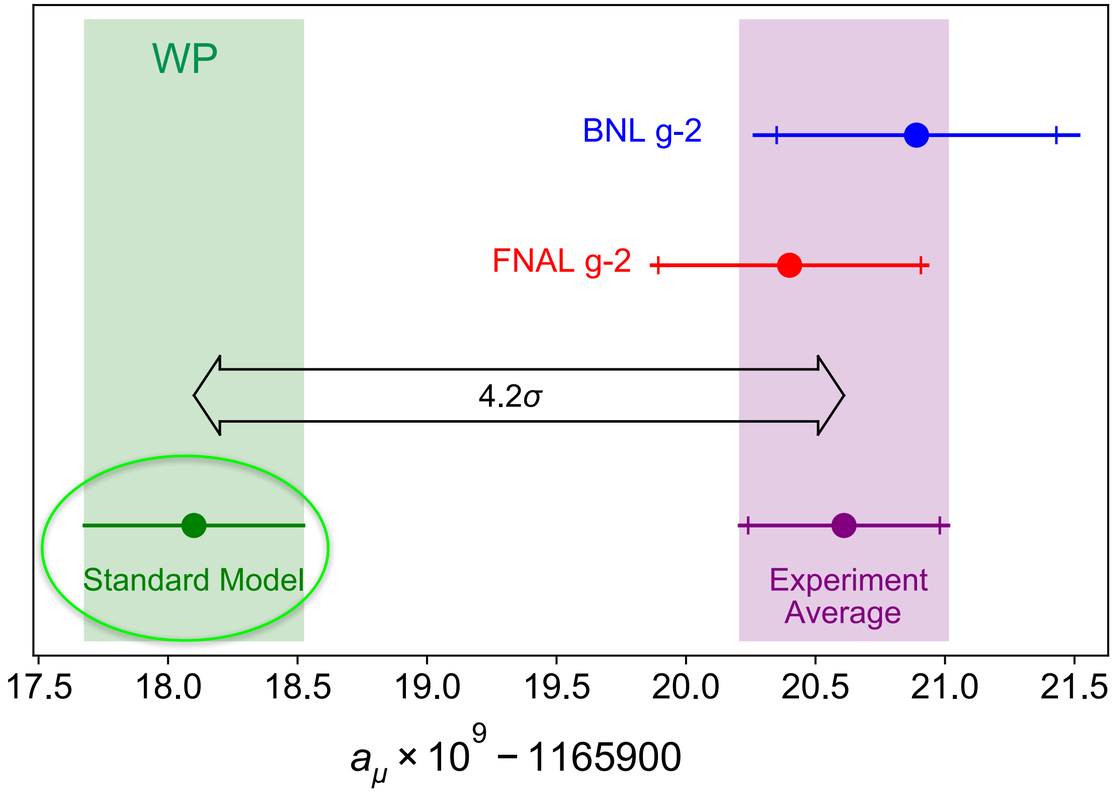
$$= (-i\,e)\,\bar{u}(p')\left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2)\right]u(p)$$

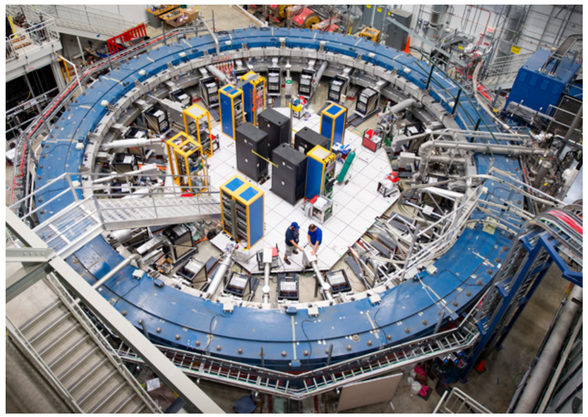
Note: $F_1(0) = 1$ and $g = 2 + 2\,F_2(0)$

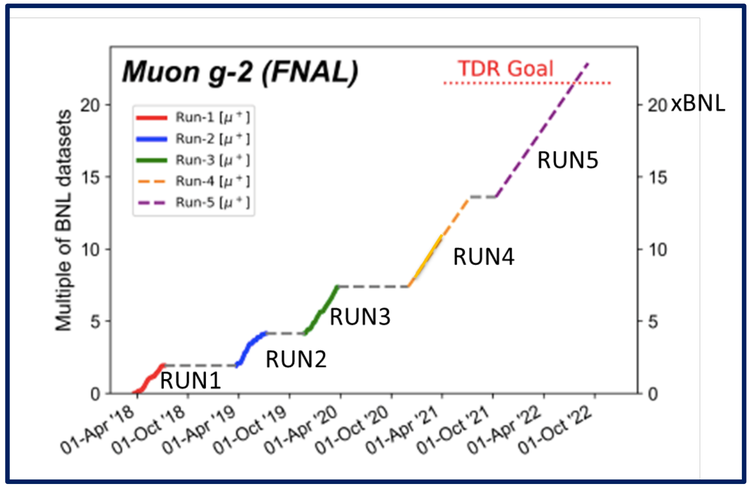
Anomalous magnetic moment: $\boxed{a \equiv \frac{g-2}{2} = F_2(0)}$

Fermilab muon g-2 experiment

- The Fermilab experiment released the measurement result from their run 1 data on 7 April 2021. [B. Abi et al, Phys. Rev. Lett. 124, 141801 (2021)]
- Analysis of runs 2 and 3 is now underway.

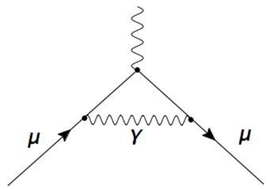
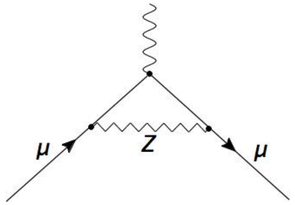
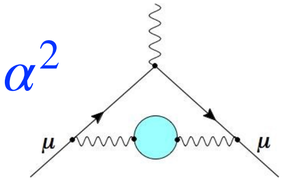
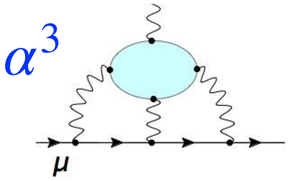






Muon g-2: SM contributions

$$a_\mu = a_\mu(\mathrm{QED}) + a_\mu(\mathrm{EW}) + a_\mu(\mathrm{hadronic})$$

QED		$+\ldots$ (5 loops)	$116\,584\,718.9\,(1) \times 10^{-11}$	0.001 ppm
EW		$+\ldots$ (2 loops)	$153.6\,(1.0) \times 10^{-11}$	0.01 ppm
Hadronic corrections				
HVP	α^2 	$+\ldots$ (NNLO)	$6845\,(40) \times 10^{-11}$ [0.6%]	0.34 ppm
HLbL	α^3 	$+\ldots$ (NLO)	$92\,(18) \times 10^{-11}$ [20%]	0.15 ppm

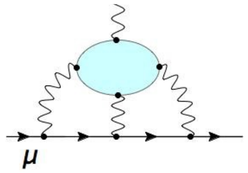
Muon g-2 Theory Initiative

Steering Committee

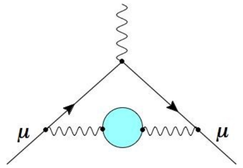
- Gilberto Colangelo (Bern)
- Michel Davier (Orsay) co-chair
- Aida El-Khadra (UIUC & Fermilab) chair
- Martin Hoferichter (Bern)
- Christoph Lehner (Regensburg University & BNL) co-chair
- Laurent Lellouch (Marseille)
- Tsutomu Mibe (KEK)
  J-PARC Muon g-2/EDM experiment
- Lee Roberts (Boston)
  Fermilab Muon g-2 experiment
- Thomas Teubner (Liverpool)
- Hartmut Wittig (Mainz)

https://muon-gm2-theory.illinois.edu

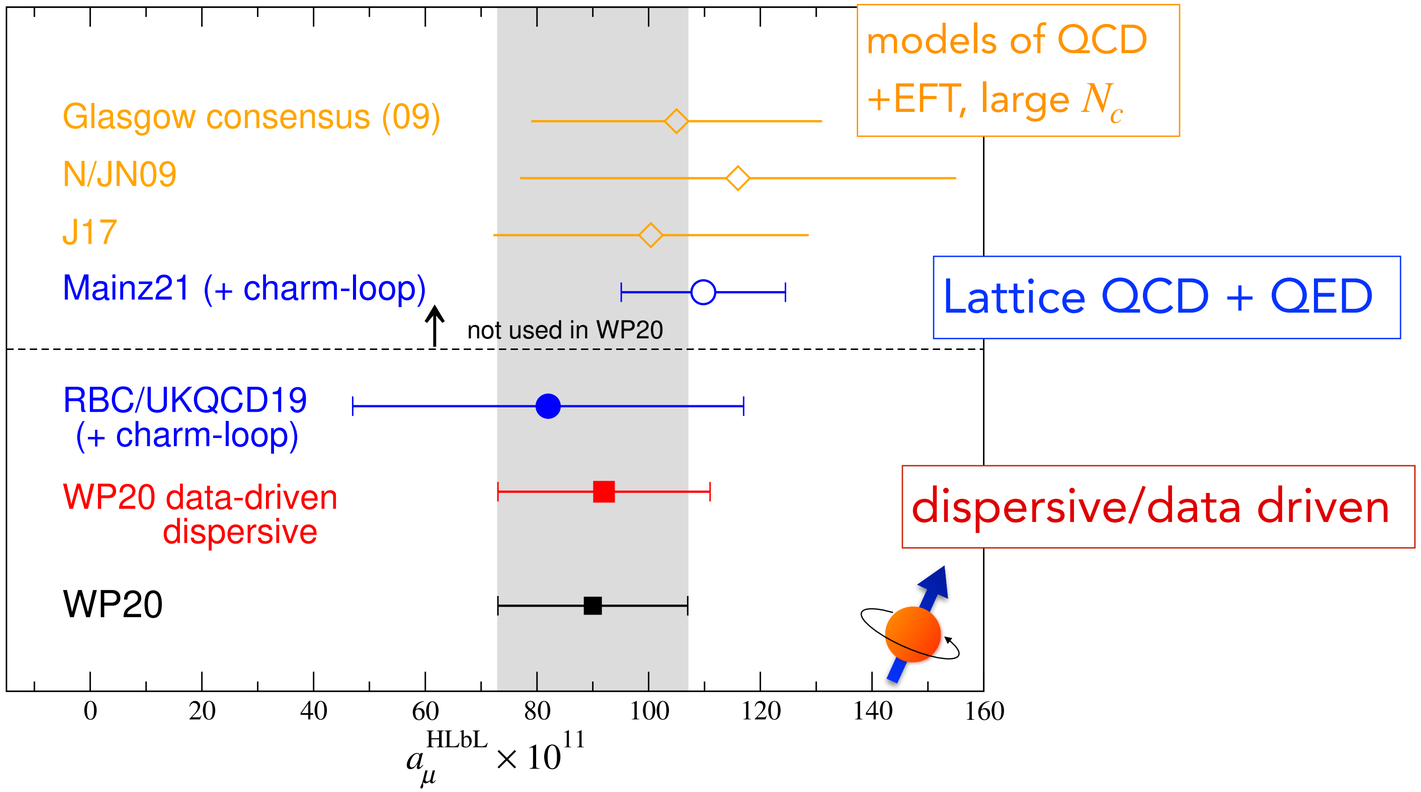
- Maximize the impact of the Fermilab and J-PARC experiments
  ➠ quantify and reduce the theoretical uncertainties on the hadronic corrections
- summarize the theory status and assess reliability of uncertainty estimates
- organize workshops to bring the different communities together:
  First plenary workshop @ Fermilab: 3-6 June 2017
  HVP workshop @ KEK: 12-14 February 2018
  HLbL workshop @ U Connecticut: 12-14 March 2018
  Second plenary workshop @ HIM (Mainz): 18-22 June 2018
  Third plenary workshop @ INT (Seattle): 9-13 September 2019
  Lattice HVP at high precision workshop (virtual): 16-20 November 2020
  Fourth plenary workshop @ KEK (virtual): 28 June - 02 July 2021
  Fifth plenary workshop @ Higgs Centre (Edinburgh): 5-9 September 2022
- 1st White Paper published in 2020 (132 authors, 82 institutions, 21 countries) [T. Aoyama et al, arXiv:2006.04822, Phys. Repts. 887 (2020) 1-166.]
- 2nd White Paper: First discussions @ KEK meeting in June 2021
  expect to develop a concrete plan @ Higgs Centre workshop



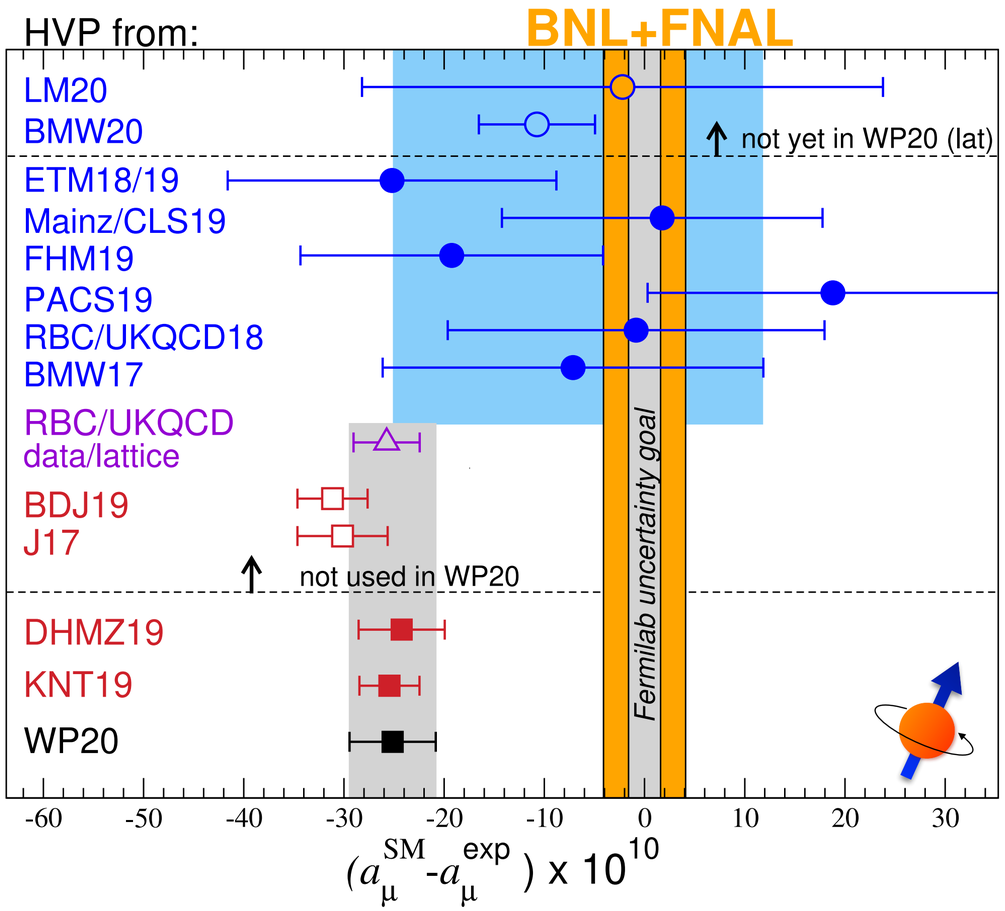
Hadronic Corrections: Comparisons

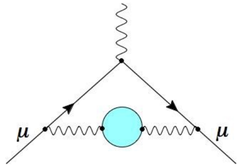


$$a_\mu^{\rm HLbL}$$

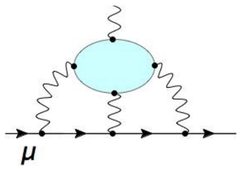


$$a_\mu^{\rm SM} \quad a_\mu^{\rm HVP} + \left[a_\mu^{\rm QED} + a_\mu^{\rm Weak} + a_\mu^{\rm HLbL}\right]$$





Hadronic Corrections

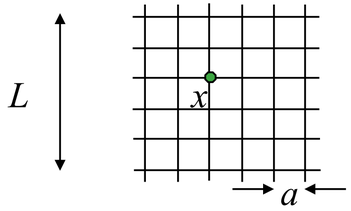


Two different, independent strategies:

1. For HVP, use dispersion relations to rewrite integral in terms of hadronic cross section:

Many experiments (over 20+ years) have measured the e^+e^-cross sections for the different channels over the needed energy range with increasing precision.
New dispersive approach developed for HLbL

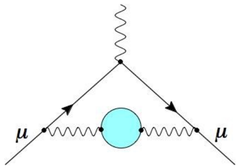
2. Direct calculation using Euclidean Lattice QCD



Approximations:
- discrete space-time (spacing a)
- finite spatial volume (L), and time extent (T)
- …

Integrals are evaluated numerically using Monte Carlo methods.

- *ab-initio* method to quantify QCD effects
- already used for simple hadronic quantities with high precision
- requires large-scale computational resources
- allows for entirely SM theory based evaluations



HVP: data-driven

In 2020 WP:

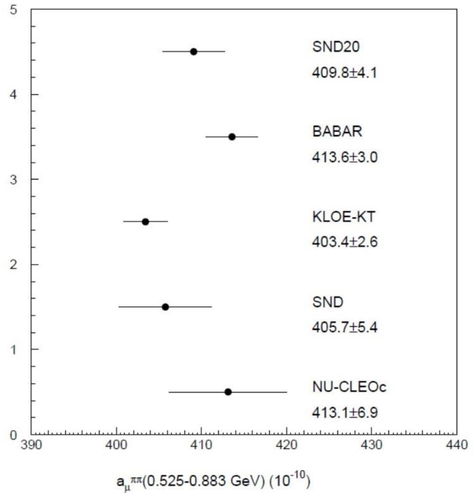
Conservative merging procedure to obtain a realistic assessment of the underlying uncertainties:

- account for tensions between data sets
- account for differences in methodologies for compilation of experimental inputs
- include correlations between systematic errors
- cross checks from unitarity & analyticity constraints
  [Colangelo et al, 2018; Anantharayan et al, 2018; Davier et al, 2019; Hoferichter et al, 2019]
- Full NLO radiative corrections [Campanario et al, 2019

$$a_\mu^{\mathrm{HVP,LO}} = 693.1\,(2.8)_{\mathrm{exp}}\,(0.7)_{\mathrm{DV+pQCD}}\,(2.8)_{\mathrm{BaBar-KLOE}} \times 10^{-10}$$

$$= 693.1\,(4.0) \times 10^{-10}$$

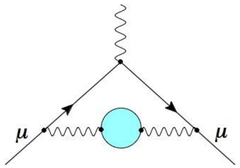
[M. Davier @ KEK workshop]



Ongoing work:

- BaBar: new analysis of large (7×) data set in $\pi\pi$ channel (1-2 years), also $\pi\pi\pi$, other channels
- SND: new results for $\pi\pi$ channel, other channels in progress
- CMD-3: ongoing analyses for $\pi\pi$ and other channels
- BESIII: new results in 2021 for $\pi\pi$ channel, continued analysis also for $\pi\pi\pi$, other channels
- Belle II: arXiv:2207.06307 (Snowmass WP)
  Better statistics than BaBar or KLOE; similar or better systematics for low-energy cross sections
- STFC: arXiv:2203.06961
- Need blind analyses to resolve the tensions (esp. for $\pi\pi$ channel)
- Developing NNLO Monte Carlo generators (STRONG 2020 workshop next week https://agenda.infn.it/event/28089/)

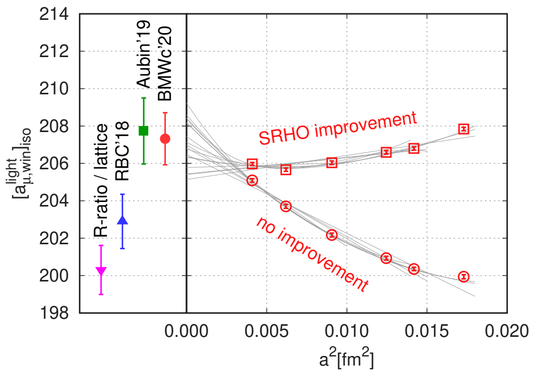
Data-driven evaluations of HVP with $\sim 0.3\,\%$ feasible by ~2025, if tensions between experiments are resolved.



HVP: lattice

In 2020 WP:

- Lattice HVP average at $2.6\,\%$ total uncertainty: $a_\mu^{\mathrm{HVP,LO}} = 711.6\,(18.4) \times 10^{10}$
- BMW 20 (published in 2021)
  first LQCD calculation with sub-percent ($0.8\,\%$) error
  in tension with data-driven HVP (2.1σ)
- Further tensions for intermediate window



-3.7σ tension with data-driven evaluation

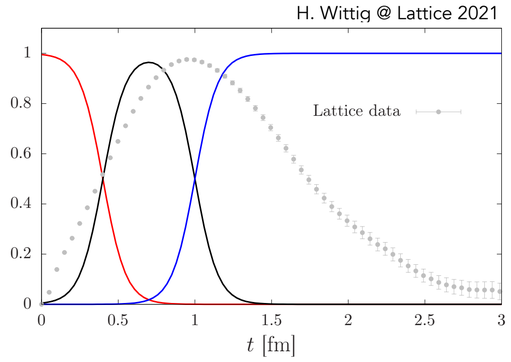
-2.2σ tension with RBC/UKQCD18

$$a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\, \tilde{w}(t)\, C(t)$$

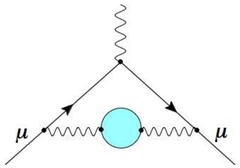
- Use windows in Euclidean time to consider the different time regions separately. [T. Blum et al, arXiv:1801.07224, 2018 PRL]

Short Distance (SD)	$t : 0 \to t_0$
Intermediate (W)	$t : t_0 \to t_1$
Long Distance (LD)	$t : t_1 \to \infty$

$$t_0 = 0.4\,\mathrm{fm}, t_1 = 1.0\,\mathrm{fm}$$



- disentangle systematics/statistics from long distance/FV and discretization effects
- intermediate window: easy to compute in lattice QCD & using disperse approach:
- Internal cross check:
  Compute each window separately (in continuum, infinite volume limits,…) and combine: $a_\mu = a_\mu^{\mathrm{SD}} + a_\mu^{\mathrm{W}} + a_\mu^{\mathrm{LD}}$



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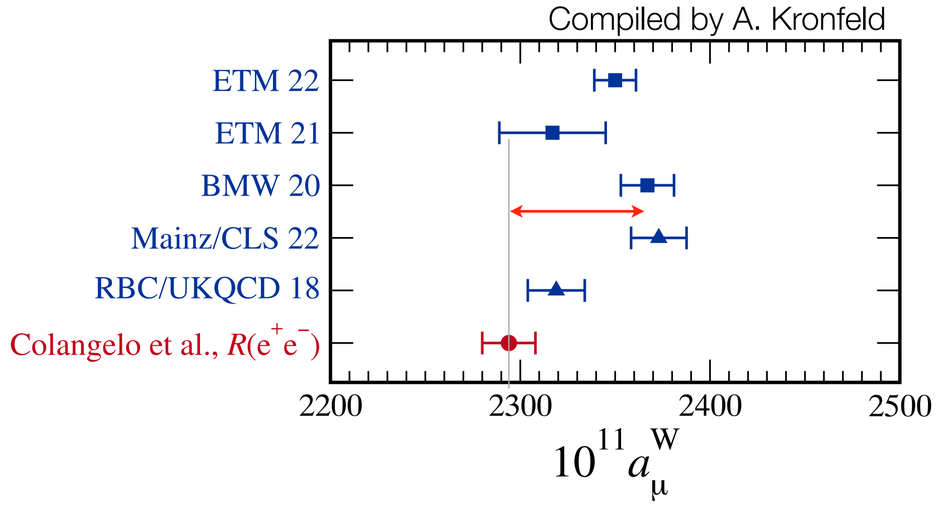


-3.7σ tension with data-driven evaluation

-2.2σ tension with RBC/UKQCD18

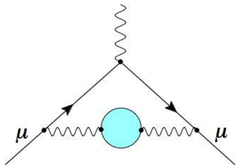
Proposals for computing more windows:

- Use linear combinations of finer windows to locate the tension (if it persists) in $\sqrt{s}$
  [Colangelo et al, arXiv:12963]

- Use larger windows, excluding the long-distance region $t \gtrsim 2\,\mathrm{fm}$ to maximize the significance of any tension
  [Davies at at, arXiv:2207.04765]

New results for intermediate windows:

- χQCD: arXiv:2204.01280
- Aubin et al: arXiv:2204.12256
- Mainz: arXiv:2206.06582
- ETM: arXiv:2206.15084 (ETM)



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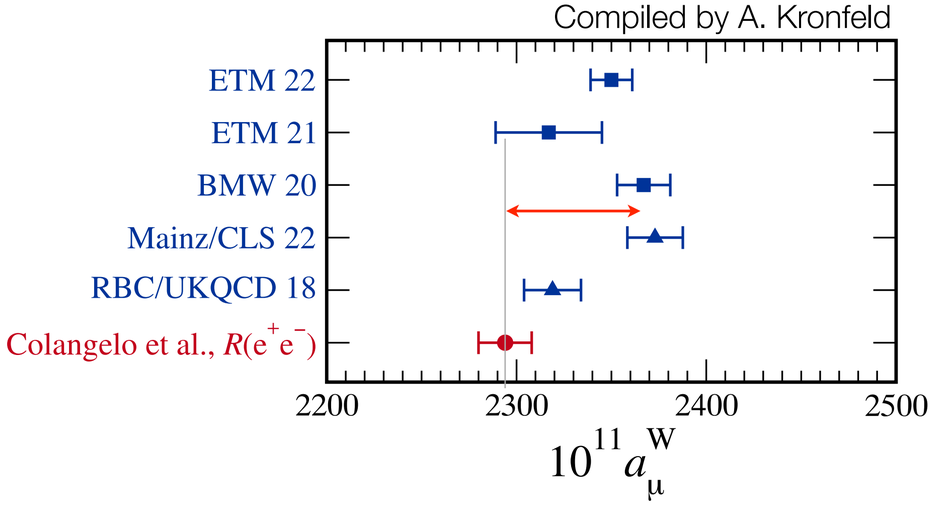


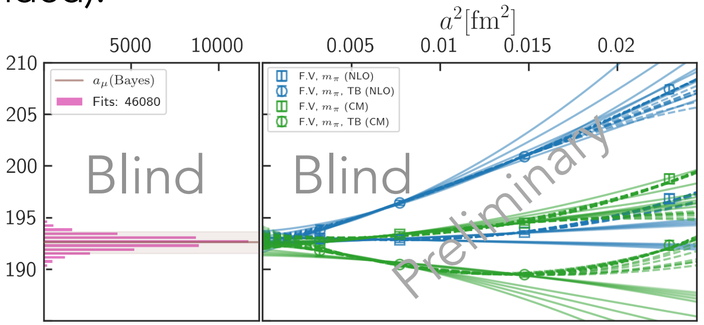
-3.7σ tension with data-driven evaluation

-2.2σ tension with RBC/UKQCD18

Ongoing work:

- Expect new results from RBC/UKQCD and FNAL/HPQCD/MILC in coming months (both are blinded):

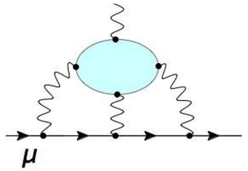
S. Lahert
(FNAL/HPQCD/MILC)



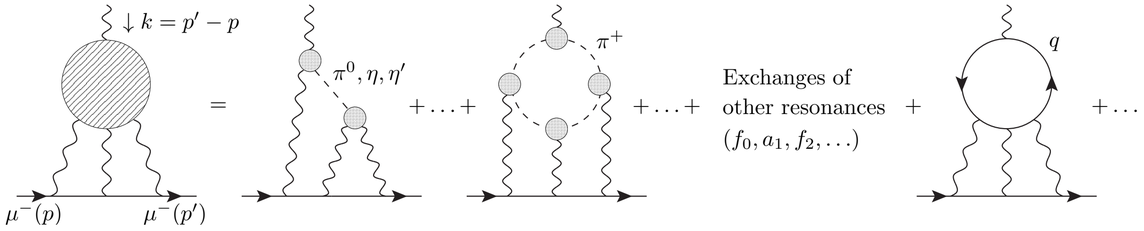
For total HVP:

- Including $\pi\pi$ states for refined long-distance computation
  (Mainz, RBC/UKQCD, FNAL/MILC)
- Developing method average for lattice HVP — started at KEK workshop (June 2021), based on detailed comparisons
  - list of sub quantities (and their definitions)
  - common prescription for separating QCD & QED
  - quality criteria for inclusion
- Most groups plan to include smaller lattice spacings to test continuum extrapolations (needs adequate computational resources)

If results are consistent, Lattice HVP (average) with $\lesssim 0.5\,\%$ errors feasible by 2025



Hadronic Light-by-light

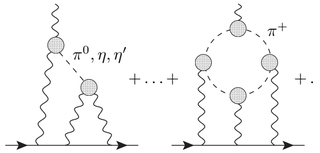


Dispersive approach:

[Colangelo at al, 2014; Pauk & Vanderhaegen 2014; …]

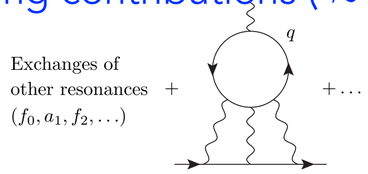
- model independent
- significantly more complicated than for HVP
- provides a framework for data-driven evaluations
- can also use lattice results as inputs

Dominant contributions ($\approx 75\,\%$ of total):



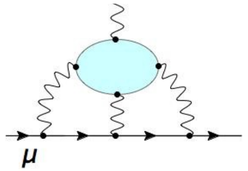
- Well quantified with $\approx 6\,\%$ uncertainty
- η, η' pole contributions: Canterbury approximants only
- Ongoing work: consolidation of η, η' pole contributions using disp. relations and LQCD

Subleading contributions ($\approx 25\,\%$ of total):



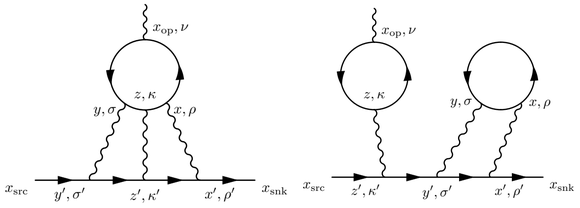
- Not yet well known
  - ➠ dominant contribution to total uncertainty
- Ongoing work:
  - Implementation of short-distance constraints (now at 2-loop)
  - DR implementation for axial vector contributions
  - BESIII ramping up $\gamma^{(*)}\gamma^*$ program

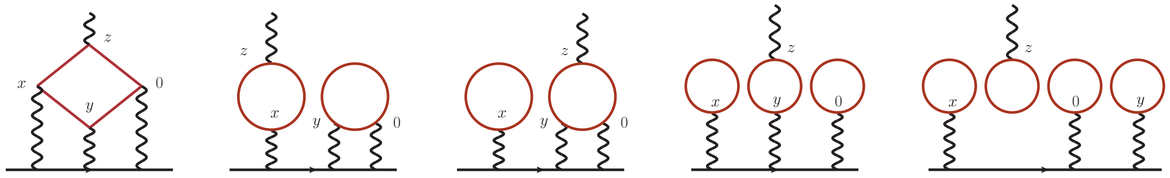
Dispersive, data-driven evaluation of HLbL with $\leq 10\,\%$ total uncertainty feasible by ~2025.



Hadronic Light-by-light

Lattice QCD+QED: Two independent and complete direct calculations of a_μ^{HLbL}





- RBC/UKQCD
  [T. Blum et al, arXiv:1610.04603, 2016 PRL; arXiv:1911.08123, 2020 PRL]
- QCD + $\mathrm{QED_L}$ (finite volume)

  DWF ensembles at/near phys mass,
  $a \approx 0.08 - 0.2\,\mathrm{fm}$, $L \sim 4.5 - 9.3\,\mathrm{fm}$

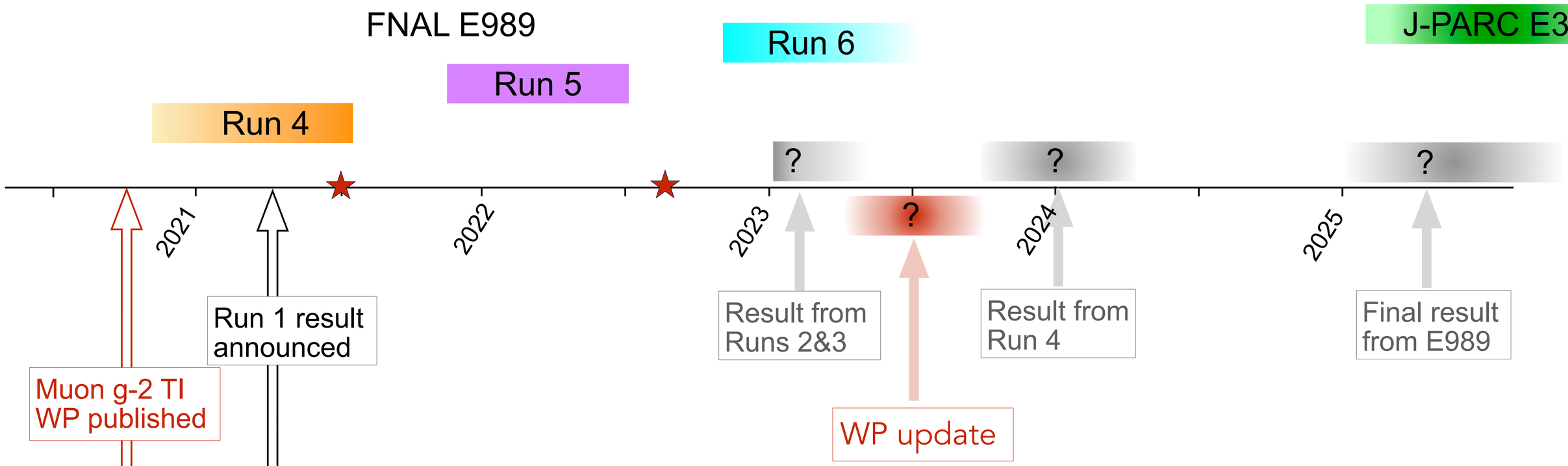
- Mainz group
  [E. Chao et al, arXiv:2104.02632]
- QCD + QED (infinite volume & continuum)

  CLS (2+1 Wilson-clover) ensembles
  $m_\pi \sim 200 - 430\,\mathrm{MeV}$, $a \approx 0.05 - 0.1\,\mathrm{fm}$, $m_\pi L > 4$

- Cross checks between RBC/UKQCD & Mainz approaches in White Paper at unphysical pion mass
- Both groups will continue to improve their calculations, adding more statistics, lattice spacings, physical mass ensemble (Mainz)

Lattice HLbL results with $10\,\%$ total uncertainty feasible by ~2025

Timeline

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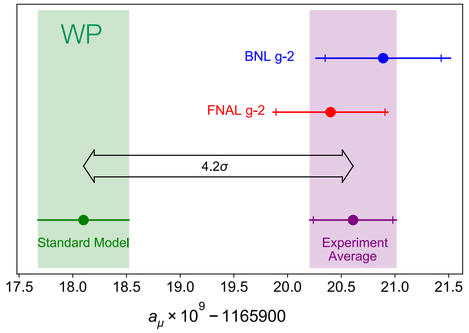
The anomalous magnetic moment of the muon in the Standard Model



Theory Initiative:

- ☆ ongoing activities: develop method average for Lattice HVP
- ☆ plan to update WP: new lattice HVP results (when available)
- ☆ main update with all available results ~ 2023

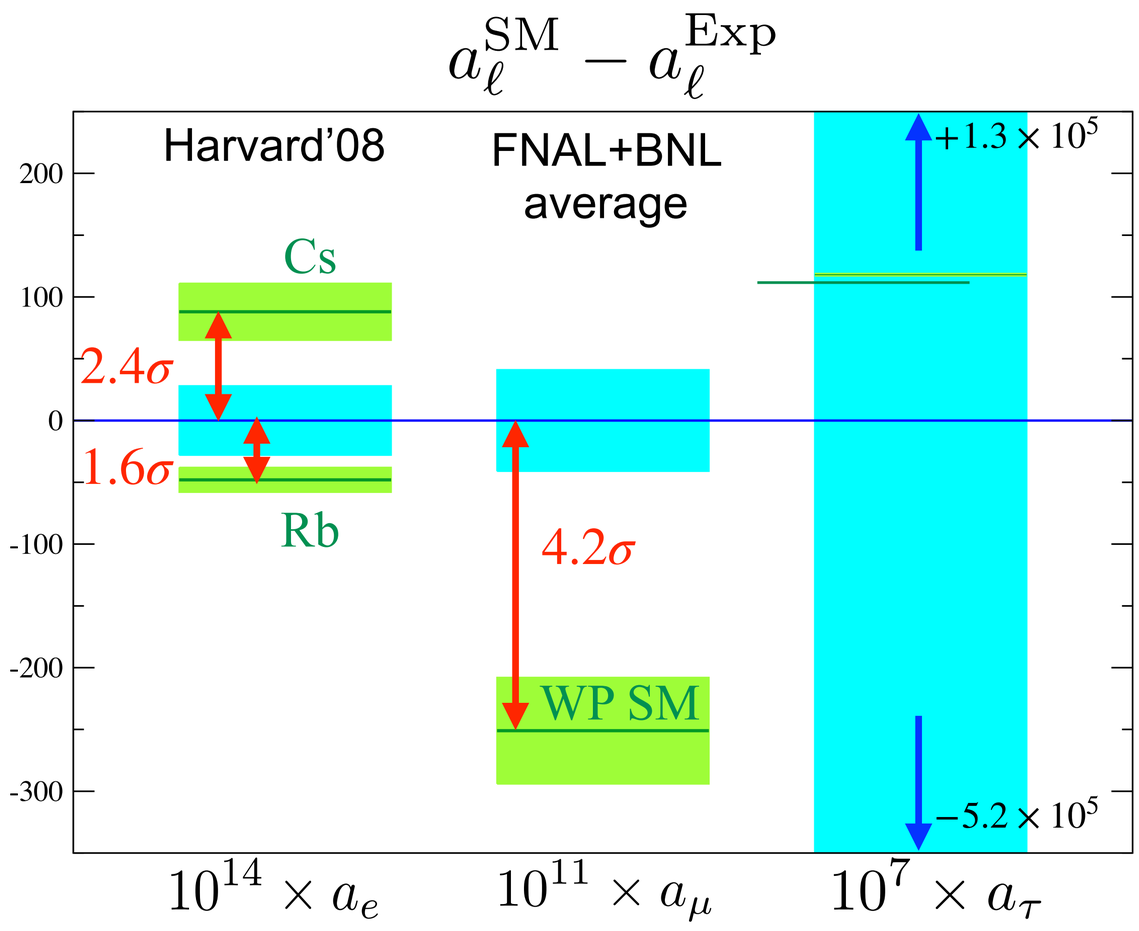
★ TI workshops: Jun 2021 @ KEK (virtual)
Sep 2022 @ Higgscentre

Lepton moments summary

Sensitivity to heavy new physics:

$$a_\ell^{\rm NP} \sim \frac{m_\ell^2}{\Lambda^2}$$

$$(m_\mu/m_e)^2 \sim 4\times 10^4$$



Chiral Belle arXiv:2205.12847

- ☆ use polarized e^- beam
- ☆ with $40ab^{-1}$ measurement of a_τ at 10^{-5} feasible
- ☆ with more statistics measurement at 10^{-6} possible

Cs: α from Berkeley group [Parker et al, Science 360, 6385 (2018)]

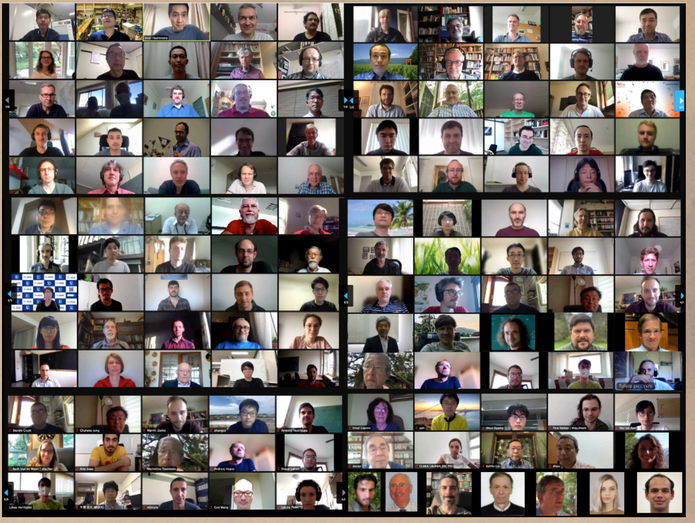
Rb: α from Paris group [Morel et al, Nature 588, 61–65(2020)]

Summary

- Theory Initiative:
  - Concrete plans for WP updates @ next workshop (update on lattice HVP if new results are available)
  - method averages for lattice HVP, HLbL
- Programs and plans in place to improve by 2025:
  - data-driven HVP $\sim 0.3\,\%$ (if tension between KLOE and BaBar is resolved)
  - lattice HVP $\lesssim 0.5\,\%$ (if tension between BMW and RBC/UKQCD is resolved)
  - dispersive HLbL and lattice HLbL: $\sim 10\,\%$
- **If tensions between data-driven HVP and lattice HVP are resolved,** SM prediction will likely match precision goal of the Fermilab experiment.
- If not, will need detailed comparisons, explore connections between HVP, $\sigma(e^+e^-)$, $\Delta\alpha$, global EW fits.
  - MUonE (space-like momentum measurement of $\Delta\alpha$) will provide more information/cross checks.

Outlook

- Experimental program beyond 2025:
  - J-PARC: Muon g-2/EDM, MuSEUM, COMET, DeeMe
  - Fermilab: Mu2e, future muon campus experiments?
  - PSI: MEG, MuMass,Mu3e, MuEDM, MUSE, CREMA, HIMB?
  - Belle II, MUonE, BESIII, Novosibirsk
  - STCF (?), Chiral Belle (?)
- Data-driven/dispersive program beyond 2025:
  - further uncertainty reductions from new measurements
    STCF, τ decay data, …
  - development of NNLO MC generators
  - for HLbL, improved experimental/lattice inputs together with further development of dispersive approach
- Lattice QCD program beyond 2025:
  - access to future computational resources (coming Exascale) will enable improvements of all errors (statistical and systematic)
  - concurrent development of better methods and algorithms (gauge-field sampling, noise reduction) will accelerate progress
    ➠ continued coordination by Muon g-2 Theory Initiative



Fermilab

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Thank you!

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HIM HELMHOLTZ Helmholtz-Institut Mainz

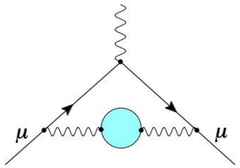
SFB 1044 THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

PRISMA

UCONN DEPARTMENT OF PHYSICS

JG|U JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Appendix



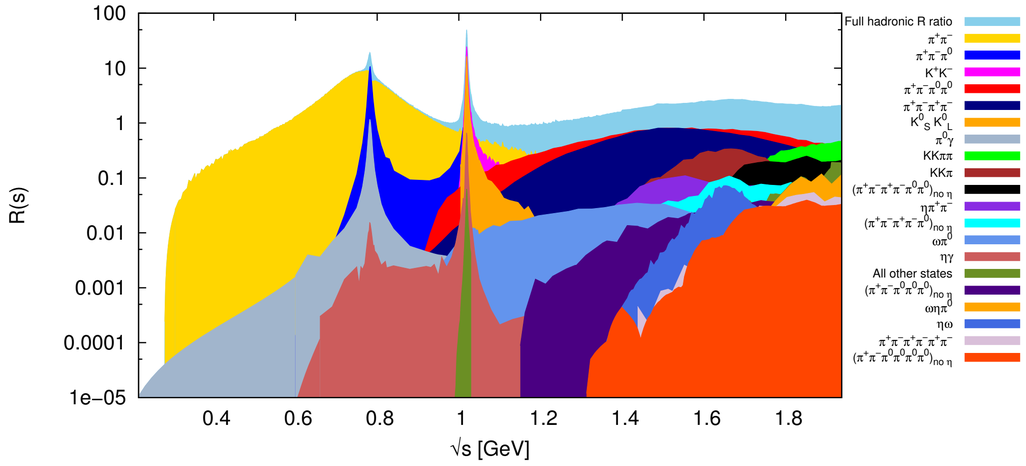
HVP: data-driven

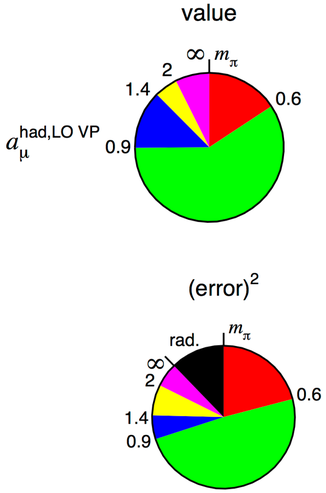
Z. Zhang for DHMZ @ INT g-2 workshop

[M. Davier et al, arXiv:1908.00921]

Channel	$a_\mu^{\text{had, LO}}[10^{-10}]$
$\pi^0\gamma$	$4.29 \pm 0.06 \pm 0.04 \pm 0.07$
$\eta\gamma$	$0.65 \pm 0.02 \pm 0.01 \pm 0.01$
$\pi^+\pi^-$	$507.80 \pm 0.83 \pm 3.19 \pm 0.60$
$\pi^+\pi^-\pi^0$	$46.20 \pm 0.40 \pm 1.10 \pm 0.86$
$2\pi^+2\pi^-$	$13.68 \pm 0.03 \pm 0.27 \pm 0.14$
$\pi^+\pi^-2\pi^0$	$18.03 \pm 0.06 \pm 0.48 \pm 0.26$
$2\pi^+2\pi^-\pi^0$ (η excl.)	$0.69 \pm 0.04 \pm 0.06 \pm 0.03$
$\pi^+\pi^-3\pi^0$ (η excl.)	$0.49 \pm 0.03 \pm 0.09 \pm 0.00$
$3\pi^+3\pi^-$	$0.11 \pm 0.00 \pm 0.01 \pm 0.00$
$2\pi^+2\pi^-2\pi^0$ (η excl.)	$0.71 \pm 0.06 \pm 0.07 \pm 0.14$
$\pi^+\pi^-4\pi^0$ (η excl., isospin)	$0.08 \pm 0.01 \pm 0.08 \pm 0.00$
$\eta\pi^+\pi^-$	$1.19 \pm 0.02 \pm 0.04 \pm 0.02$
$\eta\omega$	$0.35 \pm 0.01 \pm 0.02 \pm 0.01$
$\eta\pi^+\pi^-\pi^0$(non-ω, ϕ)	$0.34 \pm 0.03 \pm 0.03 \pm 0.04$
$\eta 2\pi^+2\pi^-$	$0.02 \pm 0.01 \pm 0.00 \pm 0.00$
$\omega\eta\pi^0$	$0.06 \pm 0.01 \pm 0.01 \pm 0.00$
$\omega\pi^0$ ($\omega \to \pi^0\gamma$)	$0.94 \pm 0.01 \pm 0.03 \pm 0.00$
$\omega(\pi\pi)^0$ ($\omega \to \pi^0\gamma$)	$0.07 \pm 0.00 \pm 0.00 \pm 0.00$
ω (non-$3\pi, \pi\gamma, \eta\gamma$)	$0.04 \pm 0.00 \pm 0.00 \pm 0.00$
K^+K^-	$23.08 \pm 0.20 \pm 0.33 \pm 0.21$
K_SK_L	$12.82 \pm 0.06 \pm 0.18 \pm 0.15$
ϕ (non-$K\overline{K}, 3\pi, \pi\gamma, \eta\gamma$)	$0.05 \pm 0.00 \pm 0.00 \pm 0.00$
$K\overline{K}\pi$	$2.45 \pm 0.05 \pm 0.10 \pm 0.06$
$K\overline{K}2\pi$	$0.85 \pm 0.02 \pm 0.05 \pm 0.01$
$K\overline{K}3\pi$ (estimate)	$-0.02 \pm 0.01 \pm 0.01 \pm 0.00$
$\eta\phi$	$0.33 \pm 0.01 \pm 0.01 \pm 0.00$
$\eta K\overline{K}$ (non-ϕ)	$0.01 \pm 0.01 \pm 0.01 \pm 0.00$
$\omega K\overline{K}$ ($\omega \to \pi^0\gamma$)	$0.01 \pm 0.00 \pm 0.00 \pm 0.00$
$\omega 3\pi$ ($\omega \to \pi^0\gamma$)	$0.06 \pm 0.01 \pm 0.01 \pm 0.01$
7π ($3\pi^+3\pi^-\pi^0$ + estimate)	$0.02 \pm 0.00 \pm 0.01 \pm 0.00$
J/ψ (BW integral)	6.28 ± 0.07
$\psi(2S)$ (BW integral)	1.57 ± 0.03
R data $[3.7 - 5.0]$ GeV	$7.29 \pm 0.05 \pm 0.30 \pm 0.00$
R_{QCD} $[1.8 - 3.7\text{ GeV}]_{uds}$	$33.45 \pm 0.28 \pm 0.65_{\text{dual}}$
R_{QCD} $[5.0 - 9.3\text{ GeV}]_{udsc}$	6.86 ± 0.04
R_{QCD} $[9.3 - 12.0\text{ GeV}]_{udscb}$	1.21 ± 0.01
R_{QCD} $[12.0 - 40.0\text{ GeV}]_{udscb}$	1.64 ± 0.00
R_{QCD} $[> 40.0\text{ GeV}]_{udscb}$	0.16 ± 0.00
R_{QCD} $[> 40.0\text{ GeV}]_t$	0.00 ± 0.00
Sum	$693.9 \pm 1.0 \pm 3.4 \pm 1.6 \pm 0.1_\psi \pm 0.7_{\text{QCD}}$

[A. Keshavarzi et al, arXiv:1802.02995]

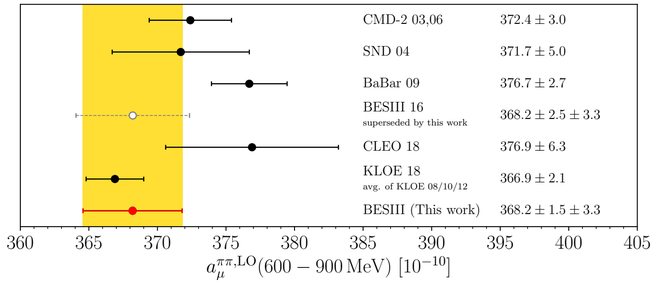




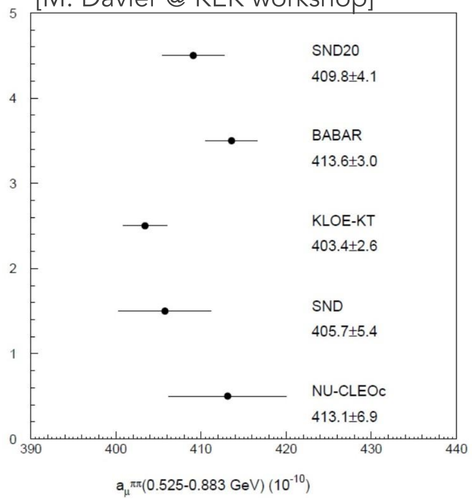
Tensions between BaBar and KLOE data sets:

- Cross checks using analyticity and unitarity relating pion form factor to $\pi\pi$ scattering
- Combinations of data sets affected by tensions
  ➠ conservative merging procedure

[M. Ablikim et al (BES III), arXiv:2009.05011]

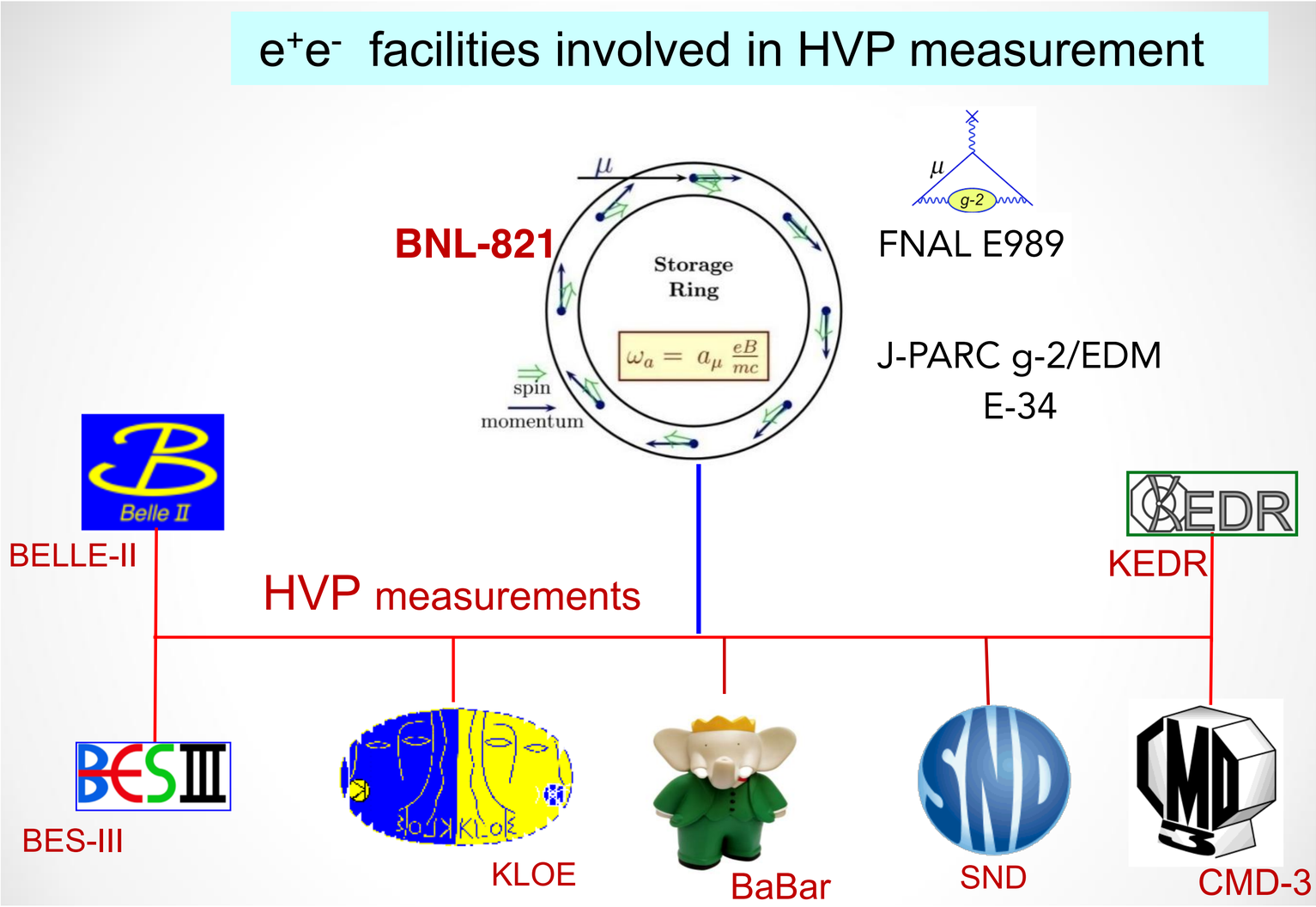


[M. Davier @ KEK workshop]



Experimental Inputs to HVP

S. Serednyakov (for SND) @ HVP KEK workshop



Connections

$$\sigma(e^+e^- \to \text{hadrons}) \quad \Leftrightarrow \quad a_\mu^{\text{HVP}} \quad \Leftrightarrow \quad \Delta\alpha_{\text{had}}(M_Z^2)$$

- $\Delta\alpha_{\text{had}}(M_Z^2)$ also depends on the hadronic vacuum polarization function, and can be written as an integral over $\sigma(e^+e^- \to \text{hadrons})$, but weighted towards higher energies.
- a shift in a_μ^{HVP} also changes $\Delta\alpha_{\text{had}}(M_Z^2)$: ➠ EW fits
  [Passera, et al, 2008, Crivellin et al 2020, Keshavarsi et al 2020, Malaescu & Scott 2020]
  If the shift in a_μ^{HVP} is in the low-energy region ($\lesssim 1\,\text{GeV}$), the impact on $\Delta\alpha_{\text{had}}(M_Z^2)$ and EW fits is small.
- A shift in a_μ^{HVP} from low ($\lesssim 2\,\text{GeV}$) energies
  ➠ $\sigma(e^+e^- \to \pi\pi)$
  must satisfy unitarity & analyticity constraints ➠ $F_\pi^V(s)$
  can be tested with lattice calculations
  [Colangelo, Hoferichter, Stoffer, arXiv:2010.07943]

Connections

$$\sigma(e^+e^- \to \text{hadrons}) \quad \Leftrightarrow \quad a_\mu^{\text{HVP}} \quad \Leftrightarrow \quad \Delta\alpha_{\text{had}}(M_Z^2)$$

Martin Hoferichter @ Lattice HVP workshop

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Hadronic running of α and global EW fit

	e^+e^- KNT, DHMZ	EW fit HEPFit	EW fit GFitter	guess based on BMWc
$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2) \times 10^4$	276.1(1.1)	270.2(3.0)	271.6(3.9)	277.8(1.3)
difference to e^+e^-		-1.8σ	-1.1σ	$+1.0\sigma$

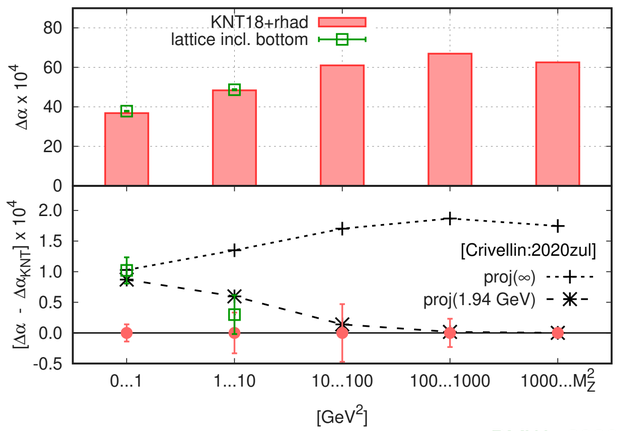
- **Time-like formulation**:

$$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2) = \frac{\alpha M_Z^2}{3\pi} P \int_{s_{\text{thr}}}^{\infty} ds \frac{R_{\text{had}}(s)}{s(M_Z^2 - s)}$$

- **Space-like formulation**:

$$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2) = \frac{\alpha}{\pi}\hat{\Pi}(-M_Z^2) + \frac{\alpha}{\pi}\left(\hat{\Pi}(M_Z^2) - \hat{\Pi}(-M_Z^2)\right)$$

- Global EW fit
  - Difference between HEPFit and GFitter implementation mainly treatment of M_W
  - Pull goes into **opposite direction**



BMWc 2020

More in talks by M. Passera, B. Malaescu (phenomenology) and K. Miura, T. San José (lattice)

Connections

$$\sigma(e^+e^- \to \text{hadrons}) \quad \Leftrightarrow \quad a_\mu^{\text{HVP}} \quad \Leftrightarrow \quad \Delta\alpha_{\text{had}}(M_Z^2)$$

- $\Delta\alpha_{\text{had}}(M_Z^2)$ also depends on the hadronic vacuum polarization function, and can be written as an integral over $\sigma(e^+e^- \to \text{hadrons})$, but weighted towards higher energies.
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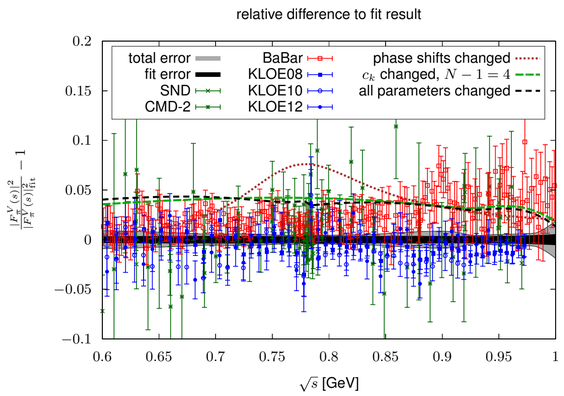
Peter Stoffer @ Lattice HVP workshop

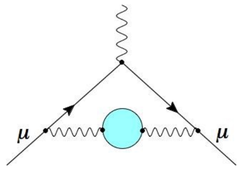
Constraints on the two-pion contribution to HVP — arXiv:2010.07943 [hep-ph]

Modifying $a_\mu^{\pi\pi}|_{\leq 1\,\text{GeV}}$

- "low-energy" scenario: local changes in cross section of $\sim 8\%$ **around ρ**
- "high-energy" scenario: impact on **pion charge radius** and space-like VFF $\Rightarrow$ chance for **independent lattice-QCD checks**
- requires **factor ~ 3 improvement** over χQCD result: $\langle r_\pi^2 \rangle = 0.433(9)(13)\,\text{fm}^2$ → arXiv:2006.05431 [hep-ph]



- Can new physics hide in the low-energy $\sigma(e^+e^- \to \pi\pi)$ cross section? ➠ No [Luzio, et al, arXiv:2112.08312]
- Neutral, long-lived hadrons, heretofore undetected? [Farrar, arXiv:2206.13460]

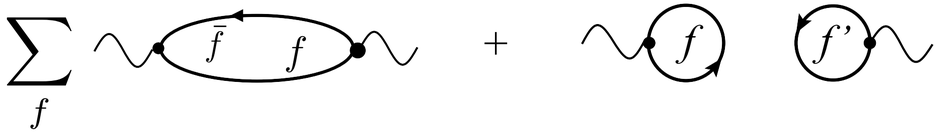


Lattice HVP: Introduction

Calculate $a_\mu^{\rm HVP}$ in Lattice QCD:

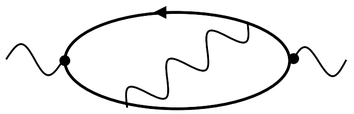
$$a_\mu^{\rm HVP,LO} = \sum_f a_{\mu,f}^{\rm HVP,LO} + a_{\mu,\rm disc}^{\rm HVP,LO}$$

- Separate into connected for each quark flavor + disconnected contributions
  (gluon and sea-quark background not shown in diagrams)
  Note: almost always $m_u = m_d$



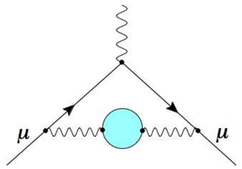
$f = ud, s, c, b$

- need to add QED and strong isospin breaking ($\sim m_u - m_d$) corrections:



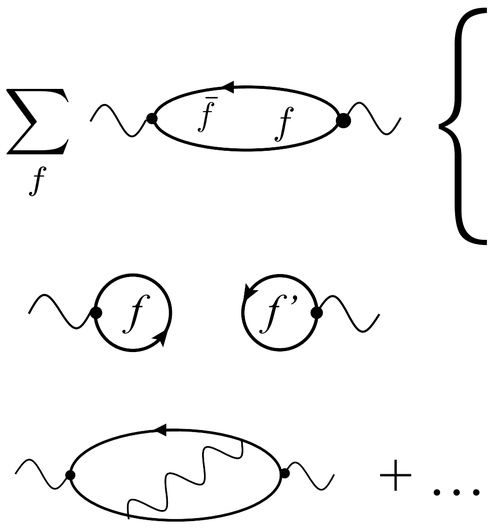
$+ \ldots$

- either perturbatively on isospin symmetric QCD background
- or by using QCD + QED ensembles with $m_u \neq m_d$



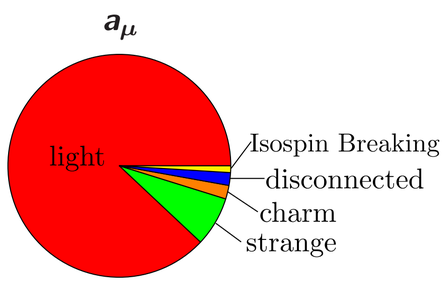
Lattice HVP: Introduction

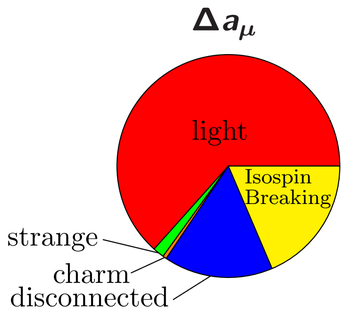
- Target: ~ 0.2% total error



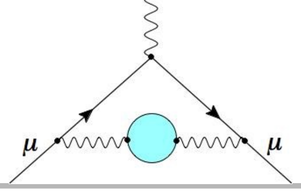
- light-quark connected contribution:
  $a_\mu^{\mathrm{HVP,LO}}(ud)$ ~90% of total
- *s*,*c*,*b*-quark contributions
  $a_\mu^{\mathrm{HVP,LO}}(s,c,b)$ ~8%, 2%, 0.05% of total
- disconnected contribution:
  $a_{\mu,\mathrm{disc}}^{\mathrm{HVP,LO}}$ ~2% of total
- Isospinbreaking (QED + $m_u \neq m_d$) corrections:
  $\delta a_\mu^{\mathrm{HVP,LO}}$ ~1% of total

V. Gülpers @ Lattice HVP workshop



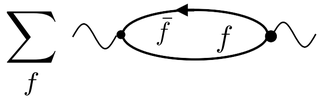


$$a_\mu^{\mathrm{HVP,LO}} = a_\mu^{\mathrm{HVP,LO}}(ud) + a_\mu^{\mathrm{HVP,LO}}(s) + a_\mu^{\mathrm{HVP,LO}}(c) + a_{\mu,\mathrm{disc}}^{\mathrm{HVP,LO}} + \delta a_\mu^{\mathrm{HVP,LO}}$$

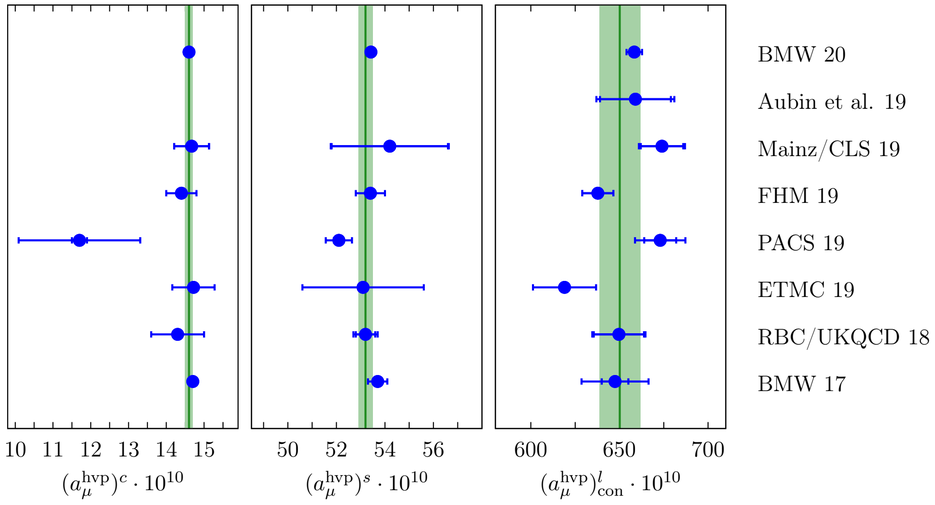


Lattice HVP: Isospin corrections

H. Wittig @ Lattice HVP workshop

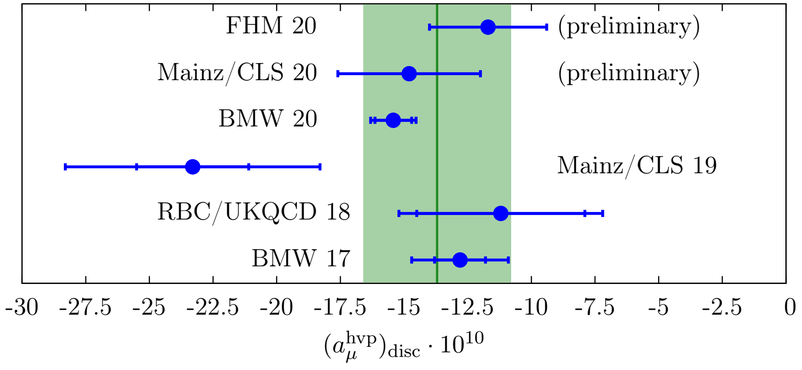


- Charm, strange contributions already well determined.
- Mild tensions for light contribution

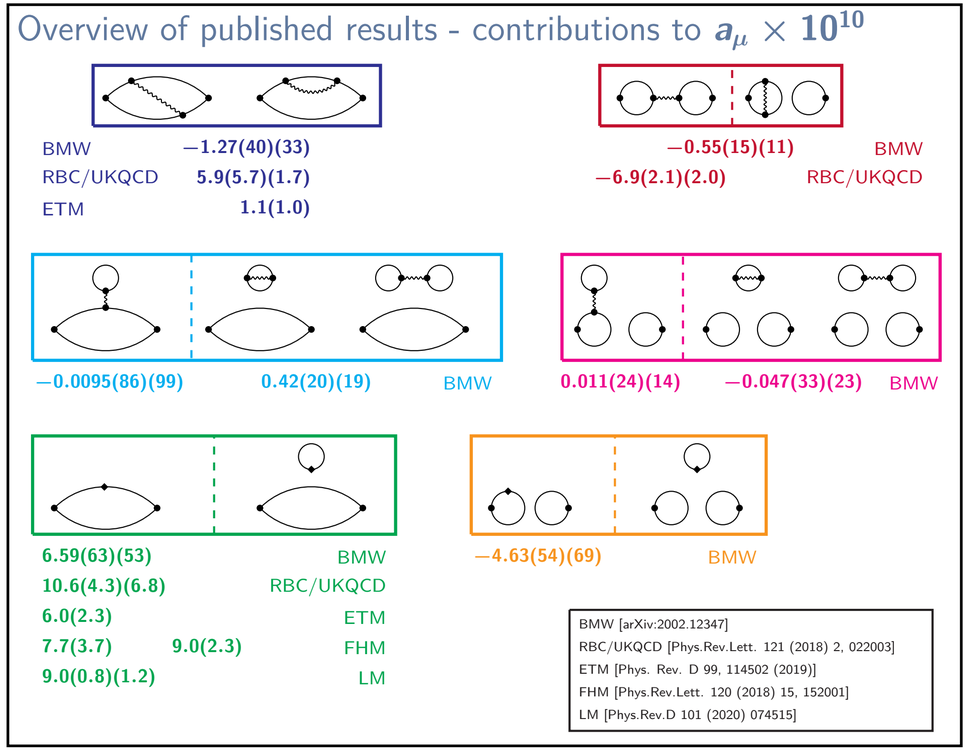


Ongoing efforts by FNAL-HPQCD-MILC RBC/UKQCD, Mainz

Consistent results with increasing precision



V. Gülpers @ Lattice HVP workshop



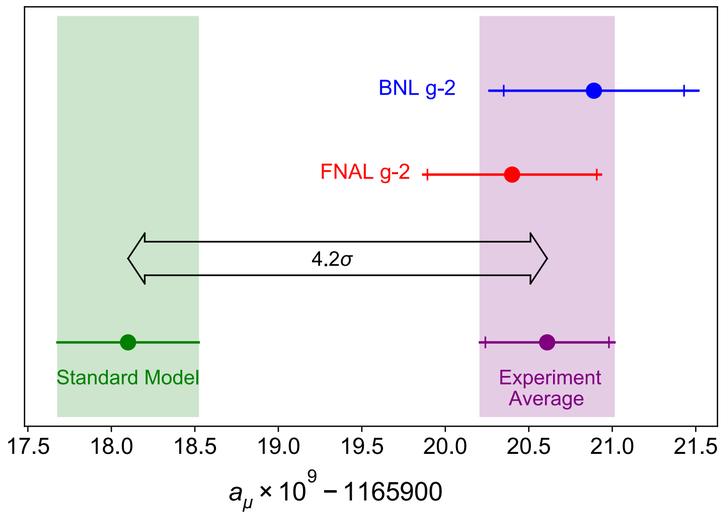
- Some tensions between lattice results for individual contributions.
- Large cancellations between individual contributions: $\delta a_\mu^{\rm IB} \lesssim 1\,\%$

Beyond the SM possibilities

a_μ is loop-induced, conserves CP & flavor, flips chirality.

The difference between Exp-SM is large:

$$\Delta a_\mu = 251(59) \times 10^{-11} \; > \; a_\mu(\mathrm{EW})$$



Generically expect:

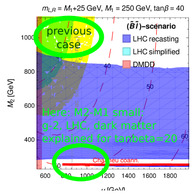
$$a_\mu^{\mathrm{NP}} \sim a_\mu^{\mathrm{EW}} \times \frac{M_W^2}{\Lambda^2} \times \text{couplings}$$

Can be accommodated by many BSM theories

D. Stöckinger @ g-2 Days (http://pheno.csic.es/g-2Days21/)

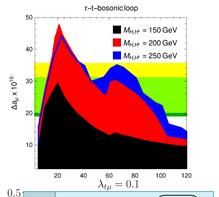
SUSY: MSSM, MRSSM

- MSugra. . . many other generic scenarios
- Bino-dark matter+some coannihil.+mass splittings
- Wino-LSP+specific mass patterns



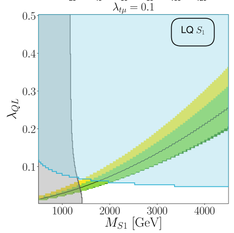
Two-Higgs doublet model

- Type I, II, Y, Type X(lepton-specific), flavour-aligned



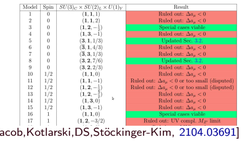
Lepto-quarks, vector-like leptons

- scenarios with muon-specific couplings to μ_L and μ_R



Simple models (one or two new fields)

- Mostly excluded
- light N.P. (ALPs, Dark Photon, Light $L_\mu - L_\tau$)



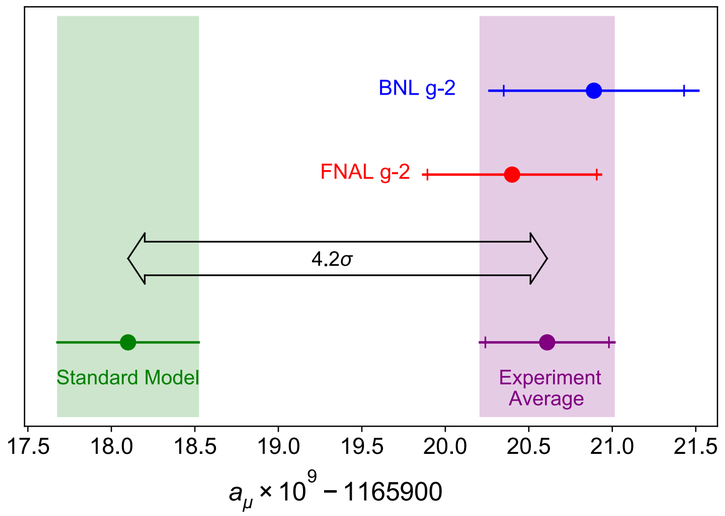
[Athron,Balazs,Jacob,Kotlarski,DS,Stöckinger-Kim, 2104.03691]

Beyond the SM possibilities

a_μ is loop-induced, conserves CP & flavor, flips chirality.

The difference between Exp-SM is large:

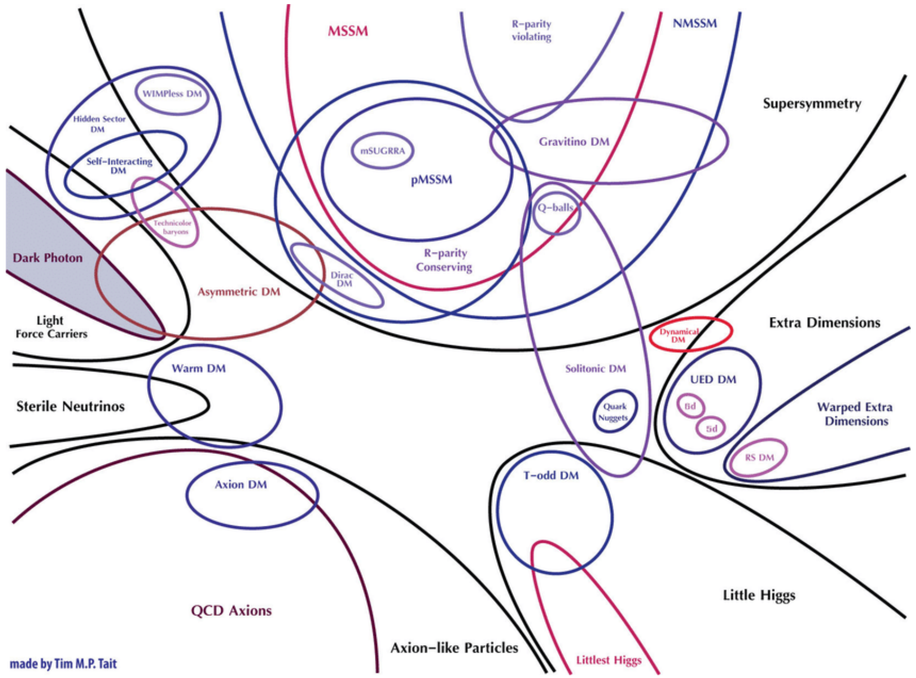
$$\Delta a_\mu = 251(59) \times 10^{-11} \; > \; a_\mu(\mathrm{EW})$$



Generically expect:

$$a_\mu^{\mathrm{NP}} \sim a_\mu^{\mathrm{EW}} \times \frac{M_W^2}{\Lambda^2} \times \text{couplings}$$

Can be accommodated by many BSM theories



other possibilities:

New boson produced resonantly at ~ 1GeV (KLOE CM energy)

[L. Darmé et al, arXiv:2112.09139]