Prospects for precise predictions of $a_\mu$ in the SM

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RPF3 Session
Snowmass CSS
University of Washington
17-26 July 2022
Outline

- Introduction
- Hadronic corrections in comparison
- HVP
- HLbL
- Timeline
- Summary and Outlook

Based on: “Prospects for precise predictions or $a_\mu$ in the SM”: arXiv:2203.15810
also: Belle II: arXiv:2207.06307 (Snowmass WP)
STFC: arXiv:2203.06961,
Chiral Belle arXiv:2205.12847
Anomalous magnetic moment

The magnetic moment of charged leptons ($e, \mu, \tau$):

$$\vec{\mu} = g \frac{e}{2m} \vec{S}$$

Dirac (leading order): $g = 2$

$$= (-ie) \bar{u}(p') \gamma^\mu u(p)$$

Quantum effects (loops):

All SM particles contribute

$$= (-ie) \bar{u}(p') \left[ \gamma^\mu F_1(q^2) + \frac{i \sigma^{\mu\nu} q_\nu}{2m} F_2(q^2) \right] u(p)$$

Note: $F_1(0) = 1$ and $g = 2 + 2 F_2(0)$

Anomalous magnetic moment:

$$a \equiv \frac{g - 2}{2} = F_2(0)$$
The Fermilab experiment released the measurement result from their run 1 data on 7 April 2021.
[B. Abi et al, Phys. Rev. Lett. 124, 141801 (2021)]
Analysis of runs 2 and 3 is now underway.

WP
• FNAL measurement confirms BNL result
• Analyzed 6% of the planned data
  – Statistically limited: 434 ppb
  – Systematics: 157 ppb
• Collected more than 50% of our planned data
  – Aim to analyze Run 2-3 for summer of 2022

Meanwhile … theory steps in: What could it all mean?
Please see talk on Muon g-2 SM and BSM theory review by Martin Hoferichter, Wednesday at 14:00
Muon g-2: SM contributions

\[ a_\mu = a_\mu^{\text{(QED)}} + a_\mu^{\text{(EW)}} + a_\mu^{\text{(hadronic)}} \]

- **QED**
  - +… (5 loops)
  - 116 584 718.9 (1) \( \times 10^{-11} \)
  - 0.001 ppm

- **EW**
  - +… (2 loops)
  - 153.6 (1.0) \( \times 10^{-11} \)
  - 0.01 ppm

- **HVP**
  - \( \alpha^2 \)
  - +… (NNLO)
  - 6845 (40) \( \times 10^{-11} \)
  - 0.34 ppm [0.6%]

- **HLbL**
  - \( \alpha^3 \)
  - +… (NLO)
  - 92 (18) \( \times 10^{-11} \)
  - 0.15 ppm [20%]
Maximize the impact of the Fermilab and J-PARC experiments
.quantify and reduce the theoretical uncertainties on the hadronic corrections
summarize the theory status and assess reliability of uncertainty estimates
organize workshops to bring the different communities together:

- First plenary workshop @ Fermilab: 3-6 June 2017
- HVP workshop @ KEK: 12-14 February 2018
- HLbL workshop @ U Connecticut: 12-14 March 2018
- Second plenary workshop @ HIM (Mainz): 18-22 June 2018
- Third plenary workshop @ INT (Seattle): 9-13 September 2019
- Fourth plenary workshop @ KEK (virtual): 28 June - 02 July 2021
- Fifth plenary workshop @ Higgs Centre (Edinburgh): 5-9 September 2022

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2
nd White Paper: First discussions @ KEK meeting in June 2021
expect to develop a concrete plan @ Higgs Centre workshop

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Steering Committee

- Gilberto Colangelo (Bern)
- Michel Davier (Orsay) co-chair
- Aida El-Khadra (UIUC & Fermilab) chair
- Martin Hoferichter (Bern)
- Christoph Lehner (Regensburg University & BNL) co-chair
- Laurent Lellouch (Marseille)
- Tsutomu Mibe (KEK)
  - J-PARC Muon g-2/EDM experiment
- Lee Roberts (Boston)
  - Fermilab Muon g-2 experiment
- Thomas Teubner (Liverpool)
- Hartmut Wittig (Mainz)

https://muon-gm2-theory.illinois.edu

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A. El-Khadra

Snowmass CSS, 17-26 July 2022
Hadronic Corrections: Comparisons

\[ a_{\mu}^{SM} = <a_{\mu}^{HVP} + a_{\mu}^{QED} + a_{\mu}^{Weak} + a_{\mu}^{HLbL}> \]

Glasgow consensus (09)
N/JN09
J17
Mainz21 (+ charm-loop)
not used in WP20
RBC/UKQCD19
(+ charm-loop)
WP20 data-driven
dispersive
WP20

Lattice QCD + QED

dispersive/data driven

models of QCD + EFT, large \( N_c \)

HVP from:

BNL+FNAL

LM20
BMW20
ÉTM18/19
Mainz/CLS19
FHM19
PACS19
RBC/UKQCD18
BMW17
RBC/UKQCD
data/lattice
BDJ19
J17

not used in WP20

DHMZ19
KNT19
WP20

(\( a_{\mu}^{SM} - a_{\mu}^{exp} \) \times 10^{10})

not yet in WP20 (lat)
Hadronic Corrections

Two different, independent strategies:

1. For HVP, use dispersion relations to rewrite integral in terms of hadronic cross section:

   \[ \text{Im} \left[ \ldots \right] \sim \left| \text{hadrons} \right|^2 \]

   Many experiments (over 20+ years) have measured the \( e^+e^- \) cross sections for the different channels over the needed energy range with increasing precision.
   New dispersive approach developed for HLbL

2. Direct calculation using Euclidean Lattice QCD

   \[ L \]

   Approximations:
   - discrete space-time (spacing \( a \))
   - finite spatial volume (\( L \)), and time extent (\( T \))

   ... 

   \( ab\text{-initio} \) method to quantify QCD effects
   - already used for simple hadronic quantities with high precision
   - requires large-scale computational resources
   - allows for entirely SM theory based evaluations

   Integrals are evaluated numerically using Monte Carlo methods.
In 2020 WP:
Conservative merging procedure to obtain a realistic assessment of the underlying uncertainties:
- account for tensions between data sets
- account for differences in methodologies for compilation of experimental inputs
- include correlations between systematic errors
- cross checks from unitarity & analyticity constraints
  [Colangelo et al, 2018; Anantharayan et al, 2018; Davier et al, 2019; Hoferichter et al, 2019]
- Full NLO radiative corrections  [Campanario et al, 2019]
  \[ a_{\mu}^{\text{HVP,LO}} = 693.1 \times (2.8)_{\exp} (0.7)_{\text{DV+pQCD}} (2.8)_{\text{BaBar-KLOE}} \times 10^{-10} \]
  \[ = 693.1 (4.0) \times 10^{-10} \]
  [M. Davier @ KEK workshop]

Ongoing work:
- BaBar: new analysis of large (7X) data set in $\pi\pi$ channel (1-2 years), also $\pi\pi\pi$, other channels
- SND: new results for $\pi\pi$ channel, other channels in progress
- CMD-3: ongoing analyses for $\pi\pi$ and other channels
- BESIII: new results in 2021 for $\pi\pi$ channel, continued analysis also for $\pi\pi\pi$, other channels
- Belle II: arXiv:2207.06307 (Snowmass WP) Better statistics than BaBar or KLOE; similar or better systematics for low-energy cross sections
- Need blind analyses to resolve the tensions (esp. for $\pi\pi$ channel)
- Developing NNLO Monte Carlo generators (STRONG 2020 workshop next week https://agenda.infn.it/event/28089/)

Data-driven evaluations of HVP with $\sim 0.3\%$ feasible by $\sim 2025$, if tensions between experiments are resolved.
In 2020 WP:

- Lattice HVP average at 2.6% total uncertainty:
  \[ a_{\mu}^{HVP,LO} = 711.6 (18.4) \times 10^{10} \]
- BMW 20 (published in 2021)
  first LQCD calculation with sub-percent (0.8%) error in tension with data-driven HVP (2.1σ)
- Further tensions for intermediate window

\[ a_{\mu}^{HVP,LO} = \left( \frac{\alpha}{\pi} \right)^2 \int_0^\infty dt \bar{w}(t) C(t) \]

Use windows in Euclidean time to consider the different time regions separately. [T. Blum et al, arXiv:1801.07224, 2018 PRL]

Short Distance (SD) \( t : 0 \rightarrow t_0 \)
Intermediate (W) \( t : t_0 \rightarrow t_1 \)
Long Distance (LD) \( t : t_1 \rightarrow \infty \)

\( t_0 = 0.4 \text{fm}, t_1 = 1.0 \text{fm} \)

disentangle systematics/statistics from long distance/FV and discretization effects

intermediate window: easy to compute in lattice QCD & using disperse approach:

Internal cross check:
Compute each window separately (in continuum, infinite volume limits, ...) and combine:

\[ a_{\mu} = a_{\mu}^{SD} + a_{\mu}^{W} + a_{\mu}^{LD} \]
In 2020 WP:

- Lattice HVP average at 2.6% total uncertainty:
  \[ a_\mu^{\text{HVP,LO}} = 711.6 (18.4) \times 10^{10} \]
- BMW 20 (published April 2021)
  first LQCD calculation with sub-percent (0.8%) error
  in tension with data-driven HVP (2.1\sigma)
- Further tensions for intermediate window:
  - tension with data-driven evaluation
  - tension with RBC/UKQCD18

Proposals for computing more windows:

- Use linear combinations of finer windows to locate the tension (if it persists) in \( \sqrt{s} \)
  [Colangelo et al, arXiv:12963]

- Use larger windows, excluding the long-distance region
  \( t \gtrsim 2 \text{ fm} \) to maximize the significance of any tension
  [Davies at at, arXiv:2207.04765]

New results for intermediate windows:

- \( \chi^\text{QCD} \): arXiv:2204.01280
- Mainz: arXiv:2206.06582
- ETM: arXiv:2206.15084 (ETM)

-3.7\sigma tension with data-driven evaluation
-2.2\sigma tension with RBC/UKQCD18
In 2020 WP:
- Lattice HVP average at 2.6% total uncertainty:
  \[ a^{\text{HVP,LO}}_\mu = 711.6 (18.4) \times 10^{10} \]
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Further tensions for intermediate window:
-3.7\sigma tension with data-driven evaluation
-2.2\sigma tension with RBC/UKQCD18

Ongoing work:
- Expect new results from RBC/UKQCD and FNAL/HPQCD/MILC in coming months (both are blinded):
  S. Lahert
  (FNAL/HPQCD/MILC)

For total HVP:
- Including \( \pi \pi \) states for refined long-distance computation
  (Mainz, RBC/UKQCD, FNAL/MILC)
- Developing method average for lattice HVP — started at KEK workshop (June 2021), based on detailed comparisons
  - list of sub quantities (and their definitions)
  - common prescription for separating QCD & QED
  - quality criteria for inclusion
- Most groups plan to include smaller lattice spacings to test continuum extrapolations (needs adequate computational resources)

If results are consistent, Lattice HVP (average) with \( \lesssim 0.5\% \) errors feasible by 2025
Dispersive approach:
[Colangelo at al, 2014; Pauk & Vanderhaegen 2014; ...]

- model independent
- significantly more complicated than for HVP
- provides a framework for data-driven evaluations
- can also use lattice results as inputs

Dominant contributions (≈ 75% of total):

- Well quantified with ≈ 6% uncertainty
- \( \eta, \eta' \) pole contributions: Canterbury approximants only
- Ongoing work: consolidation of \( \eta, \eta' \) pole contributions using disp. relations and LQCD

Subleading contributions (≈ 25% of total):

- Not yet well known
  - dominant contribution to total uncertainty
- Ongoing work:
  - Implementation of short-distance constraints (now at 2-loop)
  - DR implementation for axial vector contributions
  - BESIII ramping up \( \gamma^*(\gamma^*) \) program

Dispersive, data-driven evaluation of HLbL with ≤ 10% total uncertainty feasible by ~2025.
Lattice QCD+QED: Two independent and complete direct calculations of $a_{\mu}^{\text{HLbL}}$

- **RBC/UKQCD**
  
  
  QCD + QED$_L$ (finite volume)

  DWF ensembles at/near phys mass, $a \approx 0.08 - 0.2$ fm, $L \sim 4.5 - 9.3$ fm

- **Mainz group**


  QCD + QED (infinite volume & continuum)

  CLS (2+1 Wilson-clover) ensembles

  $m_\pi \sim 200 - 430$ MeV, $a \approx 0.05 - 0.1$ fm, $m_\pi L > 4$

- **Cross checks between RBC/UKQCD & Mainz approaches in White Paper at unphysical pion mass**

- **Both groups will continue to improve their calculations, adding more statistics, lattice spacings, physical mass ensemble (Mainz)**

Lattice HLbL results with 10% total uncertainty feasible by ~2025
Timeline

FNAL E989

Run 4

Run 5

Run 6

J-PARC E34

Muon g-2 TI WP published

Run 1 result announced

? 2021

2022

2023

2024

2025

Result from Runs 2&3

WP update

Result from Run 4

Final result from E989

Theory Initiative:

🌟 ongoing activities: develop method average for Lattice HVP

🌟 plan to update WP: new lattice HVP results (when available)

🌟 main update with all available results ~ 2023

🌟 TI workshops: Jun 2021 @ KEK (virtual)

Sep 2022 @ Higgscentre
Lepton moments summary

Sensitivity to heavy new physics:

\[ a^{NP}_\ell \sim \frac{m^2_\ell}{\Lambda^2} \]

\[(m_\mu/m_e)^2 \sim 4 \times 10^4\]

Cs: \(\alpha\) from Berkeley group [Parker et al, Science 360, 6385 (2018)]

Rb: \(\alpha\) from Paris group [Morel et al, Nature 588, 61–65(2020)]

Chiral Belle  arXiv:2205.12847

☆ use polarized \(e^-\) beam

☆ with 40\(ab^{-1}\) measurement of \(a_\tau\) at \(10^{-5}\) feasible

☆ with more statistics measurement at \(10^{-6}\) possible
Summary

⭐ Theory Initiative:
- Concrete plans for WP updates @ next workshop (update on lattice HVP if new results are available)
- method averages for lattice HVP, HLbL

⭐ Programs and plans in place to improve by 2025:
- data-driven HVP $\sim 0.3\%$ (if tension between KLOE and BaBar is resolved)
- lattice HVP $\lesssim 0.5\%$ (if tension between BMW and RBC/UKQCD is resolved)
- dispersive HLbL and lattice HLbL: $\sim 10\%$

⭐ If tensions between data-driven HVP and lattice HVP are resolved, SM prediction will likely match precision goal of the Fermilab experiment.
⭐ If not, will need detailed comparisons, explore connections between HVP, $\sigma(e^+e^-)$, $\Delta\alpha$, global EW fits.
- MUonE (space-like momentum measurement of $\Delta\alpha$) will provide more information/cross checks.
Outlook

☆ **Experimental program beyond 2025:**
  - **J-PARC:** Muon g-2/EDM, MuSEUM, COMET, DeeMe
  - **Fermilab:** Mu2e, future muon campus experiments?
  - **PSI:** MEG, MuMass, Mu3e, MuEDM, MUSE, CREMA, HIMB?
  - **Belle II, MUonE, BESIII, Novosibirsk**
  - **STCF (?)**, **Chiral Belle (?)**

☆ **Data-driven/dispersive program beyond 2025:**
  - further uncertainty reductions from new measurements
    - STCF, $\tau$ decay data, …
  - development of NNLO MC generators
  - for HLbL, improved experimental/lattice inputs together with further development of dispersive approach

☆ **Lattice QCD program beyond 2025:**
  - access to future computational resources (coming Exascale) will enable improvements of all errors (statistical and systematic)
  - concurrent development of better methods and algorithms (gauge-field sampling, noise reduction) will accelerate progress
  - continued coordination by Muon g-2 Theory Initiative
Thank you!
Appendix
The muon g-2 theory initiatives, Seattle, Sept 9-13, 2019

Tensions between BaBar and KLOE data sets:

- Cross checks using analyticity and unitarity relating pion form factor to $\pi\pi$ scattering
- Combinations of data sets affected by tensions ➔ conservative merging procedure
Experimental Inputs to HVP

S. Serednyakov (for SND) @ HVP KEK workshop

e^+e^- facilities involved in HVP measurement

BNL-821
FNAL E989
J-PARC g-2/EDM
E-34

HVP measurements

BELLE-II
BES-III
KLOE
BaBar
SND
CMD-3

A. El-Khadra
Snowmass CSS, 17-26 July 2022
Connections

\[ \sigma(e^+e^- \rightarrow \text{hadrons}) \Leftrightarrow a_H^{\text{HVP}} \Leftrightarrow \Delta\alpha_{\text{had}}(M_Z^2) \]

\( \Delta\alpha_{\text{had}}(M_Z^2) \) also depends on the hadronic vacuum polarization function, and can be written as an integral over \( \sigma(e^+e^- \rightarrow \text{hadrons}) \), but weighted towards higher energies.

A shift in \( a_H^{\text{HVP}} \) also changes \( \Delta\alpha_{\text{had}}(M_Z^2) \): \( \Rightarrow \) EW fits


If the shift in \( a_H^{\text{HVP}} \) is in the low-energy region (\( \lesssim 1 \text{ GeV} \)), the impact on \( \Delta\alpha_{\text{had}}(M_Z^2) \) and EW fits is small.

A shift in \( a_H^{\text{HVP}} \) from low (\( \lesssim 2 \text{ GeV} \)) energies

\( \Rightarrow \sigma(e^+e^- \rightarrow \pi\pi) \)

must satisfy unitarity & analyticity constraints \( \Rightarrow F_V^\pi(s) \)

can be tested with lattice calculations

\[ \sigma(e^+e^- \rightarrow \text{hadrons}) \iff a_{\mu}^{\text{HVP}} \iff \Delta \alpha_{\text{had}}(M_Z^2) \]

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\[ \Rightarrow \sigma(e^+e^- \rightarrow \pi\pi) \]

must satisfy unitarity & analyticity constraints \( \Rightarrow F^\pi(s) \)

can be tested with lattice calculations [Colangelo, Hoferichter, Stoffer, arXiv:2010.07943]

**Time-like formulation:**

\[ \Delta \alpha_{\text{had}}^{(5)}(M_Z^2) = \frac{\alpha M_Z^2}{3 \pi} \int_{s_{\text{thr}}}^{\infty} ds \frac{R_{\text{had}}(s)}{s(M_Z^2 - s)} \]

**Space-like formulation:**

\[ \Delta \alpha_{\text{had}}^{(5)}(M_Z^2) = \frac{\alpha}{\pi} [\hat{n}(M_Z^2) - \hat{n}(M_Z^2) - \hat{n}(M_Z^2)] \]

**Global EW fit**

- Difference between HEPFit and GFitter implementation mainly treatment of \( M_W \)
- Pull goes into opposite direction

### Hadronic running of \( \alpha \) and global EW fit

<table>
<thead>
<tr>
<th>( e^+e^- )</th>
<th>KNT, DHMZ</th>
<th>EW fit HEPFit</th>
<th>EW fit GFitter</th>
<th>guess based on BMWc</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta \alpha_{\text{had}}^{(5)}(M_Z^2) \times 10^4 )</td>
<td>276.1(1.1)</td>
<td>270.2(3.0)</td>
<td>271.6(3.9)</td>
<td>277.8(1.3)</td>
</tr>
</tbody>
</table>

Difference to \( e^+e^- \): 
- \( -1.8\sigma \)
- \( -1.1\sigma \)
- \( +1.0\sigma \)

**Connections**

Martin Hoferichter @ Lattice HVP workshop

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A. El-Khadra

Snowmass CSS, 17-26 July 2022
Connections

\( \sigma(e^+e^- \rightarrow \text{hadrons}) \Leftrightarrow a^\text{HVP}_\mu \Leftrightarrow \Delta \alpha_{\text{had}}(M_Z^2) \)

\( \Delta \alpha_{\text{had}}(M_Z^2) \) also depends on the hadronic vacuum polarization function, and can be written as an integral over \( \sigma(e^+e^- \rightarrow \text{hadrons}) \), but weighted towards higher energies.

A shift in \( a^\text{HVP}_\mu \) also changes \( \Delta \alpha_{\text{had}}(M_Z^2) \): \( \Rightarrow \) EW fits (Passera, et al, 2008, Crivellin et al 2020, Keshavarsi et al 2020, Malaeescu & Scott 2020)

If the shift in \( a^\text{HVP}_\mu \) is in the low-energy region (\( \lesssim 1 \text{ GeV} \)), the impact on \( \Delta \alpha_{\text{had}}(M_Z^2) \) and EW fits is small.

A shift in \( a^\text{HVP}_\mu \) from low (\( \lesssim 2 \text{ GeV} \)) energies \( \Rightarrow \sigma(e^+e^- \rightarrow \pi\pi) \)

must satisfy unitarity & analyticity constraints \( \Rightarrow F^V_\pi(s) \)
can be tested with lattice calculations [Colangelo, Hoferichter, Stoffer, arXiv:2010.07943]

Can new physics hide in the low-energy \( \sigma(e^+e^- \rightarrow \pi\pi) \) cross section? \( \Rightarrow \) No [Luzio, et al, arXiv:2112.08312]

Neutral, long-lived hadrons, heretofore undetected? [Farrar, arXiv:2206.13460]

### Constraints on the two-pion contribution to HVP


#### Modifying \( a^\pi\pi_\mu \) \( \leq 1 \text{ GeV} \)

- **“low-energy” scenario:** local changes in cross section of \( \sim 8\% \) around \( \rho \)
- **“high-energy” scenario:** impact on pion charge radius and space-like VFF \( \Rightarrow \) chance for independent lattice-QCD checks

#### Requires factor \( \sim 3 \) improvement over QCD result:

\( \chi^2_\text{QCD} \) result:

\( \langle r^2_\pi \rangle = 0.433(9)(13) \text{ fm}^2 \) 

\( \rightarrow \) arXiv:2006.05431 [hep-ph]
Calculate $a^\text{HVP}_\mu$ in Lattice QCD:

$$a^\text{HVP,LO}_\mu = \sum_f a^\text{HVP,LO}_{\mu,f} + a^\text{HVP,LO}_{\mu,\text{disc}}$$

- Separate into connected for each quark flavor + disconnected contributions (gluon and sea-quark background not shown in diagrams)

  Note: almost always $m_u = m_d$

  $$\sum_f \begin{array}{c} \includegraphics[width=2cm]{diagram1} \\ \includegraphics[width=2cm]{diagram2} \end{array} + \begin{array}{c} \includegraphics[width=2cm]{diagram3} \\ \includegraphics[width=2cm]{diagram4} \end{array} \quad f = \text{ud, s, c, b}$$

- need to add QED and strong isospin breaking ($\sim m_u - m_d$) corrections:

  $$\begin{array}{c} \includegraphics[width=2cm]{diagram5} \\ \includegraphics[width=2cm]{diagram6} \end{array} + \ldots$$

  - either perturbatively on isospin symmetric QCD background
  - or by using QCD + QED ensembles with $m_u \neq m_d$
**Target:** \( \sim 0.2\% \) total error

\[
\sum_f f_f \quad \text{light-quark connected contribution:} \quad a_{\mu}^{\text{HVP,LO}}(ud) \sim 90\% \text{ of total}
\]

\[
\text{s, c, b-quark contributions} \quad a_{\mu}^{\text{HVP,LO}}(s, c, b) \sim 8\%, 2\%, 0.05\% \text{ of total}
\]

\[
\sum_f f_f' \quad \text{disconnected contribution:} \quad a_{\mu,\text{disc}}^{\text{HVP,LO}} \sim 2\% \text{ of total}
\]

\[
+ \ldots \quad \text{Isospin breaking (QED + } m_u \neq m_d \text{) corrections:} \quad \delta a_{\mu}^{\text{HVP,LO}} \sim 1\% \text{ of total}
\]

\[
a_{\mu}^{\text{HVP,LO}} = a_{\mu}^{\text{HVP,LO}}(ud) + a_{\mu}^{\text{HVP,LO}}(s) + a_{\mu}^{\text{HVP,LO}}(c) + a_{\mu,\text{disc}}^{\text{HVP,LO}} + \delta a_{\mu}^{\text{HVP,LO}}
\]
Lattice HVP: Isospin corrections

H. Wittig @ Lattice HVP workshop

- Charm, strange contributions already well determined.
- Mild tensions for light contribution

Consistent results with increasing precision

V. Gülkers @ Lattice HVP workshop

Overview of published results - contributions to $\Delta a_\mu \times 10^{10}$

- BMW 20
- Anbin et al. 19
- Mainz/CLS 19
- FHM 19
- PACS 19
- ETMC 19
- RBC/UKQCD 18
- BMW 17

- Some tensions between lattice results for individual contributions.
- Large cancellations between individual contributions: $\Delta a_\mu^{1B} \lesssim 1\%$

Ongoing efforts by FNAL-HPQCD-MILC, RBC/UKQCD, Mainz

A. El-Khadra

Snowmass CSS, 17-26 July 2022
Beyond the SM possibilities

\( a_\mu \) is loop-induced, conserves CP & flavor, flips chirality.

The difference between Exp-SM is large:
\[ \Delta a_\mu = 251(59) \times 10^{-11} > a_\mu(\text{EW}) \]

Can be accommodated by many BSM theories

D. Stöckinger @ g-2 Days (http://pheno.csic.es/g-2Days21/)

SUSY: MSSM, MRSSM
- MSugra...many other generic scenarios
- Bino-dark matter+some coannihil.+mass splittings
- Wino-LSP+specific mass patterns

Two-Higgs doublet model
- Type I, II, Y, Type X(lepton-specific), flavour-aligned

Lepto-quarks, vector-like leptons
- scenarios with muon-specific couplings to \( \mu_L \) and \( \mu_R \)

Simple models (one or two new fields)
- Mostly excluded
- light N.P. (ALPs, Dark Photon, Light \( L_\mu - L_\tau \))

Generically expect:
\[ a_\mu^{\text{NP}} \sim a_\mu^{\text{EW}} \times \frac{M_W^2}{\Lambda^2} \times \text{couplings} \]
Beyond the SM possibilities

$a_\mu$ is loop-induced, conserves CP & flavor, flips chirality.

The difference between Exp-SM is large:

$$\Delta a_\mu = 251(59) \times 10^{-11} > a_\mu(\text{EW})$$

Generically expect:

$$a_\mu^{\text{NP}} \sim a_\mu^{\text{EW}} \times \frac{M_W^2}{\Lambda^2} \times \text{couplings}$$

Can be accommodated by many BSM theories

other possibilities:

New boson produced resonantly at $\sim 1\text{GeV}$ (KLOE CM energy)