Theory of Charged Lepton Flavor Violation

Why we should keep searching for CLFV.

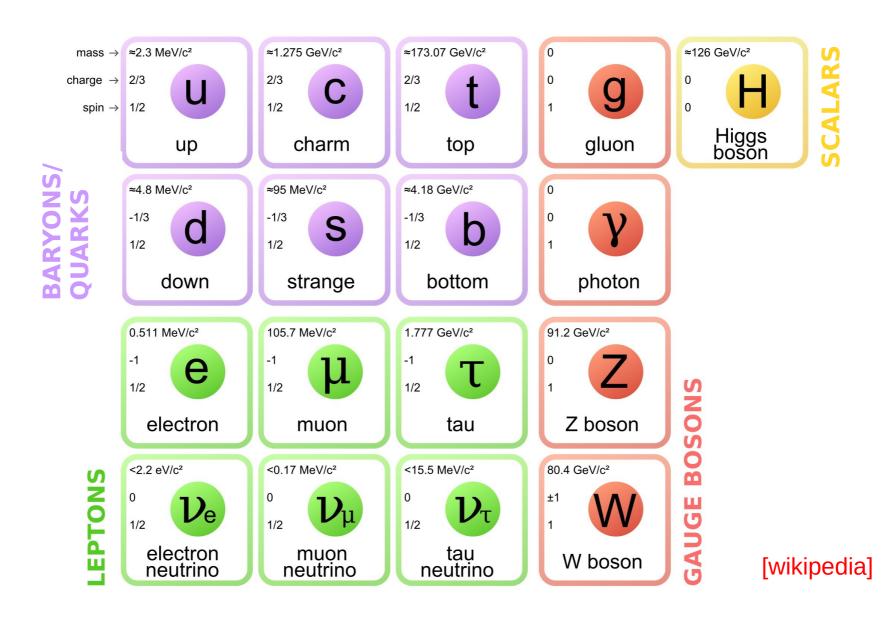
Julian Heeck

Snowmass Community Summer Study, Seattle

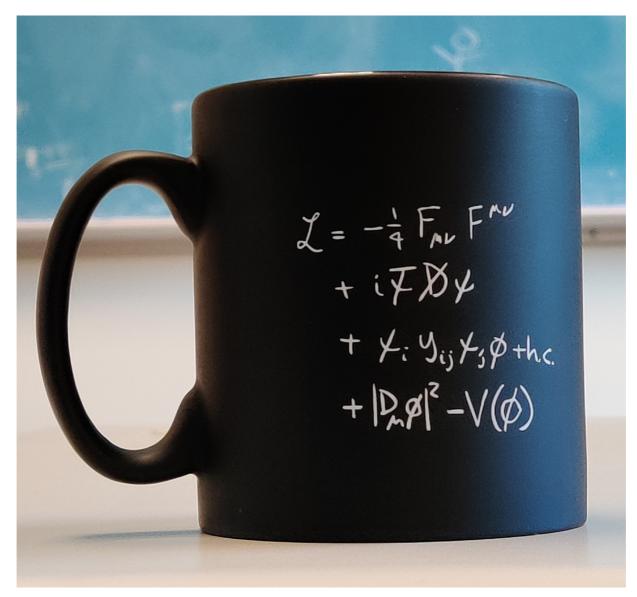
07/22/2022



Elementary particles



The Standard Model



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Symmetries of the Standard Model

• Rephasing lepton and quark fields:

$$\begin{array}{c} U(1)_B \times U(1)_{L_e} \times U(1)_{L_{\mu}} \times U(1)_{L_{\tau}} \\ = \\ U(1)_{B+L} \times U(1)_{B-L} \times U(1)_{L_{\mu}-L_{\tau}} \times U(1)_{L_{\mu}+L_{\tau}-2L_e} \,. \end{array}$$

- Broken non-perturbatively, but unobservable. ['t Hooft, PRL '76]
- True accidental global symmetry:

$$\mathsf{U}(1)_{\mathsf{B}-\mathsf{L}} imes \mathsf{U}(1)_{\mathsf{L}_{\mu}-\mathsf{L}_{ au}} imes \mathsf{U}(1)_{\mathsf{L}_{\mu}+\mathsf{L}_{ au}-2\mathsf{L}_{\mathsf{e}}}.$$

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- True accidental global symmetry:

Why look for CLFV?

- SM prediction: $U(1)_{L_{\mu}-L_{\tau}} \times U(1)_{L_{\mu}+L_{\tau}-2L_{e}}$, i.e. no LFV!
 - Background-free searches, high sensitivity.

Flavor violating decays



- Prime example: $\mu \rightarrow e\gamma$ @ MEG.
- Observation = new particles.
- $\mu \rightarrow e$ conversion @ Mu2e can probe scales up to 10^7GeV .

	LFV	process	current	future	\exp
-		$\mu ightarrow e \gamma$	4.2×10^{-13}	6×10^{-14}	MEG-II
	- <i>π</i> ,	$\mu ightarrow e \bar{e} e$	1.0×10^{-12}	10^{-16}	Mu3e
	ΔL_{μ}	$\mu \rightarrow e \text{ conv.}$	$\mathcal{O}(10^{-12})$	10^{-16}	Mu2e, COMET
		$h \to e \bar{\mu}$	6.1×10^{-5}	10^{-5}	LHC
	$L_{e} $	$Z \to e \bar{\mu}$	$7.5 imes 10^{-7}$	10^{-10}	FCC-ee
_	$\overline{\nabla}$	$had \rightarrow e\bar{\mu}(had)$	4.7×10^{-12}	10^{-12}	NA62

See following talks

Flavor violating decays



- Produce tauons at B factories (BaBar, Belle (II), LHCb).
- Observation = new particles.
- $\tau \rightarrow e^-e^+e^-$ @ Belle II will probe scales up to $2 \times 10^4 \text{GeV}$.

m LFV	process	current	future	\exp
=	$ au o e\gamma$	3.3×10^{-8}	10^{-9}	Belle II
	$ au o e \bar{\ell} \ell$	2.7×10^{-8}	10^{-9}	Belle II
ΔL	$\tau \to e \mathrm{had}$	$\mathcal{O}(10^{-8})$	10^{-9}	Belle II
	$h \to e \bar{\tau}$	$4.7 imes 10^{-3}$	10^{-4}	LHC
$L_{e} $	$Z \to e \bar{\tau}$	$9.8 imes 10^{-6}$	10^{-9}	FCC-ee
$\overline{\nabla}$	$had \rightarrow e\bar{\tau}(had)$	$\mathcal{O}(10^{-6})$	—	Belle II

Swagato Banerjee

See talk by

Neutrino oscillations = flavor violation

• Observations of $v_{\alpha} \rightarrow v_{\beta}$ prove that $M_{\nu} \neq 0$ and $U(1)_{L_{\mu}-L_{\tau}} \times U(1)_{L_{\mu}+L_{\tau}-2L_{e}}$ is broken!

• Amplitudes for *charged* lepton flavor violation are suppressed:

$$\mathcal{A}(\ell_{\alpha}^{-} \to \ell_{\beta}^{-}) \propto \frac{(\mathsf{M}_{\nu}\mathsf{M}_{\nu}^{\dagger})_{\alpha\beta}}{\mathsf{M}_{\mathsf{W}}^{2}} < 10^{-24} \,. \quad \textbf{Great goalpost for Snowmass 3000!}$$

- Most (neutrino mass) models also generate CLFV rates unsuppressed by $\rm M_{v}$ that could be observable.

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Why look for CLFV?

- SM prediction: $U(1)_{L_{\mu}-L_{\tau}} \times U(1)_{L_{\mu}+L_{\tau}-2L_{e}}$, i.e. no LFV!
 - Background-free searches, high sensitivity.
- Neutrino oscillations = LFV!
 - Induced CLFV tiny: CLFV is complementary.

Scalar-triplet seesaw

[Konetschny & Kummer '77; Magg & Wetterich, '80; Schechter & Valle '80; Cheng & Li, '80; Mohapatra & Senjanovic, '81]

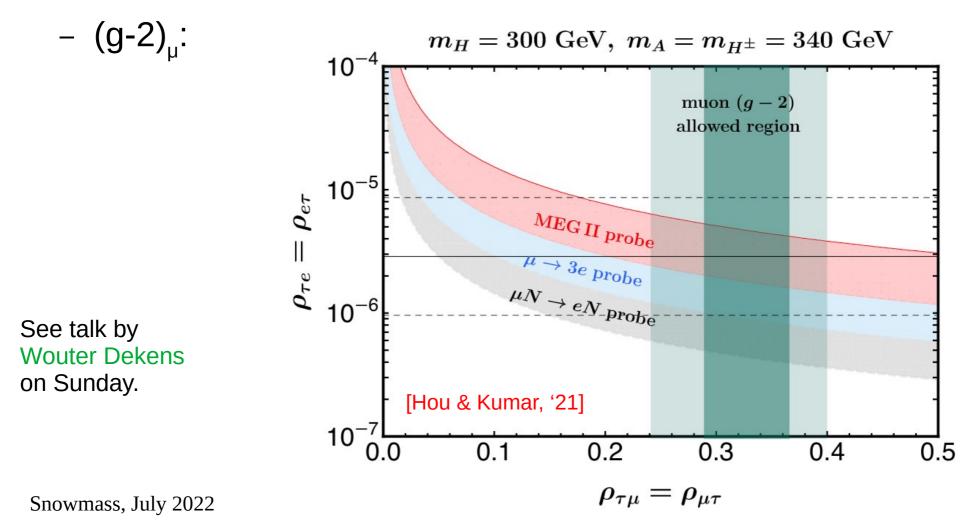
$$\begin{split} \mathsf{L} &= \mathsf{L}_{\mathrm{SM}} + |\mathsf{D}_{\alpha}\Delta|^{2} - (\mathsf{y}_{\alpha\beta}\overline{\mathsf{L}}_{\alpha}^{\mathsf{c}}\Delta\mathsf{L}_{\beta} + \mu\mathsf{H}\Delta\mathsf{H} + \mathsf{h.c.}) \\ &\stackrel{\langle H \rangle}{\xrightarrow{}} \stackrel{\langle H \rangle}{\xrightarrow{}} \stackrel{\langle H \rangle}{\xrightarrow{}} \stackrel{\langle H \rangle}{\xrightarrow{}} \stackrel{\langle I \rangle}{\xrightarrow{}} \stackrel{\langle$$

 $\label{eq:Cleanest prediction} \mbox{Cleanest prediction}: \ \mbox{BR}(\tau \to \mu \gamma) \simeq 23 \ \mbox{BR}(\tau \to {\rm e} \gamma) \simeq 3.5 \ \mbox{BR}(\mu \to {\rm e} \gamma) \,.$

CDF's W-mass first hint for this triplet with O(100 GeV) mass? [Heeck, 2204.10274]

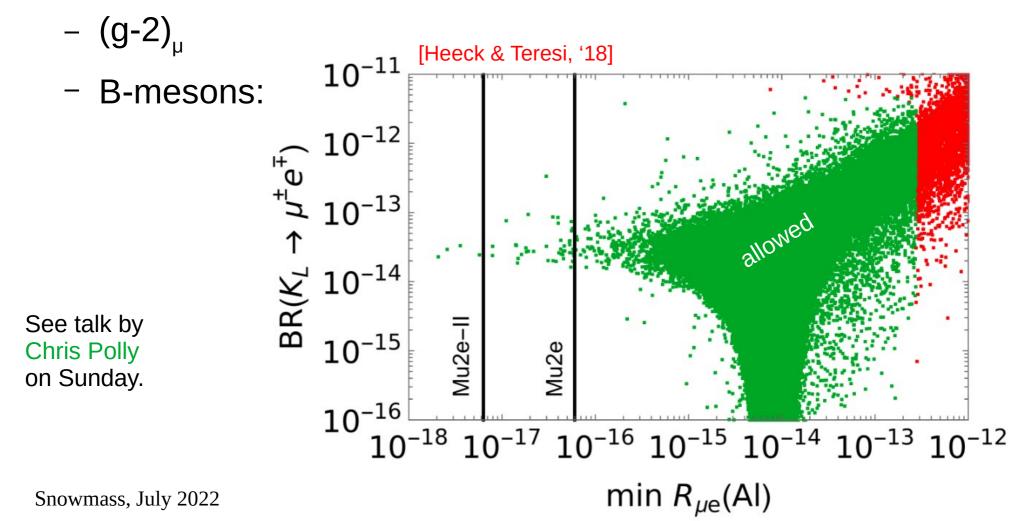
Predictions of LFV rates

- Need flavor structure (e.g. from neutrino mass) and a new mass scale to predict LFV rates.
- Tie LFV to anomalies



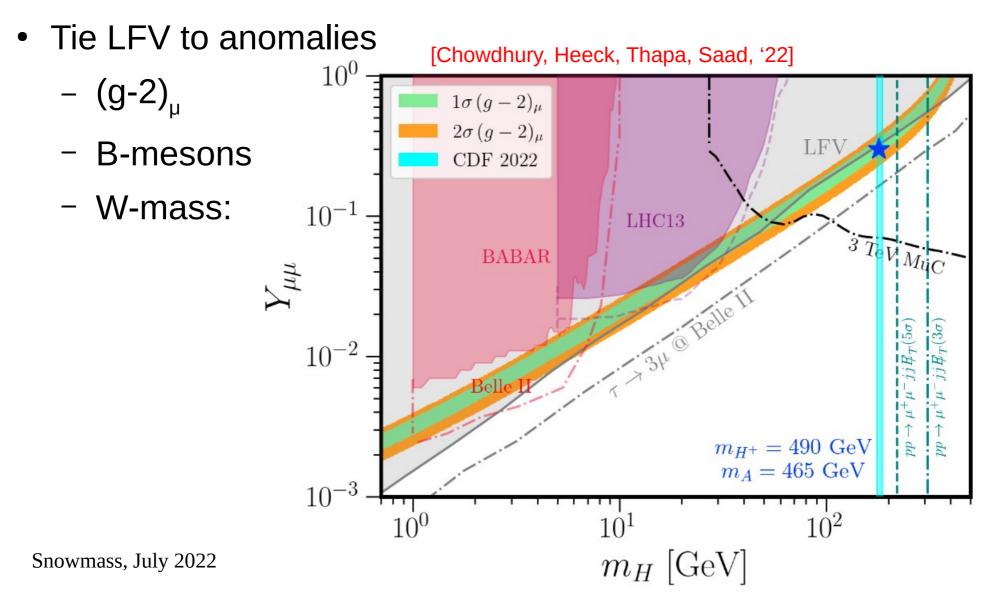
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- SM prediction: $U(1)_{L_{\mu}-L_{\tau}} \times U(1)_{L_{\mu}+L_{\tau}-2L_{e}}$, i.e. no LFV!
 - Background-free searches, high sensitivity.
- Neutrino oscillations = LFV!
 - Induced CLFV tiny: CLFV is *complementary*.
- New physics generically and easily gives testable CLFV.
 - Predictions require fixed flavor structure (PMNS, CKM) and new scale (g-2, LFUV in B, W-mass, DM,...).

Model independent: SMEFT

• 888 CLFV operators at d=6:

$$\frac{C_{ijnm}}{\Lambda^2}\ell_i^c\ell_j\ell_n^c\ell_m\,,\frac{C_{ijnm}}{\Lambda^2}\ell_i^c\ell_jd_n^cd_m\,,\frac{C_{ij}}{\Lambda^2}\ell_i^c\sigma_{\alpha\beta}\ell_j\mathsf{F}^{\alpha\beta}\,,\ldots$$

[Weinberg '79; Buchmüller & Wyler, '86; Grzadkowski++, '10; Fonseca, '17]

• Model-dependent coefficients; can get testable rates:

$$\ell \to \ell' \gamma, \ \ell \to \ell' \ell'' \ell''', \ \mu \to e \text{ conv.}, \ h \to \ell \ell', \ had \to \ell \ell', \ \dots$$

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$$\frac{C_{ijnm}}{\Lambda^2}\ell_i^c\ell_j\ell_n^c\ell_m\,,\frac{C_{ijnm}}{\Lambda^2}\ell_i^c\ell_jd_n^cd_m\,,\frac{C_{ij}}{\Lambda^2}\ell_i^c\sigma_{\alpha\beta}\ell_j\mathsf{F}^{\alpha\beta}\,,\ldots$$

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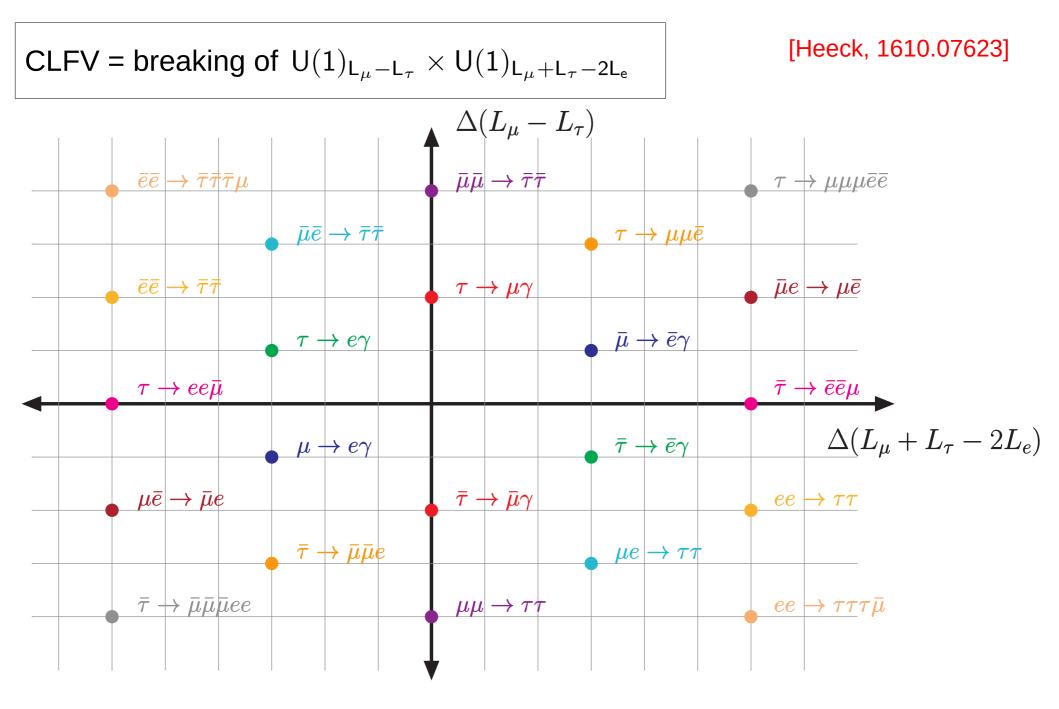
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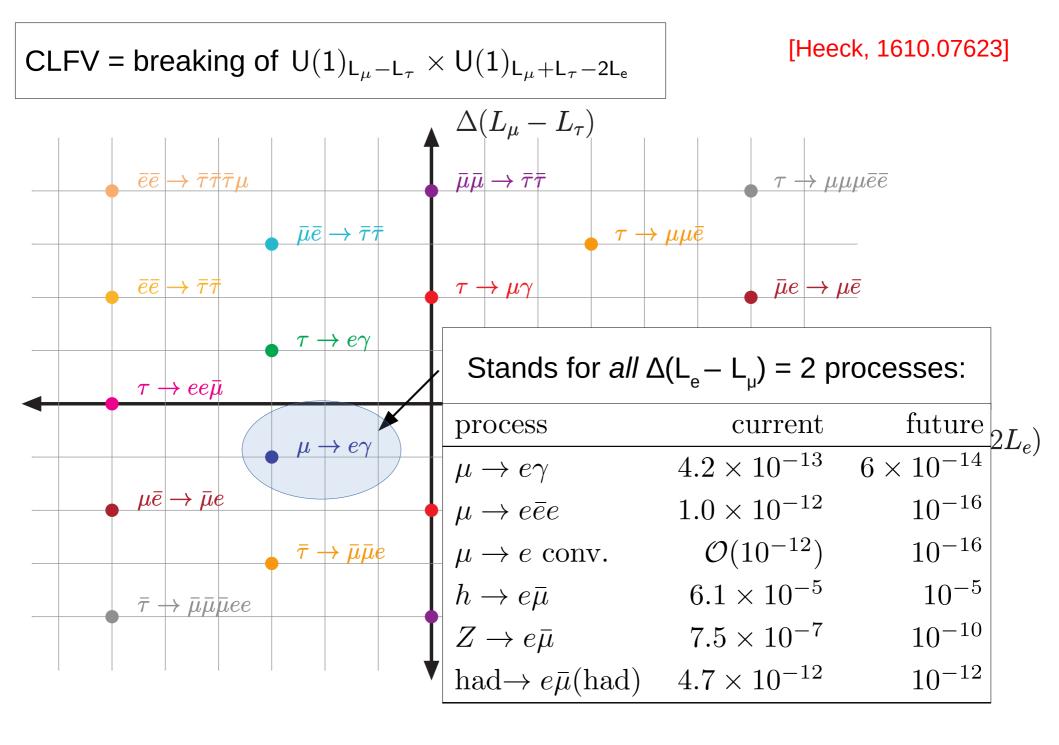
$$\ell \to \ell' \gamma, \ \ell \to \ell' \ell'' \ell''', \ \mu \to e \text{ conv.}, \ h \to \ell \ell', \ had \to \ell \ell', \ \dots$$

- Not all constrained, e.g. $\Delta L_{T} = 2$ operators.
- CLFV even sensitive to some d=8 operators, e.g. $\frac{L_{\mu}He_{R}GG}{\Lambda^{4}}$ [Davidson, Kuno, Uesaka, Yamanaka, 2007.09612; Ardu & Davidson, 2103.07212]
- Not clear if / how $U(1)_{L_{\mu}-L_{\tau}} \times U(1)_{L_{\mu}+L_{\tau}-2L_{e}}$ is broken in CLFV.

[Lew & Volkas, 9410277; Heeck, 1610.07623]

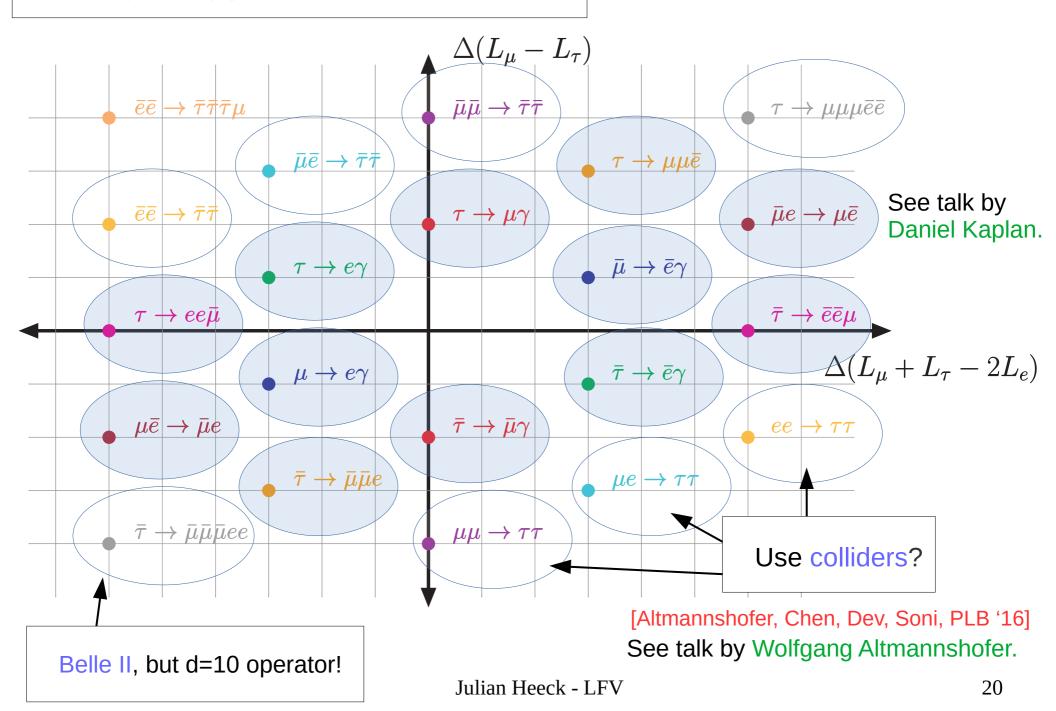
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Currently being probed:





Future:

Why look for CLFV?

- SM prediction: $U(1)_{L_{\mu}-L_{\tau}} \times U(1)_{L_{\mu}+L_{\tau}-2L_{e}}$, i.e. no LFV!
 - Background-free searches, high sensitivity.
- Neutrino oscillations = LFV!
 - Induced CLFV tiny: CLFV is *complementary*.
- New physics generically and easily gives testable CLFV.
 - Predictions require fixed flavor structure (PMNS, CKM) and new scale (g-2, LFUV in B, W-mass, DM,...).
- Huge landscape, must observe μ and τ CLFV to check if/how

$$\mathsf{U}(1)_{\mathsf{L}_{\mu}-\mathsf{L}_{\tau}}\times\mathsf{U}(1)_{\mathsf{L}_{\mu}+\mathsf{L}_{\tau}-2\mathsf{L}_{\mathsf{e}}}$$

is broken in charged-lepton sector.

Probing *light* particles

- SMEFT only works for *heavy* new particles!
- *Light* new particles X give new signatures:
- $\mu \rightarrow e \ X \text{ or } \tau \rightarrow \ell \ X$, followed by (displaced) $X \rightarrow \ell^+ \ell^-$, $\gamma \gamma$? [Heeck & Rodejohann, PLB '18; Cheung++, JHEP '21]
- Mu3e and Belle II can improve limits, maybe others too? [i Tormo++, PRD '11; Uesaka, PRD '20; Calibbi, Redigolo, Ziegler, Zupan, JHEP '21]
- Light particles as mediators change rate expectations.
- X = axion/ALP/majoron/familon/Z', connected to DM?
- Or: SMEFT + X.

[Georgi, Kaplan, Randall, '86; Brivio++, '17; Dror, Lasenby, Pospelov, '17 & '19]

Far from finished!

Summary

- Charged LFV gives info *complementary* to v oscillations.
- Generically predicted by BSM, could be around the corner.
- Difficult to predict LFV rates, needs
 - Fixed flavor structure (neutrino mass, CKM?)
 - Fixed new physics scale (DM, anomalies?)
- Light new physics open new avenues.
- Hope for sign in Mu3e, MEG-II, Belle-II, Mu2e, LHC(b),...

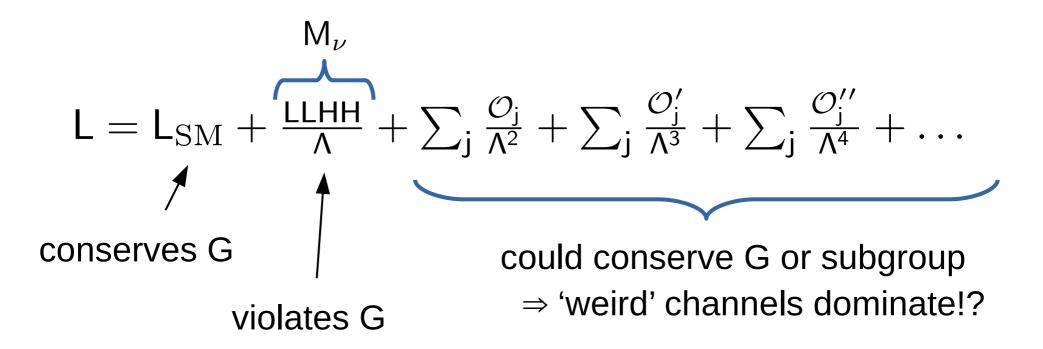
Explore every corner of our lamppost!

Backup

-

Effective field theory view

- SM symmetry: $G = U(1)_{B-L} \times U(1)_{L_{\mu}-L_{\tau}} \times U(1)_{L_{\mu}+L_{\tau}-2L_{e}}$.
- Effective field theory with Majorana v:



Upcoming CLFV

LORENZO CALIBBI and GIOVANNI SIGNORELLI

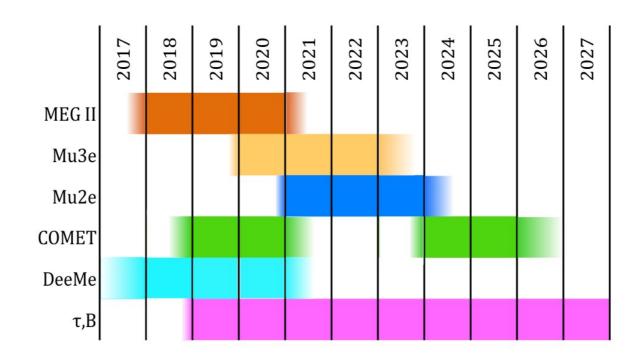
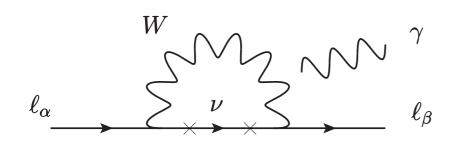


Figure 47. – Projected time lines for different projects searching for CLFV decays. MEG IIis expected to start data taking in 2018 after an engineering run in 2017; Mu3e magnet and detectors are expected at the end of 2019; Mu2e foresees three years of data taking starting in 2021; COMET Phase-I is expected to start commissioning and data taking in 2018 for two-three years, followed by a stop to develop and deploy the beamline and detectors for Phase-II; DeeMe is expected to start soon and take data with graphite and silicon carbide targets in sequence; Belle II is schedule to start data taking at end 2018.

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Neutrino mass \Rightarrow charged LFV?

• SM + Dirac neutrinos: $L = L_{SM} - (y\overline{L}H\nu_R + h.c.) + i\overline{\nu}_R \partial \!\!\!/ \nu_R$



$$\begin{array}{l} (\mathsf{yLH}\nu_{\mathsf{R}} + \mathsf{h.c.}) + \mathsf{I}\nu_{\mathsf{R}} \phi \nu_{\mathsf{R}} \\ \mathsf{m}_{\nu} = \mathsf{y} \langle \mathsf{H} \rangle \\ = \mathsf{U} \operatorname{diag}(\mathsf{m}_{1}, \mathsf{m}_{2}, \mathsf{m}_{3}) \mathsf{V}_{\mathsf{R}} \\ \overset{!}{\lesssim} \mathsf{eV} \end{array}$$

• All CLFV is GIM suppressed:

$$\frac{\Gamma(\ell_{\alpha} \to \ell_{\beta} \gamma)}{\Gamma(\ell_{\alpha} \to \ell_{\beta} \nu_{\alpha} \overline{\nu}_{\beta})} \simeq \frac{3\alpha_{\rm EM}}{32\pi} \left| \sum_{j=2,3} U_{\alpha j} \frac{\Delta m_{j1}^2}{M_W^2} U_{j\beta}^{\dagger} \right|^2 < 5 \times 10^{-53}$$

[1977: Petcov; Bilenky, Petcov, Pontecorvo; Marciano, Sanda; Lee, Pakvasa, Shrock, Sugawara]

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Seesaw mass \Rightarrow charged LFV?

• SM + seesaw neutrinos: $L = L_{SM} + i\overline{N}_R \partial N_R$

$$-\left(\frac{1}{2}M_{R}\overline{N}_{R}^{c}N_{R}+y\overline{L}HN_{R}+h.c.\right)$$
• Violates ΔL = 2. For large M_{R} :
 $m_{D}\overline{\nu}_{L}N_{R}$

 $M_N\simeq M_R\,,\qquad M_\nu\simeq -m_D M_R^{-1}m_D^T=U^*\operatorname{diag}(m_1,m_2,m_3)U^\dagger.$

• Majorana neutrinos!

• LFV:
$$\frac{\Gamma(\ell_{\alpha} \to \ell_{\beta} \gamma)}{\Gamma(\ell_{\alpha} \to \ell_{\beta} \nu_{\alpha} \overline{\nu}_{\beta})} \simeq \frac{3\alpha_{\rm EM}}{8\pi} |(m_{\rm D} M_{\rm R}^{-2} m_{\rm D}^{\dagger})_{\alpha\beta}|^{2}.$$
[Cheng & Li '80]
$$\mathcal{O}(M_{\nu}^{4}/m_{\rm D}^{4})$$
Not true with fine-tuning or structure in $m_{\rm D}$.

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Seesaw parameters

 $L = L_{\rm SM} + i \overline{N}_R \partial \!\!\!/ N_R - (\tfrac{1}{2} M_R \overline{N}_R^c N_R + m_D \overline{\nu}_L N_R + h.c.)$

 $\Rightarrow \quad \mathsf{M}_{\nu} \simeq -\mathsf{m}_{\mathsf{D}}\mathsf{M}_{\mathsf{R}}^{-1}\mathsf{m}_{\mathsf{D}}^{\mathsf{T}} \quad \& \quad \mathrm{BR}(\ell_{\alpha} \to \ell_{\beta}\gamma) \propto |(\mathsf{m}_{\mathsf{D}}\mathsf{M}_{\mathsf{R}}^{-2}\mathsf{m}_{\mathsf{D}}^{\dagger})_{\alpha\beta}|^{2}.$

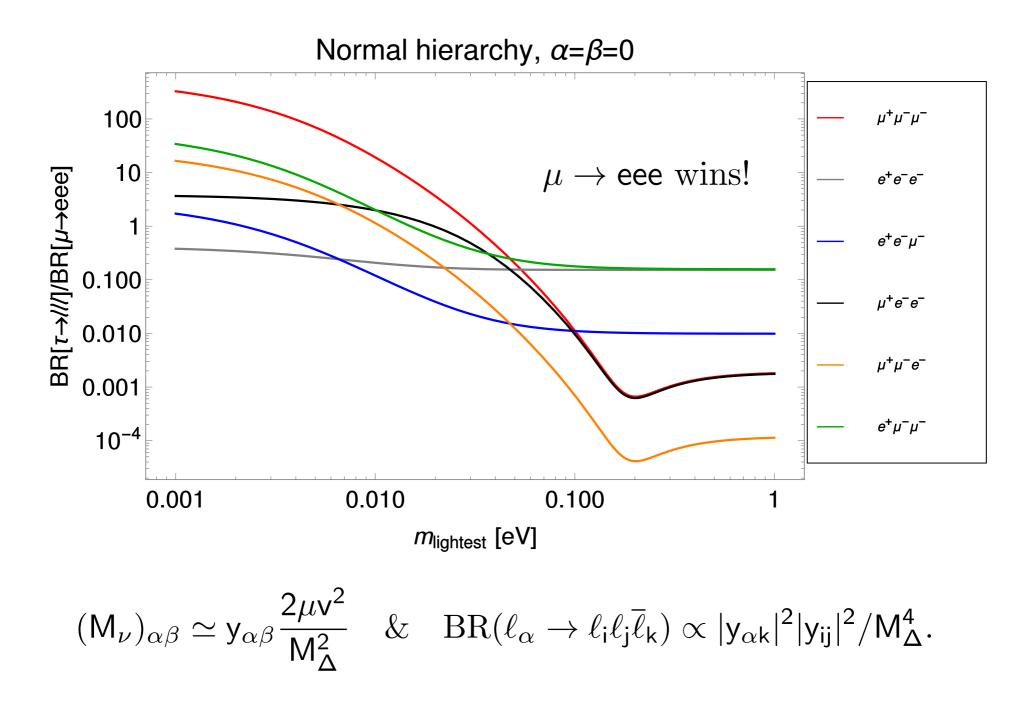
• One to one correspondence

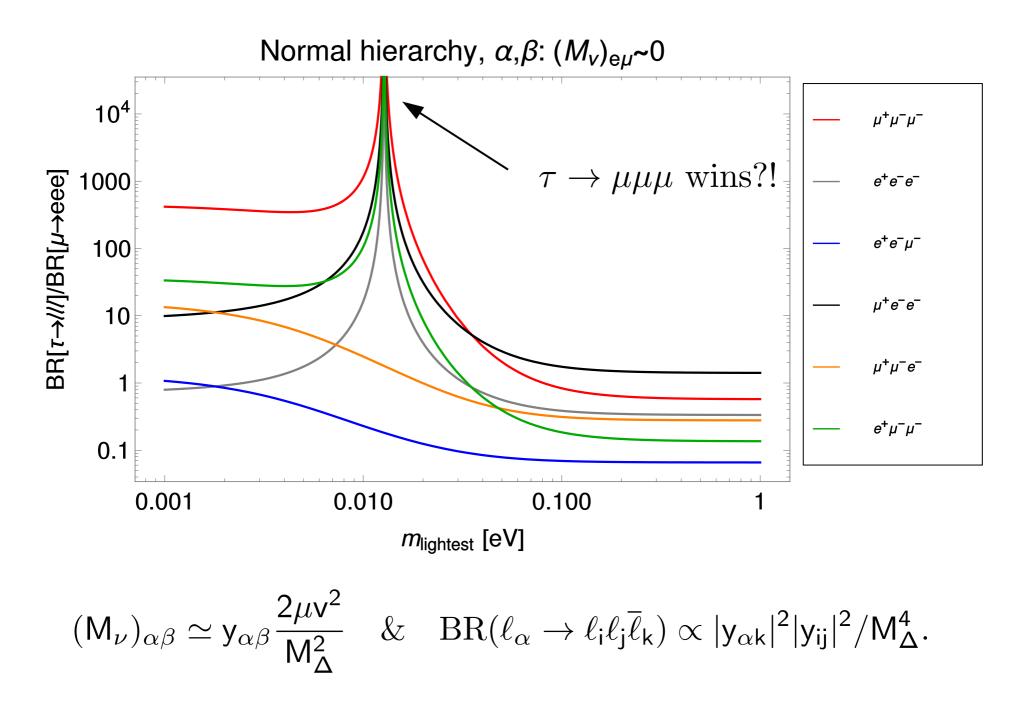
 $\{m_D, M_R\} \leftrightarrow \{M_\nu, m_D M_R^{-2} m_D^{\dagger}\}.$

[Broncano, Gavela, Jenkins, hep-ph/0210271]

- Or: unique d=6 operator $(yM_R^{-2}y^{\dagger})(\overline{L}H)(i\partial)(H^{\dagger}L)$.
- Gives LFV and non-unitary PMNS.

LFV *complementary* to M["].



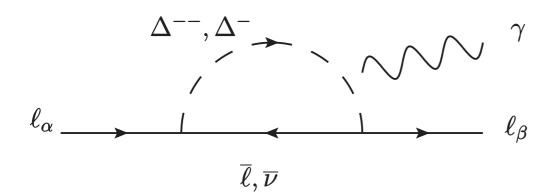


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Scalar-triplet seesaw

$$(\mathsf{M}_{\nu})_{\alpha\beta} \simeq \mathsf{y}_{\alpha\beta} \frac{2\mu \mathsf{v}^2}{\mathsf{M}_{\Delta}^2} \quad \& \quad \mathrm{BR}(\ell_{\alpha} \to \ell_i \ell_j \overline{\ell}_k) \propto |\mathsf{y}_{\alpha k}|^2 |\mathsf{y}_{ij}|^2 / \mathsf{M}_{\Delta}^4.$$

• But at loop level:



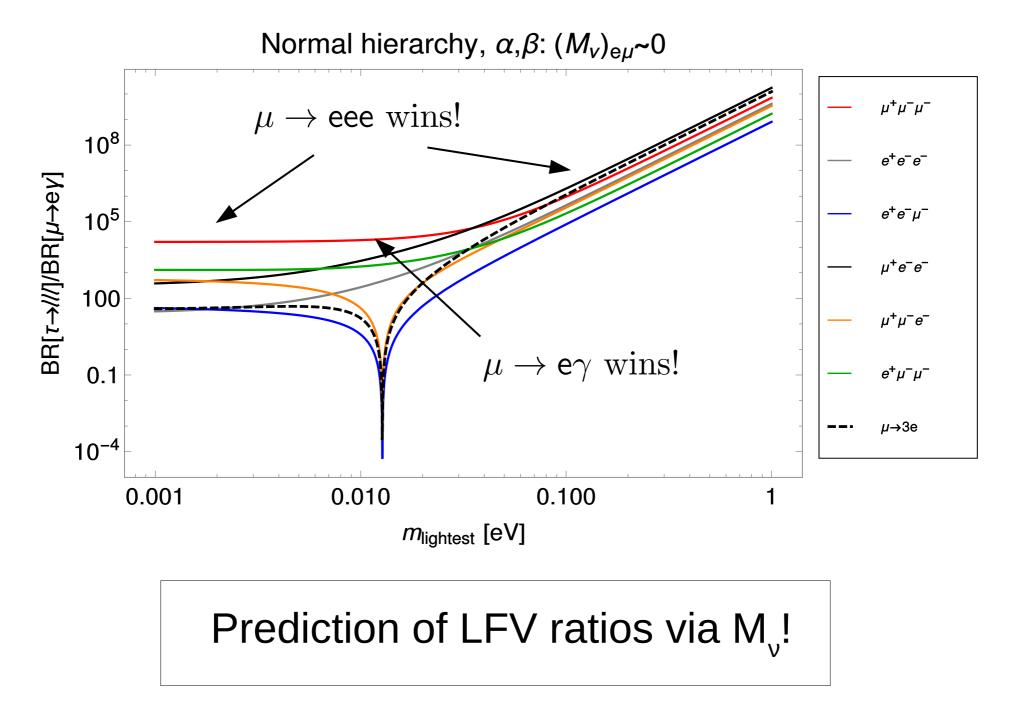
$$\mathrm{BR}(\ell_{lpha}
ightarrow \ell_{eta} \gamma) \propto rac{|(\mathbf{y}^{\dagger}\mathbf{y})_{lphaeta}|^2}{\mathsf{M}_{\Delta}^4}.$$

[Pich, Santamaria, Bernabeu, '84]

• $\mu \rightarrow$ 3e could be 0, but $\mu \rightarrow$ ey cannot (since θ_{13}).

[Chakrabortty++, 1204.1000]

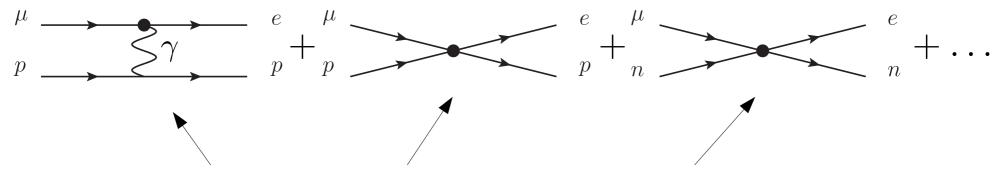
Prediction of LFV ratios via M_v!



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The inverse problem

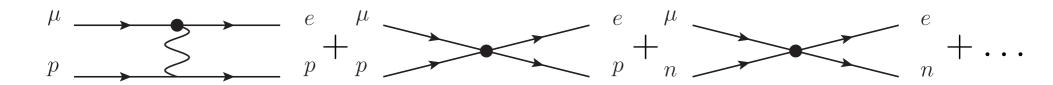
- If we see CLFV, can we pin down the underlying operator?
 - In many cases: Yes! (e.g. $\mu \rightarrow e\gamma \leftrightarrow dipole$)
 - $\mu \rightarrow e$ conversion in nucleus: No!



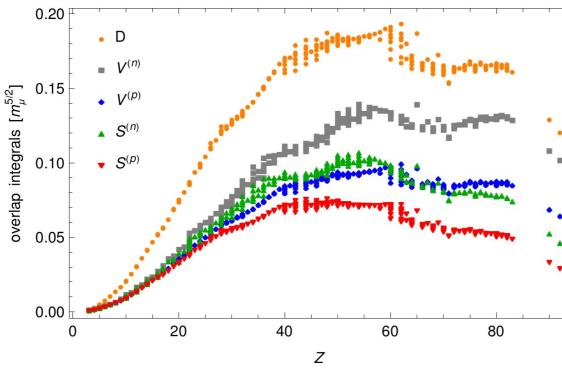
Relative contributions depend on nucleus: Z, N, spin!

Need to observe µ → e conversion in different nuclei!
 [Kitano, Koike, Okada, PRD '07; Cirigliano++, PRD '09; Davidson++, '18]

 $\mu \rightarrow e$ conversion



 Assuming spin-independent conversion:

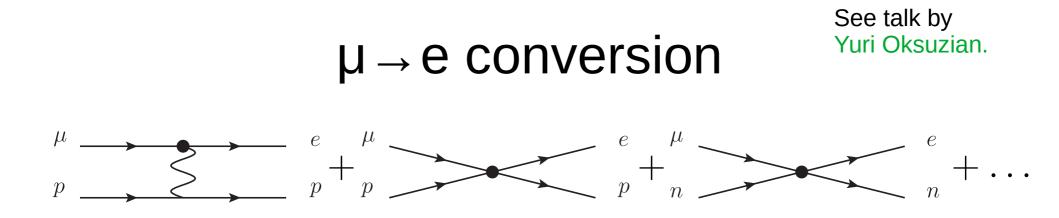


 $BR_{SI} = \frac{32G_F^2}{\Gamma_{capture}} \begin{bmatrix} |\boldsymbol{v} \cdot \boldsymbol{C}_L|^2 + |\boldsymbol{v} \cdot \boldsymbol{C}_R|^2 \end{bmatrix}$ $(\mathbf{v} \cdot \mathbf{c}_R)^2 = \mathbf{v} \cdot \mathbf{c}_R^2$ $(\mathbf{v} \cdot \mathbf{c}_R)^2 = \mathbf{v} \cdot \mathbf{c}_R^2$

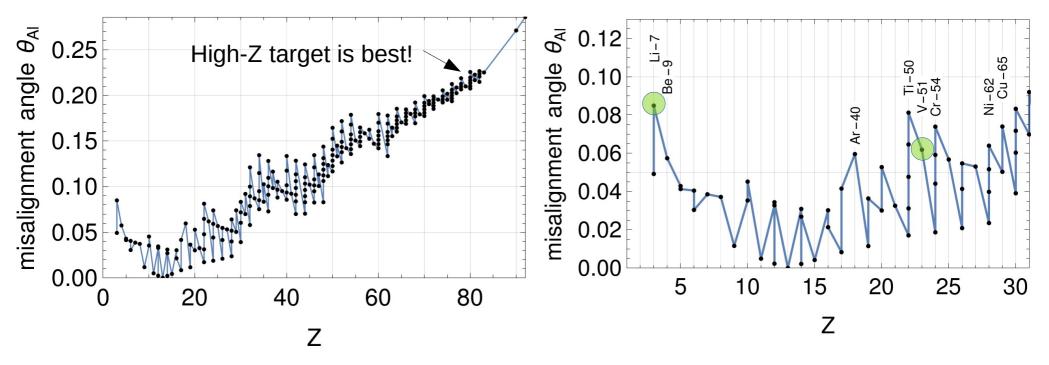
$$oldsymbol{v}\equiv\left(rac{D}{4},V^{(p)},S^{(p)},V^{(n)},S^{(n)}
ight)$$

To measure the Wilson coefficients, use nuclei whose **v** are maximally misaligned. [Davidson, Kuno, Yamanaka, PLB '19]

[Kitano, Koike, Okada, PRD '07; Heeck, Szafron, Uesaka, NPB '22]



• Misalignment with aluminium (target in COMET & Mu2e):

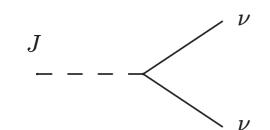


• At low Z, Li-7 and V-51 can distinguish proton/neutron.

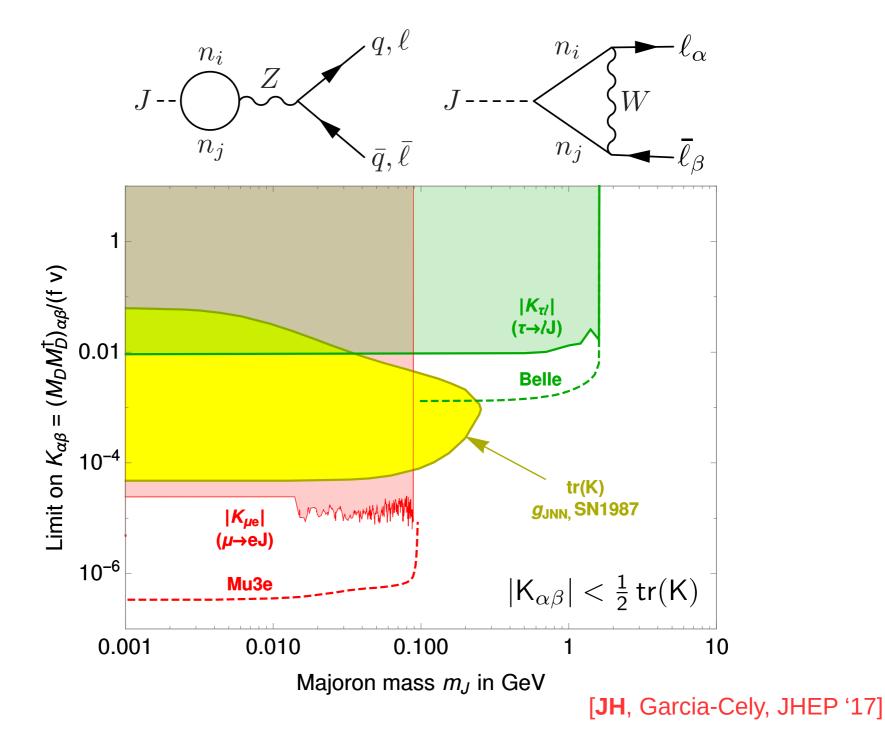
[Davidson, Kuno, Yamanaka, PLB '19; Heeck, Szafron, Uesaka, NPB '22]

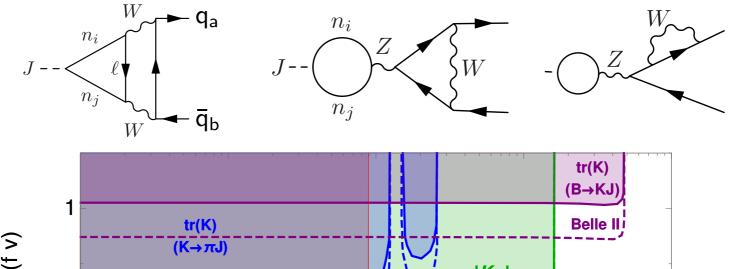
Probing light particles

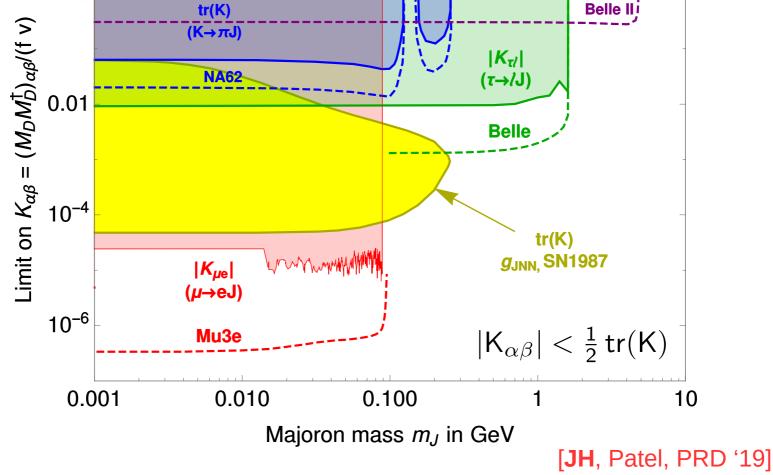
- Mu3e: BR($\mu \rightarrow e X$) from 10^{-6} to 10^{-8} .
- Belle II: BR($\tau \rightarrow \ell X$) from 10^{-3} to 10^{-5} . [JH, PLB '16]
- Followed by (displaced) $X \rightarrow \ell^+ \ell^-$, $\gamma \gamma$? [JH, Rodejohann, PLB '18]
- Example: Majoron.
 - Pseudo-Goldstone boson of lepton number.
 - Potential dark matter candidate. [JH, Garcia-Cely, JHEP '17]
 - Tree-level coupling only to neutrinos.



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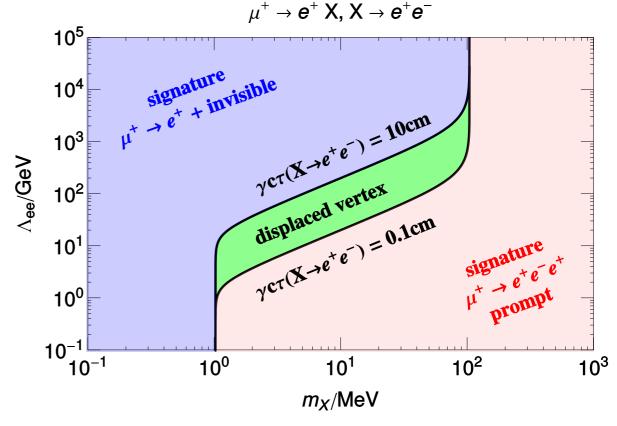


$\mu \rightarrow e \ X \ with \ X \rightarrow \ visible$

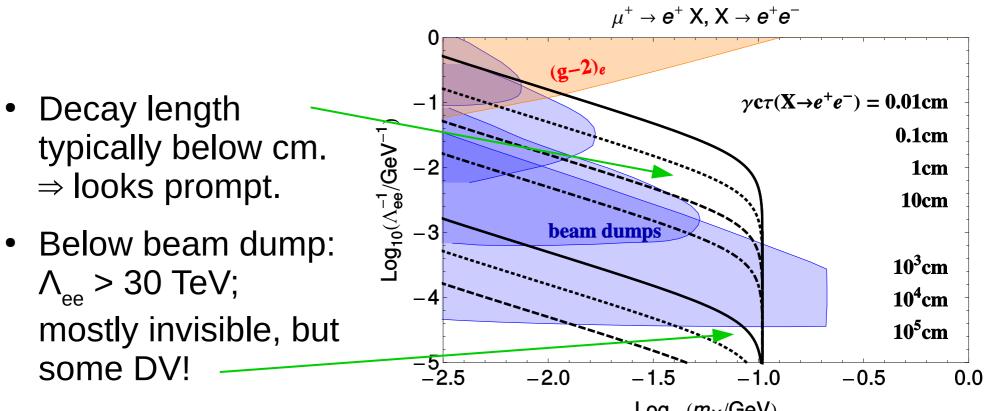
- Take $X e \gamma_5 e m_e / \Lambda_{ee}$.
- Decay length determines signature.
- Displaced vertex gives new observable.
 [JH, Rodejohann, PLB '18]
- Muon at rest:

$$\gamma c au \simeq rac{\pi m_{\mu} \Lambda_{ee}^2}{m_e^2 m_X^2} \simeq 2.5 \, {
m cm} \left(rac{\Lambda_{ee}}{100 \, {
m GeV}}
ight)^2 \left(rac{10 \, {
m MeV}}{m_X}
ight)^2.$$

Sub-GeV X with ee coupling allowed?



 $\mu \rightarrow e X$ with $X \rightarrow e e$

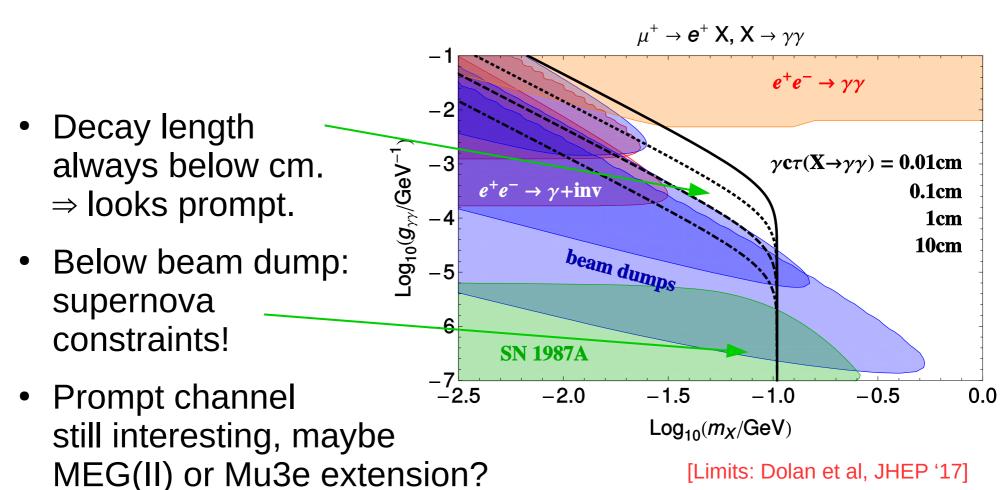


 $\text{Log}_{10}(m_X/\text{GeV})$

$$\begin{split} \mathrm{BR}(\mu \to \mathsf{eX}) \mathrm{BR}(\mathsf{X} \to \mathsf{ee}) (1 - \mathsf{P}(\mathsf{I}_{\mathrm{dec}})) \\ \simeq \mathrm{BR}(\mu \to \mathsf{eX}) \frac{\mathsf{I}_{\mathrm{dec}}}{\gamma \mathsf{c} \tau} \,. \end{split}$$

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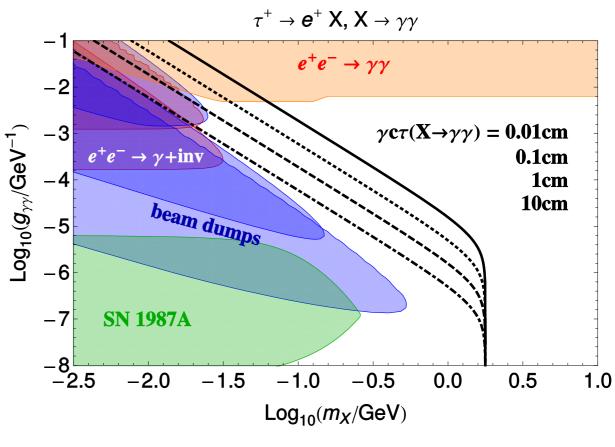
 $\mu \rightarrow e X$ with $X \rightarrow \gamma \gamma$



Muons difficult, taus easier.

$\tau \rightarrow e \ X \ with \ X \rightarrow \ visible$

- Tau at rest, higher X boost.
- Arbitrary decay lengths possible.
- Similar for $X \rightarrow ee, \mu\mu, \mue.$
- Worthwhile in LHCb and Belle (II).



[Limits: Dolan et al, JHEP '17]

New signatures from light physics!