Theory of Charged Lepton Flavor Violation

Why we should keep searching for CLFV.

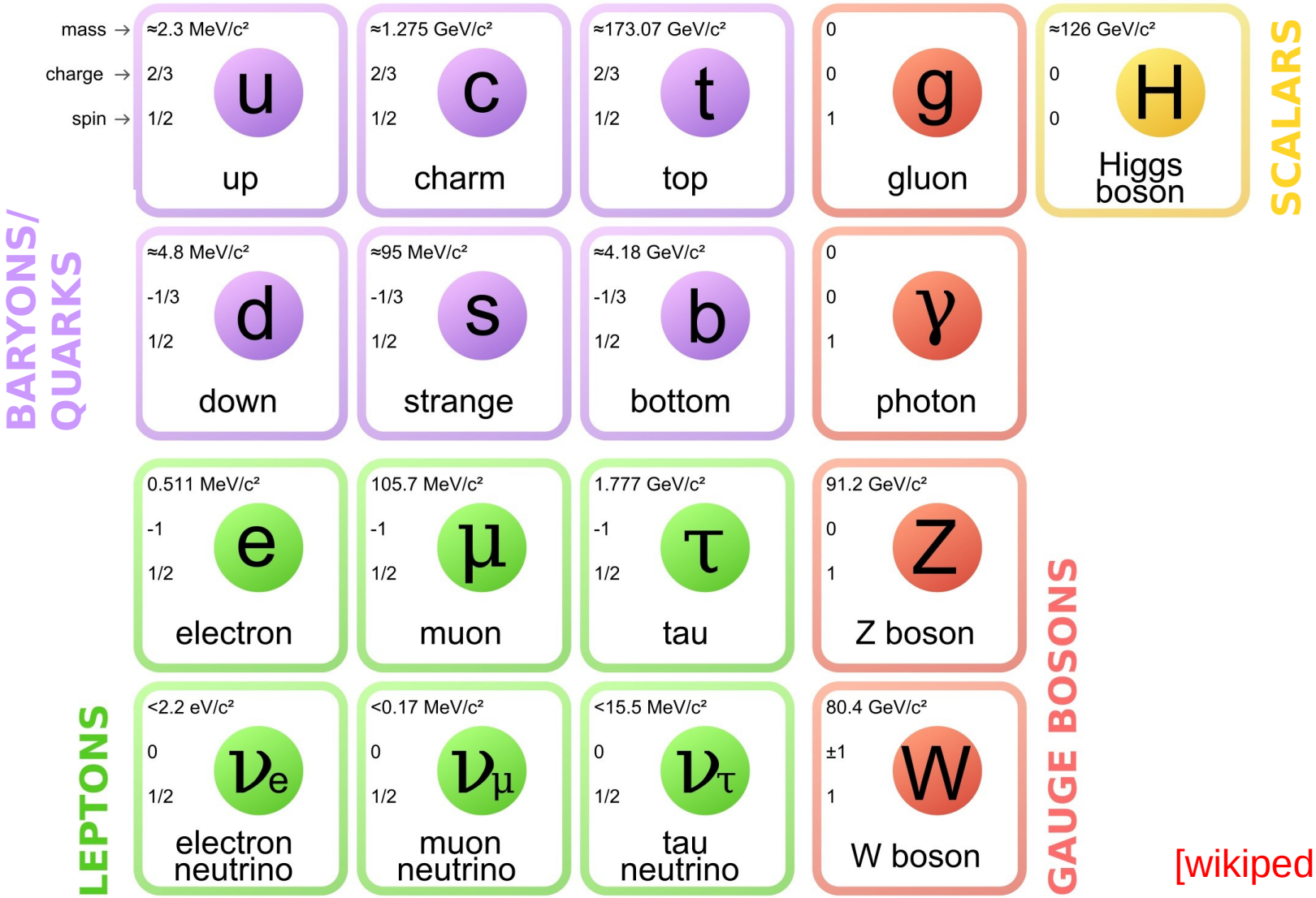
Julian Heeck

Snowmass Community Summer Study, Seattle

07/22/2022

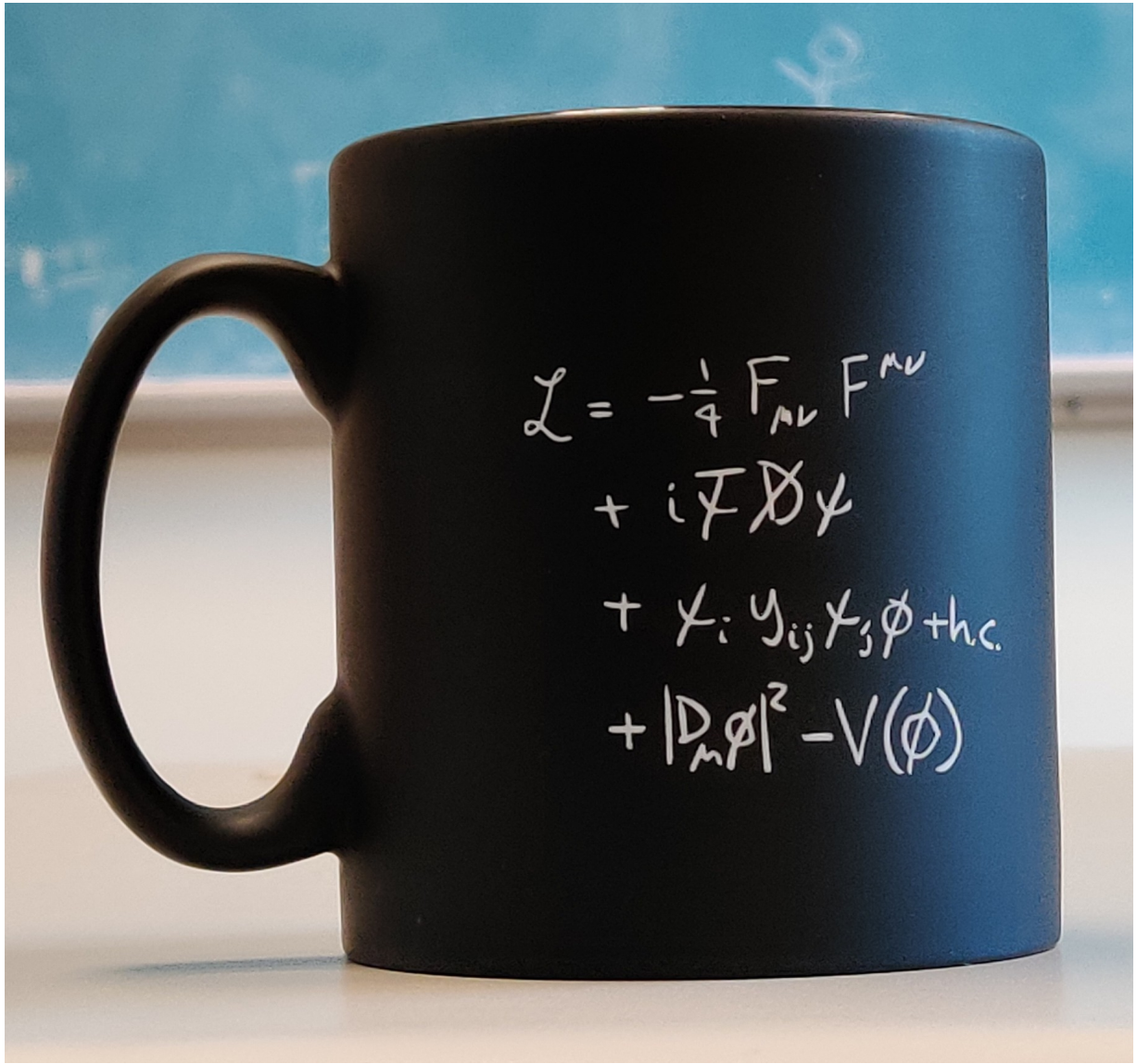
University of Virginia

Elementary particles



[wikipedia]

The Standard Model



Symmetries of the Standard Model

- Rephasing lepton and quark fields:

$$\mathrm{U}(1)_{\mathrm{B}} \times \mathrm{U}(1)_{\mathrm{L_e}} \times \mathrm{U}(1)_{\mathrm{L}_\mu} \times \mathrm{U}(1)_{\mathrm{L}_\tau}$$
$$=$$
$$\cancel{\mathrm{U}(1)}_{\mathrm{B+L}} \times \mathrm{U}(1)_{\mathrm{B-L}} \times \mathrm{U}(1)_{\mathrm{L}_\mu - \mathrm{L}_\tau} \times \mathrm{U}(1)_{\mathrm{L}_\mu + \mathrm{L}_\tau - 2\mathrm{L_e}} \,.$$

- Broken non-perturbatively, but unobservable. ['t Hooft, PRL '76]

- True accidental global symmetry:

$$\boxed{\mathrm{U}(1)_{\mathrm{B-L}} \times \mathrm{U}(1)_{\mathrm{L}_\mu - \mathrm{L}_\tau} \times \mathrm{U}(1)_{\mathrm{L}_\mu + \mathrm{L}_\tau - 2\mathrm{L_e}} \,.}$$

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RF4

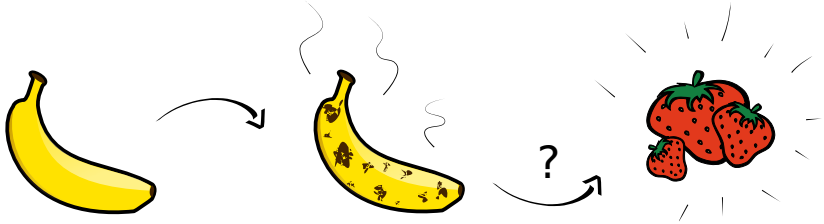
Lepton flavor conservation!

Prediction of Standard Model.

Why look for CLFV?

- SM prediction: $\mathrm{U}(1)_{\mathrm{L}_\mu - \mathrm{L}_\tau} \times \mathrm{U}(1)_{\mathrm{L}_\mu + \mathrm{L}_\tau - 2\mathrm{L}_e}$, i.e. no LFV!
  - Background-free searches, high sensitivity.

Flavor violating decays

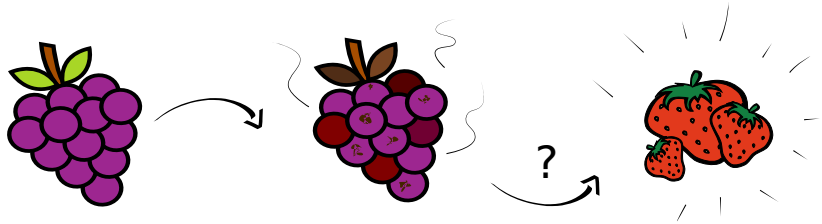


- Prime example: $\mu \to e\gamma$ @ MEG.
- Observation = new particles.
- $\mu \to e$ conversion @ Mu2e can probe scales up to 10^7 GeV.

See following talks.

LFV	process	current	future	exp
$\lvert\Delta L_\mu\rvert = 1$	$\mu \to e\gamma$	4.2×10^{-13}	6×10^{-14}	MEG-II
	$\mu \to e\bar{e}e$	1.0×10^{-12}	10^{-16}	Mu3e
	$\mu \to e$ conv.	$\mathcal{O}(10^{-12})$	10^{-16}	Mu2e, COMET
$\lvert\Delta L_e\rvert = \lvert\Delta L_\mu\rvert$	$h \to e\bar{\mu}$	6.1×10^{-5}	10^{-5}	LHC
	$Z \to e\bar{\mu}$	7.5×10^{-7}	10^{-10}	FCC-ee
	had$\to e\bar{\mu}$(had)	4.7×10^{-12}	10^{-12}	NA62

Flavor violating decays



- Produce tauons at B factories (BaBar, Belle (II), LHCb).
- Observation = new particles.
- $\tau^- \to e^- e^+ e^-$ @ Belle II will probe scales up to 2×10^4GeV.

See talk by Swagato Banerjee.

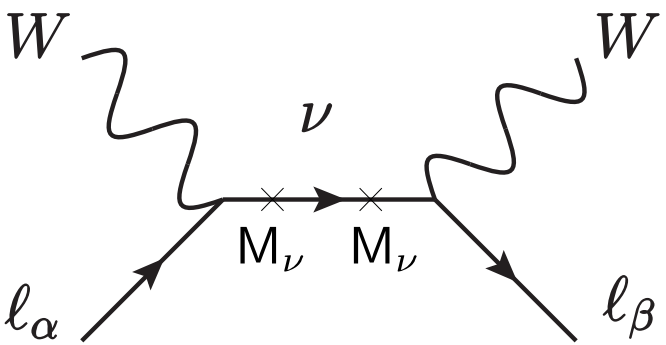
LFV	process	current	future	exp
$\lvert\Delta L_e\rvert = \lvert\Delta L_\tau\rvert = 1$	$\tau \to e\gamma$	3.3×10^{-8}	10^{-9}	Belle II
	$\tau \to e\bar{\ell}\ell$	2.7×10^{-8}	10^{-9}	Belle II
	$\tau \to e\,\mathrm{had}$	$\mathcal{O}(10^{-8})$	10^{-9}	Belle II
	$h \to e\bar{\tau}$	4.7×10^{-3}	10^{-4}	LHC
	$Z \to e\bar{\tau}$	9.8×10^{-6}	10^{-9}	FCC-ee
	$\mathrm{had} \to e\bar{\tau}(\mathrm{had})$	$\mathcal{O}(10^{-6})$	–	Belle II

Neutrino oscillations = flavor violation

- Observations of $\nu_\alpha \to \nu_\beta$ prove that $M_\nu \neq 0$ and

  $U(1)_{L_\mu - L_\tau} \times U(1)_{L_\mu + L_\tau - 2L_e}$

  is broken!



- Amplitudes for *charged* lepton flavor violation are suppressed:

$$\mathcal{A}(\ell_\alpha^- \to \ell_\beta^-) \propto \frac{(M_\nu M_\nu^\dagger)_{\alpha\beta}}{M_W^2} < 10^{-24}\,.$$

Great goalpost for Snowmass 3000!

- Most (neutrino mass) models also generate CLFV rates unsuppressed by M_ν that could be observable.

Why look for CLFV?

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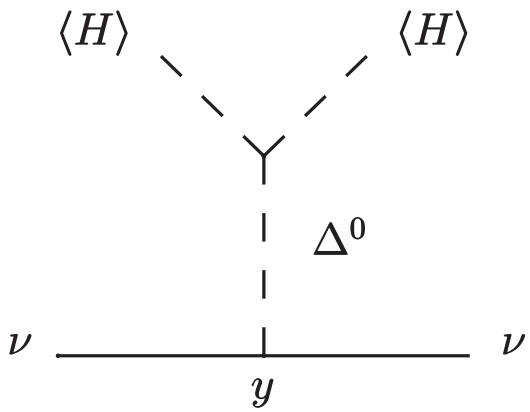
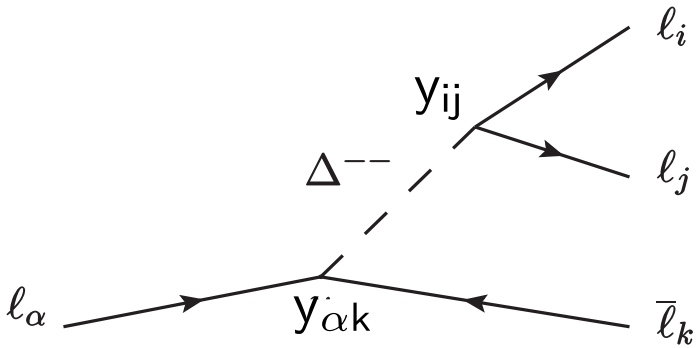
  - Background-free searches, high sensitivity.
- Neutrino oscillations = LFV!
  - Induced CLFV tiny: CLFV is *complementary*.

Scalar-triplet seesaw

[Konetschny & Kummer '77; Magg & Wetterich, '80; Schechter & Valle '80; Cheng & Li, '80; Mohapatra & Senjanovic, '81]

$$L = L_{SM} + |D_\alpha \Delta|^2 - (y_{\alpha\beta} \overline{L}^c_\alpha \Delta L_\beta + \mu H \Delta H + h.c.)$$





$$\Rightarrow \quad (M_\nu)_{\alpha\beta} \simeq y_{\alpha\beta} \frac{2\mu v^2}{M_\Delta^2} \quad \& \quad BR(\ell_\alpha \to \ell_i \ell_j \bar{\ell}_k) \propto |(M_\nu)_{\alpha k}|^2 |(M_\nu)_{ij}|^2 .$$

[Pich, Santamaria, Bernabeu, '84; Abada++, 0707.4058]

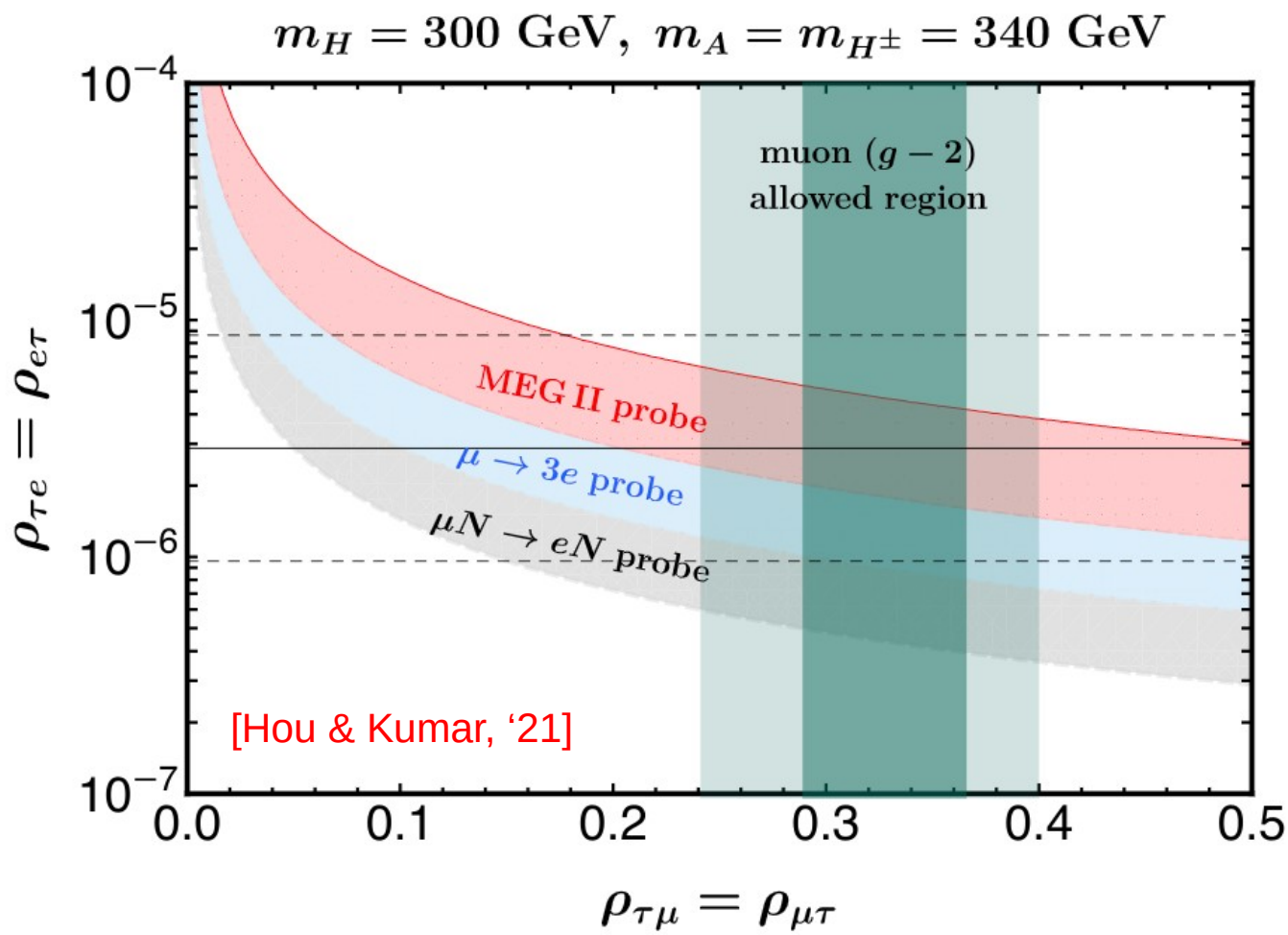
Cleanest prediction : $BR(\tau \to \mu\gamma) \simeq 23\ BR(\tau \to e\gamma) \simeq 3.5\ BR(\mu \to e\gamma)$.

Prediction of LFV *ratios* via M_ν!

CDF's W-mass first hint for this triplet with O(100 GeV) mass? [Heeck, 2204.10274]

Predictions of LFV *rates*

- Need flavor structure (e.g. from neutrino mass) and a new mass scale to predict LFV rates.
- Tie LFV to anomalies
  - $(g\text{-}2)_\mu$:



See talk by Wouter Dekens on Sunday.

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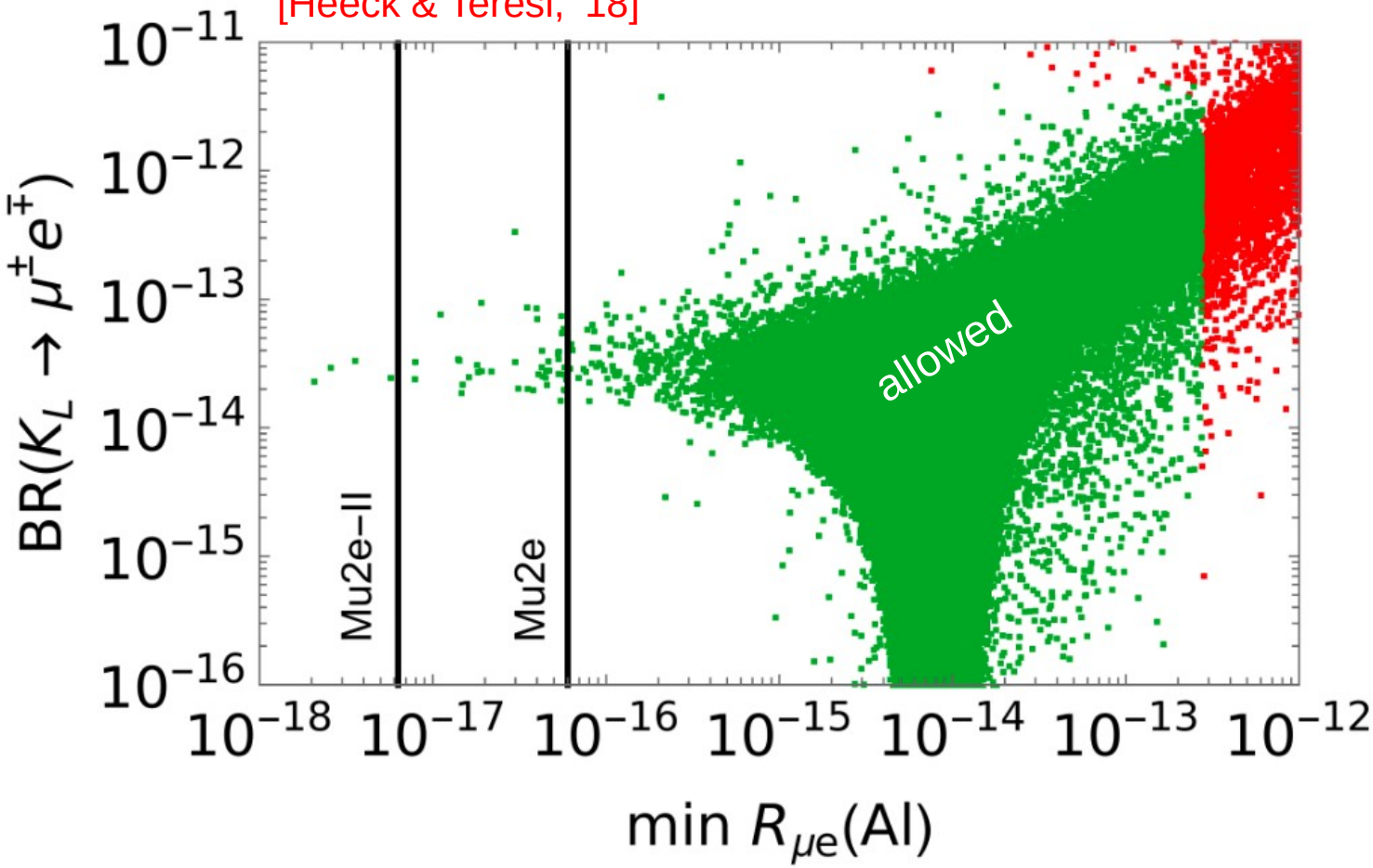
  - B-mesons:

[Heeck & Teresi, '18]



See talk by Chris Polly on Sunday.

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- Need flavor structure (e.g. from neutrino mass) and a new mass scale to predict LFV rates.
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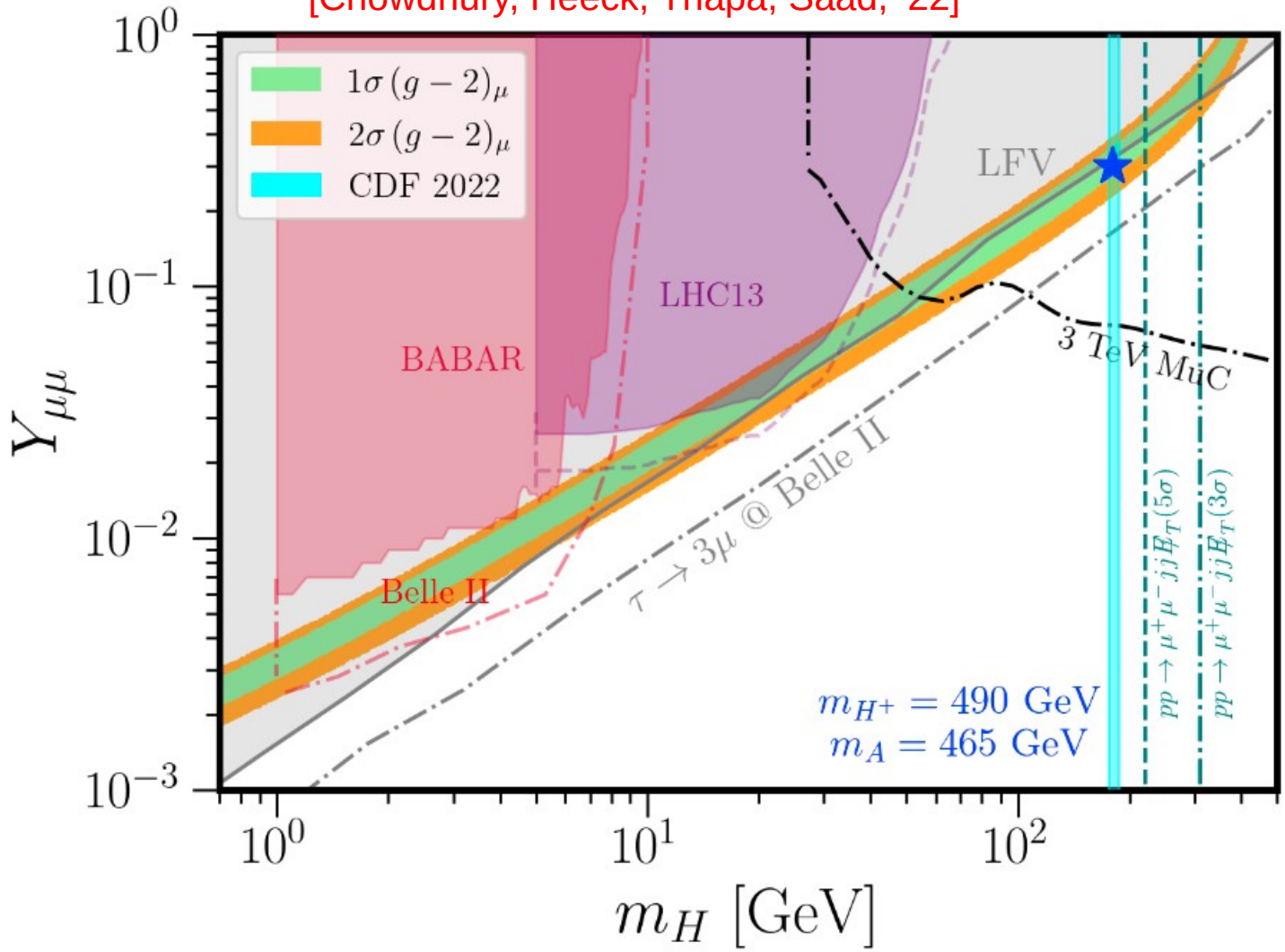
  - B-mesons
  - W-mass:

[Chowdhury, Heeck, Thapa, Saad, '22]



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  - Background-free searches, high sensitivity.
- Neutrino oscillations = LFV!
  - Induced CLFV tiny: CLFV is *complementary*.
- New physics generically and easily gives testable CLFV.
  - Predictions require fixed flavor structure (PMNS, CKM) and new scale (g-2, LFUV in B, W-mass, DM,…).

Model independent: SMEFT

- 888 CLFV operators at d=6:

$$\frac{C_{ijnm}}{\Lambda^2}\ell_i^c\ell_j\ell_n^c\ell_m\,,\ \frac{C_{ijnm}}{\Lambda^2}\ell_i^c\ell_j d_n^c d_m\,,\ \frac{C_{ij}}{\Lambda^2}\ell_i^c\sigma_{\alpha\beta}\ell_j F^{\alpha\beta}\,,\ldots$$

[Weinberg ‘79; Buchmüller & Wyler, ‘86; Grzadkowski++, ‘10; Fonseca, ‘17]

- Model-dependent coefficients; can get testable rates:

$$\ell\to\ell'\gamma,\ \ell\to\ell'\ell''\ell''',\ \mu\to e\ \mathrm{conv.},\ h\to\ell\ell',\ \mathrm{had}\to\ell\ell',\ \ldots$$

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- Not all constrained, e.g. $\Delta L_\tau = 2$ operators.
- CLFV even sensitive to some d=8 operators, e.g. $\frac{\bar{L}_\mu H e_R\, GG}{\Lambda^4}$.

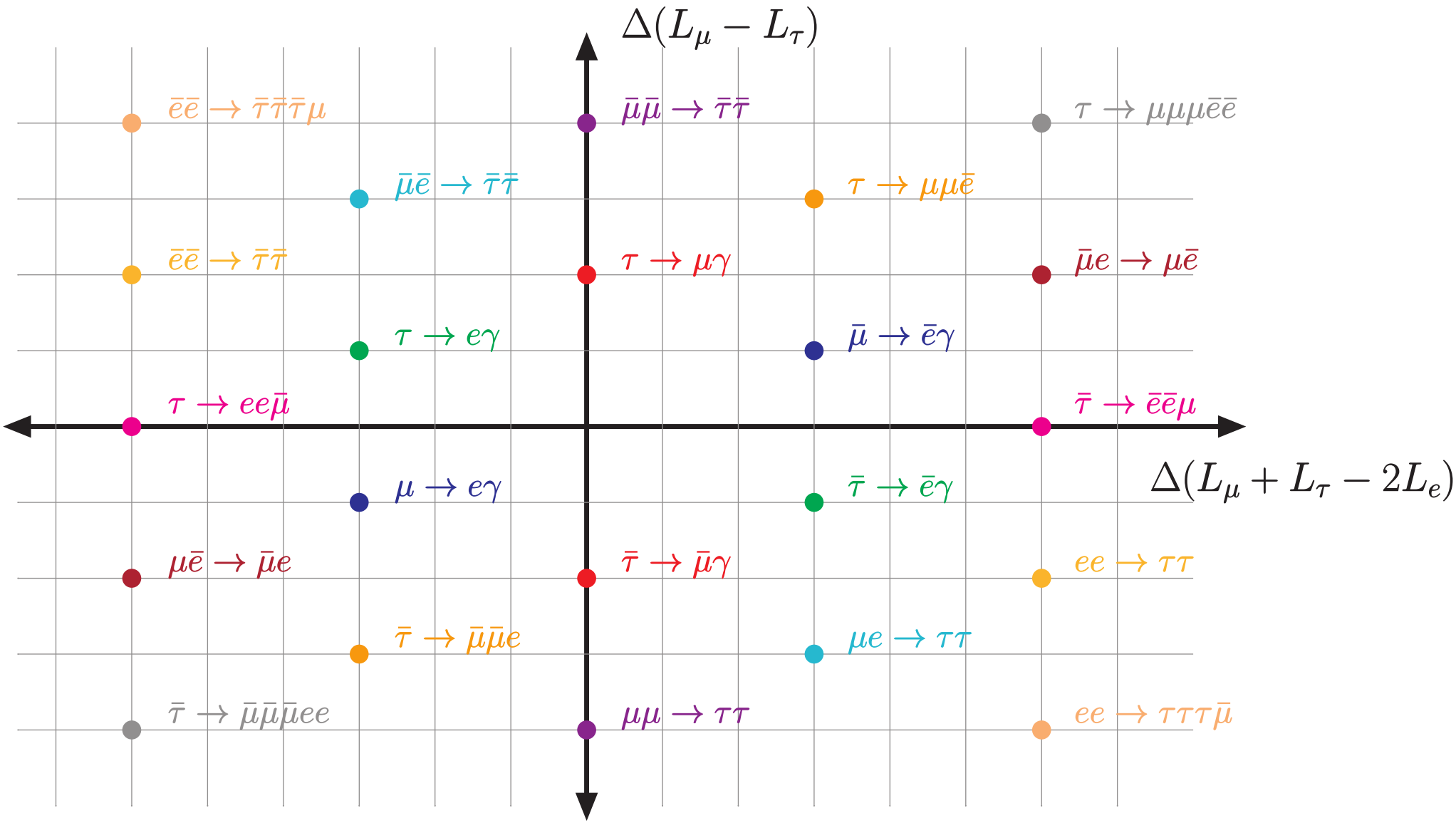
  [Petrov & Zhuridov, '14; Davidson++ 2007.09612; Ardu & Davidson '21; At LHC: Bhattacharya++ '18]

- Not clear if / how $U(1)_{L_\mu - L_\tau}\times U(1)_{L_\mu+L_\tau-2L_e}$ is broken in CLFV.

  [Lew & Volkas, 9410277; Heeck, 1610.07623]

CLFV = breaking of $U(1)_{L_\mu - L_\tau} \times U(1)_{L_\mu + L_\tau - 2L_e}$

[Heeck, 1610.07623]



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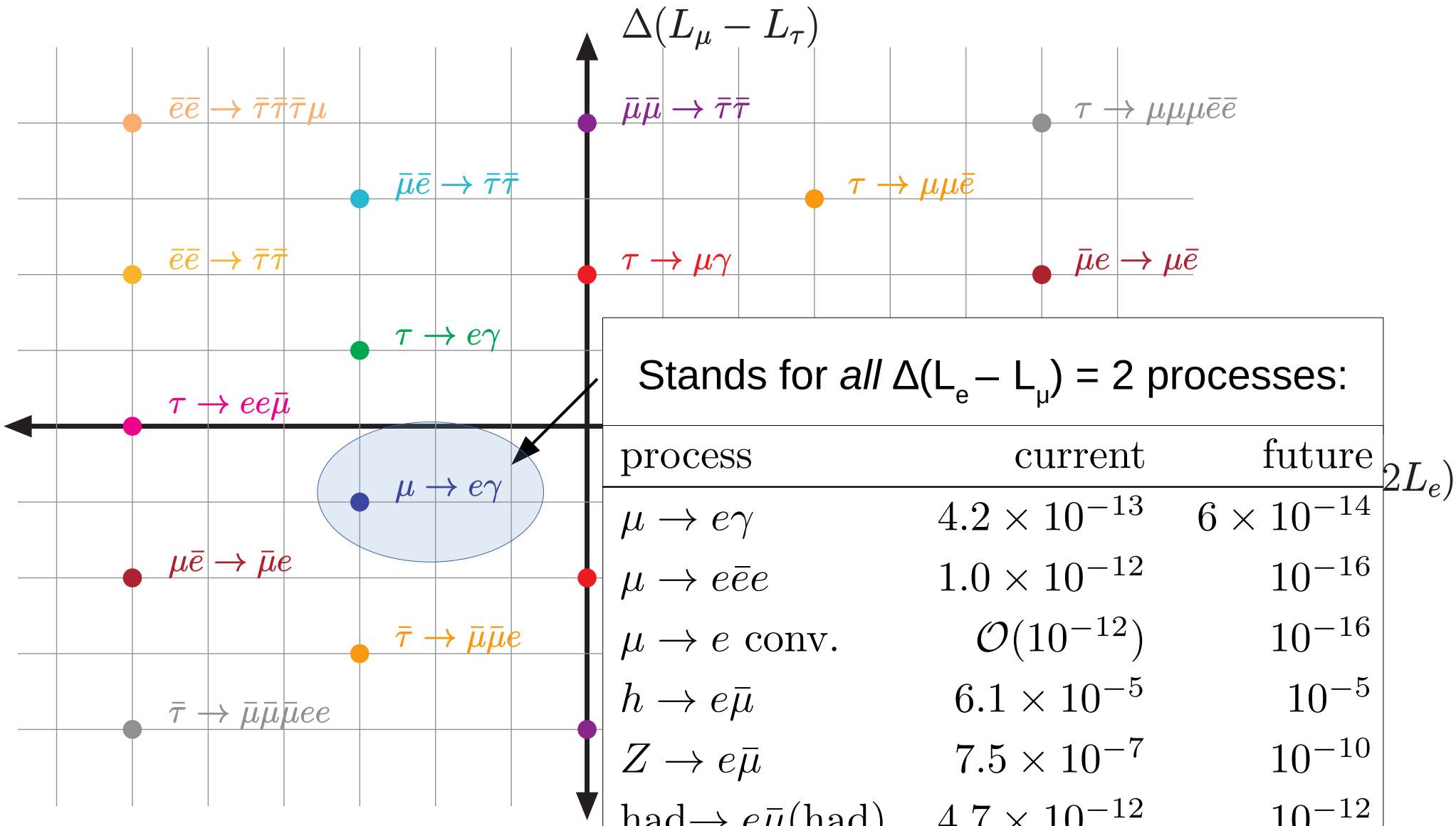

Stands for *all* $\Delta(L_e - L_\mu) = 2$ processes:

process	current	future
$\mu \to e\gamma$	4.2×10^{-13}	6×10^{-14}
$\mu \to e\bar{e}e$	1.0×10^{-12}	10^{-16}
$\mu \to e$ conv.	$\mathcal{O}(10^{-12})$	10^{-16}
$h \to e\bar{\mu}$	6.1×10^{-5}	10^{-5}
$Z \to e\bar{\mu}$	7.5×10^{-7}	10^{-10}
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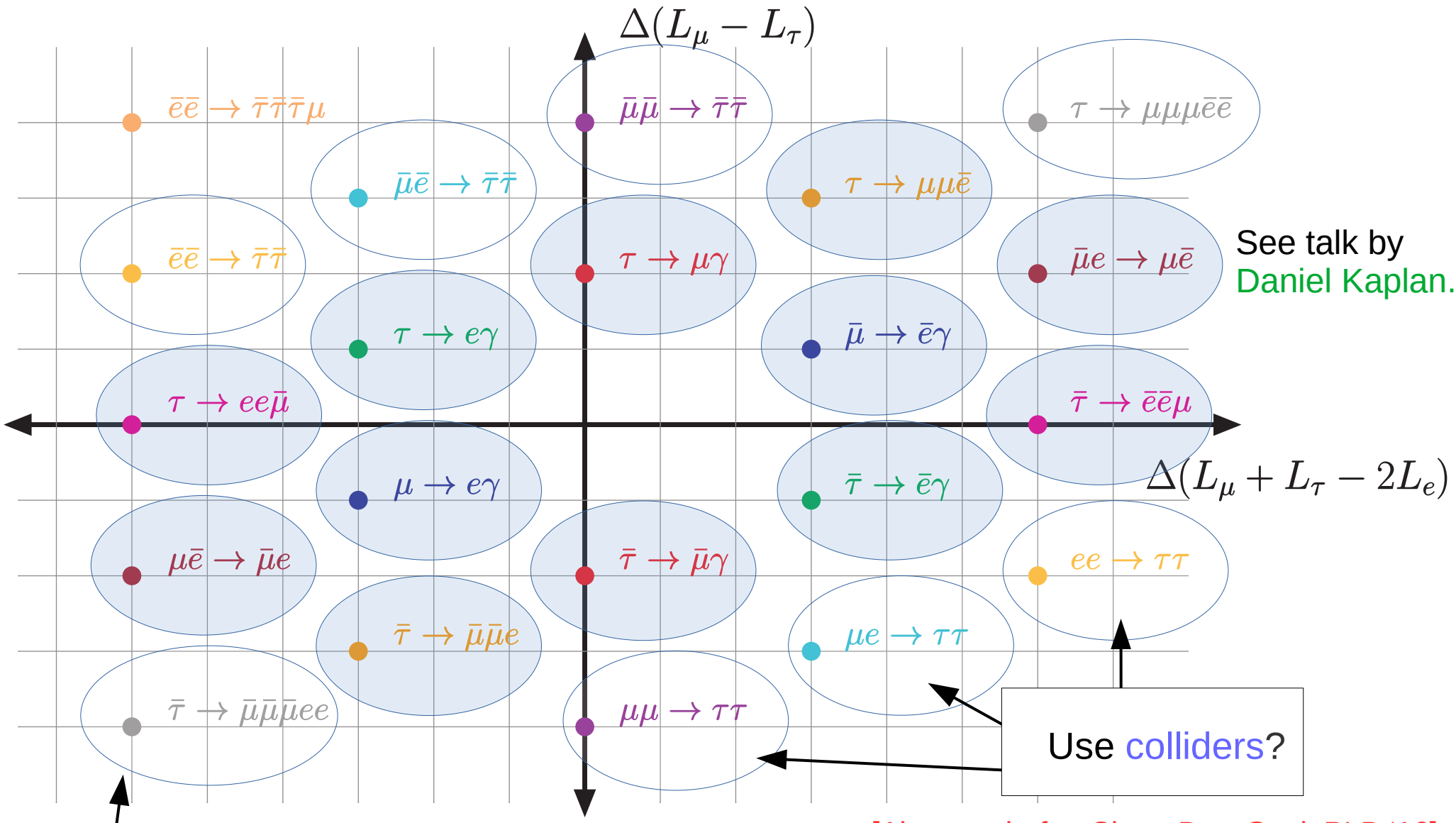
Currently being probed: Future:

[Heeck, 1610.07623]



See talk by Daniel Kaplan.

Use colliders?

[Altmannshofer, Chen, Dev, Soni, PLB '16]

See talk by Wolfgang Altmannshofer.

Belle II, but d=10 operator!

Why look for CLFV?

- SM prediction: $U(1)_{L_\mu - L_\tau} \times U(1)_{L_\mu + L_\tau - 2L_e}$, i.e. no LFV!
  - Background-free searches, high sensitivity.
- Neutrino oscillations = LFV!
  - Induced CLFV tiny: CLFV is *complementary*.
- New physics generically and easily gives testable CLFV.
  - Predictions require fixed flavor structure (PMNS, CKM) and new scale (g-2, LFUV in B, W-mass, DM,…).
- Huge landscape, must observe ***μ and τ*** CLFV to check if/how

$$U(1)_{L_\mu - L_\tau} \times U(1)_{L_\mu + L_\tau - 2L_e}$$

is broken in charged-lepton sector.

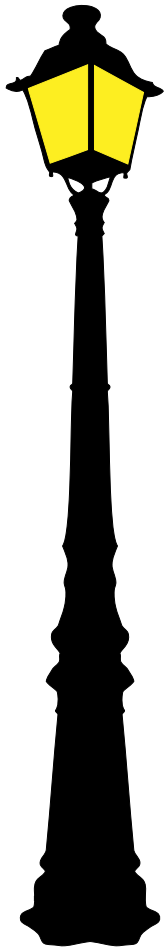
Probing *light* particles

- SMEFT only works for *heavy* new particles!
- *Light* new particles X give new signatures:
- $\mu \to e\, X$ or $\tau \to \ell\, X$, followed by (displaced) $X \to \ell^+\ell^-$, $\gamma\gamma$?
  [Heeck & Rodejohann, PLB '18; Cheung++, JHEP '21]
- Mu3e and Belle II can improve limits, maybe others too?
  [i Tormo++, PRD '11; Uesaka, PRD '20; Calibbi, Redigolo, Ziegler, Zupan, JHEP '21]
- Light particles as mediators change rate expectations.
- X = axion/ALP/majoron/familon/Z', connected to DM?
- Or: SMEFT + X.
  [Georgi, Kaplan, Randall, '86; Brivio++, '17; Dror, Lasenby, Pospelov, '17 & '19]

Far from finished!

Summary

- Charged LFV gives info *complementary* to ν oscillations.
- Generically predicted by BSM, could be around the corner.
- Difficult to predict LFV *rates*, needs
  - Fixed flavor structure (neutrino mass, CKM?)
  - Fixed new physics scale (DM, anomalies?)
- Light new physics open new avenues.
- Hope for sign in Mu3e, MEG-II, Belle-II, Mu2e, LHC(b),...



Explore every corner of our lamppost!

Backup

Effective field theory view

- SM symmetry: $G = U(1)_{B-L} \times U(1)_{L_\mu - L_\tau} \times U(1)_{L_\mu + L_\tau - 2L_e}$.
- Effective field theory with Majorana ν:

$$L = L_{SM} + \overbrace{\frac{LLHH}{\Lambda}}^{M_\nu} + \underbrace{\sum_j \frac{\mathcal{O}_j}{\Lambda^2} + \sum_j \frac{\mathcal{O}'_j}{\Lambda^3} + \sum_j \frac{\mathcal{O}''_j}{\Lambda^4} + \ldots}_{\text{could conserve G or subgroup}}$$

conserves G

violates G

⇒ ‘weird’ channels dominate!?

Upcoming CLFV

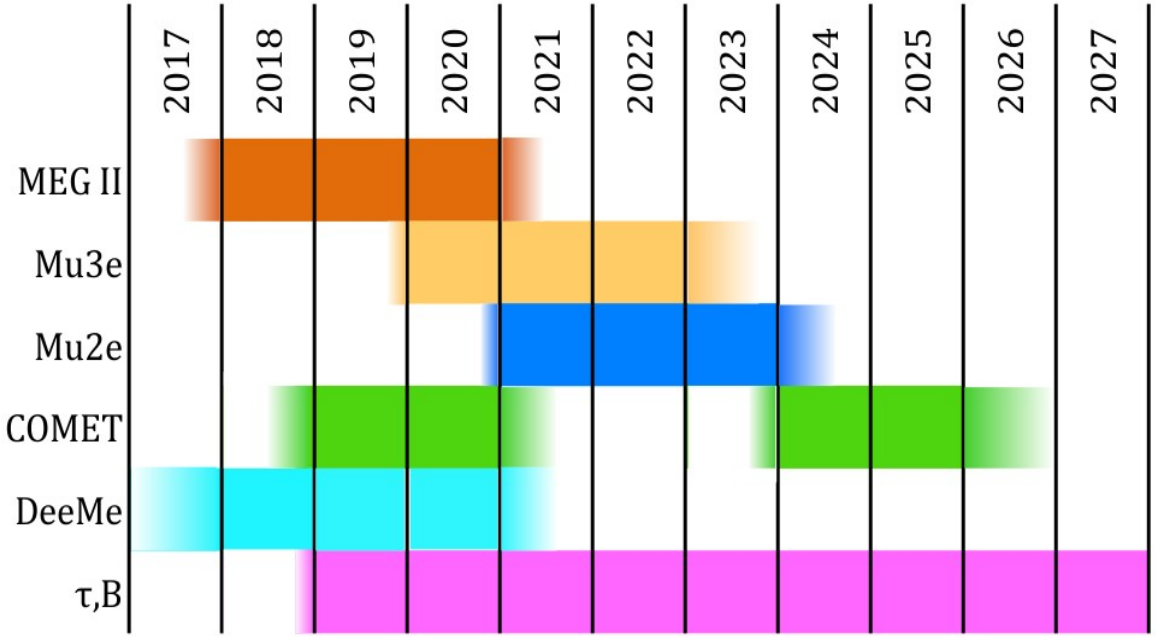
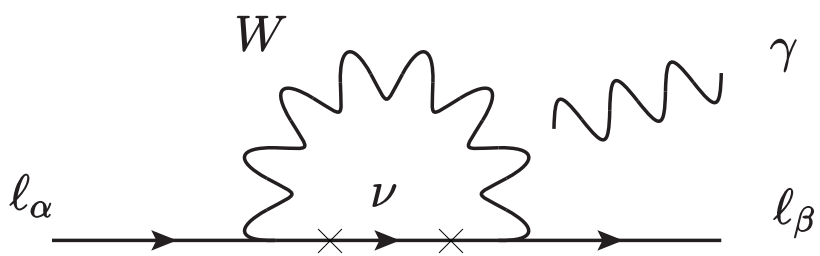


Figure 47. – Projected time lines for different projects searching for CLFV decays. MEG IIis expected to start data taking in 2018 after an engineering run in 2017; Mu3e magnet and detectors are expected at the end of 2019; Mu2e foresees three years of data taking starting in 2021; COMET Phase-I is expected to start commissioning and data taking in 2018 for two-three years, followed by a stop to develop and deploy the beamline and detectors for Phase-II; DeeMe is expected to start soon and take data with graphite and silicon carbide targets in sequence; Belle II is schedule to start data taking at end 2018.

[Calibbi & Signorelli, 1709.00294]

Neutrino mass ⇒ charged LFV?

- SM + Dirac neutrinos: $\mathcal{L} = \mathcal{L}_{\rm SM} - \underbrace{(y\overline{L}H\nu_R + \text{h.c.})}_{} + i\overline{\nu}_R \partial\!\!\!/ \nu_R$



$$m_\nu = y\langle H\rangle = U\,\mathrm{diag}(m_1, m_2, m_3)V_R \overset{!}{\lesssim} \text{eV}$$

- *All* CLFV is GIM suppressed:

$$\frac{\Gamma(\ell_\alpha \to \ell_\beta\gamma)}{\Gamma(\ell_\alpha \to \ell_\beta\nu_\alpha\overline{\nu}_\beta)} \simeq \frac{3\alpha_{\rm EM}}{32\pi}\left|\sum_{j=2,3} U_{\alpha j}\frac{\Delta m_{j1}^2}{M_W^2}U_{j\beta}^\dagger\right|^2 < 5\times 10^{-53}\,.$$

[1977: Petcov; Bilenky, Petcov, Pontecorvo; Marciano, Sanda; Lee, Pakvasa, Shrock, Sugawara]

Seesaw mass ⇒ charged LFV?

- SM + seesaw neutrinos: $L = L_{SM} + i\overline{N}_R \partial\!\!\!/ N_R$

$$- \left(\tfrac{1}{2} M_R \overline{N}_R^c N_R + \underbrace{y\overline{L}HN_R}_{m_D \overline{\nu}_L N_R} + \text{h.c.}\right)$$

- Violates $\Delta L = 2$. For large M_R:

$$M_N \simeq M_R\,, \qquad M_\nu \simeq -m_D M_R^{-1} m_D^T = U^* \,\mathrm{diag}(m_1, m_2, m_3) U^\dagger .$$

- Majorana neutrinos!
- LFV:

$$\frac{\Gamma(\ell_\alpha \to \ell_\beta \gamma)}{\Gamma(\ell_\alpha \to \ell_\beta \nu_\alpha \overline{\nu}_\beta)} \simeq \frac{3\alpha_{EM}}{8\pi} \underbrace{|(m_D M_R^{-2} m_D^\dagger)_{\alpha\beta}|^2}_{\mathcal{O}(M_\nu^4/m_D^4)} .$$

[Cheng & Li '80]

Not true with fine-tuning or structure in m_D.

Seesaw parameters

$$L = L_{SM} + i\overline{N}_R \partial\!\!\!/ N_R - (\tfrac{1}{2} M_R \overline{N}_R^c N_R + m_D \overline{\nu}_L N_R + h.c.)$$

$$\Rightarrow \quad M_\nu \simeq -m_D M_R^{-1} m_D^T \quad \& \quad \mathrm{BR}(\ell_\alpha \to \ell_\beta \gamma) \propto |(m_D M_R^{-2} m_D^\dagger)_{\alpha\beta}|^2.$$

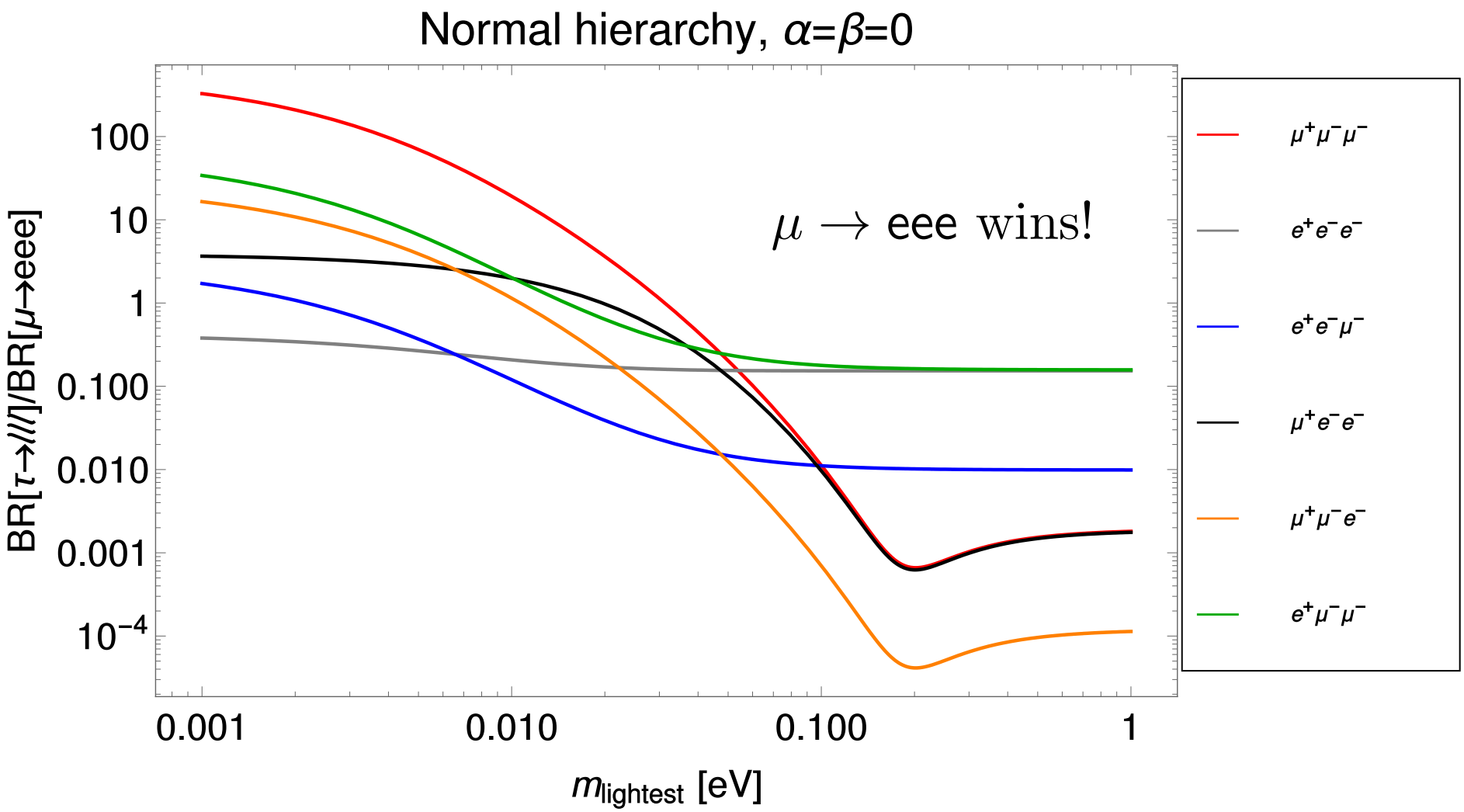
- One to one correspondence

$$\{m_D, M_R\} \leftrightarrow \{M_\nu, m_D M_R^{-2} m_D^\dagger\}.$$

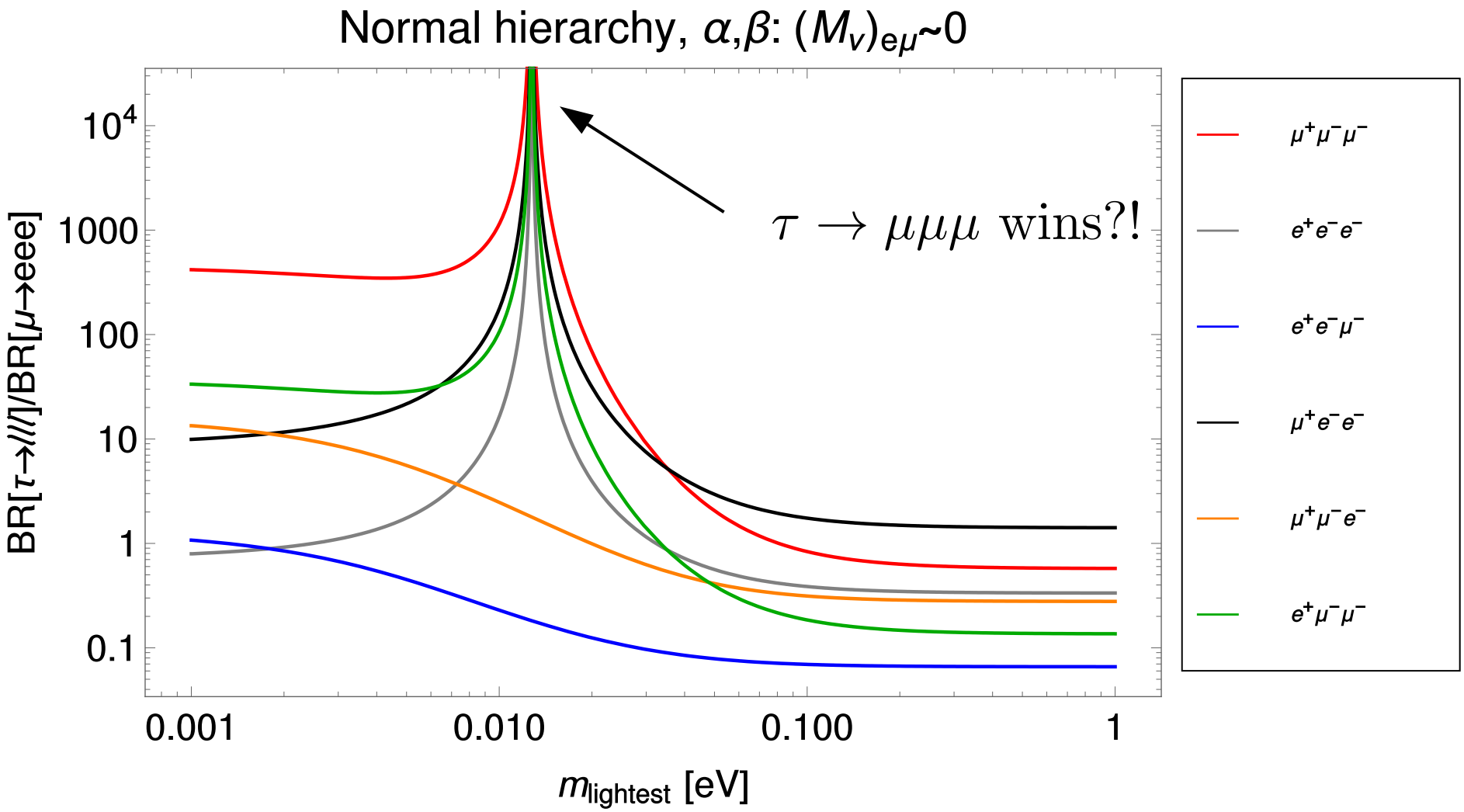
[Broncano, Gavela, Jenkins, hep-ph/0210271]

- Or: unique d=6 operator $(y M_R^{-2} y^\dagger)(\overline{L}H)(i\partial\!\!\!/)(H^\dagger L)$.
- Gives LFV and non-unitary PMNS.

LFV *complementary* to M_ν!



$$(M_\nu)_{\alpha\beta} \simeq y_{\alpha\beta}\frac{2\mu v^2}{M_\Delta^2} \quad \& \quad \mathrm{BR}(\ell_\alpha \to \ell_i \ell_j \bar{\ell}_k) \propto |y_{\alpha k}|^2 |y_{ij}|^2 / M_\Delta^4 .$$



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Scalar-triplet seesaw

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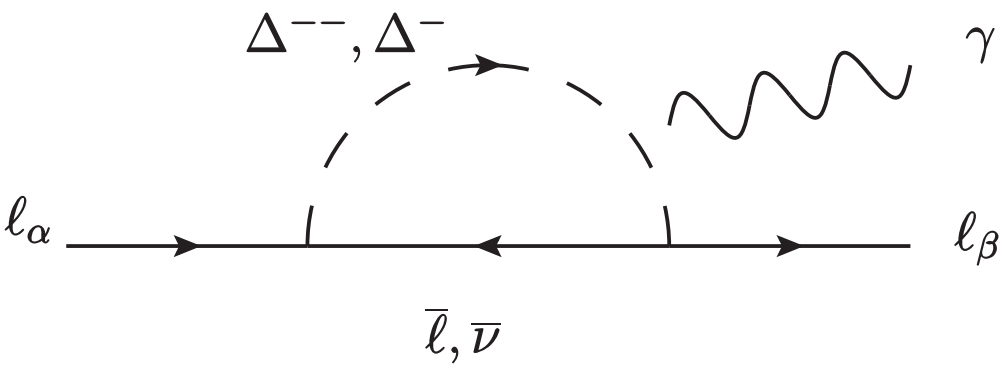
- But at loop level:



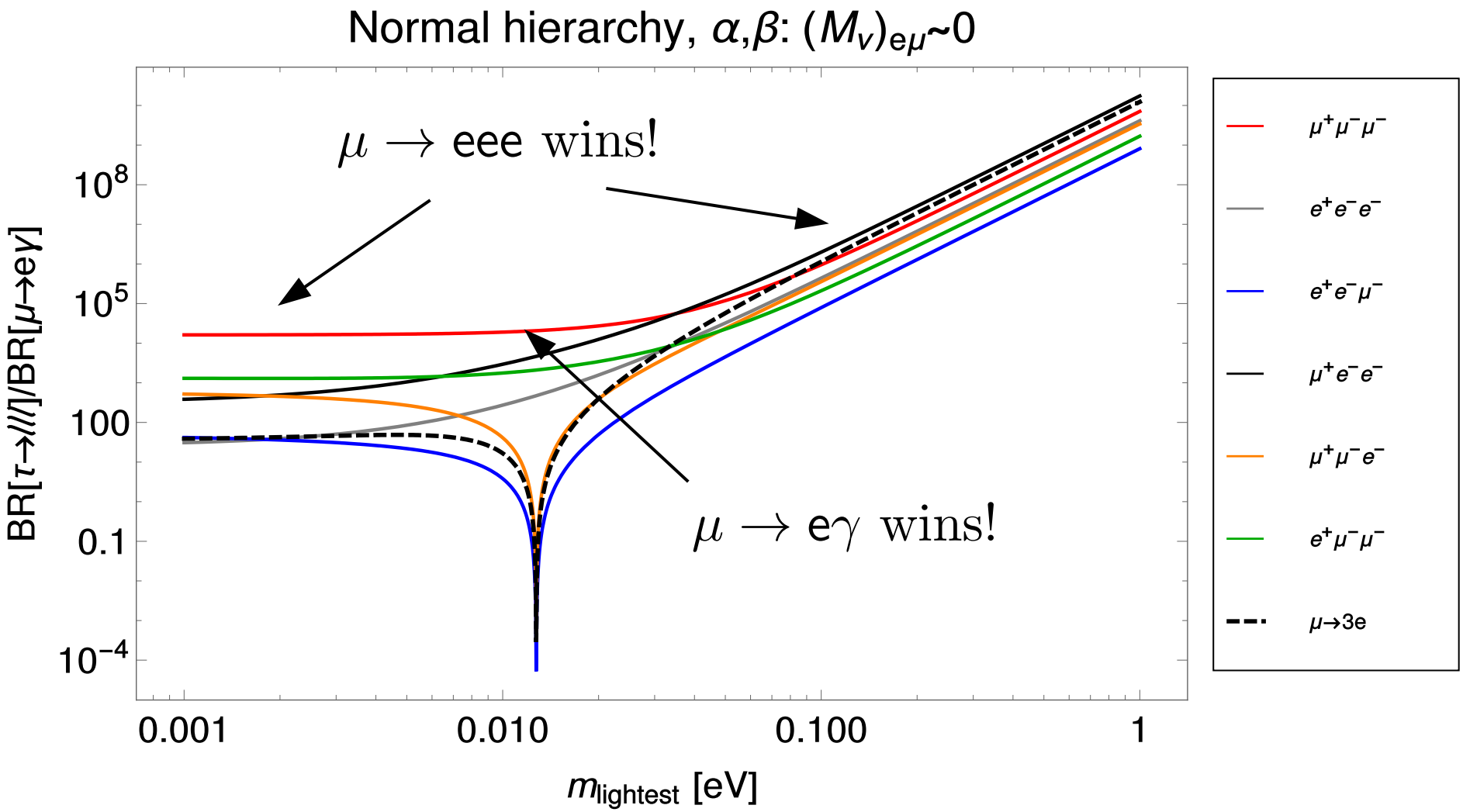
$$\mathrm{BR}(\ell_\alpha \to \ell_\beta \gamma) \propto \frac{|(y^\dagger y)_{\alpha\beta}|^2}{M_\Delta^4} .$$

[Pich, Santamaria, Bernabeu, '84]

- $\mu \to 3e$ could be 0, but $\mu \to e\gamma$ cannot (since θ_{13}).

[Chakrabortty++, 1204.1000]

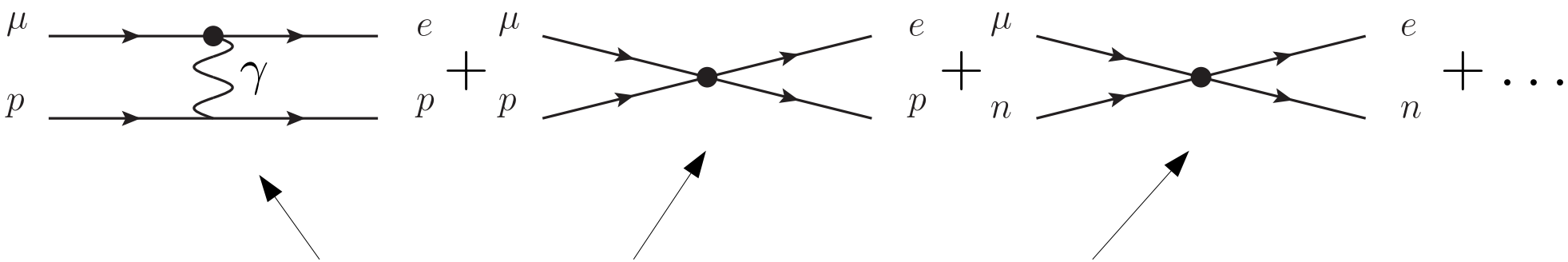
Prediction of LFV ratios via M_ν!



Prediction of LFV ratios via M_ν!

The inverse problem

- If we see CLFV, can we pin down the underlying operator?
  - In many cases: Yes! (e.g. $\mu \to e\gamma \leftrightarrow$ dipole)
  - $\mu \to e$ conversion in nucleus: No!

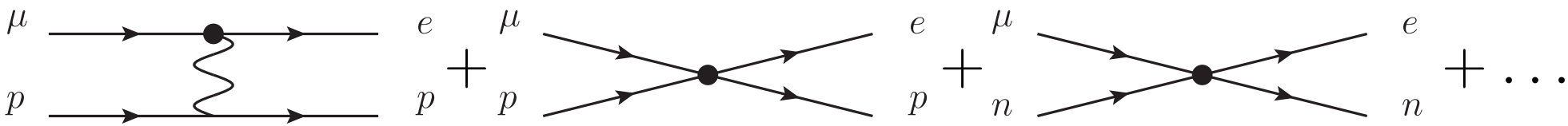


Relative contributions depend on nucleus: Z, N, spin!

- Need to observe $\mu \to e$ conversion in different nuclei!

[Kitano, Koike, Okada, PRD '07; Cirigliano++, PRD '09; Davidson++, '18]

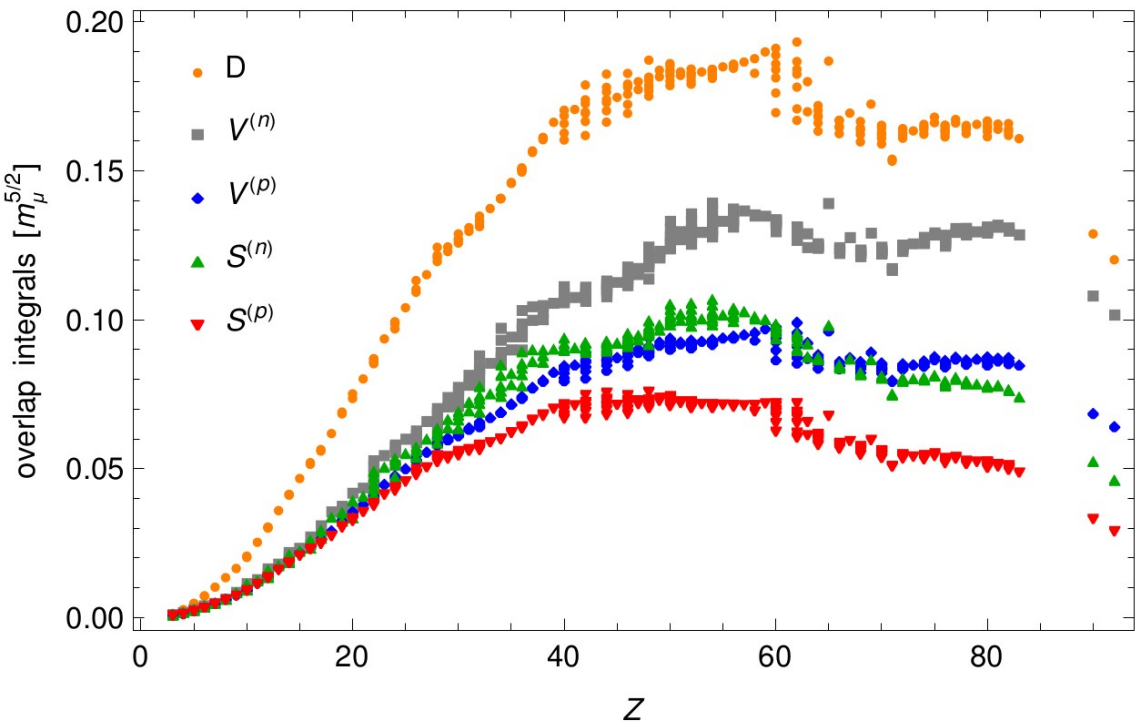
μ → e conversion



- Assuming spin-*independent* conversion:

$$\mathrm{BR}_{\mathrm{SI}} = \frac{32 G_F^2}{\Gamma_{\mathrm{capture}}} \left[|\boldsymbol{v} \cdot \boldsymbol{C}_L|^2 + |\boldsymbol{v} \cdot \boldsymbol{C}_R|^2 \right]$$

Overlap integrals Wilson coefficients



$$\boldsymbol{v} \equiv \left(\frac{D}{4}, V^{(p)}, S^{(p)}, V^{(n)}, S^{(n)} \right)$$

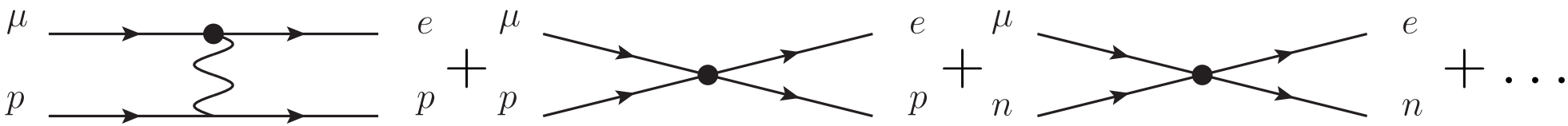
To measure the Wilson coefficients, use nuclei whose **v** are maximally misaligned.

[Davidson, Kuno, Yamanaka, PLB '19]

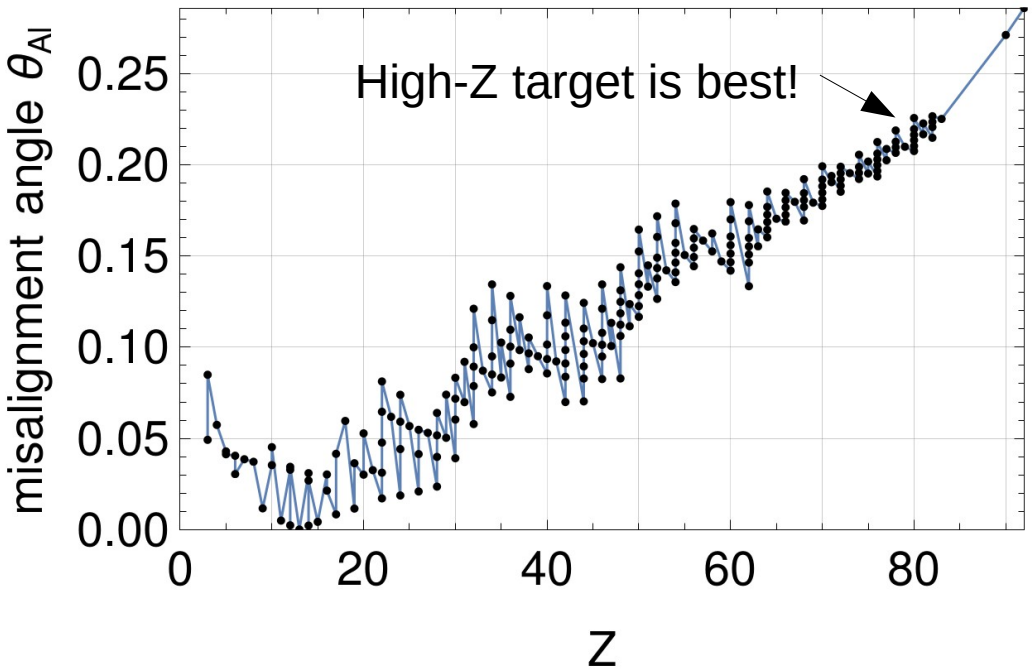
[Kitano, Koike, Okada, PRD '07; Heeck, Szafron, Uesaka, NPB '22]

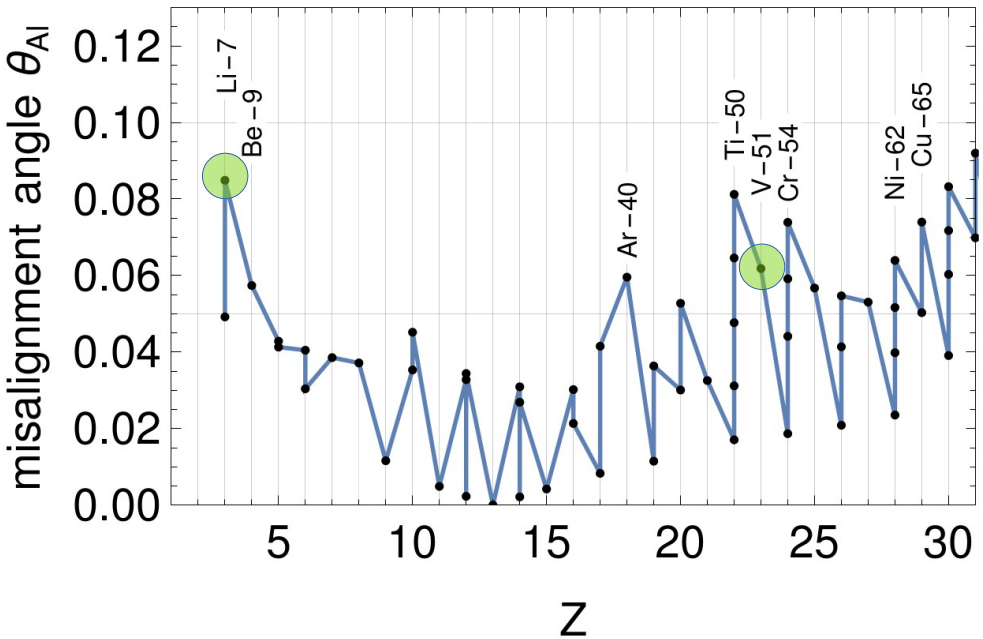
μ → e conversion

See talk by
Yuri Oksuzian.



- Misalignment with aluminium (target in COMET & Mu2e):



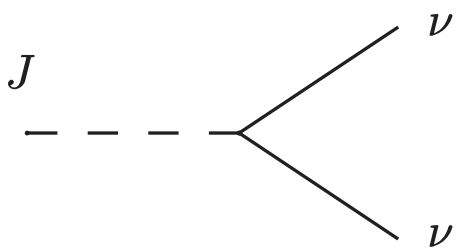


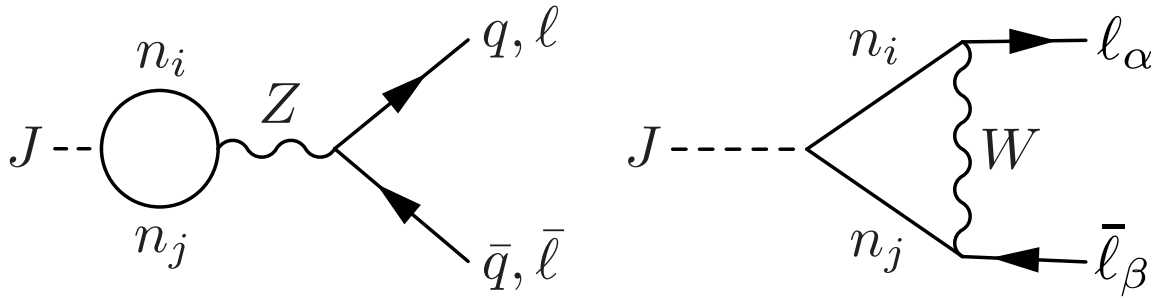
- At low Z, Li-7 and V-51 can distinguish proton/neutron.

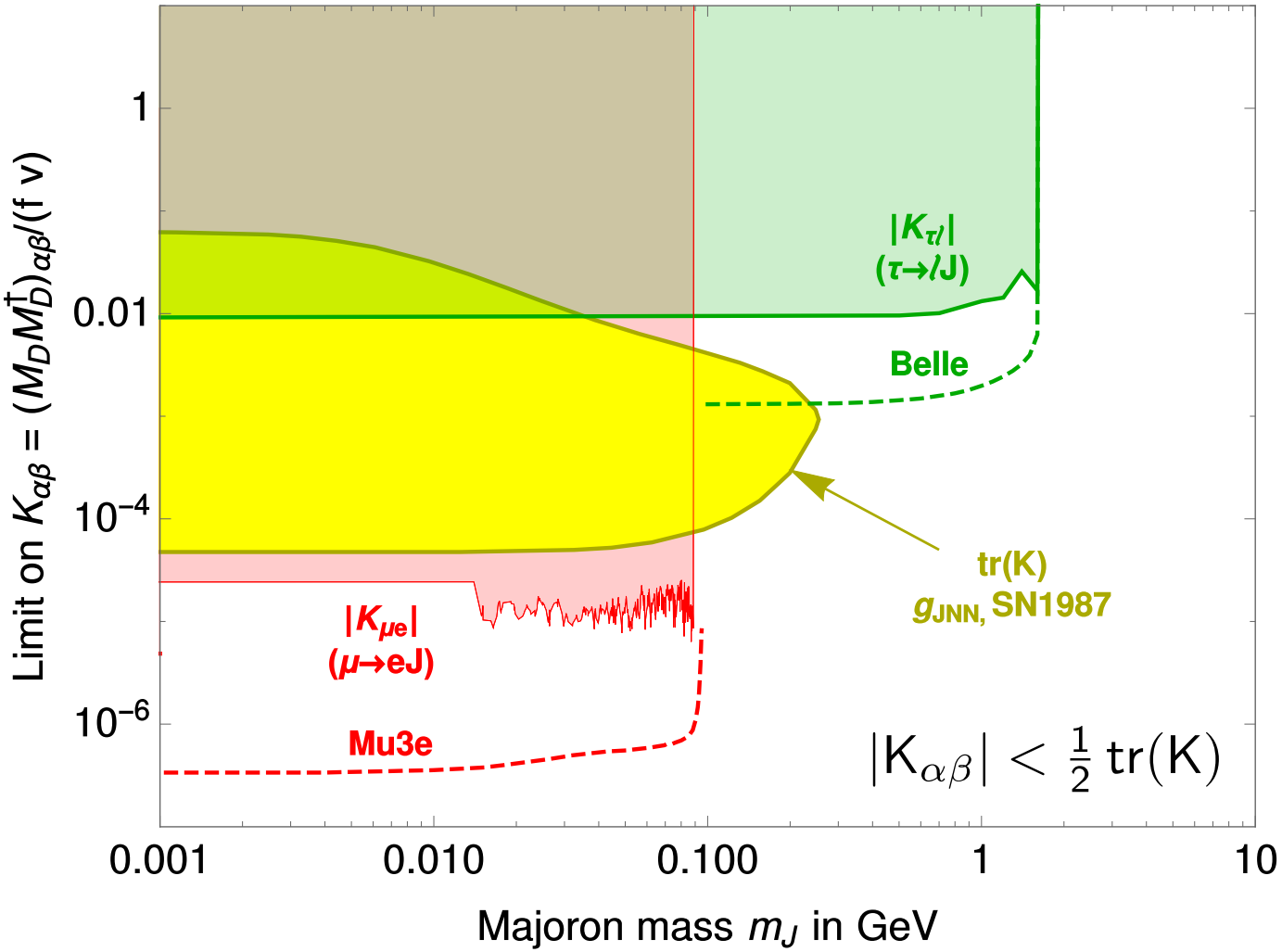
[Davidson, Kuno, Yamanaka, PLB '19; Heeck, Szafron, Uesaka, NPB '22]

Probing light particles

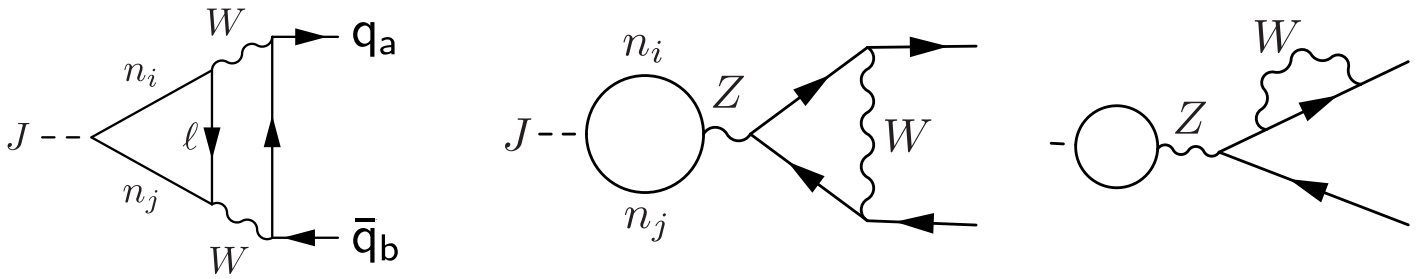
- Mu3e: BR($\mu \to e\, X$) from 10^{-6} to 10^{-8}.
- Belle II: BR($\tau \to \ell\, X$) from 10^{-3} to 10^{-5}. [**JH**, PLB '16]
- Followed by (displaced) $X \to \ell^+\ell^-$, $\gamma\gamma$? [**JH**, Rodejohann, PLB '18]
- Example: Majoron.
  - Pseudo-Goldstone boson of lepton number.
  - Potential dark matter candidate. [**JH**, Garcia-Cely, JHEP '17]
  - Tree-level coupling only to neutrinos.

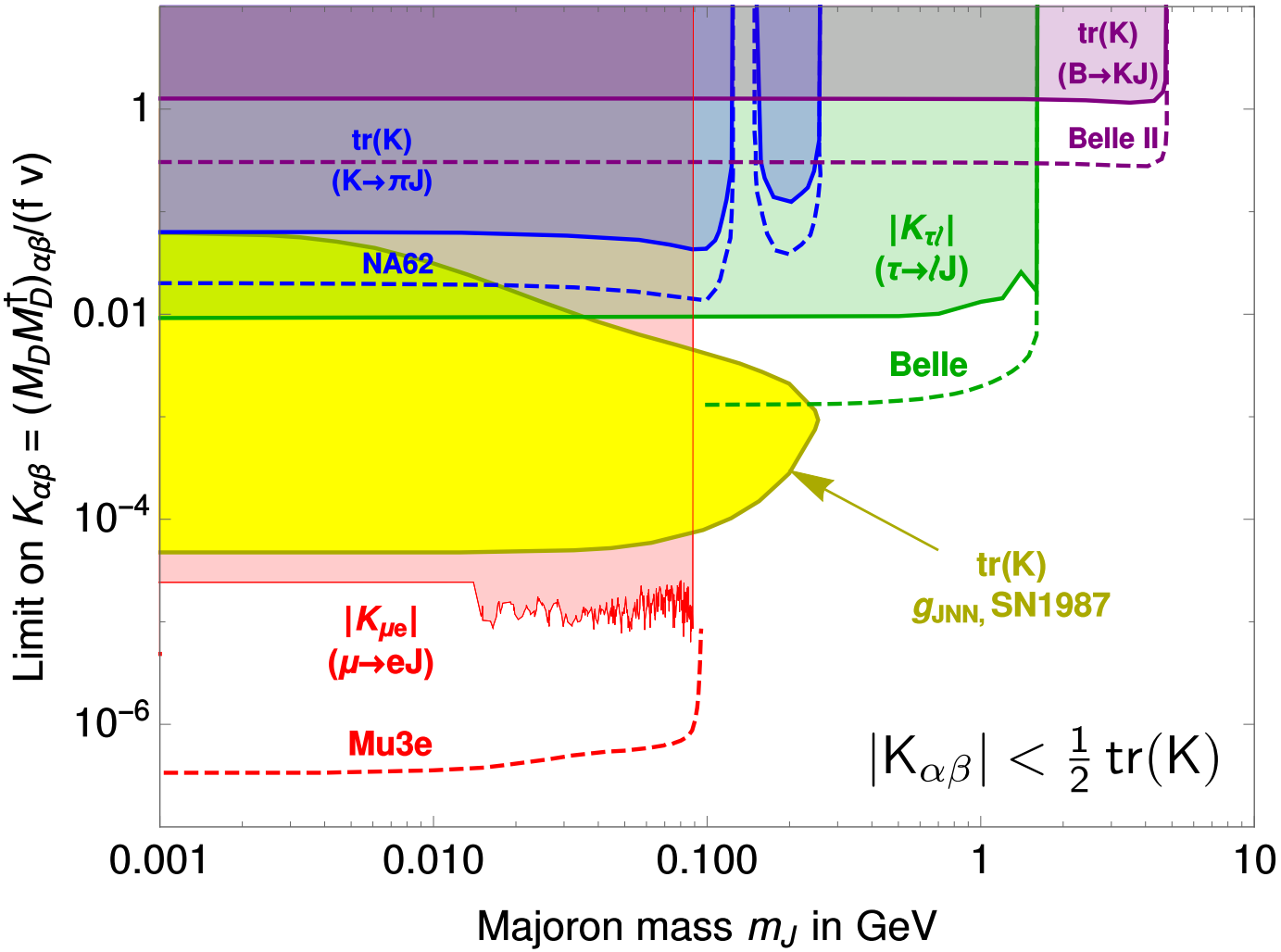






[**JH**, Garcia-Cely, JHEP '17]

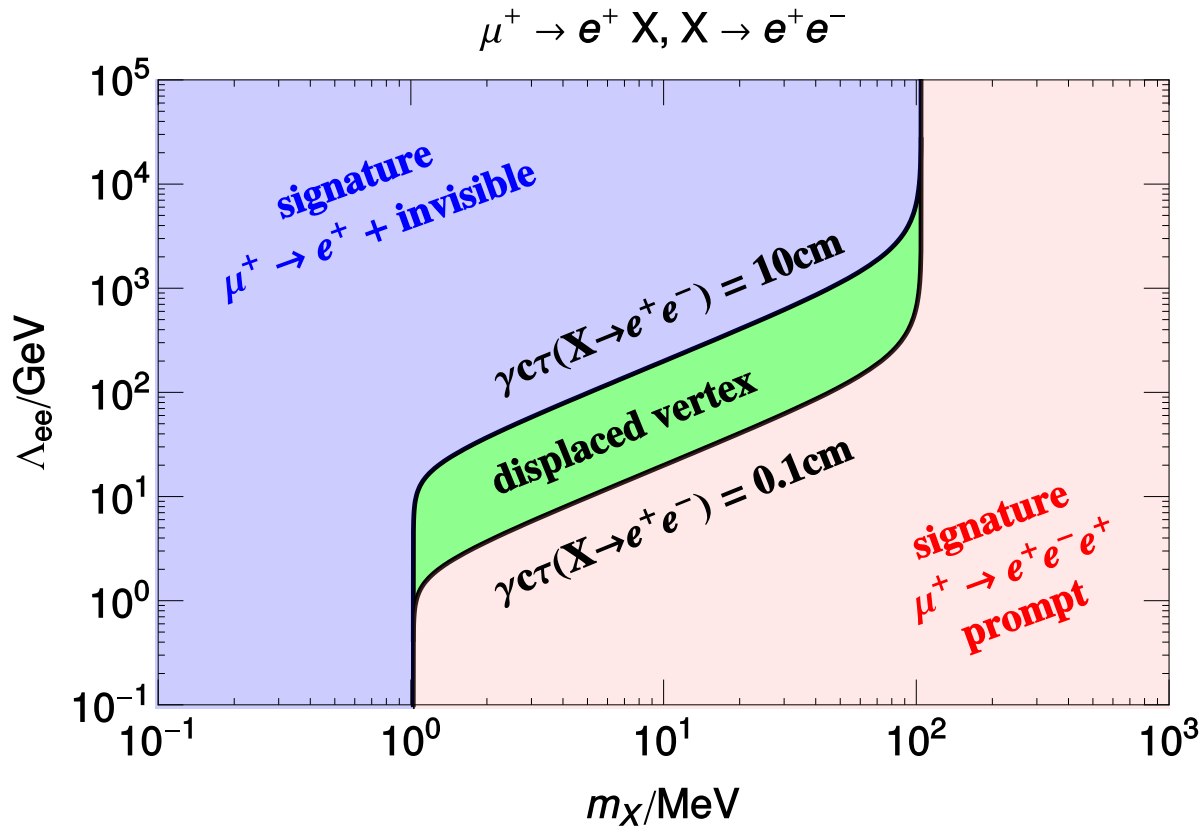




[**JH**, Patel, PRD '19]

$\mu \to e\, X$ with $X \to$ visible

- Take $X\bar{e}\gamma_5 e\; m_e/\Lambda_{ee}$.
- Decay length determines signature.
- Displaced vertex gives new observable.
  [**JH**, Rodejohann, PLB '18]



- Muon at rest:

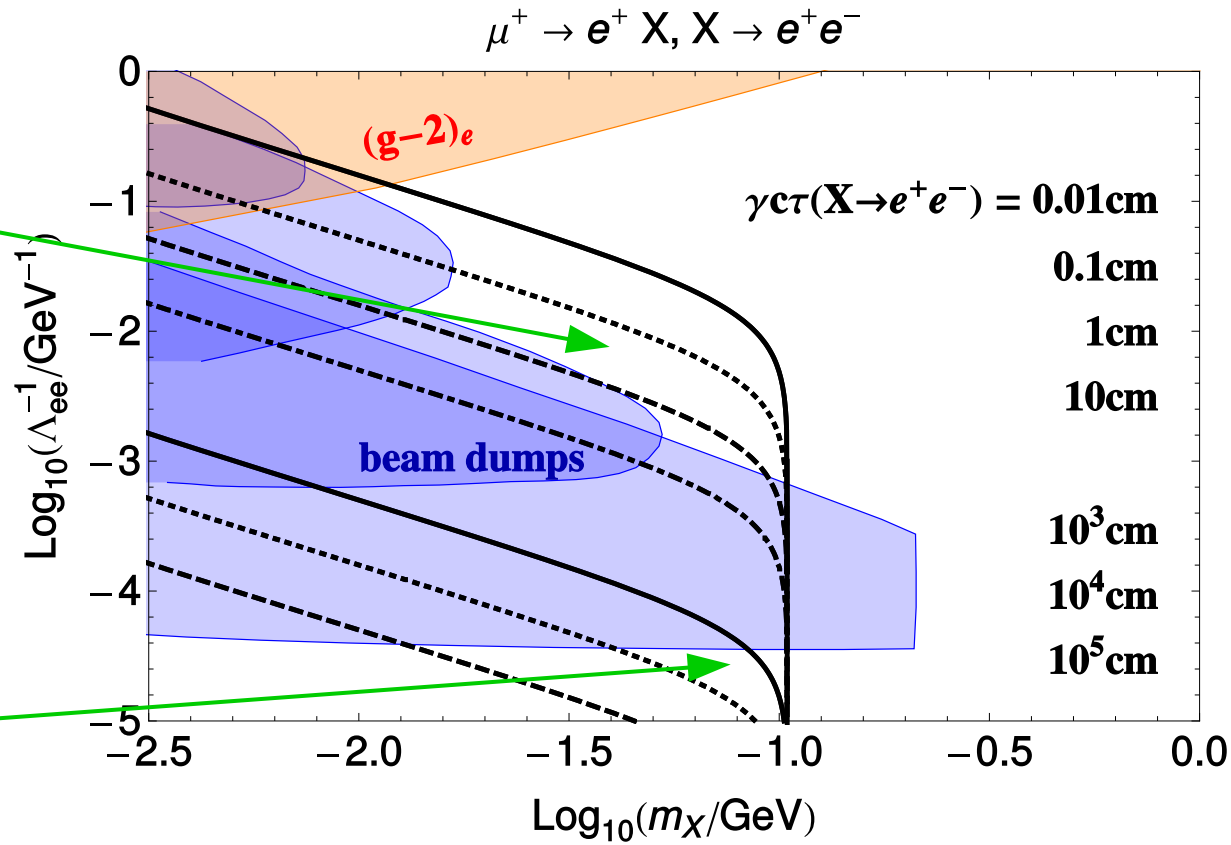
$$\gamma c\tau \simeq \frac{\pi m_\mu \Lambda_{ee}^2}{m_e^2 m_X^2} \simeq 2.5\,\mathrm{cm}\left(\frac{\Lambda_{ee}}{100\,\mathrm{GeV}}\right)^2 \left(\frac{10\,\mathrm{MeV}}{m_X}\right)^2 .$$

Sub-GeV X with ee coupling allowed?

μ → e X with X → $\bar{e}e$

[**JH**, Rodejohann, PLB '18]

- Decay length typically below cm. ⇒ looks prompt.
- Below beam dump: Λ_{ee} > 30 TeV; mostly invisible, but some DV!



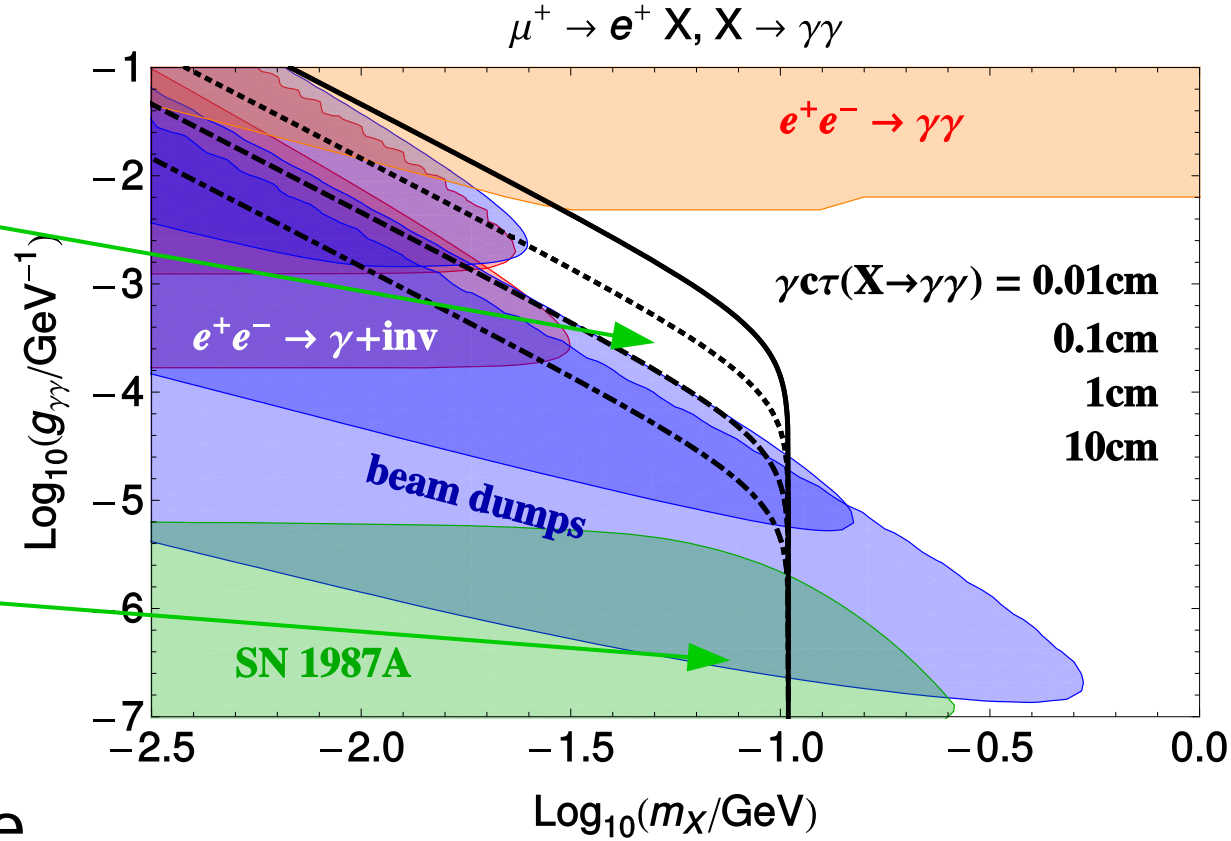
$$\mathrm{BR}(\mu \to eX)\mathrm{BR}(X \to ee)(1 - P(l_{dec})) \simeq \mathrm{BR}(\mu \to eX)\frac{l_{dec}}{\gamma c\tau}\,.$$

Possible in Mu3e!

μ → e X with X → γγ

[**JH**, Rodejohann, PLB '18]

- Decay length always below cm. ⇒ looks prompt.
- Below beam dump: supernova constraints!
- Prompt channel still interesting, maybe MEG(II) or Mu3e extension?



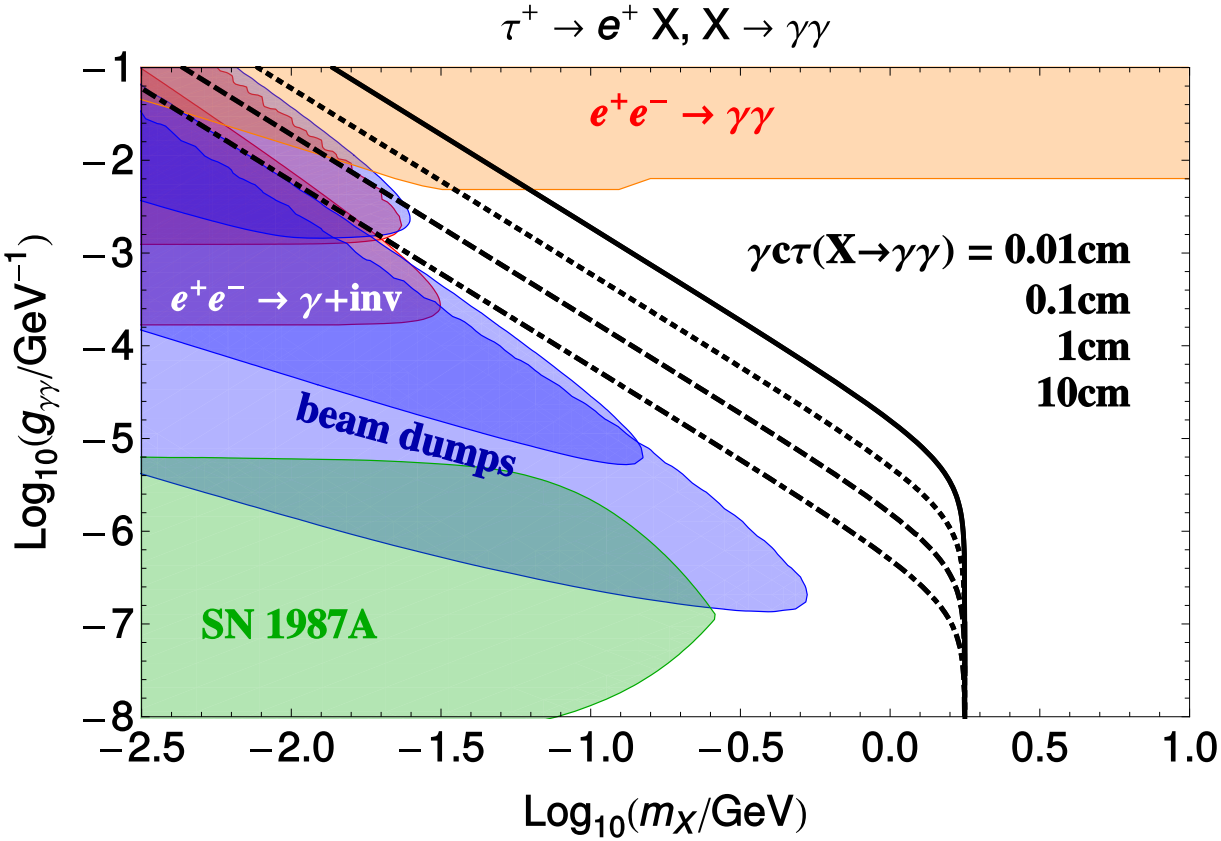
[Limits: Dolan et al, JHEP '17]

Muons difficult, taus easier.

τ → e X with X → visible

[**JH**, Rodejohann, PLB '18]

- Tau at rest, higher X boost.
- Arbitrary decay lengths possible.
- Similar for X → ee, μμ, μe.
- Worthwhile in LHCb and Belle (II).



[Limits: Dolan et al, JHEP '17]

New signatures from light physics!