



Precision calculations for e+e- colliders: future directions

Samuel Abreu CERN & The University of Edinburgh

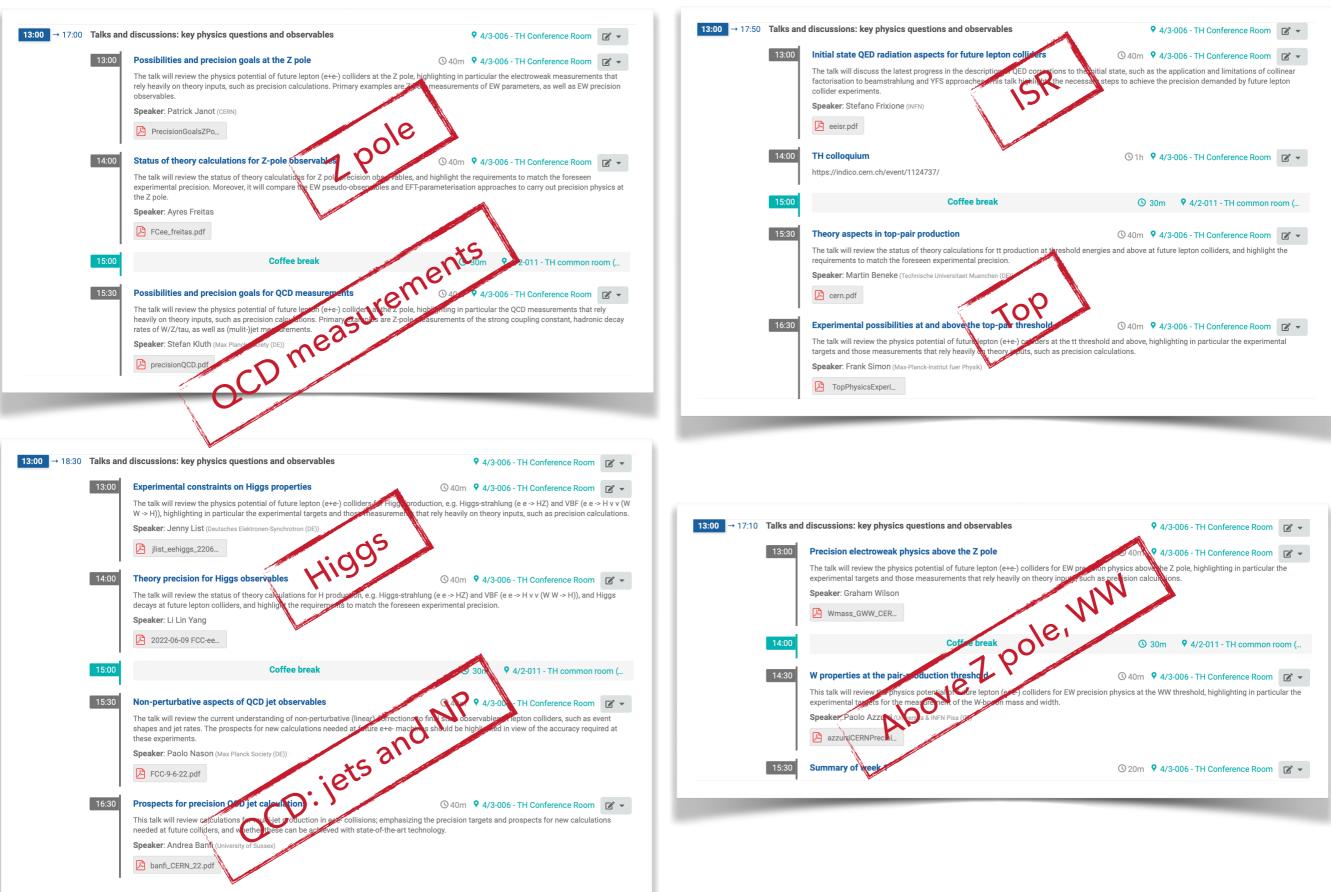
Summary of Precision calculations for future e+e- colliders: targets and tools

https://indico.cern.ch/event/1140580/

Organisation committee: S. Abreu, J. Alcaraz, J. Alimena, P. Azzi, D. D'Enterria, A. Freitas, G. Heinrich, A. Huss, M. Mangano, M. McCullough, P. Monni, J. Usovitsch, M. Vos

Snowmass CSS Workshop — 19th of July 2022

Targets ...



.. and Tools (for multi-loop calculations)

→ 17:00 Talks and discussions: multi-loop computational techniques		→ 17:00 Talks and discussions: multi-loop computational techniques	
13:00 Latest IBP reduction techniques	③ 40m ♀ 4/3-006 - TH Conference Room 🕝 🗸	13:00 Local unitarity: perspectives for future lepton colliders	③ 40m ♀ 4/3-006 - TH Conference Room
The talk will review the status and future prospects of IBP reduction to reconstruction, multivariate partial fractioning and syzygys should be		The talk will discuss the status and future prospects of the Loop-Tree Duality a relevant for future e+e- colliders (both for QCD and EW corrections).	approach for the calculation of higher-order corrections to cross sect
Speaker: Tiziano Peraro (University of Bologna and INFN)	105	Speaker: Valentin Hirschi (CERN)	
Tiziano_Peraro.pdf	discussed.	ECC_ee_workshop	
14:00 Modern calculation techniques for multi-scale loop amplitu	ides ③ 40m 24/3-006 • TH 201 € mice Room 🖉 ▼	Integrand subtraction & numerical integration	③ 40m ♀ 4/3-006 - TH Conference Room
The talk will review modern techniques to compute multi-leg/loop am and numerical techniques will be compared.	Ides () 40m 2+4/3-006 TH Coldenate Room () + plitudes, as well as future rospects for applications in e+e- collisions. Analytic	The talk will discuss the status and future prospects of integrand-subtraction in applications to future e+e- colliders.	nethods to construct locally finite two-loop amplitudes, focusing on
Speaker: Vasily Sotnikov (University of Zurich (UZH))	ric latic	Speaker: Charalampos Anastasiou	
Sotnikov_CERN_ee 13:00	Talks appidicussions: multi-loop computational equiniques	♀ 4/0 006 - TH Conference Room	
15:00 Coffee	Numerical evaluation of QLD virtual corrections with top quarks collisions	in e+e- ③ 40m ♀ 4/3-106 - TH Conference R 💽 ▾	③ 30m ♀ 4/2-011 - TH common room
15:30 OpenLoops @ 2 loops	The talk will discuss recent (unit-loop calculations involving heavy quarks, in	gyighting how these calculations and the technology adopted can be	③ 40m ♀ 4/3-006 - TH Conference Room
The talk will discuss the status and future prosp applications to future e+e- colliders.	exploited for multicol computations at future e+e- colliders. Speaker Log Chen		of the four-loop form factor, highlighting how these calculations ar
Speaker: Max Zoller (PSI)	14:00 TH colloquium https://indico.cern.ch/seit.org/51/82/	versity)	on colliders.
Zoller_CERN_ee_20	14:00 TH colloquium		
	https://indico.cern.ch/qeliving5.82/	© 30m ♥ 4/2-011 - TH common yoom (
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✓ 222 participants (50 in person), mix of theory and experiment especially on week 1

 We are aiming at producing a shopping list of important target and calculations, collecting input from the speakers/participants of the workshop

First target & Template: Z-pole physics

Summary: Theory inputs for Z lineshape observables

Numbers are given here for FCC-ee (best prospects)

Observables	Present value	FCC-ee stat.	FCC-ee current syst.	FCC-ee ultimate syst.	Theory input (not exhaustive)
m _z (keV)	91187500 ± 2100	4	100	10?	Lineshape QED unfolding Relation to measured quantities
$\Gamma_{ m Z}$ (keV)	2495500 ± 2300 [*]	4	25	5?	Lineshape QED unfolding Relation to measured quantities
σ^{0}_{had} (pb)	41480.2 ± 32.5 [*]	0.04	4	o.8	Bhabha cross section to 0.01% $e^+e^- \rightarrow \gamma\gamma$ cross section to 0.002%
$N_{\nu}(\times 10^3)$ from σ_{had}	2996.3 ± 7.4	0.007	1	0.2	Lineshape QED unfolding $(\Gamma_{vv}/\Gamma_{\ell\ell})_{SM}$
R_{ℓ} (×10 ³)	20766.6 ± 24.7	0.04	1	0.2 ?	Lepton angular distribution (QED ISR/FSR/IFI, EW corrections)
$\alpha_{\text{s}}(m_{Z})(\times 10^{4})$ from R_{ℓ}	1196 ± 30	0.1	1.5	0.4?	Higher order QCD corrections for $\Gamma_{ t had}$
R _b (×10 ⁶)	216290 ± 660	0.3	?	< 60 ?	QCD (gluon radiation, gluon splitting, fragmentation, decays,)

• And also sophisticated and state of the art MC generators (signal and backgrounds)

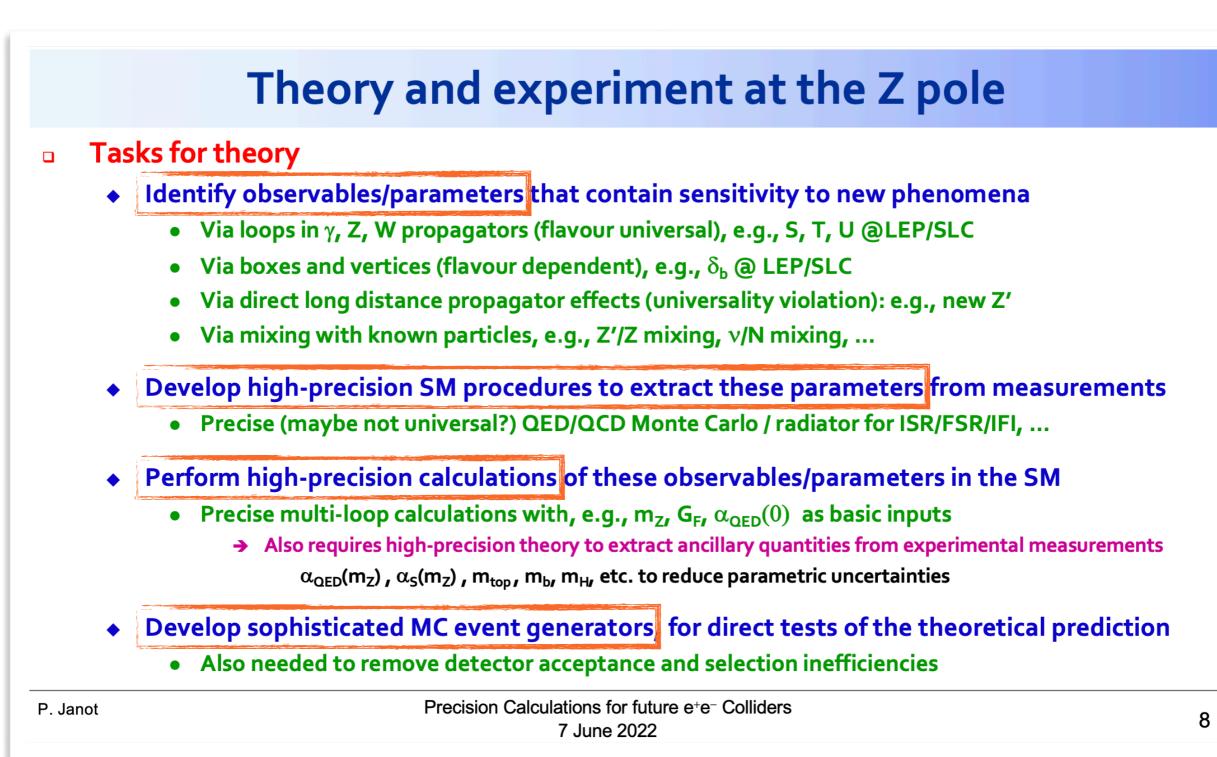
• Plus, maybe, redefined EW Precision Parameters (EWPP) and extraction procedures ?

P. Janot	Precision Calculations for future e⁺e⁻ Colliders 7 June 2022	[*] <u>https://arxiv.org/abs/1912.02067</u>	20

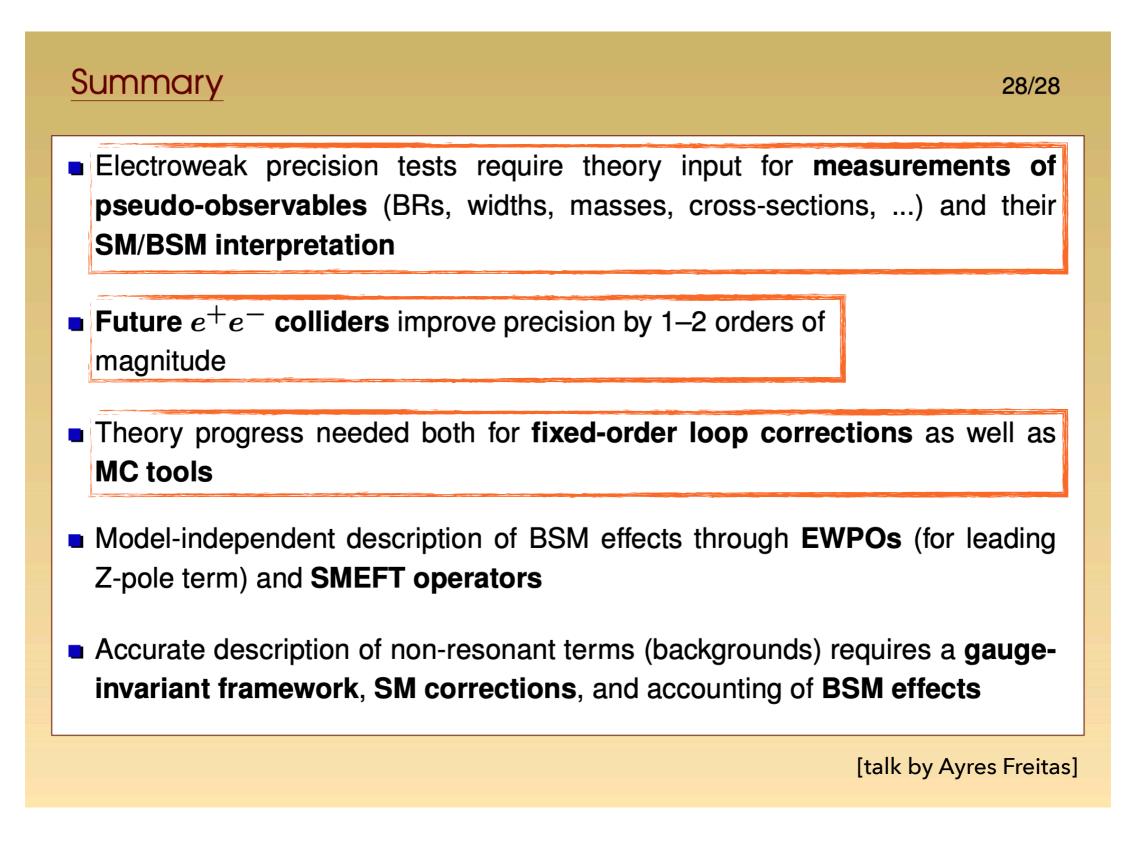
- Take statistical uncertainties as ultimate target
- Theory enters in several ways, e.g.:
 - Calibration: Initial State Radiation (ISR)
 - Parametric uncertainties: masses and couplings
 - Interpretation of results: ElectroWeak Precision Observables (EWPO)

First target & Template: Z-pole physics

What experimentalists are asking from theory:



First target & Template: Z-pole physics

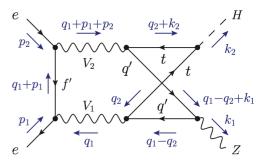


Initial State Radiation

- Talk by S. Frixione, based on Snowmass 21 contribution 2203.12557
- Model QED ISR: crucial for any measurement at lepton collider
- Process/Observable dependent
- NLL description gives sizeable correction
- More progress needed, build on recent developments in QCD:
 - NNLL description collinear radiation: within reach
 - All-order resummation of soft and collinear logarithms (e.g. ttbar threshold)
 - Implementation in accurate Monte Carlo codes: several tools exist, must be improved

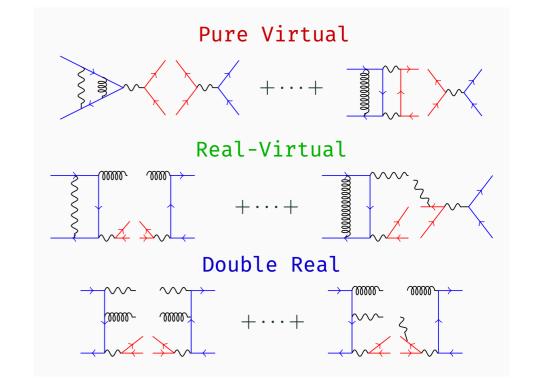
Fixed order: different types of processes/observables

- 1. Numbers: masses and other SM parameters (ρ , ...)
- 2. Multi-leg processes: QCD corrections to $e^+e^- \rightarrow n$ jets/photons
- 3. Multi-scale (e.g., masses) processes: (Mixed QCD-) EW corrections
- 4. Combination of the above: $e^+e^- \rightarrow t\bar{t}H$, ...



[EW corrections to ZH, talk by Yang]

- Multi-loop calculations:
 - Reduce amplitudes to minimal set of Feynman integrals
 - Compute Feynman integrals
 - Combine real and virtual corrections



Fixed order: different types of processes/observables

- Choose right tool for each job:
 - Fully numerical: slow evaluation but generic, ideal for type 1. [talks by Maheria, Gluza, Liu]
 - Analytic calculations: fast evaluation but hard to get, ideal for type 2.

[talks by Sotnikov, Peraro, von Manteuffel, Weinzierl]

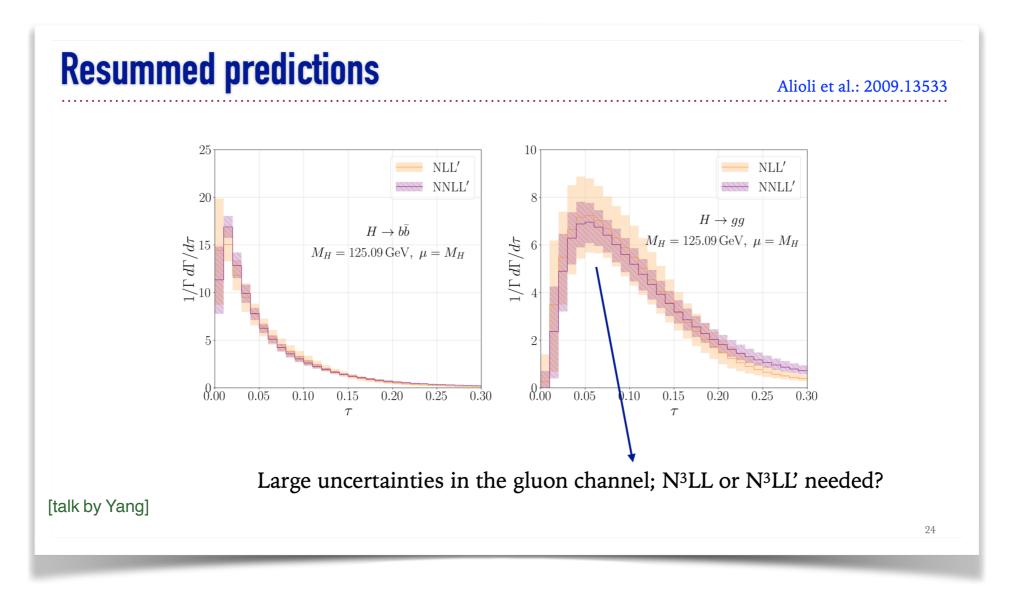
- Semi-analytic calculations, expansions: process specific, easier than full analytic, ideal for type 3.
- New directions:
 - Combine real and virtual: progress in Loop/Tree duality, integrand subtraction
 - Towards automation: progress in OpenLoops2

[talk by Zoller]

- Build on new developments in LHC calculations and new ideas from neighbouring fields (amplitudes, pure mathematics, ...)
- * FCee targets: very challenging, but substantial progress achievable with current tools (e.g., QCD corr. $e^+e^- \rightarrow 4$ jets at 2 loops vs EW corr. $e^+e^- \rightarrow H\nu\bar{\nu}$ at 2 loops)

Resummation

- Fixed-order calculations insufficient, need to resum large logarithms
 - e.g., event-shape distributions in Higgs physics

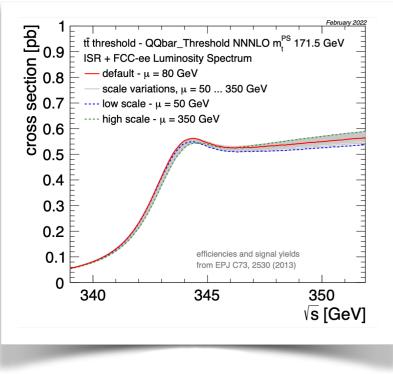


 Required resummation achievable in the coming decade, sufficient to reduce perturbative uncertainties at the ~% level

Beyond standard perturbation theory — **Top Physics**

[talks by Beneke and Simon]

✓ e⁺e⁻ colliders are precision tools for top quark physics (few permille/percent -level on cross section)



A challenge for theory: Understanding parameters on a level comparable to expected experimental precision. Theory is a / the leading systematic for many measurements - for the mass it is the leading uncertainty overall.
 Advances in theory directly translate into improvements of overall precision.

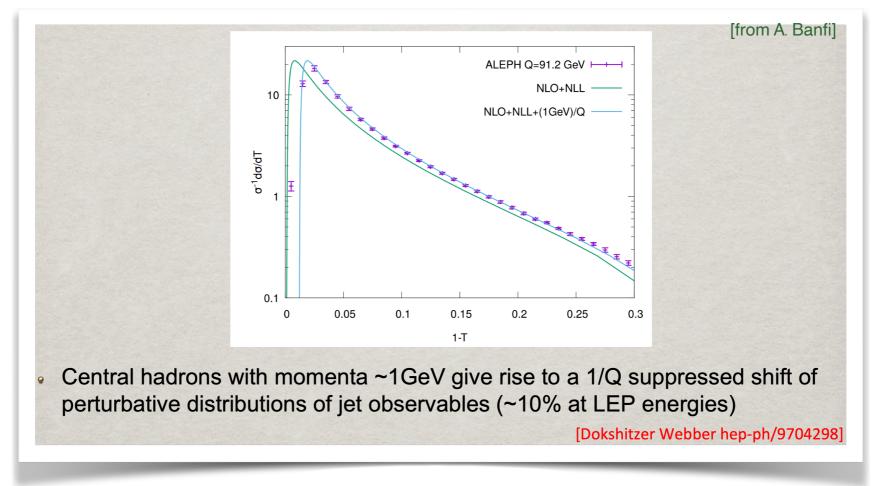
- pNRQCD known to N³LO (also including EW+non-resonant effects @ NNLO)
- ✓ TH uncertainty in top mass ~ 40MeV (Beneke et al.) while EXP target is 20MeV
 - ✓ matching of N³LO+NNLL (ongoing, NNLL from Hoang et al.)
 - NLL ISR QED, including soft limit
 - ✓ N⁴LO in pNRQCD needed (currently out of reach)

New challenges — NP effects

✓ Jet Physics: Non-Perturbative corrections are serious limitation of TH accuracy!

[talks by Kluth, Nason and Banfi]

 Better understanding of hadronisation (for even shapes, jet rates, jet substructure)



- Compute new observables with reduced NP sensitivity
- First steps in understanding linear 1/Q corrections in multi-jet final states: see P.Nason's talk for preliminary results

Conclusion and outlook

- Future lepton colliders to bring drastic reduction of experimental uncertainties
- Theory precision likely to be among main bottlenecks
- Many technical challenges, with some calculations currently beyond reach: build upon tools developed for LHC calculations, new ideas and a lot of work
- Some conceptual issues, such as Non-Perturbative corrections to hadronisation
- Not covered here in detail but very important: Monte Carlo generators for both QCD and QED, resummation, jets and parton showers, ...
- Collect theory targets into a shopping list to develop and motivate theory community
- New editions of the workshop focusing on other important theory aspects (MC, resummation, ...): we encourage everyone to attend!

THANK YOU!