α_s from Lattice QCD

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Snowmass Community Summer Study | Seattle
July 18, 2022

Strong Coupling $\alpha_s(\mu)$—Common Features

- Determinations α_s of entail

$$\mathscr{R}(Q) = R(Q) + Q^{-p} S(M, Q)$$

$$R(Q) = \sum_{l=0}^{\infty} R_l(Q/\mu)\alpha_s^l(\mu) = \sum_{l=0}^{\infty} R_l \alpha_s^l(Q)$$

$$\alpha_{\mathscr{R}}(Q) \equiv \frac{\mathscr{R}(Q) - R_0}{R_1}$$

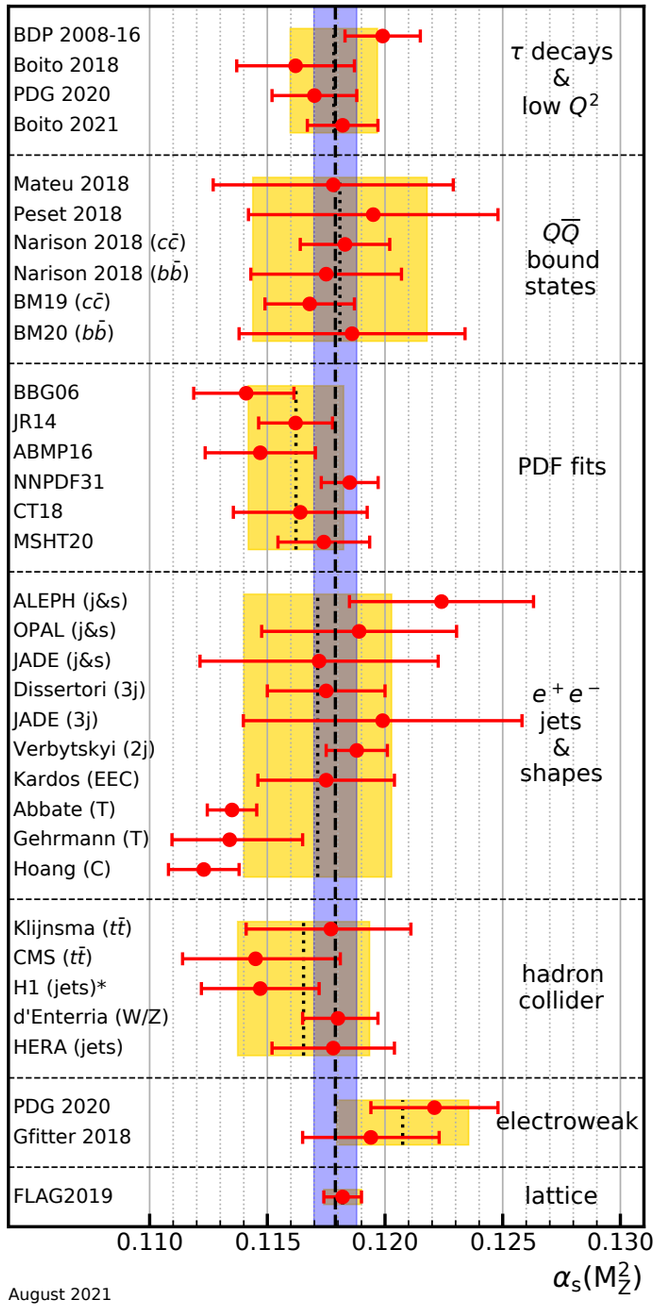
$$\frac{d\alpha_{\mathscr{R}}}{d\ln Q} = -2\beta(\alpha_{\mathscr{R}})$$

  vary Q to check running as predicted by perturbation theory ↔ tame power corrections (suppression, fitting).

- Resources: SnowWP on α_s, FLAG 2021 chapter on α_s [arXiv:2111.09849].

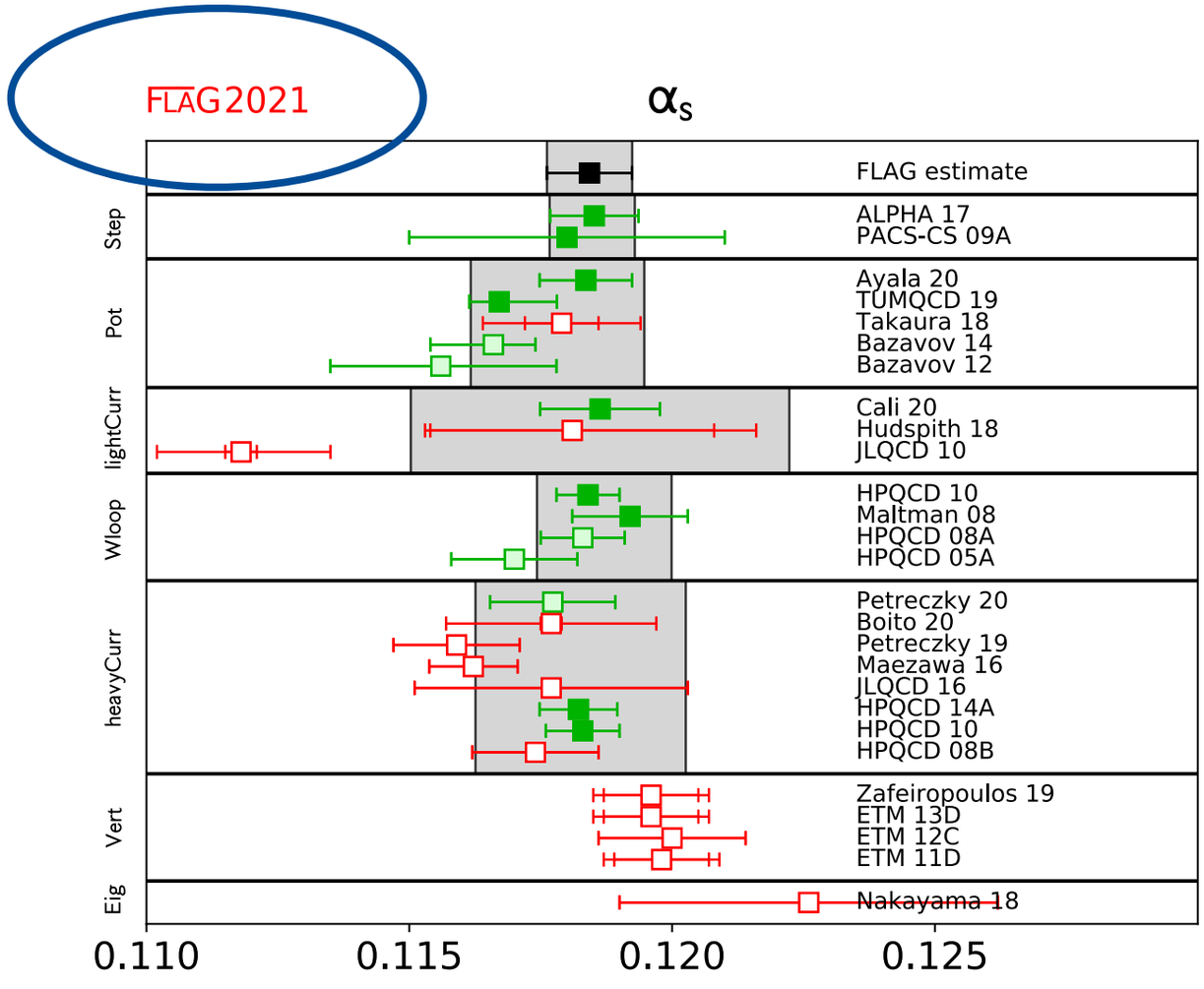
Strong Coupling $\alpha_s(\mu)$—High Energy ⊕ pQCD

- Numerous methods:
  - different observables;
  - different probes;
  - different systematics.
- Consider “QQ bound states” ☛
  - actually, moments of $R(e^+e^-)$;
  - same PT as in heavy current-current correlation functions (“heavyCurr”).



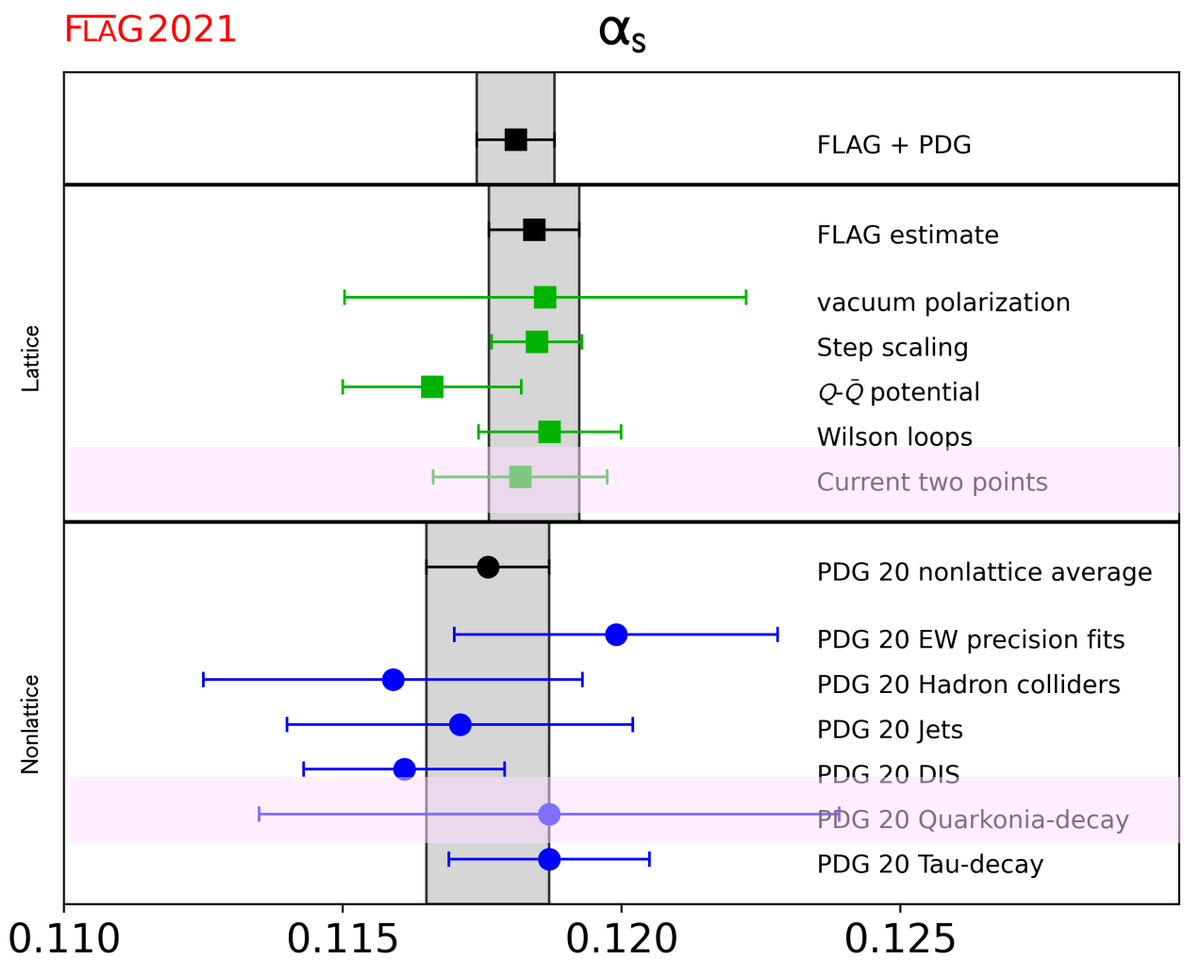
Strong Coupling $\alpha_s(\mu)$—Lattice QCD ⊕ pQCD

- **Step** scaling aka “QCD in a can” of diameter L;
- Static **Pot**ential (aka static energy) with separation r;
- **light**-quark current-**Curr**ent correlators at space-like Q;
- Small **W**ilson **loop**s of size ~a;
- Moments of **heavy**-quark current-**Curr**ent correlators of mass m_Q;
- Ghost-gluon vertex, momentum Q;
- **Eig**envalues of the Dirac operator.



Ingredient	Small Wilson loops	LGT with $a \to 0$	DIS scaling violation
Obtain $\mathscr{R}(Q)$	Compute from QCD Lagrangian		Measure e-p scattering
Large energy scale	a^{-1}	L^{-1}, $2m_Q$, ...	Momentum transfer Q
Scale separation	OPE	Various	OPE
Perturbation theory	Lattice (NLO, maybe NNLO)	Dimensional regularization (NNLO, N^{3}LO)	
Number of quantities	Several	one or few	Several
Electroweak	Omitted by construction		Included in data and theory
BSM	Omitted by construction		Unknown/omitted
Units in GeV	Hadronic quantity, viz., $Q = M_{\rm PDG}[(Qa)/(Ma)]_{\rm lat}$		Detector calibration

Strong Coupling $\alpha_s(\mu)$—Summary of Results



Strong Coupling $\alpha_s(\mu)$—Needs for Lattice QCD

- Numerical lattice gauge theory:
  - higher statistics;
  - finer lattices.
- Perturbation theory (*e.g.*):
  - full four-loop static energy;
  - four-loop quarkonium;
  - step-scaling observables in MS;
  - Wilson loops N^3LO or N^4LO?

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exascale computing

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Does precision permit fitting not truncating?