Modern Approach to Scattering Amplitudes

July 22, 2022
Snowmass
Seattle

Zvi Bern

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Outline

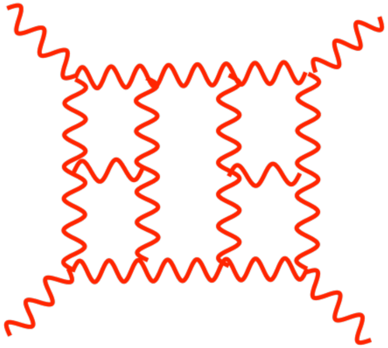
- Scattering amplitudes dates back to Rutherford's gold foil experiment.
- During 1960s the fundamentals of analyticity, causality and unitarity formulated.
- Today we combine these basic ideas with a remarkable set of new ideas.

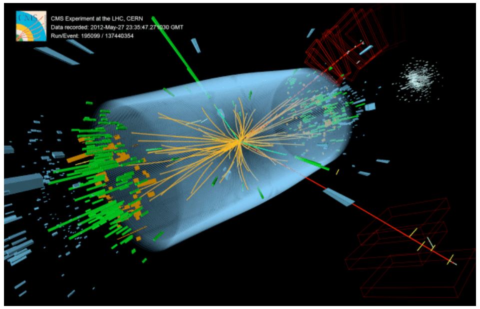
Modern scattering amplitudes is a thriving subfield of particle theory

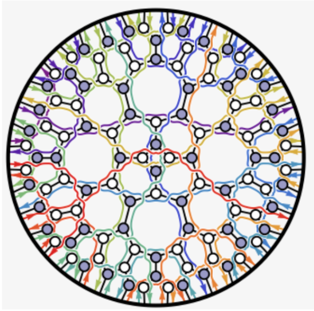
Here we give an brief overview of some of the ideas and applications

Three additional talks:

1. **Amplitudes and fundamental physics** — **Henriette Elvang**
2. **Application to collider physics** — **Fernando Febres Cordero**
3. **Applications to gravitational wave physics** — **Enrico Herrmann**

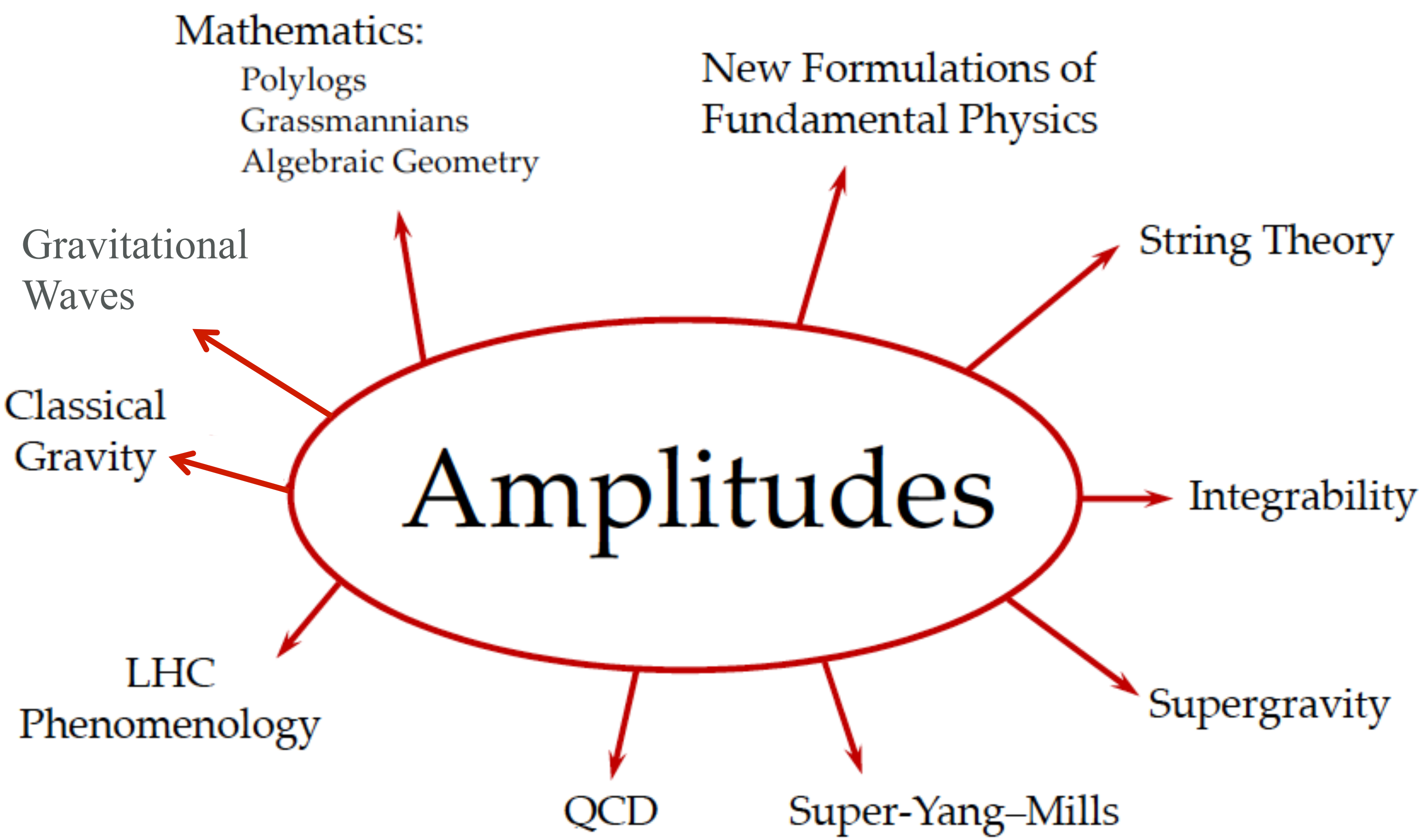






Many Directions

Amplitudes has blossomed in many directions.



White Papers

I want to thank all the white paper authors who contributed to relevant topics:

1. **Computational Challenges for Multi-Loop Collider Phenomenology**
   F. F. Cordero, A. von Manteuffel and T. Neumann

2. **Functions Beyond Multiple Polylogarithms for Precision Collider Physics**
   J. L. Bourjaily, J. Broedel, E. Chaubey, C. Duhr, H. Frellesvig, M. Hidding, R. Marzucca, A. J. McLeod, M. Spadlin, L. Tancredi, C. Vergu, M. Volk, A. Volovich, M. von Hippel, S. Weinzierl, M. Wilhelm, C. Zhang

3. **Solving Scattering in $\mathcal{N} = 4$ Super-Yang-Mills Theory**
   N. Arkani-Hamed, B. Basso, L. J. Dixon, A. J. McLeod, M. Spradlin, J. Trnka, A. Volovich.

4. **The Double Copy and its Applications**
   T. Adamo, J. J. M. Carrasco, M. Carrillo-Gonzalez, M. Chiodaroli, H. Elvang, H. Johansson, D. O'Connell, R. Roiban, O. Schlotterer

5. **Gravitational Waves and Scattering Amplitudes**
   A. Buonanno, M. Khalil, D. O'Connell, R. Roiban, M. P. Solon and M. Zeng.

6. **SMEFT at the LHC and Beyond** W. Shepherd

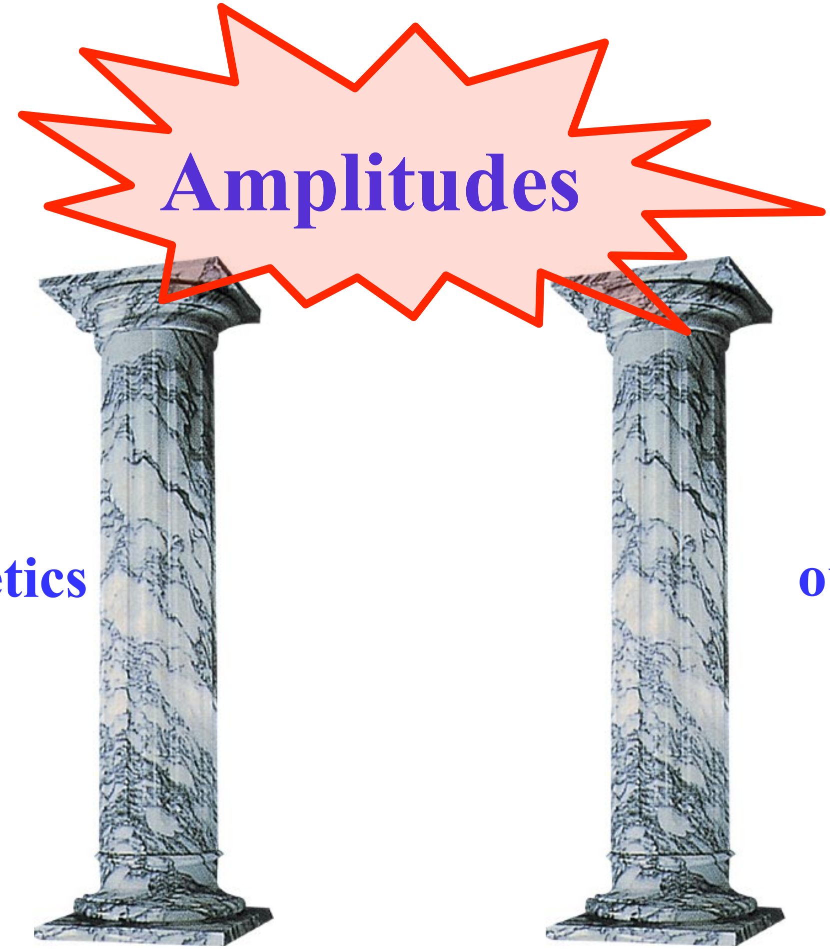
7. **String Perturbation Theory** N. Berkovits, E. D'Hoker, M. B. Green, H. Johansson, O. Schlotterer

White Papers

8. **UV Constraints on IR Physics** C. de Rham, S. Kundu, M. Reece, A. J. Tolley, S.-Y. Zhou
9. **S-matrix Bootstrap** M. Kruczenski, J. Penedones, B. C. van Rees
10. **The Deepest Problem: Some Perspectives on Quantum Gravity** S. B. Giddings
11. **The Cosmological Bootstrap.**
    D. Baumann, D. Green, A. Joyce, E. Pajer, G. L. Pimentel, C. Sleight and M. Taronna.

Apologies for leaving out most references in this talk.
Please consult white papers and other talks for references.

Two Pillars of Our Field

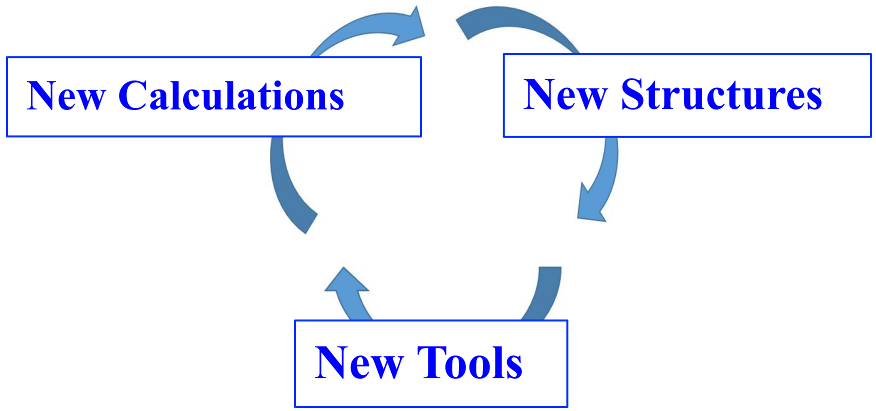


structure,
symmetry,
beauty, aesthetics

explicit
results useful
outside our field

To thrive we need the support of *both* pillars

The search for new structures.



A virtuous cycle.

Some selected examples of structure:

1. Parke-Taylor
2. Geometric interpretations. Amplituhedron
3. Double copy

Some selected applications outside the amplitudes:

1. Collider Physics, QCD.
2. Bounds on EFT Wilson coefficients.
3. Gravitational waves

New

The Birth of Modern Amplitudes

VOLUME 56, NUMBER 23 PHYSICAL REVIEW LETTERS 9 JUNE 1986

Amplitude for *n*-Gluon Scattering

Stephen J. Parke and T. R. Taylor
Fermi National Accelerator Laboratory, Batavia, Illinois 60510
(Received 17 March 1986)

A nontrivial squared helicity amplitude is given for the scattering of an arbitrary number of gluons to lowest order in the coupling constant and to leading order in the number of colors.

Motivated by the jet physics at colliders.

Parke and Taylor identified remarkable simplicity for *n* gluon scattering

$$A(1^{\pm}, 2^{+}, 3^{+}, \ldots, n^{+}) = 0$$

spinor inner product

$$A(1^{-}, 2^{-}, 3^{+}, \ldots, n^{+}) = i\frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\cdots\langle n1\rangle}$$

MHV Amplitude

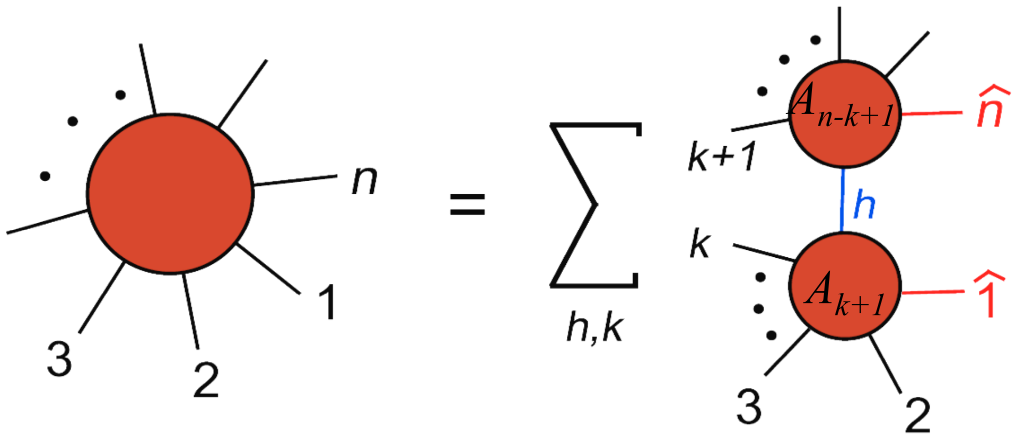
Mangano, Parke, Xu

How can something so simple be so hard to calculate?
It took a while before the significance was fully appreciated.

On-Shell Recursion

Britto, Cachazo, Feng and Witten

Work on shell where scattering is gauge invariant!



We can recursively build all tree level starting from simplest MHV ones.

A very general machinery for constructing tree-level scattering amplitudes using on-shell recursion relations.

Proof relies on so little. Power comes from generality

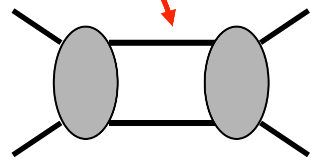
Generalized Unitarity Method

Use simpler tree amplitudes to build higher-order (loop) amplitudes.

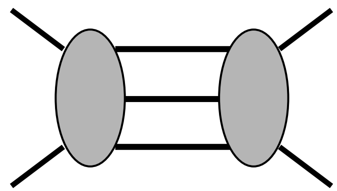
$$E^2 = \vec{p}^{\,2} + m^2$$ **on-shell**

ZB, Dixon, Dunbar and Kosower

Two-particle cut:

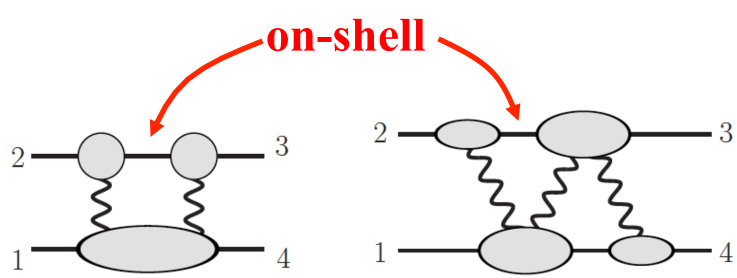


Three-particle cut:



- **Systematic assembly of complete loop amplitudes from tree amplitudes.**
- **Works for any number of particles or loops.**

Generalized unitarity as a practical tool for loops.



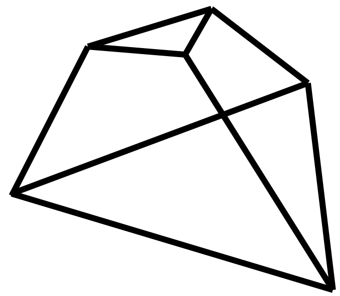
ZB, Dixon and Kosower;
ZB, Morgan;
Britto, Cachazo, Feng;
Ossala, Pittau, Papadopoulos;
Ellis, Kunszt, Melnikov;
Forde; Badger;
ZB, Carrasco, Johansson, Kosower
and many others

- **Idea used in the "NLO QCD revolution" and today at higher loops.**
- **Central to recent applications to gravitational waves.**
- **Used in $\mathcal{N} = 4$ sYM.** **See talks Elvang, Herrmann and Febres Cordero.**

Search for Foundational Principles: Amplitudes as Volumes

Scattering amplitudes have an interpretation in terms of volumes of polytopes in momentum-space version of twistor space

Hodges

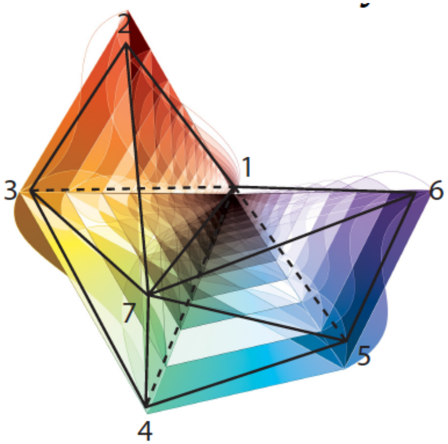
$$A_6 =$$ 

Six-point amplitude amplitude interpreted as difference of volumes of two tetrahedra!

Who ordered this?

This has been generalized to higher points and to loops. General space is called the "Amplituhedron".

$\mathcal{N} = 4$ sYM



Gluon amplitudes are volumes of the Amplituhedron

Arkani-Hamed, Trnka

Good example of the search for completely different foundational principles. No Lagrangians or principles of unitarity, just geometry.

Recent: Resummation and strong coupling Arkani-Hamed, Henn, Trnka

Structure with Applications: Double Copy

Kawai, Lewellen, Tye
ZB, Carrasco, Johansson

gauge theory:

color factor

$$\mathcal{A}_m^{\text{tree}} = g^{m-2} \sum_j \frac{c_j n_j}{D_j}$$

kinematic numerator

Feynman propagators

$$c_i = c_j - c_k \;\Rightarrow\; n_i = n_j - n_k$$

color/kinematics duality

Color Jacobi　　Kinematic Jacobi

gauge theory ⟶ gravity theory

simply take

color factor ⟶ kinematic numerator

gravity:

$$\mathcal{M}_m^{\text{tree}} = i\left(\frac{\kappa}{2}\right)^{m-2} \sum_j \frac{n_j n_j}{D_i}$$

$$c_j \to n_j$$

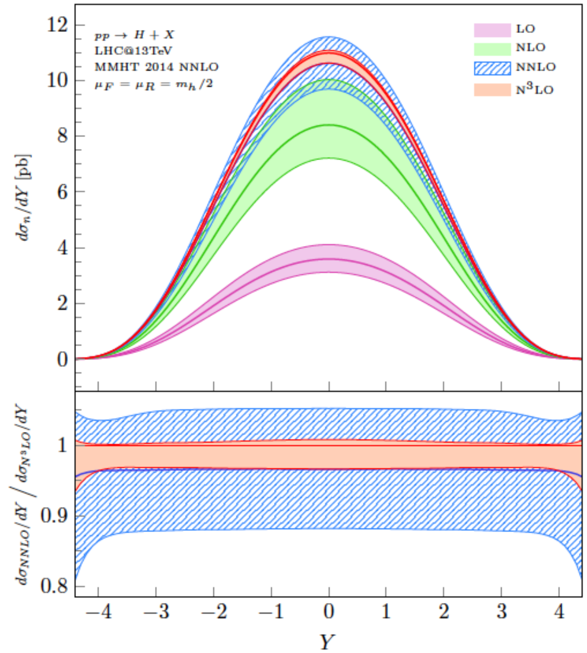
Gravity ~ (gauge theory) x (gauge theory)

Applications: supergravity, web of theories, gravitational waves

Collider Physics

The field of scatter amplitudes had its origins in collider physics
In light of difficulties to discovering new physics beyond the standard model precision measurements at the LHC are now more important than ever.

N^3LO Higgs Rapidity



Dulat, Mistlberger, Pelloni

Measurements with higher than O(7%) precision require NNLO QCD. Two loops! In certain cases percent level measurements are possible. Need N^3LO

Serious challenge, which has led to an enormous theoretical effort.

Many advances:

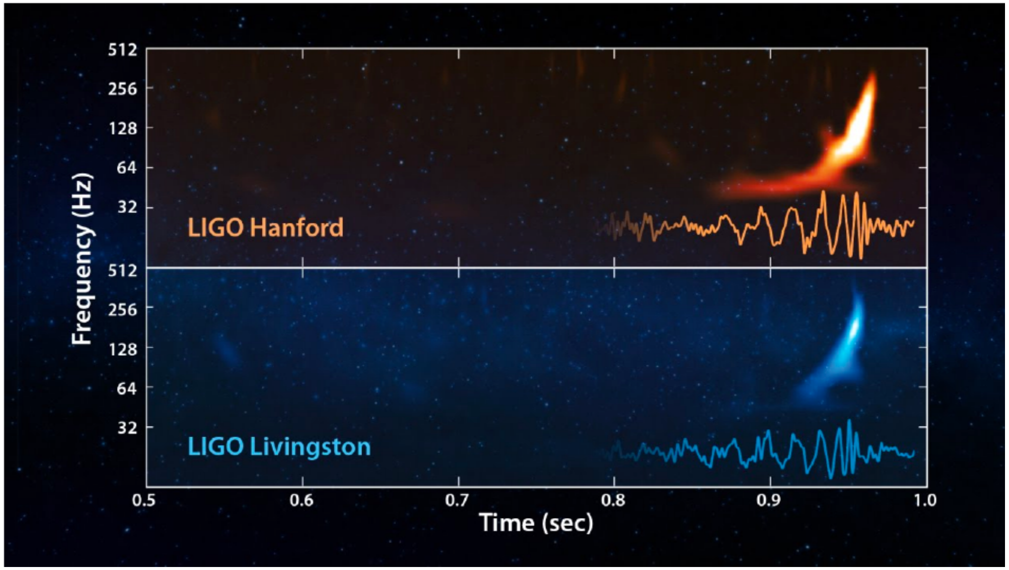
- **Multiloop integrand constructions.**
- **Improved integration.**
- **Improved IR subtraction.**
- **Better understand of the functions appearing in the amplitudes**

See Fernando Febres Coredero's talk

For more details and refs. see white papers: arXiv:2204.04200 and arXiv:2203.07088

Recent Application to Gravitational Waves

Era of gravitational-wave astronomy has begun.



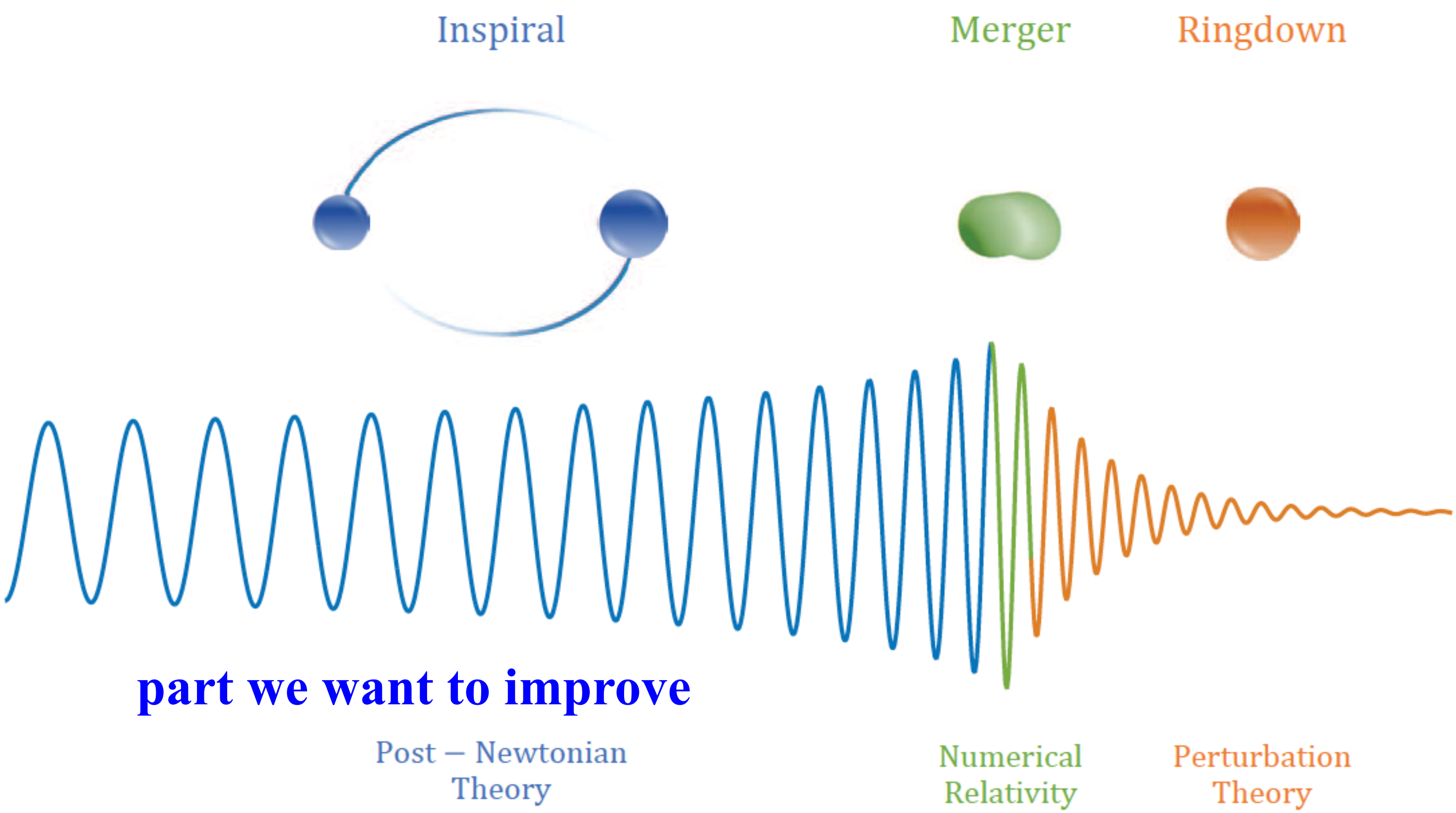
For an instant brighter in gravitational radiation than all the stars in the visible universe are in EM radiation!

The amplitudes community is now helping LIGO/Virgo theorists with the core mission.

Key Two Body Problem

See Enrico Herrmann's talk

From Antelis and Moreno, arXiv:1610.03567



part we want to improve

- **Small errors accumulate. Need for high precision.**
- **Input to EOB or other modeling to reliably approach merger.**

Buonanno and Damour

High-energy gravitational scattering and the general relativistic two-body problem

Thibault Damour*
Institut des Hautes Etudes Scientifiques, 35 route de Chartres, 91440 Bures-sur-Yvette, France
(Dated: October 31, 2017)

A technique for translating the classical scattering function of two gravitationally interacting bod-

tum gravi... ...plitude of two particles, and we urge amplitude experts to use their novel techniques to compute the 2-loop scattering amplitude of scalar masses, from which one could deduce the third post-Minkowskian effective one-body Hamiltonian

"… and we urge amplitude experts to use their novel techniques to compute the 2-loop scattering amplitude of scalar masses, from which one could deduce the third post-Minkowskian effective one-body Hamiltonian."

Hard to resist an invitation with this kind of clarity!

The recent observation [1–4] of gravitational wave signals from inspiralling and coalescing binary black holes has been significantly helped, from the theoretical side, by the availability of a large bank of waveform templates, defined [5, 6] within the analytical effective one-body (EOB) formalism [7–11]. The EOB formalism combines, in a suitably resummed format, perturbative, analytical results on the motion and radiation of compact binaries, with some non-perturbative information extracted from numerical simulations of coalescing black-hole binaries (see [12] for a review of perturbative results on binary systems, and [13] for a review of the numerical relativity of binary black holes). Until recently, the perturbative results used to define the EOB conservative dynamics were mostly based on the post-Newtonian (PN) approach to the general relativistic two-body interaction. The conservative two-body dynamics was derived, successively, at the second post-Newtonian (2PN) [14, 15], third post-

...ntro- ...erive ...the (gauge-invariant) *scattering function* Φ linking (half) the center of mass (c.m.) classical gravitational scattering angle χ to the total energy, $E_{\rm real} \equiv \sqrt{s}$, and the total angular momentum, J, of the system[1]

$$\frac{1}{2}\chi = \Phi(E_{\rm real}, J; m_1, m_2, G)\,. \tag{1.1}$$

The (dimensionless) scattering function can be expressed as a function of dimensionless ratios, say

$$\frac{1}{2}\chi = \Phi(h, j; \nu)\,, \tag{1.2}$$

where we denoted

$$h \equiv \frac{E_{\rm real}}{M}\,;\; j \equiv \frac{J}{G m_1 m_2} = \frac{J}{G\mu M}\,, \tag{1.3}$$

with

$M \equiv m_1 + m_2$; $\mu \equiv \frac{m_1 m_2}{\ldots}$; $\nu \equiv \frac{\mu}{M} = \frac{m_1 m_2}{\ldots}$

Pushing the State of the Art

Amplitudes results immediately recognized by gravitational wave community.

A flood of new idea and results:

- **Rapidly pushing state of the art especially in post-Minkowskian approach. First $O(G^3)$ and now $O(G^4)$.**
- **New structures, e.g. simple mass dependence now exploited by gravitational wave theorists.**
- **Applications to broad set of problems including spin, tidal effects and radiation. Rapidly growing subfield of amplitudes.**

Amplitude methods very quickly have become a standard tool for Gravitational-wave problems. It has really been a lot of fun!

See Enrico Herrmann's talk for further details

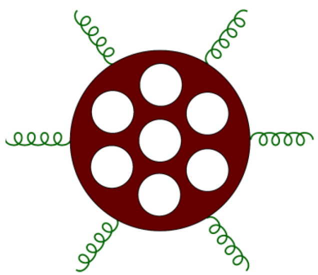
Looking to the Future: Antipodal Symmetry

Dixon, Gürdoğan, McLeod, Wilhelm

$\mathcal{N}$ = 4 super-Yang-Mills is the "hydrogen atom" of scattering amplitudes.

Elvang's talk

6pt 7 loop scattering amplitude



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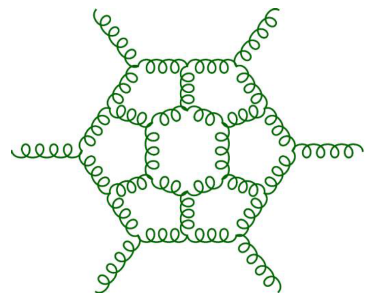
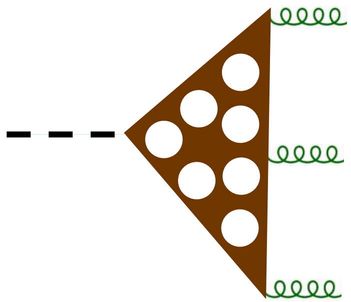


+ ~10^9 more

3 gluon 7 loop form factor



A form factor seeming looks to be completely different than an amplitude

$$\boxed{F_3^{(L)}(u,v,w) = S\left(A_6^{(L)}(\hat{u},\hat{v},\hat{w}\right)}$$

S is the antipodal map which reverses "letters" describing polylogs.

- **Who ordered this?**
- **Is this a curiosity or is it a breakthrough?**

It will be exciting to see how far this goes.

Looking to the Future: EFT Coefficient Bounds

Consider gravity EFT:

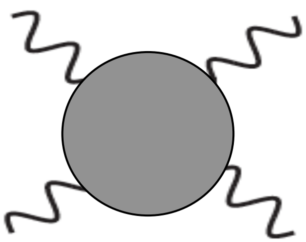
$$S_{\rm EFT} = \int d^4x\sqrt{-g}\left[-\frac{2}{\kappa^2}R + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{2\beta_\phi}{\kappa^2}\phi\, C + \frac{8}{\kappa^3}\frac{\beta_{R^3}}{3!}R^3 + \frac{2\beta_{R^4}}{\kappa^4}C^2 + \frac{2\tilde\beta_{R^4}}{\kappa^4}\tilde C^2 + \dots\right]$$

$$R^3 \equiv R^{\mu\nu\kappa\lambda}R_{\kappa\lambda\alpha\gamma}R^{\alpha\gamma}{}_{\mu\nu} \qquad C \equiv R^{\mu\nu\kappa\lambda}R_{\mu\nu\kappa\lambda} \qquad g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

What low-energy EFT coefficients are physically allowed?

Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi

To study bounds want to look at gauge-invariant quantities:



Scattering amplitudes natural language.

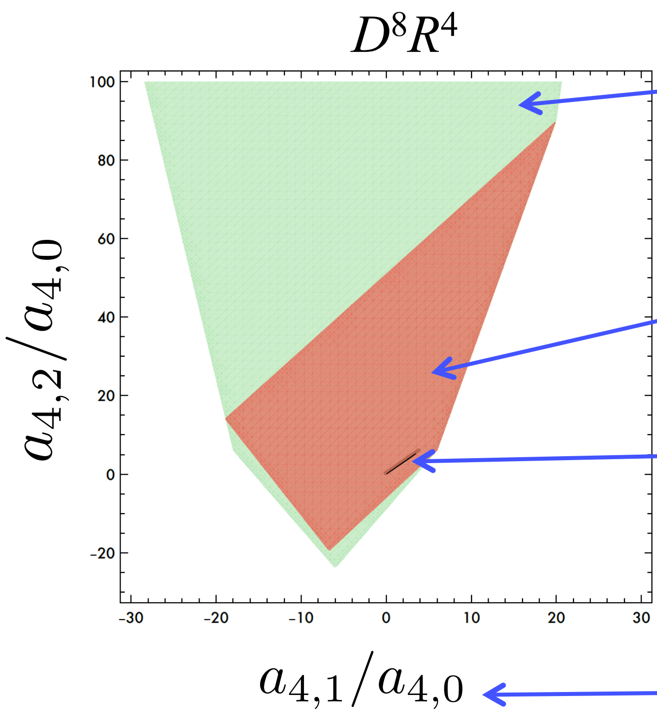
- **Avoids issues with field redefinitions and gauge fixing.**
- **Unitarity, crossing and dispersion relations natural tools.**
- **Ideas apply as well to SMEFT and other nongravitational theories.**

See Claudia de Rahm's Thurs. talk.

Looking to the Future: Bounds on Gravitational EFTs

Arkani-Hamed, Huang, Huang
ZB, Kosmopoulos, Zhiboedov

Consider a 4 graviton amplitude in an EFT



Green region from unitarity a good high energy behavior

Region is from adding crossing symmetry

Tiny black region is the observed island from known sensible theories.

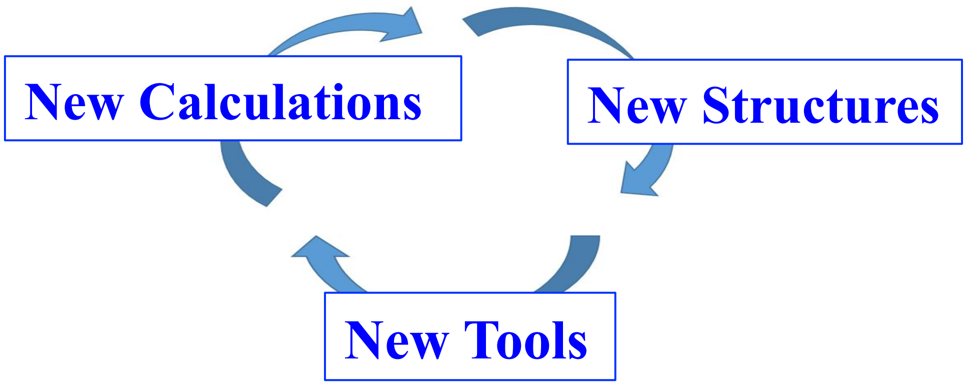
Essentially ratio of Wilson coefficients.

Observed theory islands suggest:

1) **Vast improvements are possible.**
2) **New principle: "Low spin dominance" in partial-wave expansion.**

Is this a curiosity or is it of fundamental importance?

Summary



Amplitudes is a thriving field with many direction, of which I only skimmed the surface.

Upcoming talks will present further details and examples.