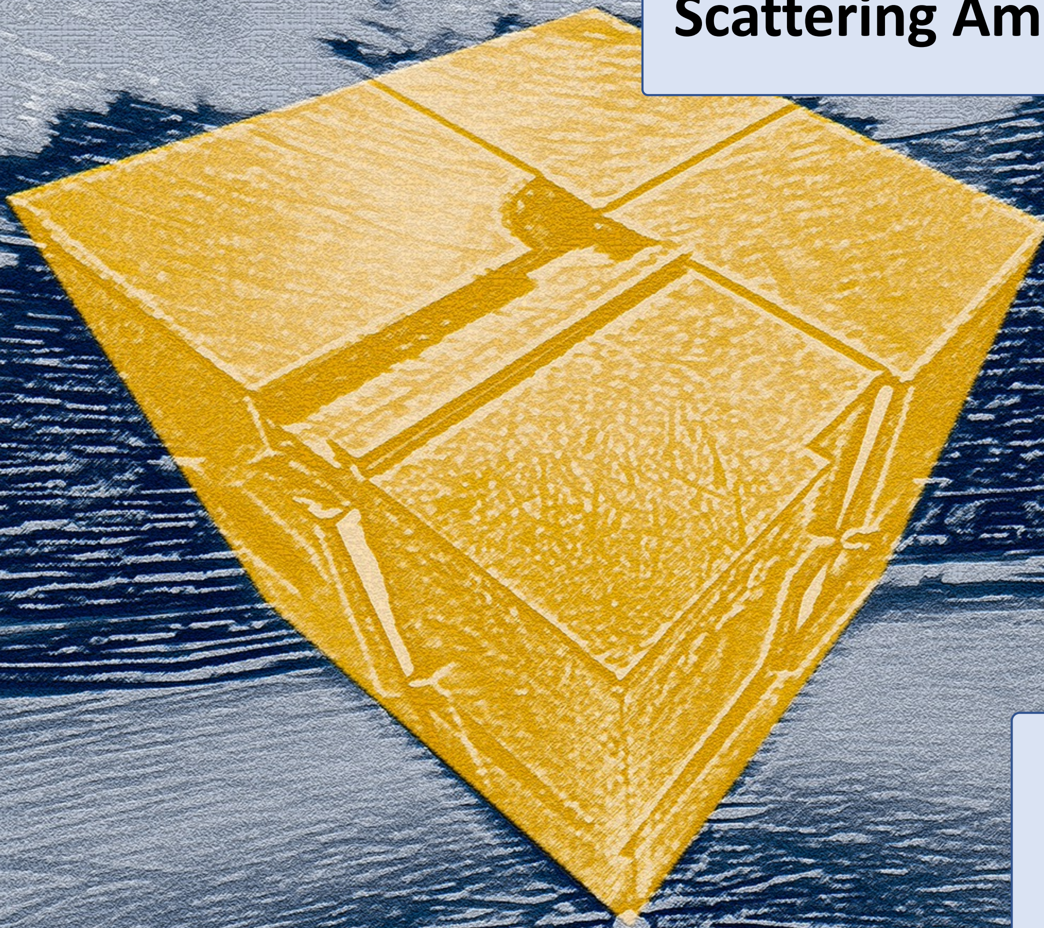
Scattering Amplitudes and Fundamental Physics



Snowmass Community Summer Study Workshop

Henriette Elvang
University of Michigan
Leinweber Center for Theoretical Physics

University of Washington
Seattle
July 17-26, 2022

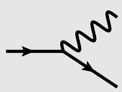
UNIVERSITY OF MICHIGAN

Traditional

Lagrangian

↓

Feynman Rules



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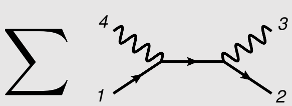
Amplitude



Modern On-shell

Model w/ spectrum of particles, symmetries

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Amplitudes constructed from analytic properties & symmetries

Modern on-shell techniques allow us to calculate scattering amplitudes efficiently

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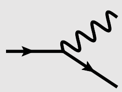

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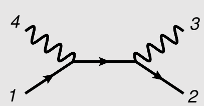
$\sum$ 

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Can also explore the space of Quantum Field Theories through the knowledge of possible scattering processes given physical states and symmetries

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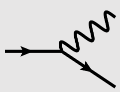

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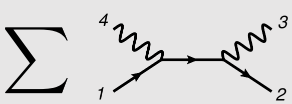

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Practical applications

Modern on-shell techniques allow us to calculate scattering amplitudes efficiently

Can also explore the space of Quantum Field Theories through the knowledge of possible scattering processes given physical states and symmetries

Gain insights into the fundamental properties of quantum field theories, effective field theories

Effective Field Theory (EFT)

EFT Principle 1
To given order in the derivative-expansion, include all higher-derivative gauge-invariant local operators permitted by the symmetries.

Lagrangian formulation: How many gauge-invariant local operators are there subject to
1) integration-by-parts *and* 2) the EOM *and* 3) field redefinitions ?

On-shell amplitudes methods are VERY efficient for such questions.

On-shell local operators in 1-1 correspondence on-shell matrix elements

Amplitudes formulation: How many independent on-shell matrix elements are there modulo momentum conservation and Bose/Fermi symmetry of identical states?

Examples

1 $\partial^{2k}\phi^4$ Abelian => Bose symmetry => symmetric degree *k* polynomials in *s,t,u* indep. under to *s+t+u*=0

Such polynomials are of the form $(stu)^{n_1}(s^2+t^2+u^2)^{n_2}$

So, count of indep. operators is number of ways to write $k = 3n_1 + 2n_2$

Example $\partial^{22}\phi^4$ k=11 n_1 odd -> n_1=1 or 3 => there are **2** such indep. operators.

So: Counting easy. Direct construction of local matrix elements easy. Basis changes easy.

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2 $D^k\bar{\psi}F^2\phi^2\psi$

Spinor-helicity formalism makes this very efficient: $\langle\psi^+\psi^-\gamma^+\gamma^-\phi\phi\rangle$ massless

k =	1,	3,	5,…
dim =	10,	12,	14,…
# operators =	2,	21,	114,…
Comp time =	0.1s,	1.6s,	5min,…

Same principles (and more machinery) for MASSIVE particles.

Application in SMEFT

Many other applications of these ideas in formal theory, such as for local counterterms for UV divergences in perturbative supergravity, higher-derivative corrections to chiral perturbation theory, Galileons, finite local counterterms in Born-Infeld, monopoles, dark matter…

Freedman, Kiermaier, Elvang; Beisert, Morales; Mitchell; Hadjiantonis, Jones, Paranjape; Bern, Parra-Martinez, Roiban; Csaki, Hong, Shirman, Telem, Terning; Falkowski, Isabella, Machado;…

Example
3- and 4-pt SMEFT operators systematically characterized by

Aoude, Durieux, Kitahara, Machado, Shadmi, Weiss (2018-21)

using the massive spinor helicity formalism of

Arkani-Hamed, Huang, and Huang (2017)

Some comparisons in certain sectors so far with Lagrangian approaches, for example w/ Henning Lu Melia Murayama. Plus in follow-up papers.

Further expanded technique and analysis by Accettulli Huber + De Angelis (2022) and De Angelis (2022).

Anomalous dimension mixing matrix

Under RG, operators mix.

Important for interpretation of experimental results to understand how.

Under RG, operators can mix.

$$16\pi^2 \frac{\partial c_i}{\partial \log \mu} = \gamma_{ij}^{\mathrm{UV}} c_j \qquad \Delta\mathcal{L} = \sum_i c_i \mathcal{O}_i$$

$\gamma_{ij}^{\mathrm{UV}}$ is the anomalous-dimension matrix

Surprising **1-loop non-renormalization results for SMEFT** dim 6 operators.
Alonso, Jenkins, Manohar (2014)
(Grojean, Jenkins, Manohar, Trott; Elias-Miro, Espinosa, Masso, Pomarol (2013))

Explained by Cheung and C-H Shen (2015) using on-shell amplitudes methods to characterize the possible local operators at dim 5 and 6.

Using on-shell unitarity methods to get anomalous dimensions and beta functions from Caron-Huot and Wilhelm (2016), new non-renormalization theorems derived for **dim 5 through 7 SMEFT operators** by Bern, Parra-Martinez, and Sawyer (2019). **2-loop** SMEFT anomalous dim's Bern, Parra-Martinez, and Sawyer (2020). Mixing matrix at Dim 8 in Accettulli Huber + De Angelis (2022).

Pushing the Loop Limit

Front-line particle physics calls for high-precision theory.

Need to push beyond NLO => need to push techniques for calculating higher-loops.

- It can be very fruitful to use theories with a high degree of symmetry as ``a lab” for developing and testing new calculational tools.

- Such a theory is **N=4 super Yang-Mills theory**.

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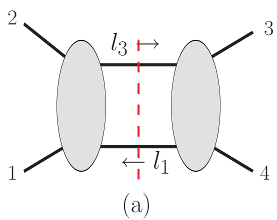
At the origin of moduli space (all scalars have vanishing VEVs).

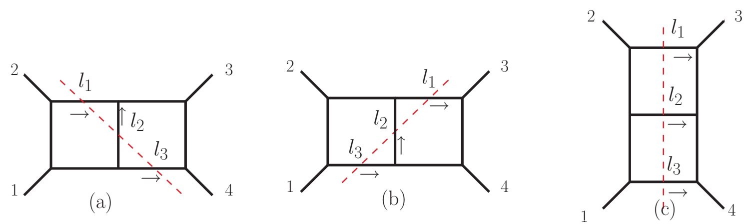
Can give scalars VEV (Coulomb branch) while preserving supersymmetry but reducing other symmetries and generate masses. A step closer to SM – but still far. Yet, still very useful.

Example of benefits

Generalized Unitarity: idea is to use unitarity cuts to sow loop-amplitudes together from trees (and lower loop).

Bern, Dixon, Kosower,…





from [Bern & Huang review (2011)]

Develop and test in N=4 SYM. Push loop order in N=4 SYM & perturbative N=8 supergravity.

Apply in pheno. [Berger, Bern, Dixon, Febres Cordero, Forde, Ita, Kosower, Maitre (2008)]

e.g. from `early days' W + 3 jets at NLO e.g. [K. Ellis, Giele, Kunszt, Melnikov, Zanderighi (2008)]

to very recent leading edge assembly of tools to get NNLO for four partons and a W boson in QCD

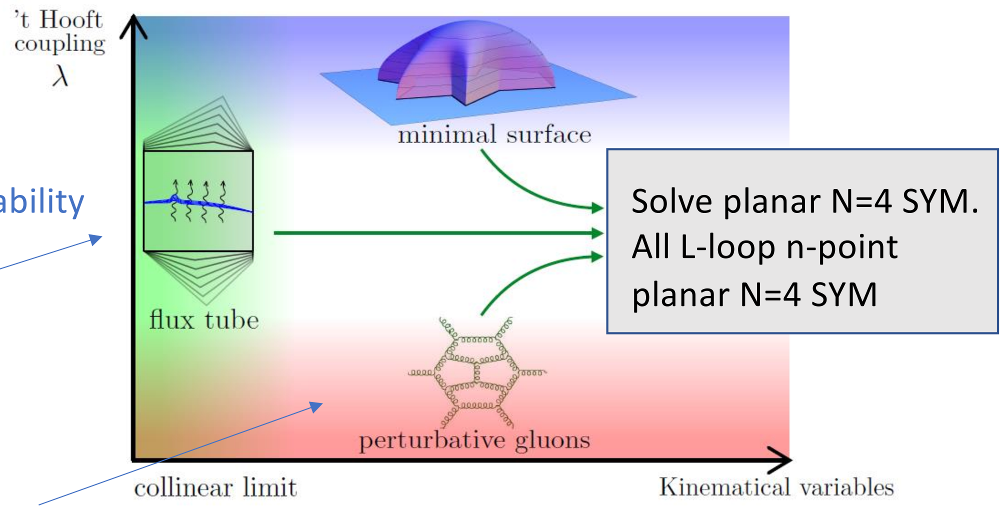
[Abreu, Febres Cordero, Ita, Klinkert, Page, Sotnikov (2022)]

See Febres Cordero's talk

How are the loop-techniques pushed in (planar) N=4 SYM?

Strong coupling

AdS/CFT



Integrability

All coupling

Weak coupling

Generalized unitarity, Amplituhedron, Amplitude bootstrap

Figure slightly modified from Fig 1 in the white paper by Arkani-Hamed, Dixon, McLeod, Spradlin, Trnka, and Volovich [2207.10636]

Amplitude bootstrap

Loop amplitudes are complicated – but not arbitrary – combinations of special mathematical functions whose arguments are functions of particular kinematic variables.

Transcendental functions: Logarithms, di-logarithms, polylogarithms…
with increasingly intricate branch cut structure.

N=4 SYM UV finite and IR divergences well-understood (BDS ansatz).

This leaves an undetermined ``remainder function”.

There are a long list of properties the combination of special functions that make up an amplitude must satisfy. That means only certain combinations can occur. The more of these constraints are imposed, the fewer free parameters.

See white paper 2207.10636

Amplitude bootstrap for planar N=4 SYM

From 2207.10636 Planar 6-point MHV at L-loops:

Space of relevant polylogarithms

Only certain letters at the end of the symbol

MRK = Multi-Regge Kinematics

LL = Leading Logarithms

Constraint	$L=1$	$L=2$	$L=3$	$L=4$	$L=5$	$L=6$
1. $\mathcal{H}^{\text{hex}}$	6	27	105	372	1214	3692?
2. Symmetry	2	7	22	66	197	567
3. Final-entry	1	4	11	30	85	236
4. Collinear	0	0	0^*	0^*	1^{*3}	6^{*2}
5. LL MRK	0	0	0	0	0^*	1^{*2}
6. NLL MRK	0	0	0	0	0^*	1^*
7. NNLL MRK	0	0	0	0	0	1
8. N^3LL MRK	0	0	0	0	0	1
9. Full MRK	0	0	0	0	0	1
10. T^1 OPE	0	0	0	0	0	1
11. T^2 OPE	0	0	0	0	0	0

For table that includes both **MHV** and **NMHV** counting,
see 1903.10890 (Caron-Hout, Dixon, Dulat, von Hippel, McLeod, Papathanasiou)]

Future directions for N=4 SYM

On-going research on understanding "letter alphabet" needed; pushing loop-order, applying to form factors, and much much more

e.g. up to and incl. **8-loop(!!)** results for 3-pt stress-tensor form factor

[Dixon, Gurdogan, McLeod, Wilhelm (2022)]

From abstract:

By the principle of maximal transcendentality, our results are expected to give the highest weight part of the gg → Hg and H → ggg amplitudes in the heavy-top limit of QCD through eight loops

Future directions for N=4 SYM

Ongoing research on understanding "letter alphabet" needed; pushing loop-order, applying to form factors, and much much more

This develops technical tools for loop-calculations but at the same time also offers invaluable insight into the intriguing mathematical structure of the observables of QFT. Very fruitful connections to mathematics (cluster algebra, Grassmannians, positive geometry…)

Future should bring *cross-fertilization between formal theory advances and pheno applications.* Several people work in both areas.

Bern, Dixon, Kosower, Mistlberger, Duhr,…

Excitingly, N=4 SYM amplitudes technology has also recently found its way into gravitational wave physics: Zvi Bern's talk and Enrico Hermann's talks.

Back to EFTs

EFT principle 2

The higher-derivative operators appear with generic coefficients naturally expected to be of order ~1 in units of the scale of the UV physics.

… so if these coefficients are not ~1 (say <<1 or >>1 or even 0), we have some explaining to do.

Study UV-completable models have constraints on the Wilson coefficients.

Examples

Exploring those bounds are the subject of the S-matrix bootstrap / EFT-hedron / weak gravity conjecture via amplitudes

Adams, Arkani-Hamed, Dubovski, Nicolis, Rattazzi; Arkani-Hamed, T-C Huang, Y-t Huang; Vafa, Ooguri; Arkani-Hamed, Y-t Huang , J-Y Liu, Cheung, Remmen, Jones, McPeak, Caron-Huot, …

Bottom-up bootstrap of string theory via amplitudes:

Arkani-Hamed, Y-t Huang, Vieira, Penedones, Guerrieri, Komargodski, Sever, Zhiboedov, Alonso, Rodina, Eberhardt, Mizera, Liu, Wang, Van Duong, Mazáč, Rastelli, Simmons-Duffin, Bellazzini, Miro, Rattazzi, Riembau, Riva, Tolley, Wang, S-Y Zhou, Parra-Martinez,…

Related: Snowmass white paper on bootstrapping string theory
Gopakumar, Perlmutter, Pufu, Yin

Examples

(see also Bern's talk for other examples)

Proposal: EFT-hedron + String Monodromy => the open string

Huang, J-Y Liu, Rodina, Wang (2008)

Evidence: Wilson coefficients of YM theory with higher-derivative operators bootstrapped at 4-pt with EFT-hedron + monodromy => open string.

Forthcoming: analysis of replacing String Monodromy with N=4 supersymmetry.

Berman, Elvang, Herderschee (arXiv:2022….)

Amplitudes and EFTs

Soft theorems in EFTs and bootstrapping exceptional EFTs

Cheung, Trnka, Elvang, Jones, Naculich, Hadjiantonis, Paranjape, Helset, Parra-Martinez, Z Yin,C-H Shen.
I. Low, Kampf, Novotný, …

Celestial amplitudes & EFTs

Arkani-Hamed, Pate, Raclariu, Strominger

Double-copy in EFTs

BCJ-based	Carrasco, Rodina, Zekioglu
KLT bootstrap	HH Chi, Elvang, Herderschee, Jones, Paranjape
Connecting (4pt)	Durieux, Grojean, Bonnefoy, Machado, Roosmale Nepveu

Double-copy white paper
[Adamo, Carrasco, Carillo-Gonzales, Chiodaroli, Elvang, Johansson, O'Connel, Roiban Schlotterer (2022)]

KLT double-copy bootstrap

Chi, Elvang, Herderschee, Jones, Paranjape (2021)

Proposed that an underlying structure, the KLT algebra, is key for generalizations of the double-copy, not only in EFT context but also more generally.

Says that regarded as a map between field theories (FT): FT x FT -> FT
the double copy has an identity element "1" with the defining relations

$$\underbrace{\mathrm{L} = \mathrm{L} \otimes \mathbf{1}\,, \quad \mathrm{R} = \mathbf{1} \otimes \mathrm{R}\,,}_{\text{KKBCJ / monodromy relations}} \quad \underbrace{\mathbf{1} = \mathbf{1} \otimes \mathbf{1}\,.}_{\textbf{KLT Bootstrap Equation}}$$

Identity uniquely linked with product rule => generalize the double-copy product rule.

In EFT, applying the bootstrap for 4- and 5-point amplitudes allows a double-copy kernel which is much more general than that of string theory (so what makes string theory special?).

However, new things begin to happen at 6-points!!

A. Sh-K. Chen, Elvang, to appear; see also my Amplitudes 2022 talk

KLT double copy bootstrap

- Insights into the fundamental inner workings of the double-copy as a map on the landscape of QFTs.
- Applications relevant for EFTs.
- Narrowing in on string theory from a bottom-up EFT bootstrap approach

Modern Amplitudes

A powerful approach to explore fundamental physics and structure of QFT

Very active and growing field of research, attracting a lot of young researchers

Impact both on the front of

Advancing our understanding of Quantum Field Theory on the formal side

And direct applications to particle physics, beyond-LO calculation, SMEFT, …

Those are the pillars of our field: the interplay between

the pursuit of the mathematical truth and beauty

&

experimental + pheno particle physics and description of Nature