Quantum Computing Theory

Seattle Snowmass Summer Meeting 2022
Joint TF10 and CF6 Fri 07/22/2022

Christian Bauer
Quantum Simulation

Quantum Simulation for High Energy Physics

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arXiv:2204.03381v1 [quant-ph]

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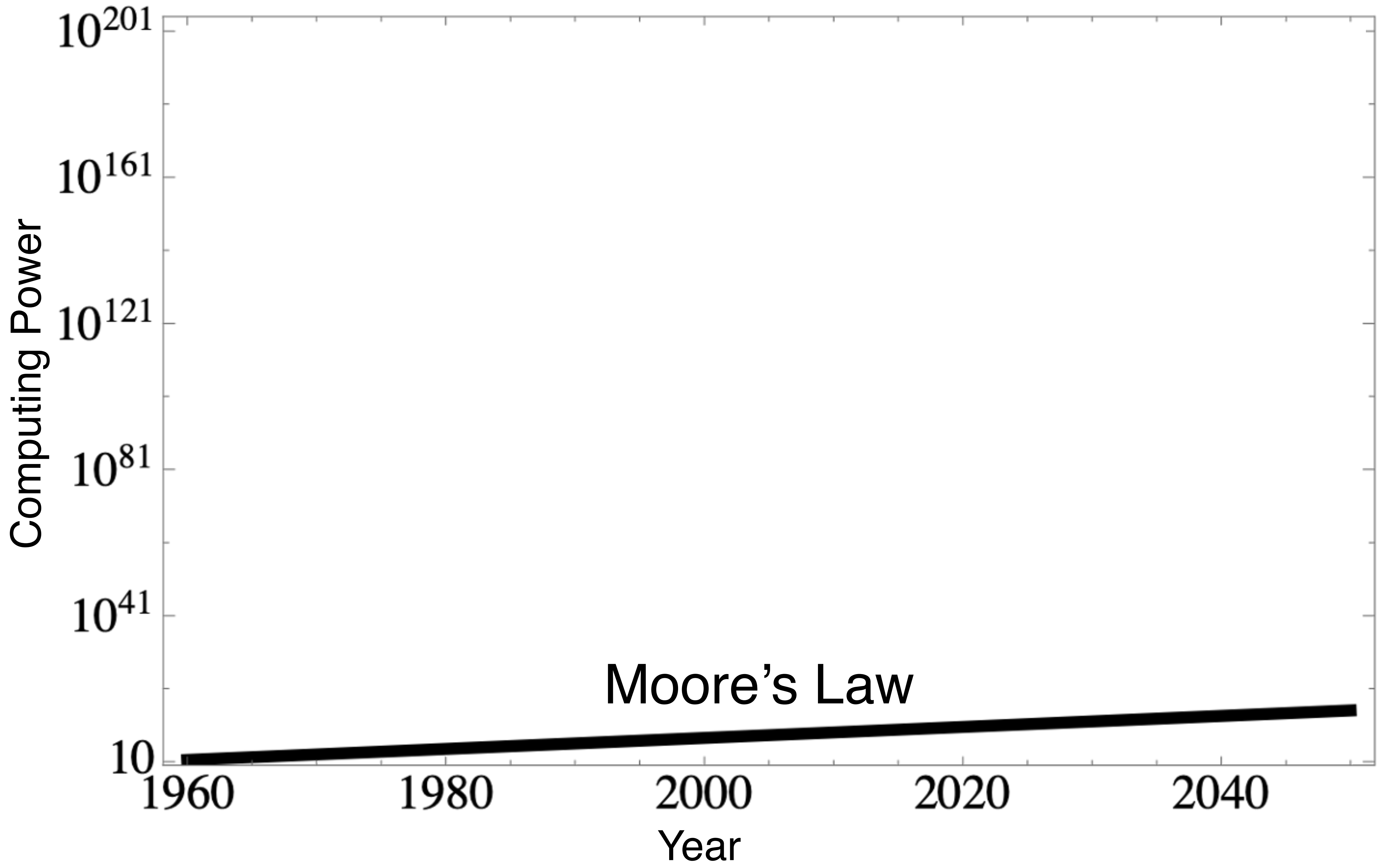

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Computing Power
Moore's Law
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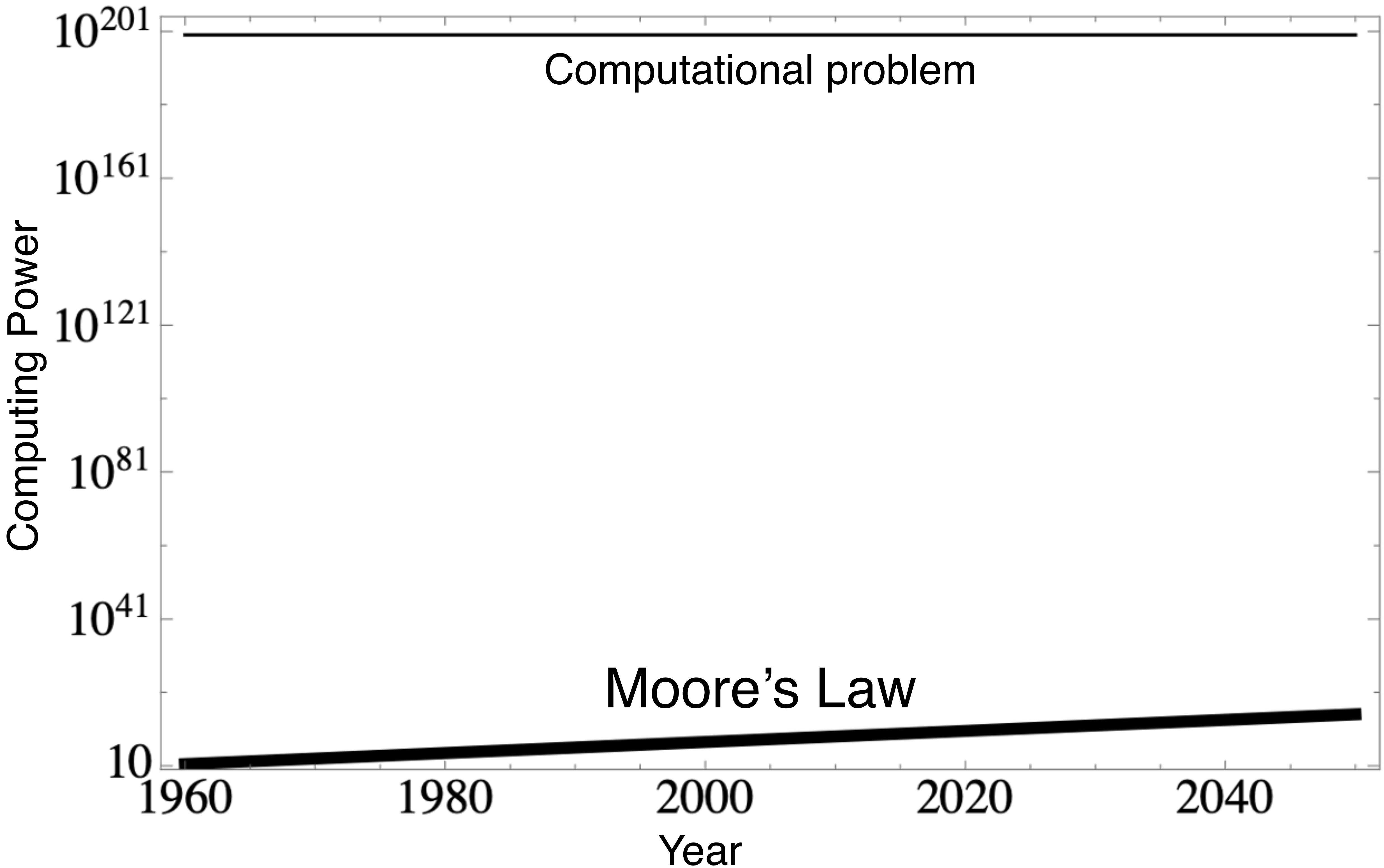
Year



Computational problem
Moore's Law
Computing Power
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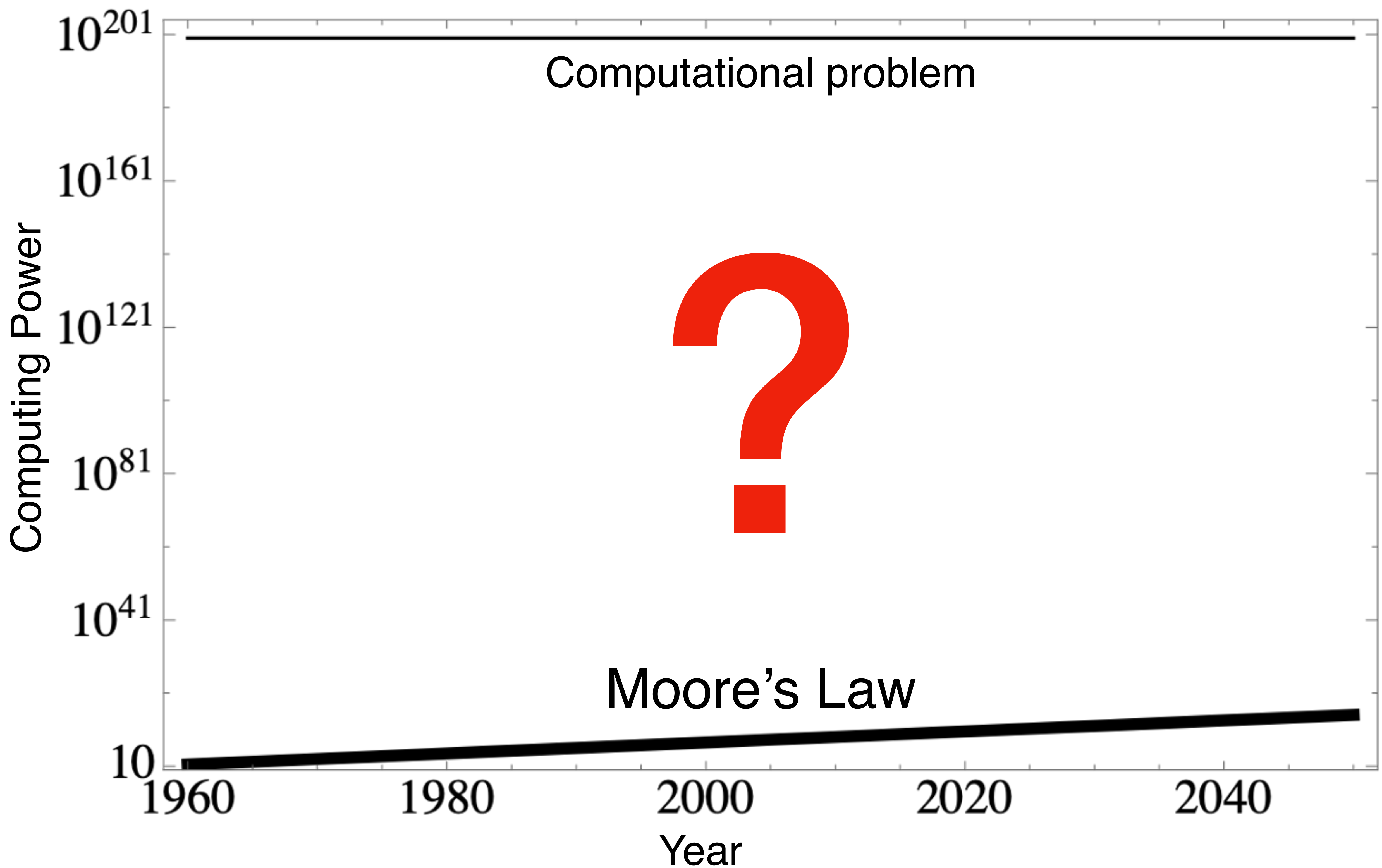
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Computational problem
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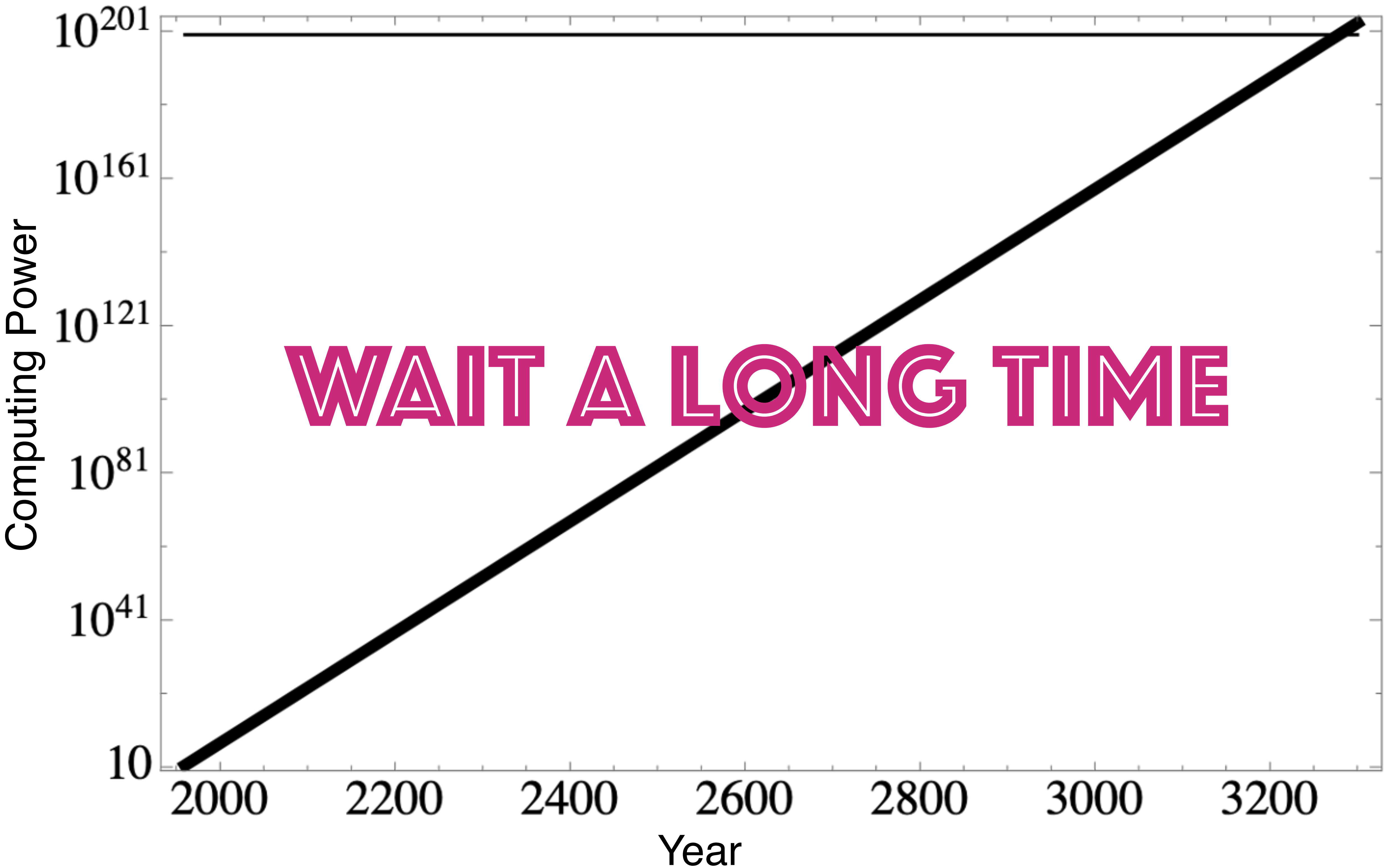

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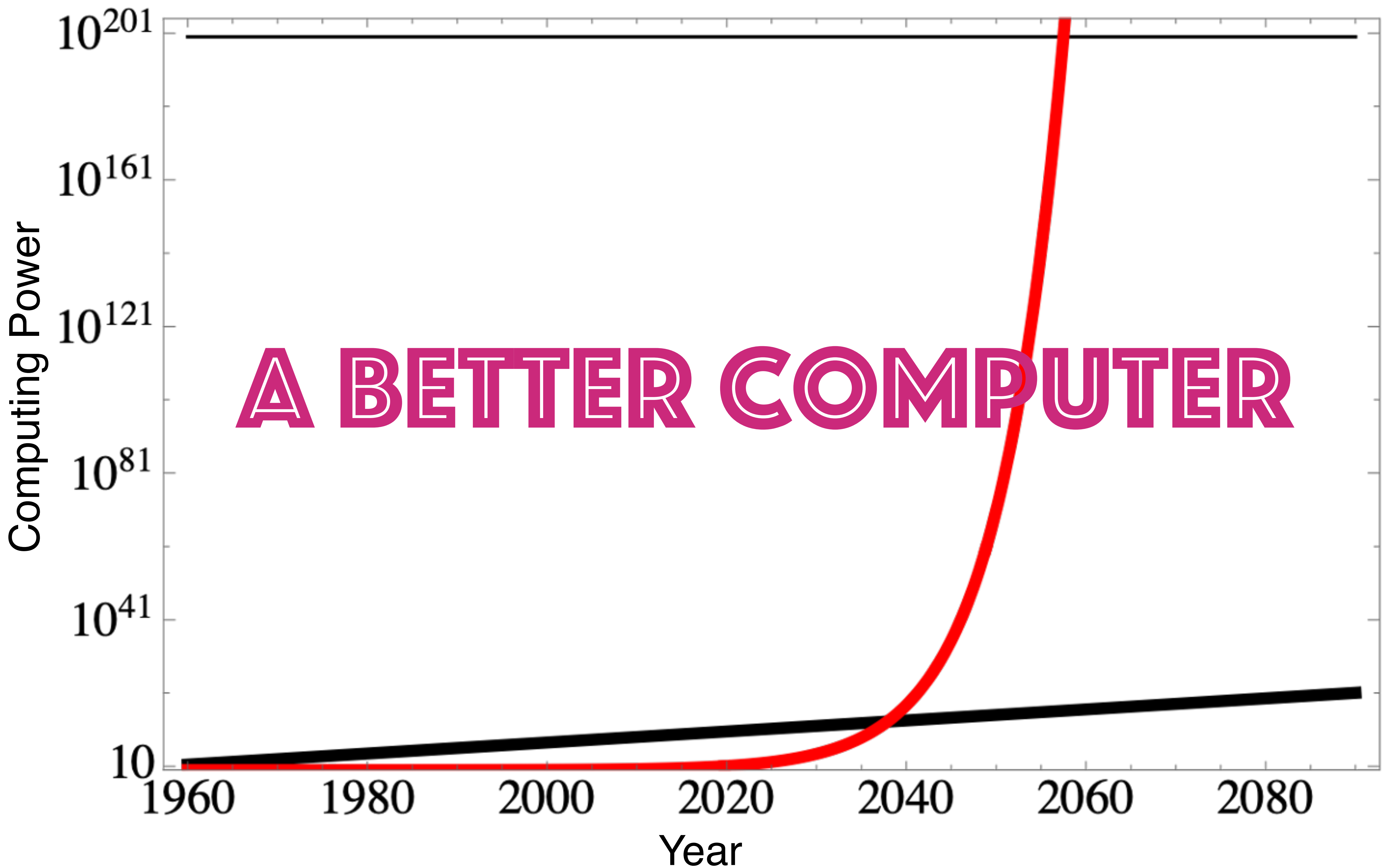
Computing Power
WAIT A LONG TIME



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Computing Power
A BETTER COMPUTER
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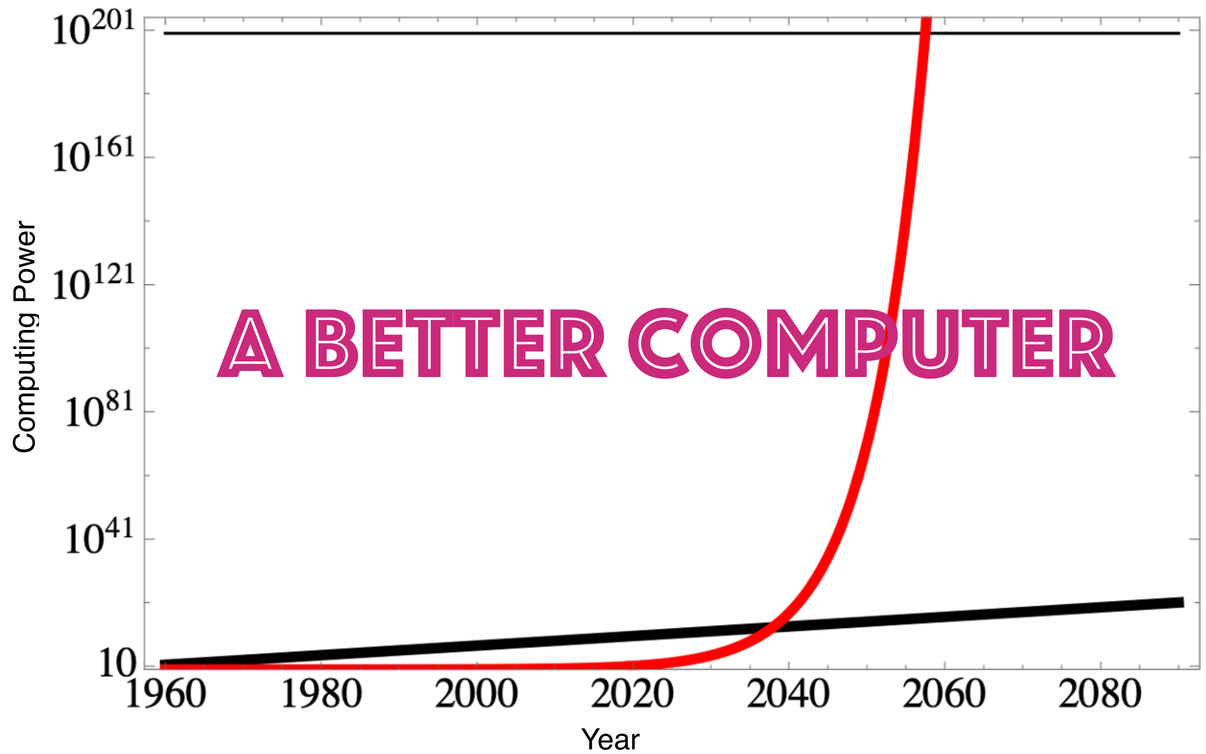
Year

Very important to scrutinize the validity of this statement for HEP and ask two important questions

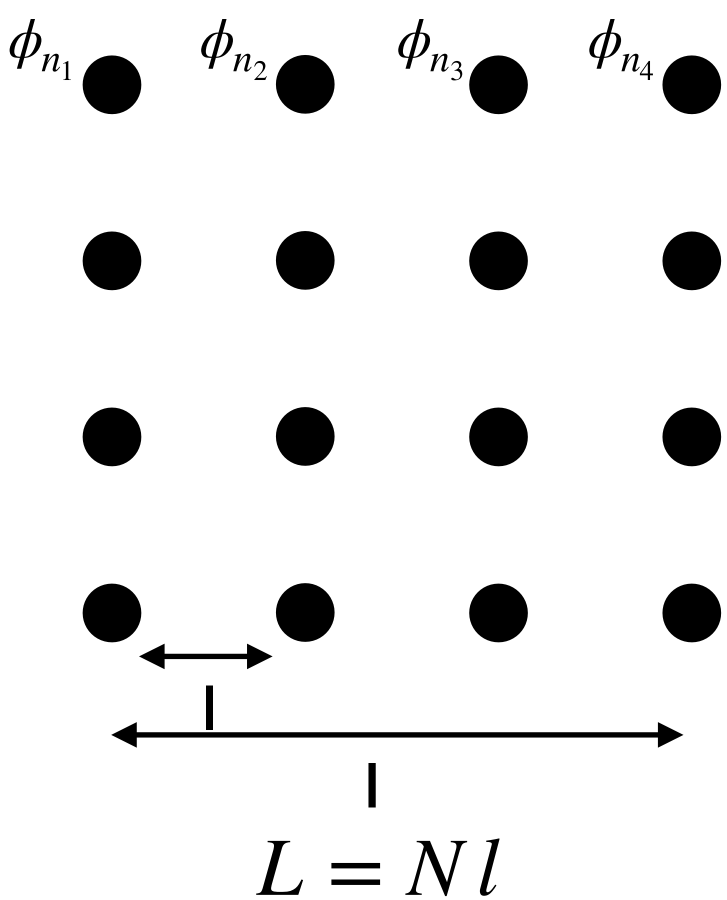


1. Does such a computational problem(s) exist, and will it alter the direction of HEP?
2. Do we really know that a quantum computer performs this calculation exponentially faster?

1. Have identified many such problems in collider physics, neutrino physics, cosmology, early universe physics, quantum gravity etc
   - One example is a first principles calculation of scattering process, such as pp->X (LHC). Will focus on this in remainder of talk
2. This problem is exponentially difficult in energy of collider, but has been shown that can be done on quantum computer polynomial in the energy

The S-matrix for a scattering process from an initial state q_I to a final state q_F can be computed using a lattice approach

Turn into finite dimensional Hilbert space by discretizing both spatial directions and field values



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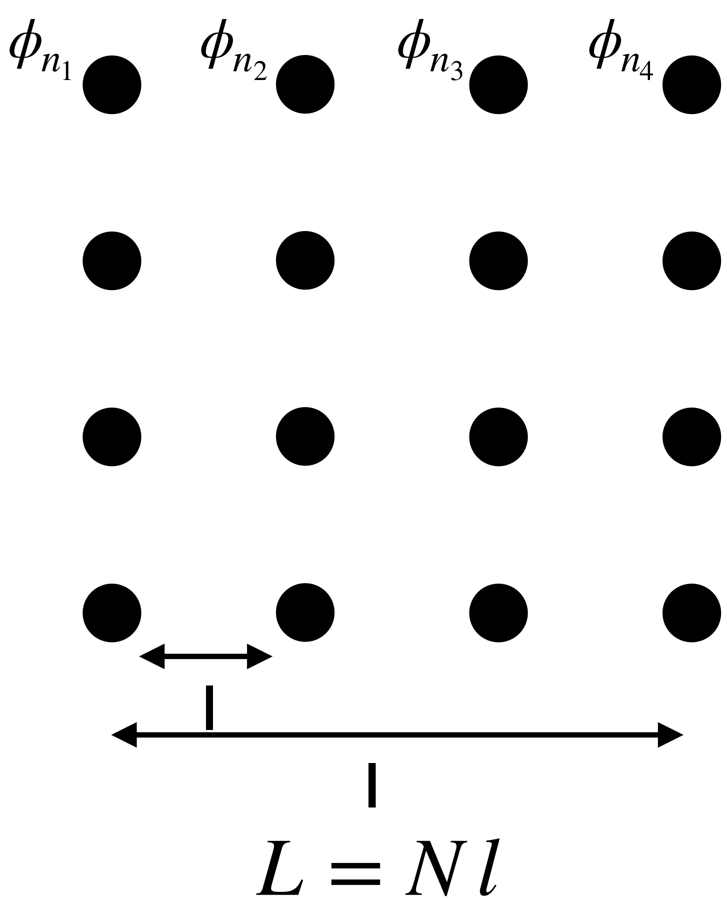

Hilbert space has dimension

$$\left(n_\phi\right)^{N^d}$$

n_ϕ : # of digitized field values
N : # of lattice points per dim
d : # of dimensions

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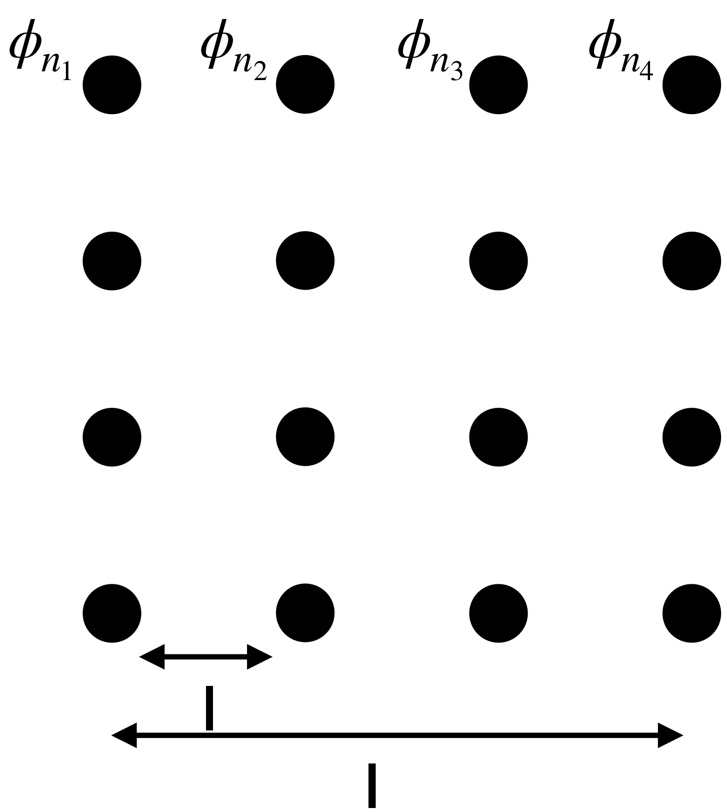

$$L = N\,l$$

Hilbert space has dimension

$$\left(n_\phi\right)^{N^d}$$

n_ϕ : # of digitized field values
N : # of lattice points per dim
d : # of dimensions

Situation more complicated for gauge theories

Can use directly the time evolution between initial and final state

$$\langle q_F(T) \,|\, e^{-iH(2T)/\hbar} \,|\, q_I(-T) \rangle$$

$$\langle q_F | U | q_I \rangle = \begin{bmatrix} \star & \star & \cdots & \star \end{bmatrix} \begin{bmatrix} \star & \star & \cdots & \star \\ \star & \star & \cdots & \star \\ & \cdots & & \\ \star & \star & \cdots & \star \end{bmatrix} \begin{bmatrix} \star \\ \star \\ \cdots \\ \star \end{bmatrix}$$

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After discretization, states are vectors in the finite dimensional Hilbert space, while Hamiltonian is a matrix

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$$\langle q_F \,|\, U \,|\, q_I \rangle = [\, \star \; \star \; \cdots \; \star \,] \begin{bmatrix} \star & \star & \cdots & \star \\ \star & \star & \cdots & \star \\ & \cdots & & \\ \star & \star & \cdots & \star \end{bmatrix} \begin{bmatrix} \star \\ \star \\ \cdots \\ \star \end{bmatrix}$$

Quantum computers can do this calculation with resources (number of qubits and number of operations) that scale logarithmically in the size of Hilbert space

The size of the lattice is determined by the energy range we try to simulate

Energy rage that can be described by lattice is given by

$$\frac{1}{Nl} \lesssim E \lesssim \frac{1}{l}$$

Assume we want to compute scattering at 100 GeV collider

$$100\,\mathrm{MeV} \lesssim E \lesssim 100\,\mathrm{GeV}$$

This needs $\mathcal{O}(1{,}000^3) \sim 10^9$ lattice sites

Assume I need at least 5 bit digitization $\Rightarrow n_\phi = 2^5 = 32$

Dimension of Hilbert space is

$$32^{10^9} \sim \infty$$

Number of qubits and required

$$5 \times 10^9$$

Less resources required to address only the most difficult aspect of the problem: non-perturbative quantities

Assume we only want to compute low energy quantity

$$100\,\mathrm{MeV} \lesssim E \lesssim 2\,\mathrm{GeV}$$

This needs $\mathcal{O}(20^3) \sim 10^4$ lattice sites

Number of qubits and required

$$5 \times 10^4$$

Groundbreaking questions can be addressed with "reasonable" resource requirements

Given this important result, the important work now is to work out details and compare different approaches

Three main directions of research are required to fully develop this program

1. Theoretical foundations to formulate QFT calculations such that they are accessible to quantum computer

2. Development of efficient algorithms that allow to run required calculations using resources optimally

3. Understand how to optimize hardware for the physics problems at hand

Given this important result, the important work now is to work out details and compare different approaches

Three main directions of research are required to fully develop this program

1. Theoretical foundations to formulate QFT calculations such that they are accessible to quantum computer

2. Development of efficient algorithms that allow to run required calculations using resources optimally

3. Understand how to run these algorithms on actual quantum hardware

Lattice formulation of the Hamiltonian of gauge theories requires careful considerations

- What are the DOF that should be used to describe fields?
- How should one digitize the fields
- How to deal with gauge redundancy?
- How to ensure that Lattice theory is in same universality class as continuum theory?
- What is the RGE of the Lattice theory?
- How does one take the continuum limit?
- …

How do we use the lattice theory to compute the S-matrix for a scattering process?

- How do we create an Eigenstate of the interacting theory?
  - variational techniques
  - create ground state and excite wave packets
  - …

- How do we exponentiate the Hamiltonian?
  - Suzuki-Trotter approximations
  - Algebraic techniques (Qubitization)
  - …

- How do we measure final states?
  - Run initial state creation backwards
  - use phase estimation
  - …

Many other questions that are important to answer, which I don't have time to go into more detail

- Repeat all above steps for problems in other fields of HEP (neutrino, cosmology, …)

- Think about alternative strategies, such as analog quantum simulations

- Can we contribute to broader questions in Quantum Computing (general QIS questons, such as entanglement, noise mitigation, software development, general algorithmic techniques)

- Find similarities of problems in HEP with other fields, such as NP, chemistry, …

- …

Quantum Computing is a field that needs to be fully explored in the next decade and beyond

- There are several "killer applications" that would revolutionize our approach to HEP
- Need to spend resources to fully investigate feasibility and find best approaches
- Very young field, and most of the interesting questions are still looking for answers
- Questions range from fundamental QFT questions to detailed implementation approaches

HEP Theory and especially young scientists can have a major impact in this newly emerging field