Production mechanisms for cold dark matter axions

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• Relic axion abundance depends on the Peccei-Quinn scale, and hence on the axion mass.

\[ \Omega_a = \Omega_a(f_a), \quad m_a \simeq 57 \mu eV \left( \frac{10^{11} \text{ GeV}}{f_a} \right) \]

• One can guess the axion DM mass from its relic density.

\[ \Omega_a h^2 = 0.12 \quad \Rightarrow \quad m_a = ??? \mu eV \]
Misalignment mechanism

Equation of motion for $\theta = a/f_a$

$$\ddot{\theta} + 3H\dot{\theta} + \frac{1}{f_a^2} \frac{\partial V_{QCD}}{\partial \theta} = 0$$

- Important input:
  - Topological susceptibility $\chi(T)$

$$m_a(T)f_a = \sqrt{\chi(T)}$$

- Late times, axion comoving number is conserved.

$$N_a \equiv \frac{\rho_a R^3}{m_a} = \text{const.}$$

$$\Omega_a \propto \frac{m_a N_a}{R^3}$$
Initial condition (I): Pre-inflationary scenario

- Assume that Peccei-Quinn (PQ) symmetry is never restored after inflation.
- Relic axion abundance depends on the initial misalignment angle.

\[ \Omega_a h^2 \approx 0.14 \theta_i^2 \left( \frac{f_a}{10^{12} \text{ GeV}} \right)^{1.17} \]
Initial condition (II): Post-inflationary scenario

- Naive calculation:
  Use the misalignment formula with angle-average. \( \theta_i^2 \rightarrow \langle \theta_i^2 \rangle = \frac{\pi^2}{3} \)

- But topological defects could have an important role. [Davis (1986)]
Production mechanisms and prediction for dark matter mass

**Misalignment mechanism [pre-inflationary]**
- Tuned $\theta_i \to 0$
- Dominant / subdominant
- Tuned $\theta_i \to \pi$
- Subdominant

**Strings and domain walls ($N_{DW} = 1$) [post-inflationary]**
- Overclosure
- Dominant (uncertainty?)
- Subdominant

**Long-lived domain walls ($N_{DW} > 1$, e.g. DFSZ models) [post-inflationary]**
[Kawasaki, KS and Sekiguchi, 1412.0789; Ringwald and KS, 1512.06436]
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**Kinetic misalignment mechanism [pre-inflationary]**
[Co and Harigaya, 1910.02080; Co, Hall and Harigaya, 1910.14152]
- Reduced to conventional misalignment mechanism
- Dominant / subdominant
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For production mechanisms and prediction for dark matter mass:

- This talk
- Co and Harigaya, 1910.02080; Co, Hall and Harigaya, 1910.14152
- Kawasaki, KS and Sekiguchi, 1412.0789; Ringwald and KS, 1512.06436
Axionic strings

\[ \mathcal{L} = |\partial_\mu \phi|^2 - V(\phi), \quad V(\phi) = \lambda \left( |\phi|^2 - \frac{f_a^2}{2} \right)^2 \]

- Form when \( U(1)_{\text{PQ}} \) symmetry is spontaneously broken.
- Disappear around the epoch of the QCD phase transition.
Axionic strings

$$\mathcal{L} = |\partial_{\mu} \phi|^2 - V(\phi), \quad V(\phi) = \lambda \left( |\phi|^2 - \frac{f_a^2}{2} \right)^2 + \chi(T) \left( 1 - \cos \left( \frac{a}{f_a} \right) \right)$$

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Simulations of axion production in the post-inflationary scenario

- Kawasaki, KS and Sekiguchi (2014) [arXiv:1412.0789]
  \[ N = 512^3 \text{ lattice} \quad m_a = 80-130 \mu eV \]

  \[ N = 1600^3 \quad m_a = 18-124 \mu eV \]

  \[ N = 2048^3 \quad m_a = 26.2 \pm 3.4 \mu eV \]

- Gorghetto, Hardy and Villadoro (2018) [arXiv:1806.04677]
  \[ N = 1250^3 \quad 0.4 \mu eV (?) \lesssim m_a \lesssim 4.4 \times 10^3 \mu eV \]

  \[ N = 2048^3 \quad m_a = 25.2 \pm 11.0 \mu eV \]

  \[ N = 4500^3 \quad m_a \sim 500 \mu eV \]

- Redondo, KS and Vaquero (2021?)
  \[ N \lesssim 10000^3 \quad \text{(to be confirmed, RAVEN at MPCDF)} \]
Difficulty in string dynamics

- Two extremely different length scales.

- String core radius \( \sim m_s^{-1} \sim f_a^{-1} \)

  \( m_s \) : mass scale of UV completion

- Hubble radius \( \sim H^{-1} \)

- String tension acquires a logarithmic correction.

\[
\mu \simeq \pi f_a^2 \log \left( \frac{f_a}{H} \right)
\]

- Realistic value

\[
f_a/H_{\text{QCD}} \sim 10^{30} \quad \Rightarrow \quad \log(f_a/H) \sim 70
\]

- Difficult to reach the large-log regime for simulations with limited dynamical range (\( f_a/H \sim 10^3 \)).
Scaling solution

- $\mathcal{O}(1)$ strings per horizon volume:

$$\rho_{\text{string}} = \xi \frac{\mu}{t^2} \sim \frac{\mu \ell}{\ell^3} \bigg|_{\ell \sim H^{-1}} \sim t$$

- The net energy density of radiated axions should be the same order.

$$\rho_a \sim \xi \frac{\mu}{t^2} \sim \xi H^2 f_a^2 \log\left(m_s/H\right)$$
Recent simulations observe a logarithmic growth of string density

(However, full consensus has not been reached, cf. [Hindmarsh et al., 1908.03522].)
“Scaling” solution suggests that the energy density of the system is of order

\[ \rho \sim 8\pi \xi \log \left( \frac{f_a}{H} \right) H^2 f_a^2. \]

If \( \xi \propto \log \), this leads to an enhancement by a factor of \( \mathcal{O}(10^4) \) than typical density \( H^2 f_a^2 \) at QCD temperatures.

Does it imply an enhancement of axion abundance (and DM mass)?

Studying the large-log regime is a challenge.

Two possible approaches:

1. [Direct approach] use some effective description.
2. [Indirect approach] rely on a careful extrapolation.
Effective theory approach

- Integrate out (unresolvable) logarithmically distributed field and only describe string “core” and low-momentum axions.

Kalb-Ramond (KR) effective action \([\text{Kalb and Ramond (1974)}]\)

\[
S = -\mu \int \sqrt{-\gamma} d^2 \zeta + \frac{1}{6} \int d^4 x H^{\mu \nu \lambda} H_{\mu \nu \lambda} + 2\pi f_a \int B_{\mu \nu} d\sigma^{\mu \nu}
\]

Nambu-Goto string \(\text{axion field}\) \(\text{string-axion coupling}\)

used in the literature to study axion radiation analytically. \([\text{Davis and Shellard (1989); Dabholkar and Quashnock (1990); Battye and Shellard (1994); Battye and Shellard (1996)}]\)
Numerical implementation of effective description

- 3D simulations of KR strings have not been performed. (But worked in 2D, [Fleury and Moore, 1602.04818].)

- Alternative method (trick):
  Theory with 1 vector field + 2 complex scalars.
  [Klaer and Moore, 1707.05566]

\[ -\mathcal{L} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \]
\[ + |(\partial_{\mu} - iq_1 e A_{\mu})\phi_1|^2 + |(\partial_{\mu} - iq_2 e A_{\mu})\phi_2|^2 \]
\[ + \lambda \left[ \left( |\phi_1|^2 - \frac{\nu^2}{2} \right)^2 + \left( |\phi_2|^2 - \frac{\nu^2}{2} \right)^2 \right] \]

- Two phases, one is eaten by vector field and the other is identified as axion with a decay constant

\[ f_a = \frac{\nu}{\sqrt{q_1^2 + q_2^2}}. \]
Tunable effective tension

\[ -\mathcal{L} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + |(\partial_{\mu} - iq_1 e A_{\mu})\phi_1|^2 + |(\partial_{\mu} - iq_2 e A_{\mu})\phi_2|^2 + \lambda \left[ \left( |\phi_1|^2 - \frac{v^2}{2} \right)^2 + \left( |\phi_2|^2 - \frac{v^2}{2} \right)^2 \right] \]

- Tension is dominated by the local string core.

\[ \mu \simeq 2\pi v^2 + \pi \frac{v^2}{q_1^2 + q_2^2} \log(v/H) \simeq 2\pi v^2 \]

- It becomes relatively high compared to \( f_a \):

\[ \kappa \equiv \frac{\mu}{\pi f_a^2} \simeq 2(q_1^2 + q_2^2) \gg 1 \]

\begin{itemize}
  \item Pure global string
    \[ \mu \simeq \pi f_a^2 \log \left( \frac{f_a}{H} \right) \quad \text{with} \quad f_a = v \]
    \[ \kappa \simeq \log \left( \frac{f_a}{H} \right) \]
  \item Hybrid string
    \[ \mu \simeq 2\pi v^2 \quad \text{with} \quad f_a = \frac{v}{\sqrt{q_1^2 + q_2^2}} \]
    \[ \kappa \simeq 2(q_1^2 + q_2^2) \]
\end{itemize}

[Klaer and Moore, 1707.05566]
Simulations at high effective tension

- String density increases.
  \[ \xi \sim 4 \]

- Axion production becomes less efficient than the estimate based on the angle-average misalignment calculation.

- Prediction for axion DM mass:
  \[ m_a = 26.2 \pm 3.4 \mu eV \]

[Klaer and Moore, 1708.07521]
Spectrum of radiated axions: a controversy?

- Differential energy transfer rate
  \[ F \left( \frac{k}{H}, \frac{m_s}{H} \right) = \frac{1}{R^3} \frac{H}{\Gamma} \frac{\partial}{\partial t} \left( R^3 \frac{\partial \rho_\alpha}{\partial k} \right), \quad \Gamma = \frac{\xi \mu}{t^3} \]

- Slope matters.

\[ F \propto k^{-q} \]

- Results of effective tension approach point to a hard spectrum \((q < 1)\).
  (unphysical) UV modes may not have decoupled, breakdown of effective theory?
Simulations with 1 complex scalar (not relying on the effective description), but limitation on the value of string tension, $\log < 8$.

- Evidence of growing spectral index.
- When extrapolated, the spectrum will turn IR dominated ($q > 1$) at large $\log$. 

Characterizing axion spectrum in the scaling regime

[Gorghetto, Hardy and Villadoro, 2007.04990]
Extrapolation to large log

Assume \( q > 1 \) at large log (spectrum is IR-dominated).

If the typical gradient of the IR field is set by \( k \sim H \),

\[
\rho_a \sim k^2 \langle a^2 \rangle \sim H^2 \langle a^2 \rangle \\
\sim 8\pi \xi \log H^2 f_a^2.
\]

\[
\frac{\langle a^2 \rangle}{f_a^2} \sim \mathcal{O}(\xi \log) \gg 1
\]

Interpreted as large field amplitude.
Energy density of radiated axions at the epoch of QCD crossover

\[ \rho_{\text{rad}} \approx 8\pi \xi \log H^2 f_a^2 \sim \mathcal{O}(10^4) H^2 f_a^2 \gg 2m_a f_a^2 \sim \rho_{\text{pot}} \]

Axion number is fixed only after \( \rho_{\text{rad}} \lesssim \rho_{\text{pot}} \) (later than \( H \simeq m_a \)).

This delay alleviates the log-enhancement of the axion number.

\[ \frac{n_a^{\text{string}}}{n_a^{\text{misalignment}}} \propto (\xi \log)^{1/2} \]

But it is still enhanced by \( (\xi \log)^{1/2} \), implying higher axion DM mass:

\[ m_a \approx 500 \mu\text{eV} \]
The axion is a compact field:
Any enhancement of the energy density should come from higher gradient or higher $k$.

$$\rho \sim |\partial_i \phi|^2 \sim |\partial_i \theta|^2 f_a^2 \sim k^2 f_a^2$$

Enhancement of the string density should shift the IR cutoff to higher momentum, $k_{IR} \sim \sqrt{\xi} H$. 

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Skepticism about an enhancement of the axion abundance

[Dine, Fernandez, Ghalsasi and Patel, 2012.13065]
Revisiting axion radiation spectrum

[Redondo, KS and Vaquero, work in progress]
Summary of axion dark matter mass prediction

- Current best guesses

- Direct calculation with a high effective tension
  \[ m_a = 26.2 \pm 3.4 \mu eV \]
  (less efficient than the angle-average misalignment)

- Extrapolation with a scaling solution
  \[ m_a \approx 500 \mu eV \]
  (more efficient than the angle-average misalignment)

- The discrepancy should be attributed to our limited knowledge on the spectrum of axions around the QCD epoch.
Conclusions

- Prediction for axion dark matter mass is strongly related to physics at the early universe.

- Uncertainty in the calculation for post-inflationary scenario:
  - Results of effective theory approach and extrapolation from the scaling solution appear to disagree.
  - Careful study of the low momentum axion distribution with improved dynamical ranges would be highly motivated (project for Beyond Gen 2 simulations).

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- subdominant

Klaer 2017

Gorghetto 2020

![Graphical representation of axion mass and frequency](image-url)