Dark Matter Halos from Parametric Resonance and their Signatures

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The Mystery of Dark Matter

- Dark Energy: ~73%
- Dark Matter: ~23%
- Other nonluminous components:
  - Intergalactic gas: 3.6%
  - Neutrinos: 0.1%
  - Supermassive BHs: 0.04%
- Luminous matter:
  - Stars and luminous gas: 0.4%
  - Radiation: 0.005%
Light Scalar Dark Matter

- Just like a harmonic oscillator

\[ \ddot{\phi} + 3H \dot{\phi} + m^2_{\phi} \phi = 0 \]

\[ \ddot{x} + \gamma \dot{x} + \omega^2 x = 0 \]

Frozen when: Hubble > \( m_{\phi} \)

Initial conditions set by inflation

*The story changes slightly if DM is a dark photon*
Light Scalar Dark Matter

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Frozen when: Hubble > \( m_\phi \)

Oscillates when: Hubble < \( m_\phi \)

\( \rho_\phi \) scales as \( a^{-3} \)

just like Dark Matter

Initial conditions set by inflation

*The story changes slightly if DM is a dark photon
Light Scalar Dark Matter Today

- If $m_\phi < 1$ eV, can still be thought of as a scalar field today

\[ m_\phi^2 \phi_0^2 \cos^2 (m_\phi t) \sim \rho_\phi \]

Coherent for $\nu_{\text{vir}}^{-2} \sim 10^6$ periods

Amplitude compared to $M_{\text{Pl}}$ in the galaxy:

\[ \kappa_0 = \frac{\sqrt{8\pi \rho_\phi}}{m_\phi M_{\text{Pl}}} = 6.4 \cdot 10^{-13} \left( \frac{10^{-18} \text{eV}}{m_\phi} \right) \]
Axion Dark Matter and the Large Misalignment Mechanism

Axions generically have attractive self-interactions

Axion self-interactions affect evolution at $H \sim m$
Structure growth due to axion self-interactions

Attractive self-interactions boost scales of size
\[ \lambda^* \sim 2\pi/m \] at the time of oscillation

\[ V(\phi) = m^2 \phi^2 \left[ 1 - \cos(\phi/f) \right] \]

or mass of:

\[ M^*_s = \frac{4\pi \rho_{DM}^0}{3} \left( \frac{\lambda^*_s}{2} \right)^3 \approx 5 \times 10^9 M_\odot \left[ \frac{10^{-22} \text{eV}}{m} \right]^{3/2} \]

or physical size of:

\[ r_s = 87 \text{ pc} \left( \frac{M_s}{5 \times 10^9 M_\odot} \right)^{1/3} \left( \frac{10^5}{B} \right)^{1/3} \]
Large Misalignment vs Small Misalignment

Axion mass in $\log_{10}$ $m_{\text{eV}}$

Axion coupling to photons in $\log_{10}$ $g_{a\gamma\gamma}$ GeV$^{-1}$

$\Theta_0 = \phi_0 / f$

SN/Hess/Fermi

HB stars

Telescope

Haloscopes

$\pi - |\Theta_0| = 0.1$

QCD axion

CAST

$\Theta_0 = \phi_0 / f$
Large Misalignment vs Small Misalignment

Axion mass in $\log_{10} \frac{m}{\text{eV}}$

Axion coupling to photons in $\log_{10} g_{\text{av}}^{-\gamma\gamma} \text{GeV}^{-1}$

Axion starts close to the origin

$\Theta_o = \phi_o / f$

SN/Hess/Fermi

HB stars

Telescope

Haloscopes

QCD axion

$\pi - |\Theta_o| = 0.1$
Large Misalignment vs Small Misalignment

Axion starts close to the top
Axion starts close to the origin

Large Misalignment is most relevant where experiments have good sensitivity
Effects of the large misalignment mechanism

- Formation of compact halos as a component of DM

- Formation of solitons as a component of DM

- Formation of structures well before matter-radiation equality

- Oscillons as an (early) component of Dark Matter

- Happens without the need of a phase transition, starting from a scale-invariant spectrum
Effects of the large misalignment mechanism

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- Formation of structures well before matter-radiation equality: Oscillons as an (early) component of Dark Matter
- Happens without the need of a phase transition, starting from a scale-invariant spectrum
Signatures of the large misalignment mechanism

\[ V(\phi) = m^2 f^2 (1 - \cos(\phi/f)) \]
Outline

- Dynamics of the large misalignment mechanism
- Signatures of the large misalignment mechanism
- Comments and future prospects
Parametric Resonance Growth

\[ \ddot{x} + \gamma \dot{x} + \omega^2 (1 + h \cos ((2\omega + \epsilon)t)) = 0 \]

Instability occurs when \( h > \frac{\gamma}{2\omega} \) and \( \epsilon \sim 0 \)
Parametric Resonance Growth

\[ \ddot{x} + \gamma \dot{x} + \omega^2 \left(1 + h \cos \left((2\omega + \epsilon)t\right) \right) = 0 \]

Instability occurs when \( h > \gamma/(2\omega) \) and \( \epsilon \sim 0 \)
Parametric Resonance Growth

For the axion field and small $\Theta_0(t) + \theta_k e^{ikx}$ ($\theta=\phi/f$):

$$\ddot{\phi}_k + \frac{3}{2tm} \dot{\phi}_k + \left( 1 - \frac{\Theta_0(t_m)^2}{4} + \tilde{k}^2 \right) \left( \frac{\Theta_0(t_m)^2}{4} \cos(2t_m) \right) \theta_k = 0$$

- Friction term
- Resonant frequency
- Driving term

$t_m$: time in units of $1/m$

$\tilde{k}$: dimensionless variable — size of the mode compared to $m$ at $t_m=1$
Parametric Resonance Growth

For the axion field and small $\Theta_0(t) + e^{ikx}$ ($\theta=\phi/f$):

$$\ddot{\theta}_k + \frac{3}{2t_m} \dot{\theta}_k + \left( 1 - \frac{\Theta_0(t_m)^2}{4} + \frac{\tilde{k}^2}{t_m} - \frac{\Theta_0(t_m)^2}{4} \cos(2t_m) \right) \theta_k = 0$$

nonrelativistic mode

$\tilde{k} \ll 1$

enters horizon when nonlinearities are small
Parametric Resonance Growth

For the axion field and small $\Theta_0(t) + \theta_k e^{i k \cdot x}$ ($\theta = \phi / f$):

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- nonrelativistic mode
  - $\tilde{k} \ll 1$
- enters horizon when nonlinearities are small

- semi-relativistic mode
  - $\tilde{k} \sim 1$
- frequency match; nonlinearity > friction
Parametric Resonance Growth

For the axion field and small $\Theta_0(t) + \Theta_k e^{ikx}$ ($\phi=\phi/f$):

$$
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$$

- **nonrelativistic mode**
  \[ \tilde{k} \ll 1 \]
  enters horizon when nonlinearities are small

- **semi-relativistic mode**
  \[ \tilde{k} \sim 1 \]
  frequency match; nonlinearity > friction

- **ultra-relativistic mode**
  \[ \tilde{k} \gg 1 \]
  frequency mismatch
Parametric Resonance Growth

For the axion field and small $\Theta_0(t) + \theta_k e^{-ikx}$ ($\theta = \phi/f$):

$$\ddot{\theta}_k + \frac{3}{2t_m} \dot{\theta}_k + \left( 1 - \frac{\Theta_0(t_m)^2}{4} + \frac{\tilde{k}^2}{t_m} - \frac{\Theta_0(t_m)^2}{4} \cos(2t_m) \right) \theta_k = 0$$

- **Nonrelativistic mode** ($\tilde{k} \ll 1$): enters horizon when nonlinearities are small.
- **Semi-relativistic mode** ($\tilde{k} \sim 1$): frequency match; nonlinearity $>$ friction.
- **Ultra-relativistic mode** ($\tilde{k} \gg 1$): frequency mismatch.

In order to overcome the friction the field needs to start close to the top to delay the oscillation time.
Delayed onset of oscillation

Starting closer and closer to the top

Boost relative to CDM \[ B \equiv \frac{\rho_s}{\rho_{sCDM}} \sim \exp\{\xi m t_{osc}\} \]

Need to start close to the top of the potential to get a non-trivial effect in structure formation
Outline

• Dynamics of the large misalignment mechanism

• Signatures of the large misalignment mechanism

• Comments and future prospects
Signatures of the Large misalignment mechanism

- Direct Gravitational Interactions
- Gravitational Waves
- Star Formation
- Direct Detection
Direct Gravitational Signatures

![Graph showing direct gravitational interactions](image-url)
Direct Gravitational Signatures

Velocity “kicks” on stars in the path of the DM compact halo
Direct Gravitational Signatures

Preferred parameter space explaining GD-1 observation
Direct Gravitational Signatures

Looking for correlations in the apparent velocities and accelerations of stars in GAIA and THEIA
Direct Gravitational Signatures

Diffraction of GWs from mergers
Modulations in rare high amplification of distant stars
Gravitational Wave Emission

- Attractive self-interactions can overcome Hubble expansion long before matter radiation equality

- Dense structures collapsing can lead to gravitational wave production
Gravitational Wave Emission

\[ \Omega_{\text{GW}} = \frac{\mathcal{L}}{\mathcal{M}_\text{P}^2} \]

- \[ 10^{-19} \text{eV} \]
- \[ 10^{-17} \text{eV} \]
- \[ 10^{-15} \text{eV} \]
- \[ 10^{-13} \text{eV} \]

Frequency (Hz)

VLBN, VLBN+Gaia, AGN, M/R Eq. Planck 2015, EUCLID, Astrometry, EPTA-5, IPTA-20, SKA-100
Compact halos jumpstarting star formation

- Necessary (but not sufficient) requirements for star formation:
  
  - Gravitational pressure from Dark Matter needs to be bigger than kinetic pressure from baryons

- Could also jumpstart black hole formation
Compact halos jumpstarting star formation

- Necessary (but not sufficient) requirements for star formation:
  
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  - Baryons need to lose energy
Compact halos jumpstarting star formation

- Necessary (but not sufficient) requirements for star formation:
  - Gravitational pressure from Dark Matter needs to be bigger than kinetic pressure from baryons
  - Baryons need to lose energy
  - Need to have enough baryons

In our case, star forming halos as small as ~ $10^5 M_{\odot}$ Could also jumpstart black hole formation
Compact halos jumpstarting star formation

- Necessary (but not sufficient) requirements for star formation:
  
  - Gravitational pressure from Dark Matter needs to be bigger than kinetic pressure from baryons
  
  - Baryons need to lose energy
  
  - Need to have enough baryons

For pressure-less cold dark matter no stars in halos less than $\sim 10^8 \, M_{\odot}$

In our case, star forming halos as small as $\sim 10^5 \, M_{\odot}$

Could also jumpstart black hole formation
Effects on Earth-bound Experiments

Axion mass in $\log_{10} \frac{m}{\text{eV}}$

$\rho_{\text{DM}}$ density relative to $\rho_{\odot}$ in $\log_{10} \mathcal{B}_\odot$

$t_{\text{cross}} = 1 \text{ min}$

$t_{\text{cross}} = 1 \text{ hr}$

$t_{\text{cross}} = 1 \text{ day}$

$QCD$ axion prediction

Once in 10 years

Once a year

Once a month

Once a week

Tidal disruption by galactic disk
Let’s examine the case of a $10^{-18}$ solar mass halo that is 1000 times more dense than CDM.
Resonant Axion Searches

Harmonic oscillator analog:

Axion Dark Matter $\rightarrow$ Driving Force
Cavity Mode $\rightarrow$ Displacement
Cavity Size $\rightarrow$ Resonant Frequency
DM coherence $\rightarrow$ Q Factor ($\sim 10^6$)

$$g_{a\gamma\gamma} a(x) \vec{E} \cdot \vec{B}$$
Effects on Earth-bound experiments

Ex. $10^{-18}$ solar mass halo
1000 times more dense than CDM

- Every ~3 years, power goes up by the over-density factor for approximately 6 hours

- Effective DM Q factor $> 10^{11}$ vs $10^6$ for CDM

- Need to make sure experiment is running at the right frequency
  Favors broadband approach
Can a resonant detector run in broadband mode?

For a harmonic oscillator

Displacement response to a constant force

Frequency

$\omega_0$
Can a resonant detector run in broadband mode?

For a harmonic oscillator
Can a resonant detector run in broadband mode?

For a harmonic oscillator

\[ \omega_0 \]

The bandwidth is determined by the detector sensitivity for displacement.
Effects on Earth-bound DM Searches

Ex. $10^{-18}$ solar mass halo
1000 times more dense than CDM
Search strategy in resonant axion searches

- Record data outside the resonance frequency
- Data taking time bins of 6 hours and look for excesses

- $Q_{DM}$ in clump $>> Q_{DM}$-diffuse
Summary of effects on earth-bound experiments

- Axion mass in Log$_{10}$ m$_{eV}$
- Axion coupling to photons in Log$_{10} g_{a\gamma\gamma}$

- QCD axion
- CAST
- SN/Hess/Fermi
- HB stars
- Telescope

\[\pi - |\Theta_0| = 0.1\]
Summary of effects on earth-bound experiments

Need to reconsider existing bounds and search strategies
What about the QCD axion?
The QCD axion

Temperature dependent mass

\[ V(\phi, T) = m^2(T) f^2 \left[ 1 - \cos \left( \frac{\phi}{f} \right) \right] \]

with

\[ m(T)^2 \equiv \chi_{QCD}(T) \ m(T = 0)^2 \]

\[ \propto T^{-8.16}, \quad T > 1 \text{ GeV} \]

The QCD axion field starts oscillating when \( H \ll m(T=0) \)

corresponding to \( \tilde{k} \approx 0.1 \)
The QCD axion: Density perturbation growth

Halo Spectra, QCD axion, $\Phi_0^2 = 2.4 \times 10^{-9}$, $\bar{\sigma} = 1/3$

Affects high-frequency QCD axion searches ($m > 10^{-4}$ eV)
Other lessons learned from the large misalignment mechanism

• Produces axions stars, miniclusters or oscillons of mass:

\[ M_*^s = \frac{4\pi \rho_{DM}^0}{3} \left( \frac{\lambda_*}{2} \right)^3 \approx 5 \times 10^9 M_\odot \left[ \frac{10^{-22} \text{eV}}{m} \right]^{3/2} \]

• Cannot produce dense axion stars without overclosing the universe with Dark Matter

• For earth-bound experiments the relevant frequency range is that of ADMX and higher

• Oscillons DM should be a possibility but exact mechanism of longevity or appropriate potential has yet to be identified
Open Questions

- How much can we really tell about star formation?

- Are there non gravitational probes of large DM halos?

- What fraction of DM is in these halos? 
  What are the optimal strategies for laboratory DM searches?
Signatures of the large misalignment mechanism

\[ V(\phi) = m^2 f^2 (1 - \cos(\phi/f)) \]
Shining Light on Dark Matter

Dark Energy ~ 73%
Dark Matter ~ 23%

Other nonluminous components:
- intergalactic gas 3.6%
- neutrinos 0.1%
- supermassive BHs 0.04%

Luminous matter:
- stars and luminous gas 0.4%
- radiation 0.0005%

ATOMIC CLOCKS

ADMX

LIGO
Back-up slides
What are solitons?
What are oscillons?

Scalar field configuration of size $R$ and total mass $M$

Hamiltonian energy density $\mathcal{H} = \frac{|\nabla\psi|^2}{2m} + V_{grav}(\psi) - \lambda|\psi|^4 + \lambda'|\psi|^6$
What are solitons?
What are oscillons?

Scalar field configuration of size R and total mass M

Hamiltonian energy density \(\frac{\nabla \psi}{2m}^2 + V_{grav}(\psi) - \lambda |\psi|^4 + \lambda' |\psi|^6\)

Integrate over volume

Energy(\(M, R\)) = \(\frac{M}{m^2 R^2} - \frac{G_N M^2}{R} - \lambda \frac{M^2}{m^2 R^3} + \lambda' \frac{M^3}{m^3 R^6}\)
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Solitons: Large ($R > 1/m$) scalar field configurations balancing kinetic pressure against gravity
What are solitons?
What are oscillons?

Scalar field configuration of size $R$ and total mass $M$

$$\psi(x,t)$$

Hamiltonian energy density $$= \frac{|
abla \psi|^2}{2m} + V_{\text{grav}}(\psi) - \lambda |\psi|^4 + \lambda' |\psi|^6$$

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**Solitons:** Large ($R > 1/m$) scalar field configurations balancing kinetic pressure against gravity

**Oscillons:** Small ($R \sim 1/m$) scalar field configurations balancing kinetic pressure against gravity
Delayed Onset of Oscillation
Growth of density perturbations as a function of scale and time

$m = 10^{-21} \text{ eV}; \pi - \Theta_0 = 10^{-12}; \Phi_0 = 5 \times 10^{-5}$

Initial conditions: scale invariant spectrum
$m = 10^{-18}\text{ eV, } |\pi - \Theta_0| = 10^{-10}, \Phi_0^2 = 2.1 \times 10^{-9}$
Compact DM Halos jumpstart star formation

Successful star forming halos need to have:
1. enough baryons
2. the ability to cool the baryons

Minimum halo size that forms stars can be as smaller than $10^8$ solar masses
What about tuning?

- Dynamics fixing $\Theta=\pi$ during inflation

- Environmental selection just like the case of the QCD axion outside the traditional axion window

- For string axions, a cosine potential is not the only option
What about tuning?

An example of “untuned” axion potential

\[ V(\Theta) \propto (1 - \cos(\Theta)) \]

\[ V(\Theta) \propto \frac{\Theta^2}{2 + \Theta^2} \]
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