Nucleon vector form factors via *z*-expansion in GENIE

Richard Hill, Gabriel Lee, Sasha Tomalak

Joint neutrino theory-expt meeting

2019/11/25

Cross sections and form factors

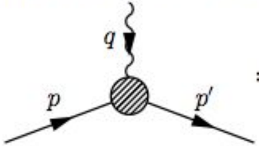
- Mott cross-section for scattering of a relativistic electron off a recoiling point-like nucleus is

$$\left(\frac{d\sigma}{d\Omega}\right)_M = \frac{Z^2\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\cos^2\frac{\theta}{2}\frac{E'}{E}\,.$$

- The Rosenbluth formula generalizes the above,

$$\left(\frac{d\sigma}{d\Omega}\right)_R = \left(\frac{d\sigma}{d\Omega}\right)_M \frac{1}{1+\tau}\left[G_E^2 + \frac{\tau}{\epsilon}G_M^2\right],\ \tau = \frac{-q^2}{4M^2},\ \epsilon = \frac{1}{1+2(1+\tau)\tan^2\frac{\theta}{2}}.$$

- The Sachs form factors $G_E(q^2), G_M(q^2)$ account for the finite size of the nucleus. In terms of the standard Dirac (F_1) and Pauli (F_2) form factors,

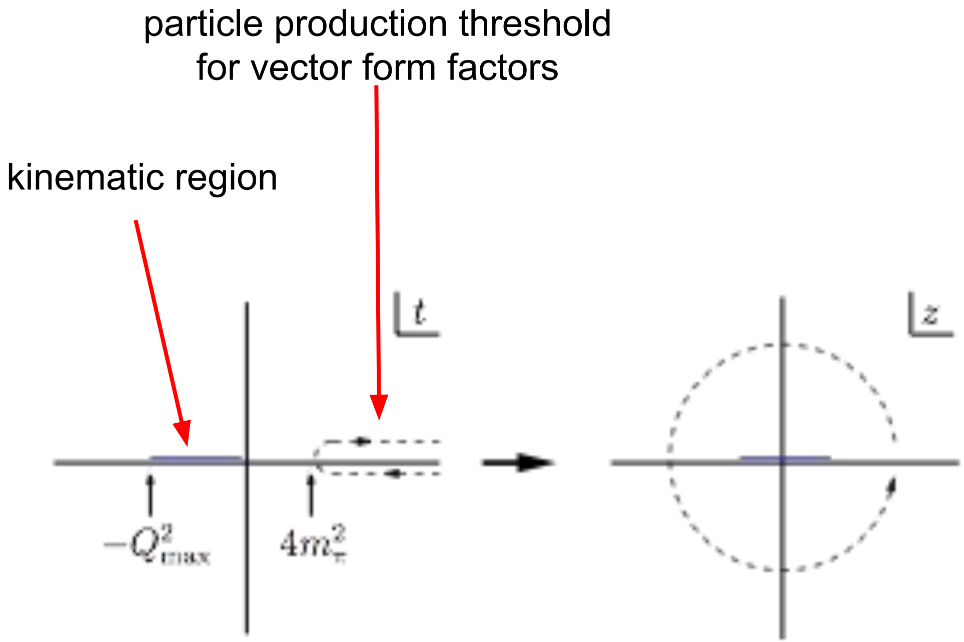


$$= \Gamma^\mu(q^2) = \underbrace{\frac{G_E+\tau G_M}{1+\tau}}_{F_1(q^2)}\gamma^\mu + \frac{i}{2M}\sigma^{\mu\nu}q_\nu \underbrace{\frac{G_M-G_E}{1+\tau}}_{F_2(q^2)}\,.$$

- The form factors are normalized at $q^2 = 0$ to the charge and anomalous magnetic moments, e.g., for the proton,

$$G_E^p(0) = 1, G_M^p(0) = \mu_p.$$

z expansion



$$F(t = q^2) = \sum_{k=0}^{\infty} \tilde{a}_k t^k$$

$$z = \frac{\sqrt{t_{\rm cut} - t} - \sqrt{t_{\rm cut} - t_0}}{\sqrt{t_{\rm cut} - t} + \sqrt{t_{\rm cut} - t_0}}$$

$$F(z) = \sum_{k=0}^{\infty} a_k z^k$$

z expansion and convergence

Q^2_{max} [GeV^2]	t_0 [GeV^2]	$\|z\|_{max}$	n_{min}
1	0	0.58	8.3
1	t_0^{opt} = -0.21	0.32	4.0
3	0	0.72	14
3	t_0^{opt} = -0.41	0.43	5.4

$$z = \frac{\sqrt{t_{\rm cut} - t} - \sqrt{t_{\rm cut} - t_0}}{\sqrt{t_{\rm cut} - t} + \sqrt{t_{\rm cut} - t_0}}$$

$$t_0^{\rm opt} = t_{\rm cut}\left(1 - \sqrt{1 + Q^2_{\rm max}/t_{\rm cut}}\right)$$

$$|z|^{\rm opt}_{\rm max} = \frac{(1 + Q^2_{\rm max}/t_{\rm cut})^{1/4} - 1}{(1 + Q^2_{\rm max}/t_{\rm cut})^{1/4} + 1}$$

Sum rules and normalization

$$F(t \to \infty) \lesssim t^{-4} \qquad\qquad F(t=0) = F_0$$

$$\sum_{k=n}^{\infty} k(k-1)(k-n+1)a_k = 0, \qquad n = 0, 1, 2, 3$$

Common convention:

- Fix coefficient for k=1...k_{max} - 4. Typically k_{max} = 8.
- First and last four coefficients from sum rules.

z expansion and axial vector FF

- 1108.0423 (Bhattacharya, Hill, Paz): use *z* expansion together with standard nuclear models to fit MiniBooNE to determine the axial form factor parameter.
- 1603.03048 (Meyer et al.): fit the global neutrino-deuteron cross sections, which yields an nucleon isovector axial radius with much larger uncertainties.

z expansion and proton radius puzzle

- 1008.4619 (Hill, Paz): perform the first fit using the z expansion to nucleon vector form factor tabulations to determine the proton charge radius.
- 1505.01489 (Arrington, Hill, Lee): reanalysis of all available (1960–2010) elastic *ep*-scattering and polarization data up to 1 GeV^2 to shed light on the proton radius puzzle.
    - "Mainz": high-statistics dataset with leading precision from 2010 A1 collaboration experiment at MAMI. After rebinning multiple data points taken under identical conditions, we are left with 657 from 1422 data points, with Q^2 up to ~ 1 GeV^2 and beam energies from 180 to 855 MeV.
    - "World": compilation of data from ~31 experiments dating back to ~1966. 562 data points with Q^2 ranging up to ~30 GeV^2. Polarization measurements are 69 points from 14 experiments with Q^2 from 0.16–8.5 GeV^2.

z expansion and nucleon vector FF

- 1707.09063 (Ye, Arrington, Hill, Lee): to provide best parameterization of nucleon vector form factors, fit to entire Q^2 range of
  - elastic *ep*-scattering cross sections;
  - polarization data;
  - and neutron form factor data.
- Ongoing work: use above to calculate contributions to uncertainties in neutrino-nucleon QE scattering cross sections from nucleon isovector form factors.
  - first fits using isospin-decomposed form factors.

Recent work

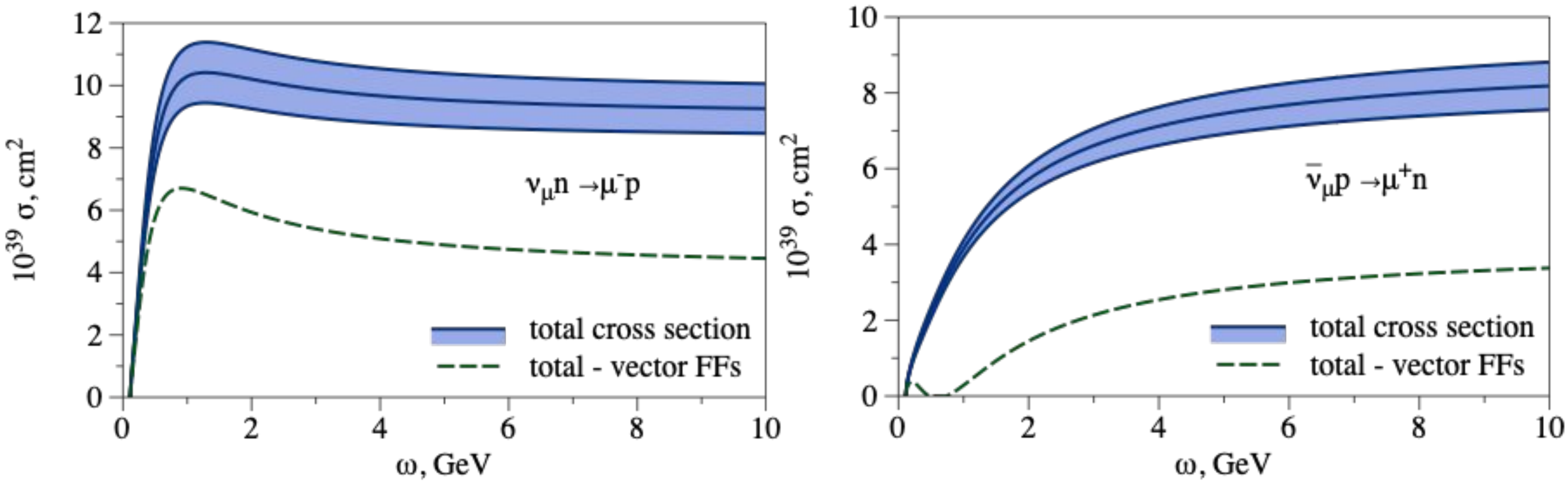
A lot of progress!

- Squashing bugs.
- Discussions of how to include uncertainties from radiative corrections and models thereof, e.g., dispersive calculations of Tomalak et al. vs. standard nuclear experimentalist's "SIFF" from Blunden et al. (2005).
- Final decisions on fit parameters and datasets:

$Q^2 < 1\ \mathrm{GeV}^2$ vs $Q^2 < 3\ \mathrm{GeV}^2$

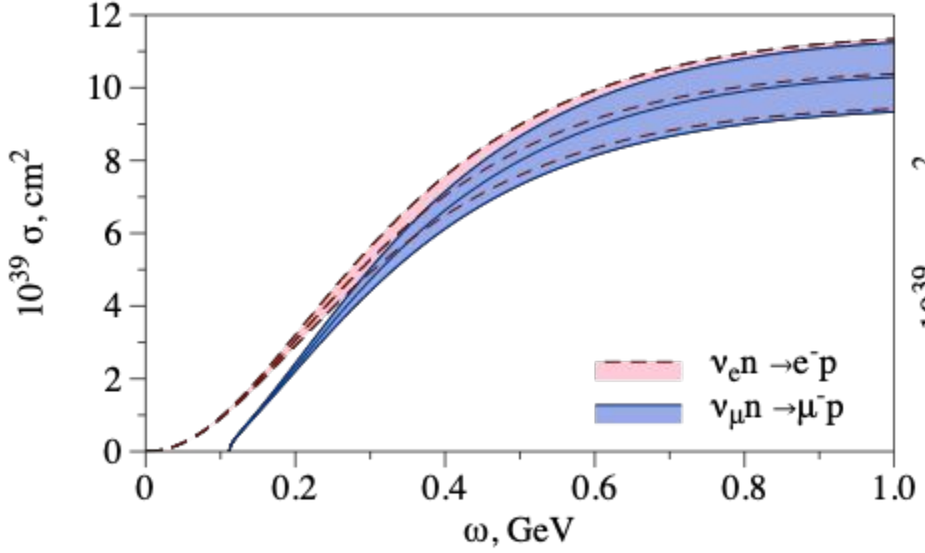
with and without constraint on proton charge radius

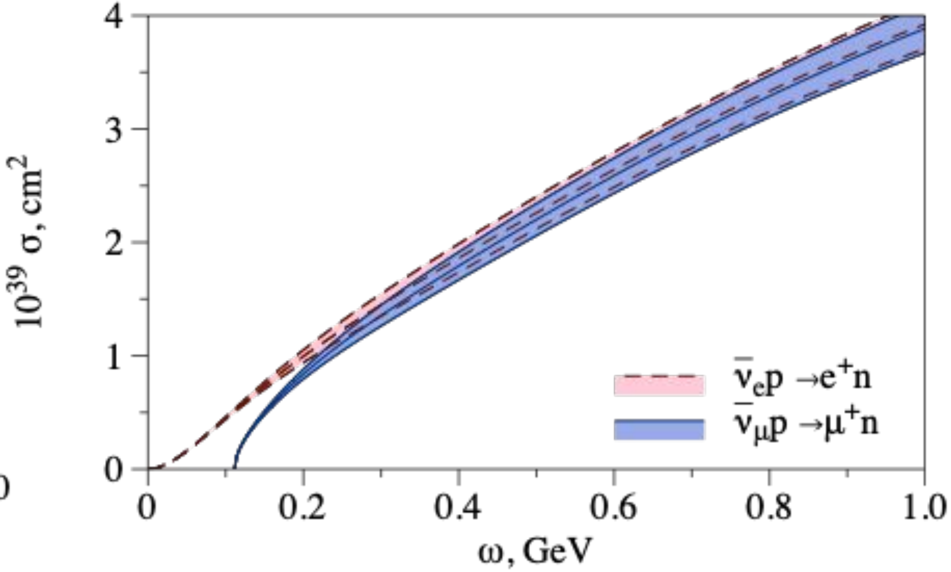
CCQE cross section



- both axial and isovector form factors important

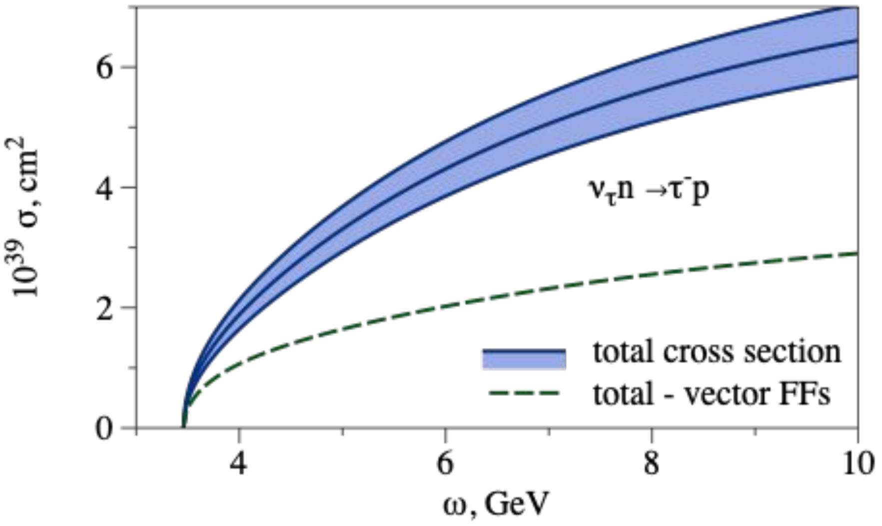
CCQE cross section

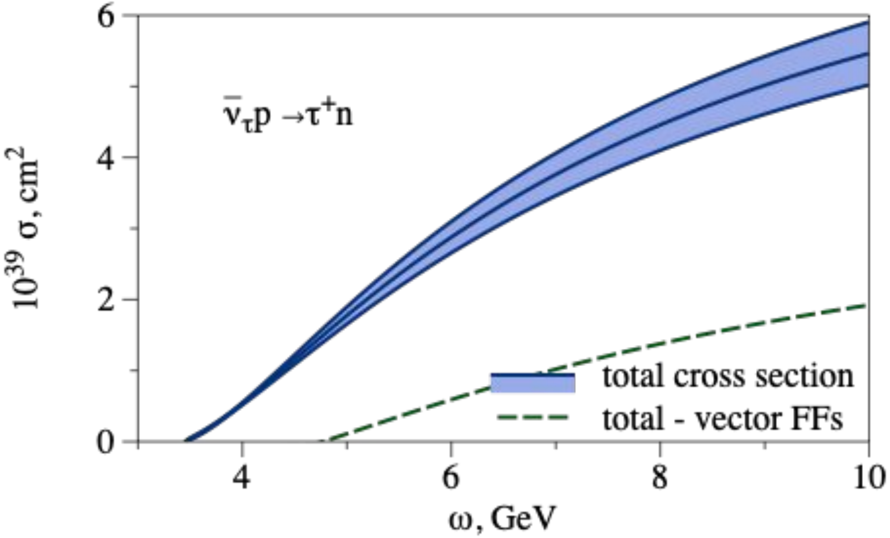




- CCQE of muon vs electron neutrino differ only below few muon masses

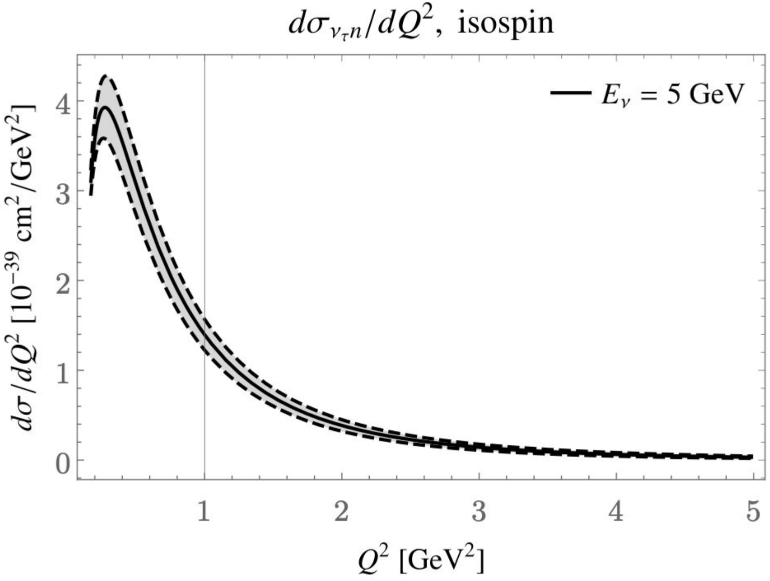
CCQE cross section

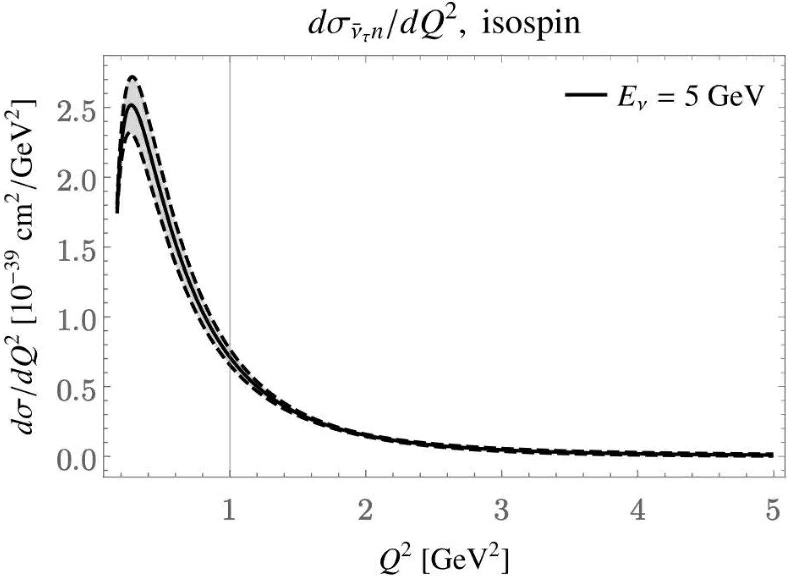




- CCQE of tau neutrino is sensitive to higher energies and Q^2

CCQE cross section





- CCQE of tau neutrino is sensitive to higher energies and Q^2

GENIE

- Results of 1603.03048 are in GENIE:

  https://indico.fnal.gov/event/12824/

- GOAL 1: to provide isovector form factors after few tests.
- GOAL 2: to include into GENIE following the same steps.