Neutrino-electron scattering and radiative corrections

RICHARD HILL, U. Kentucky and Fermilab

based on 1907.03379, 1911.01493 with O. Tomalak

cf. Fermilab W&C seminar 11/15/19,
Tomalak+MINERvA+DUNE

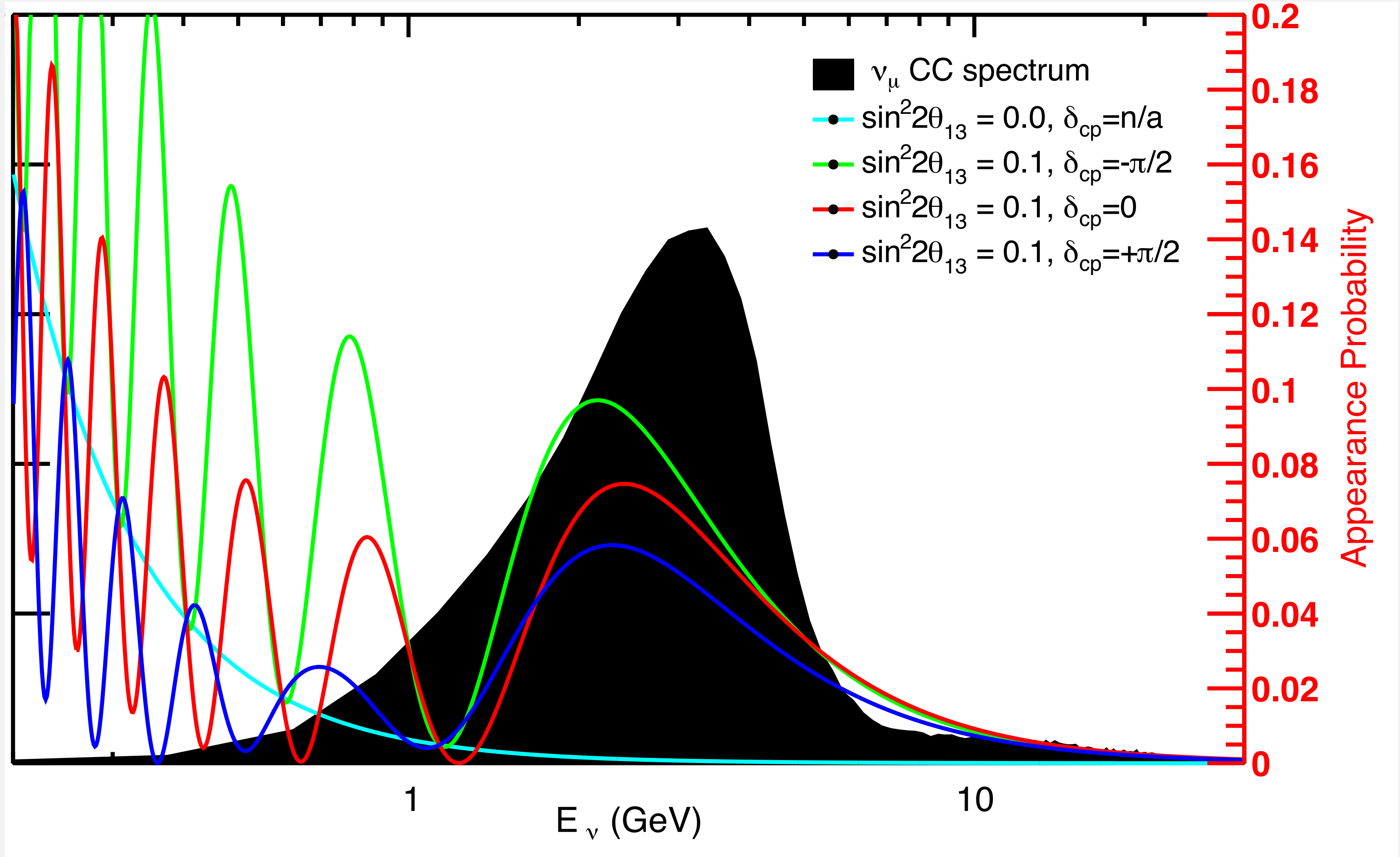
NUSTEC meeting, 10-12 December 2019

Outline

- Motivation and ν-e basics
- Four Fermi theory
- Virtual corrections
- Real radiation
- Results

ν_e appearance from a ν_μ beam

neutrino oscillation experiment is **simple in conception**:



but **difficult in practice:** rely on theory to determine cross sections: e.g. $\sigma(\nu_e)/\sigma(\nu_\mu)$ to a precision of 1%

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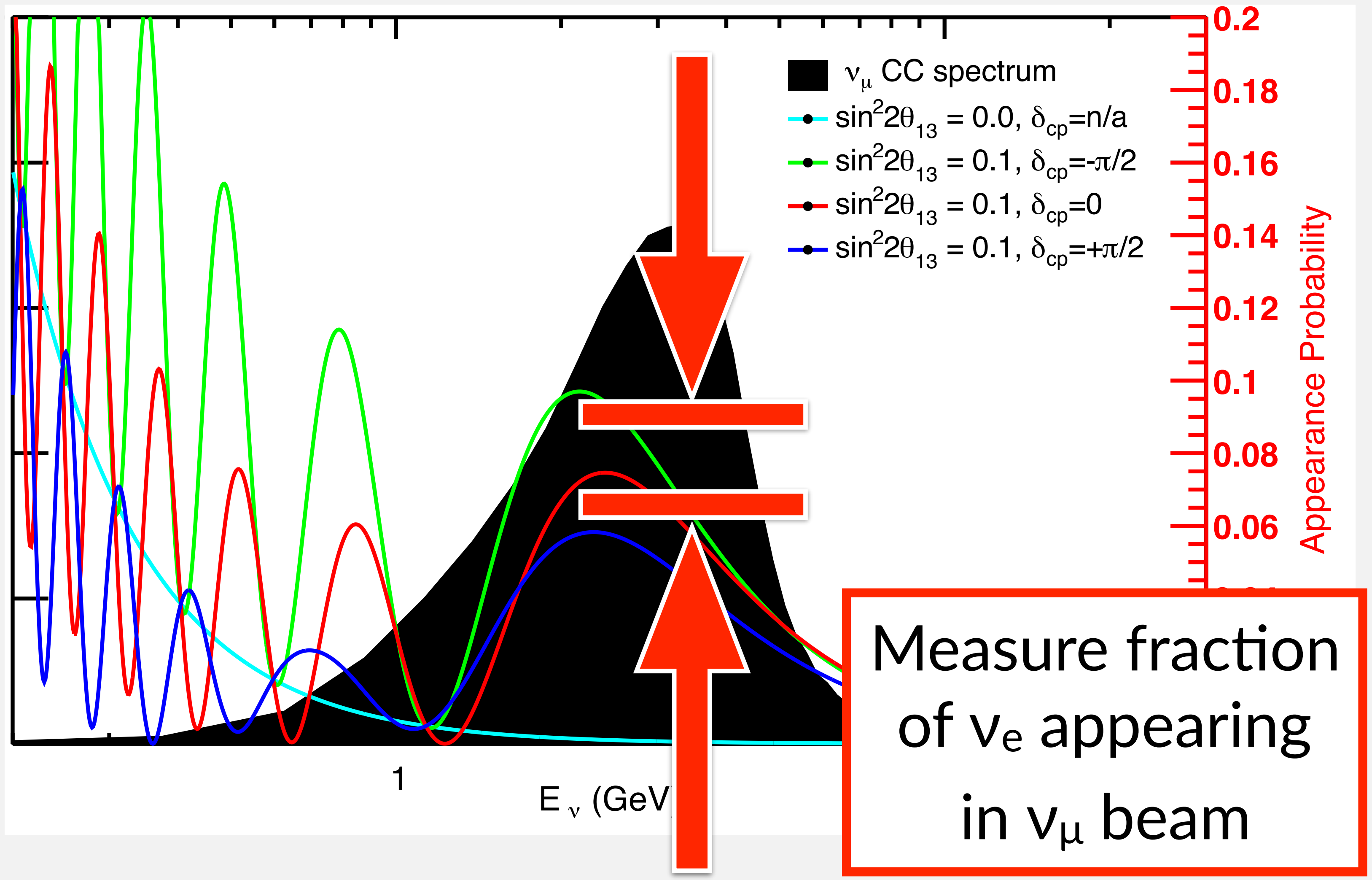

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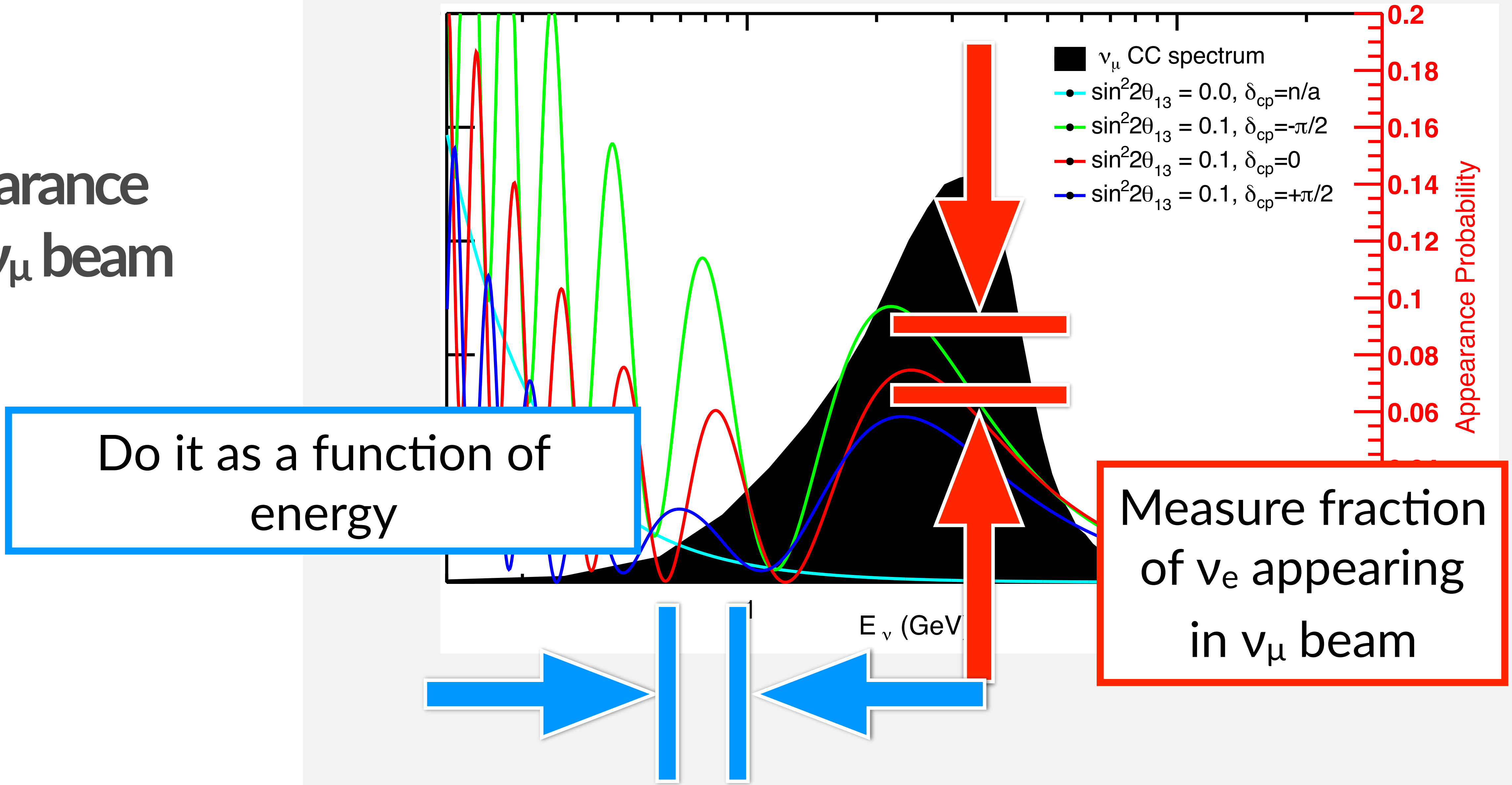
ν_e appearance from a ν_μ beam



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neutrino-electron scattering basics

- Phenomenologically important

  powerful constraint on neutrino flux: absolute normalization (current) and energy dependence (future)

  e.g. NUMI beam normalization: 7.5% → 3.9%

  MINERvA, 1906.00111

- Many features common to broader program of neutrino-nucleus scattering

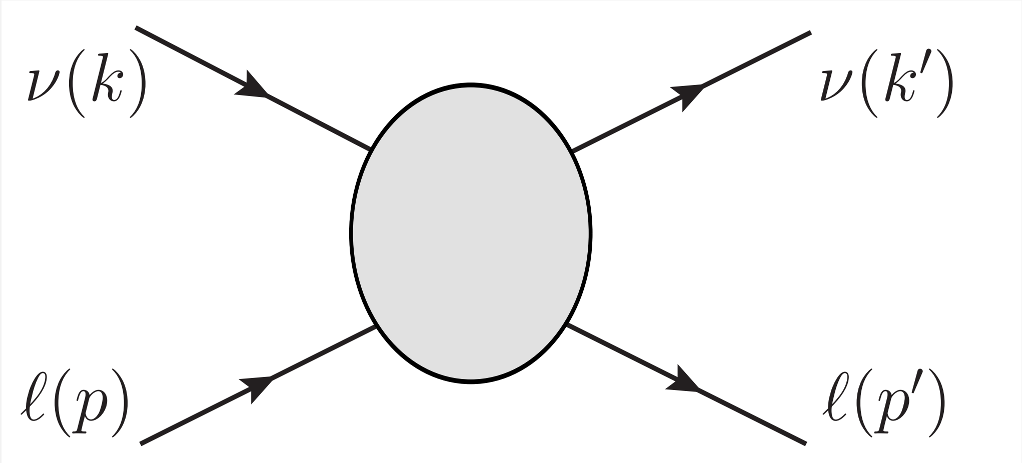
  - electroweak radiative corrections to four Fermi operator basis

  - universal "hadronic penguin" contribution *(dominates uncertainty in ν-e)*

  - analytic point particle limit for hard function in general case *(cf e-p 1605.02613)*

neutrino-electron scattering basics

- Kinematics



$$m_e \leq E'_e \leq m_e + \frac{2E_\nu^2}{m_e + 2E_\nu}$$

$$\cos\theta'_e = \frac{m_e + E_\nu}{E_\nu}\sqrt{\frac{E'_e - m_e}{E'_e + m_e}}$$

- near-forward scattering for E_e', $E_\nu \gg m_e$

- can reconstruct E_ν from E_e', θ_e'

neutrino-electron scattering basics

- Cross section

  - suppressed by lepton mass

    σ ~ G_F^2 s ~ G_F^2 m_e E_ν

$$\frac{d\sigma}{dE'_e} = \frac{m}{4\pi}\left[c_L^2 I_L + c_R^2 I_R + c_L c_R I_{LR}\right]$$

  - tree level:

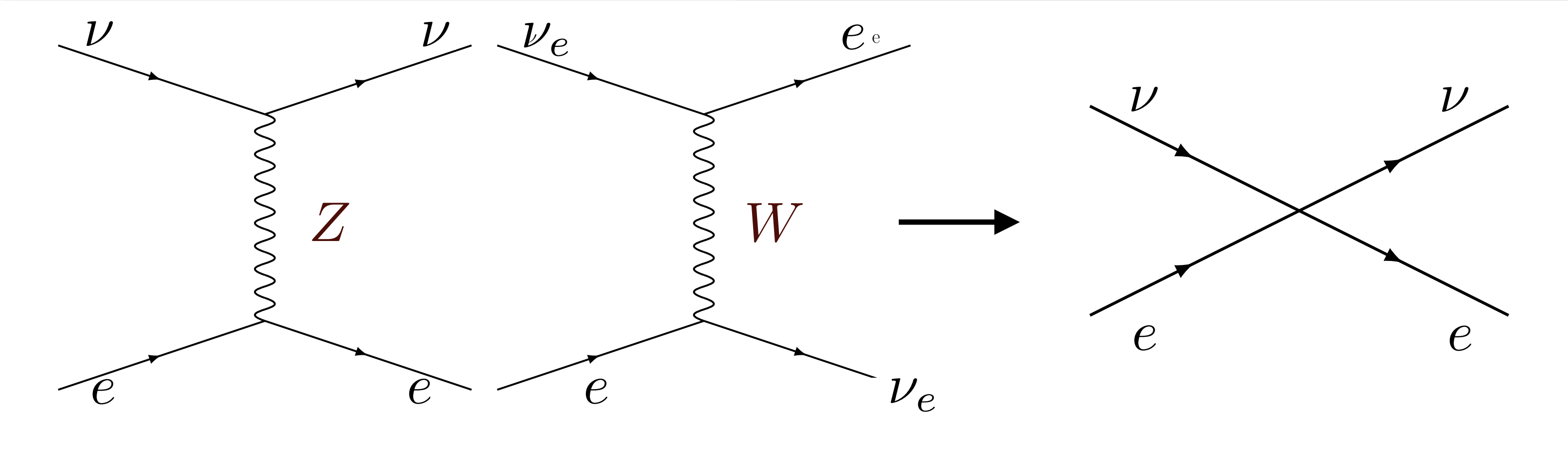
$$c_{\mathrm{L}}^{\nu_\ell \ell'} = 2\sqrt{2}\mathrm{G_F}\left(\sin^2\theta_W - \frac{1}{2} + \delta_{\ell\ell'}\right)$$

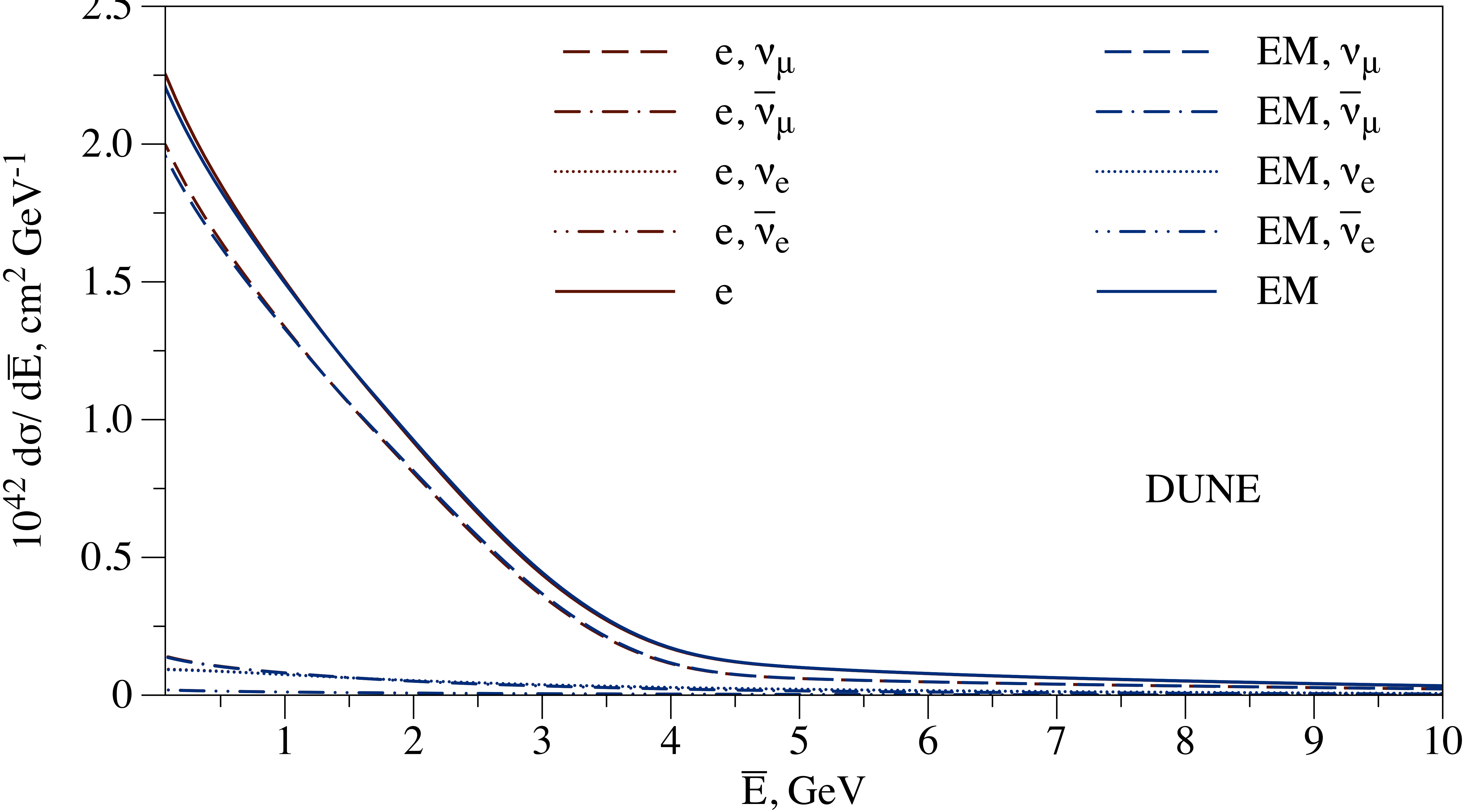
$$c_{\mathrm{R}} = 2\sqrt{2}\mathrm{G_F}\sin^2\theta_W$$

$$I_L = 1$$

$$I_R = \frac{E_\nu'^2}{E_\nu^2}$$

$$I_{LR} = -\frac{m_e}{E_\nu}\left(1 - \frac{E'_\nu}{E_\nu}\right)$$





Need: - absolute cross sections at <1%

- observable matched to detector (~EM versus e)

- corrections to ν energy reconstruction

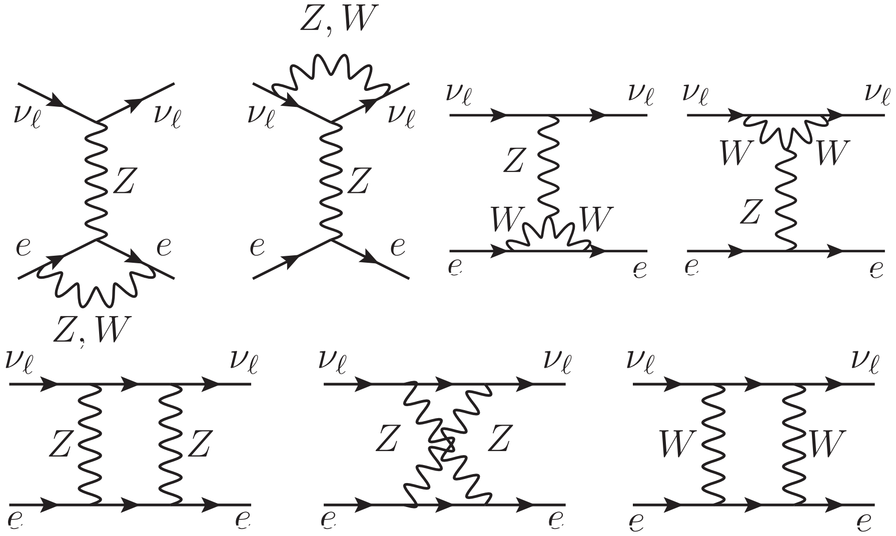
Four Fermi theory

- one loop matching at EW scale

- include two-loop mixed QED-QCD corrections for leptonic operators

- neglect fermion masses except top quark

- subsequent high-order RG evolution to hadronic scales (default μ=2 GeV, n_f=4)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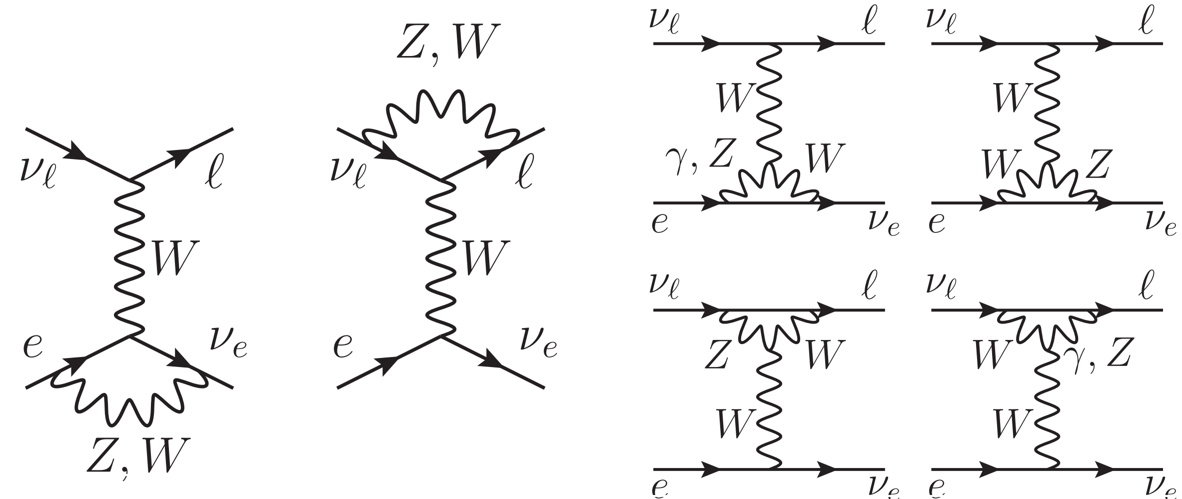
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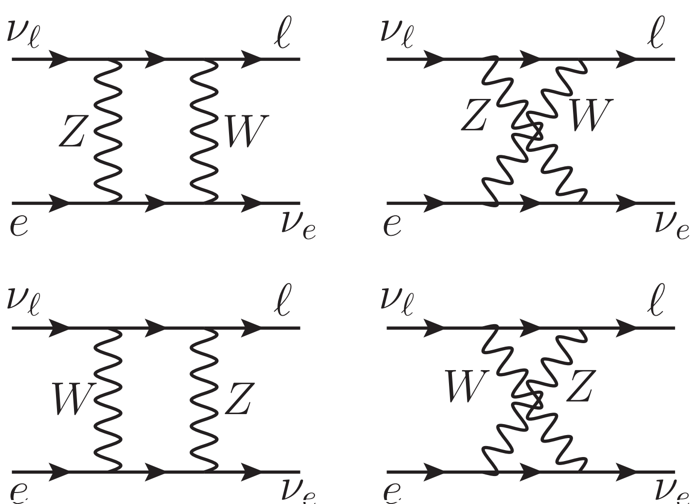
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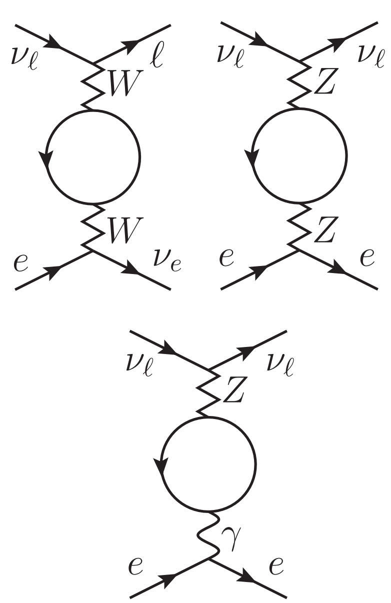
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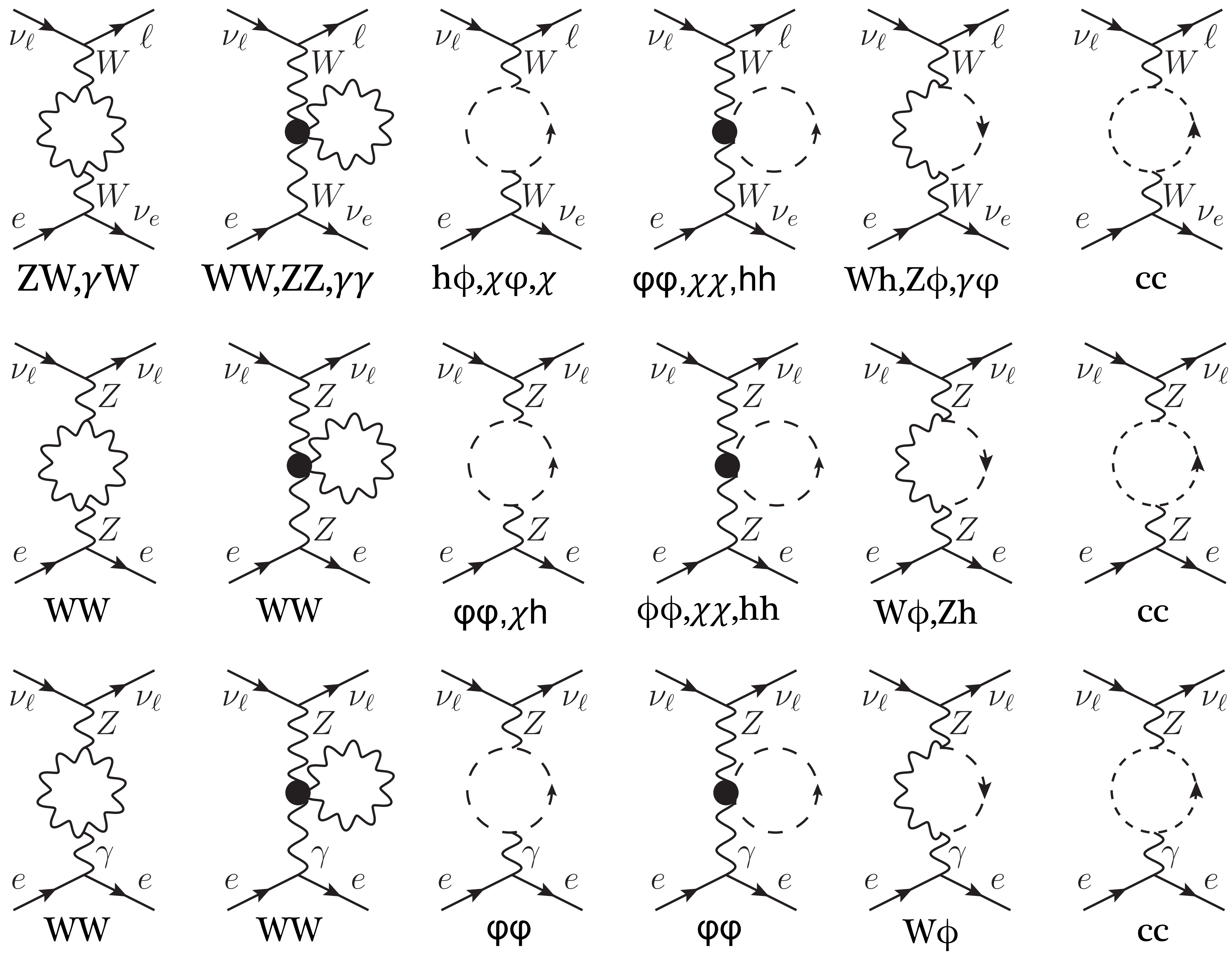
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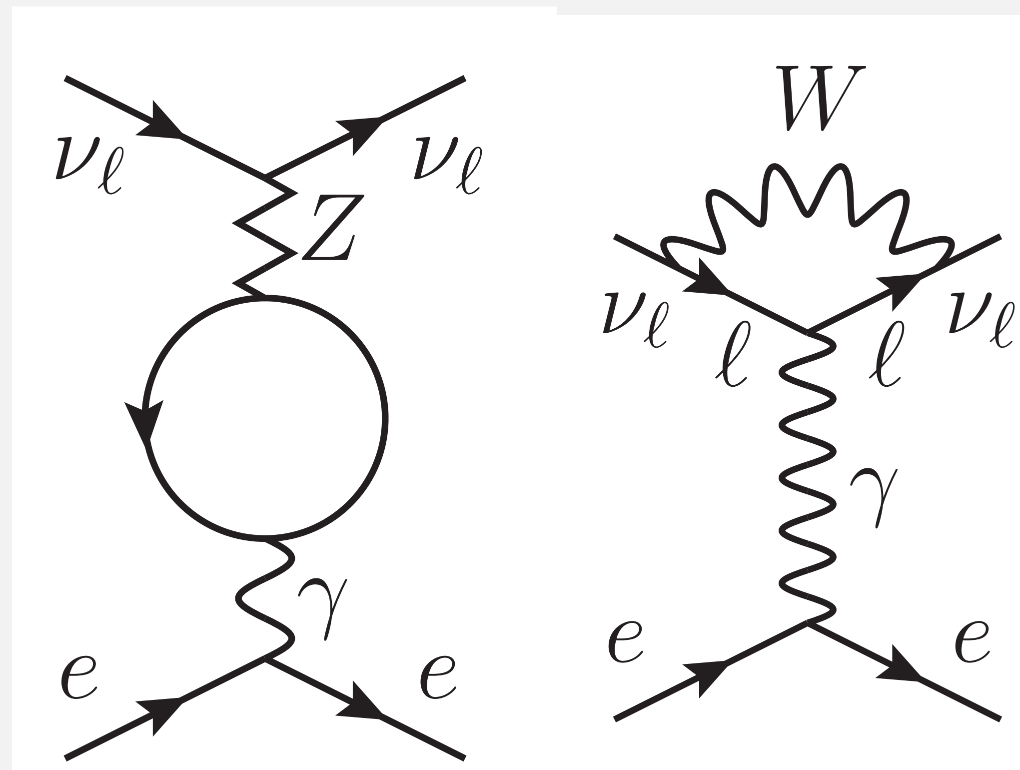
Four Fermi theory

Neutral current: matching to e.m. current operator

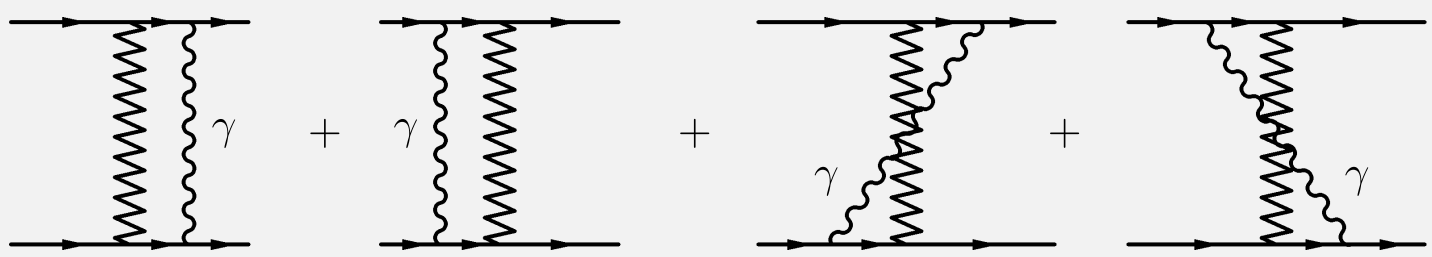


- long distance effective electromagnetic coupling to neutrino (including “charge radius”)

- associated scale dependence of 4 Fermi operators

Four Fermi theory

Charged current: scheme dependence and evanescent operator basis



$$
|\frac{m_W^{-2\epsilon}}{\epsilon(1-\epsilon)}\frac{1}{d}\Big\{-\left[Q_\ell Q_u + Q_\nu Q_d\right]\gamma^\alpha\gamma^\beta\gamma^\mu P_L \otimes \gamma_\alpha\gamma_\beta\gamma_\mu P_L + \left[Q_\ell Q_d + Q_\nu Q_u\right]\gamma^\alpha\gamma^\beta\gamma^\mu P_L \otimes \gamma_\mu P_L \gamma_\beta\gamma_\alpha\Big\}
$$

$$
\begin{aligned}
\gamma^\alpha\gamma^\beta\gamma^\mu P_L \otimes \gamma_\mu\gamma_\beta\gamma_\alpha P_L &= \sum_i (f_i + a_i\epsilon) O_i + E + \mathcal{O}(\epsilon^2) \\
&= (4-8\epsilon)\gamma^\mu P_L \otimes \gamma_\mu P_L + E
\end{aligned}
$$

(with conventional vector/axial-vector basis)

Four Fermi theory

- starting point for neutrino interaction phenomenology in current and next generation experiments

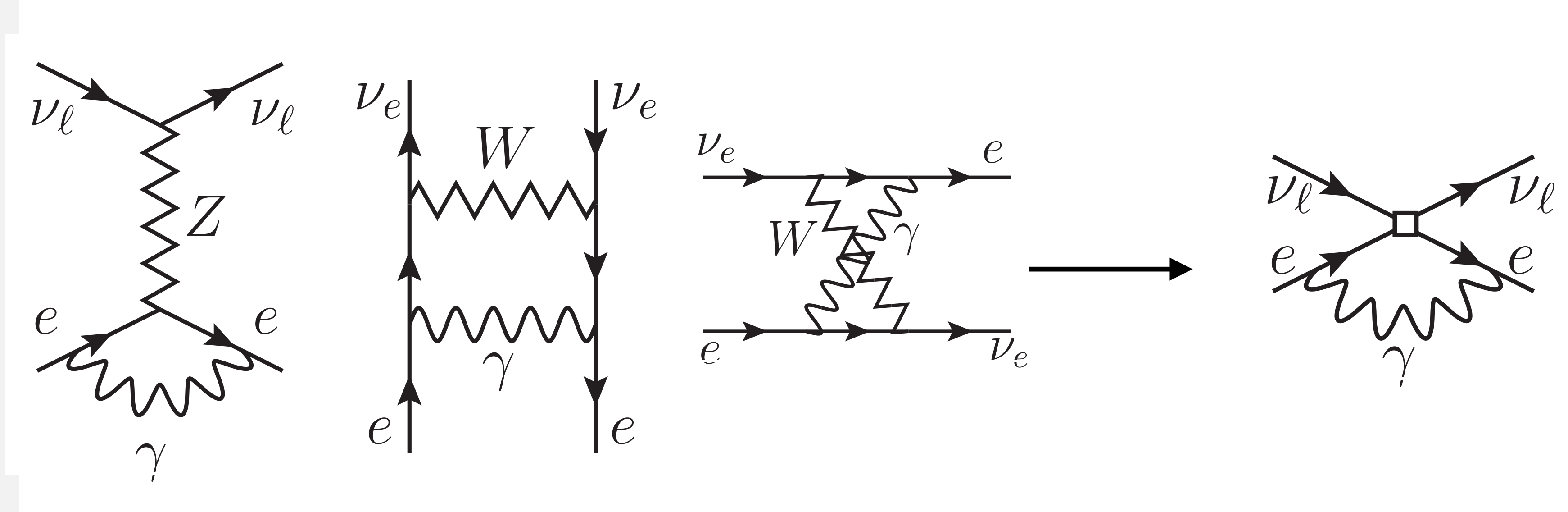
- SM check on G_F from EW precision measurements vs muon decay

- efficient separation of electroweak and hadronic effects

- scheme specification: (MS-bar, μ=2 GeV, n_f=4 and a=-8)

Virtual corrections

vertex corrections



$$\left(Z_\ell - 1\right) \mathrm{J}_\mu^{\mathrm{L,R}} + \delta \mathrm{J}_\mu^{\mathrm{L,R}} = \frac{\alpha}{\pi}\left(f_1 \mathrm{J}_\mu^{\mathrm{L,R}} + f_2 \mathrm{j}_\mu^{\mathrm{L,R}}\right)$$

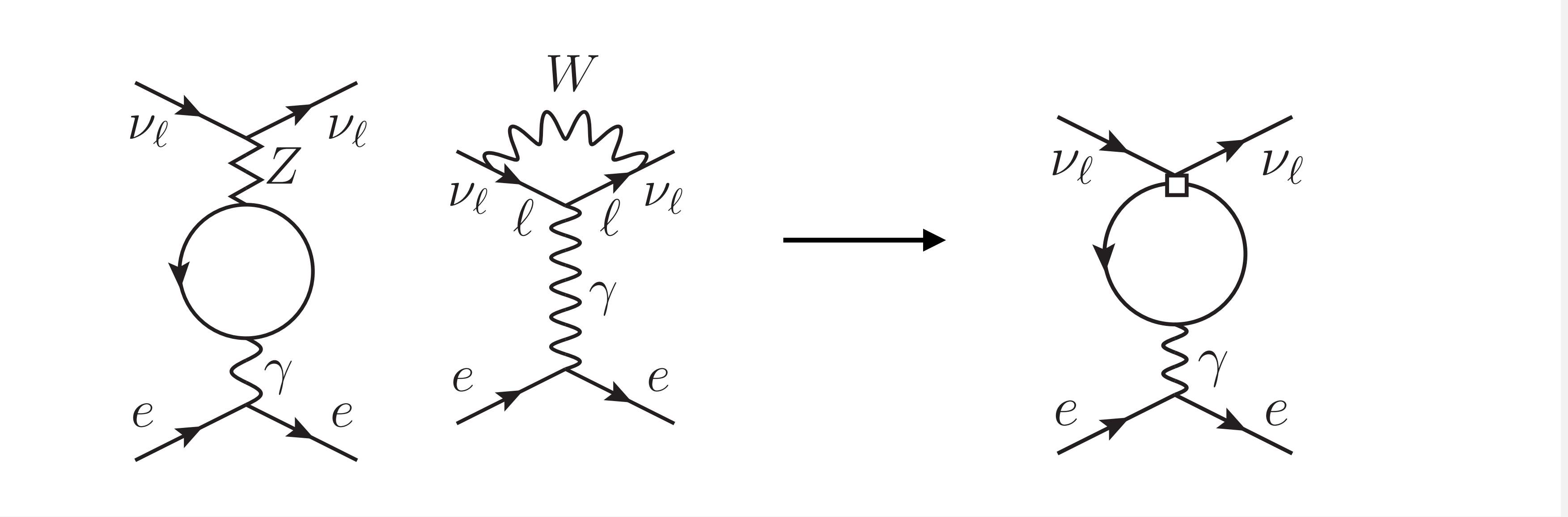
$$\mathrm{j}_\mu^{\mathrm{L}} = \frac{1}{2}\bar{e}\left(p'\right)\left(\gamma_\mu\gamma_5 + \frac{i\sigma_{\mu\nu}q^\nu}{2m}\right)e\left(p\right)$$

$$\mathrm{j}_\mu^{\mathrm{R}} = \frac{1}{2}\bar{e}\left(p'\right)\left(-\gamma_\mu\gamma_5 + \frac{i\sigma_{\mu\nu}q^\nu}{2m}\right)e\left(p\right)$$

- expressed in terms of vector current Dirac and Pauli form factors of the electron

Virtual corrections

leptonic penguins



$$\delta \mathrm{J}_\mu^{\mathrm{L}} = \delta \mathrm{J}_\mu^{\mathrm{R}} = Q_f \frac{\alpha}{2\pi} \Pi\left(q^2, m_f\right) \left(\mathrm{J}_\mu^{\mathrm{L}} + \mathrm{J}_\mu^{\mathrm{R}}\right)$$

- expressed in terms of QED vac. pol.

$$\Pi\left(q^2, m_f\right) = \frac{1}{3} \ln \frac{\mu^2}{m_f^2} + \frac{5}{9} + \frac{4m_f^2}{3q^2} + \frac{1}{3}\left(1 + \frac{2m_f^2}{q^2}\right)\sqrt{1 - \frac{4m_f^2}{q^2}} \ln \frac{\sqrt{1 - \frac{4m_f^2}{q^2}} - 1}{\sqrt{1 - \frac{4m_f^2}{q^2}} + 1},$$

Virtual corrections

heavy quark penguins

$$\Pi \to \Pi + \Pi^{\mathrm{QCD}}$$

$$\Pi^{\mathrm{QCD}} = \frac{\alpha_s}{3\pi}\left(\ln\frac{\mu^2}{m_f^2} - 4\zeta(3) + \frac{55}{12} + \frac{4m_f^2}{q^2}V_1\left(\frac{q^2}{4m_f^2}\right)\right)$$

$$\Pi^{\mathrm{QCD}}\Big|_{q^2\to -0} = \frac{\alpha_s}{3\pi}\left(\ln\frac{\mu^2}{m_f^2} + \frac{15}{4}\right)$$

- pole mass expansion poorly convergent, express as MS-bar mass

$$\begin{aligned}\Pi &= \frac{1}{3}\ln\frac{\mu^2}{\hat{m}_c^2} + \frac{\alpha_s}{3\pi}\left(-\ln\frac{\mu^2}{\hat{m}_c^2} + \frac{13}{12}\right)\\ &+ \frac{\alpha_s^2}{3\pi^2}\left(\frac{655}{144}\zeta(3) - \frac{3847}{864} - \frac{5}{6}\ln\frac{\mu^2}{\hat{m}_c^2} - \frac{11}{8}\ln^2\frac{\mu^2}{\hat{m}_c^2} + n_f\left(\frac{361}{1296} - \frac{1}{18}\ln\frac{\mu^2}{\hat{m}_c^2} + \frac{1}{12}\ln^2\frac{\mu^2}{\hat{m}_c^2}\right)\right)\end{aligned}$$

Virtual corrections

light quark penguins

- $q^2 \gg \Lambda_{QCD}^2$ a very bad approximation for kinematics of ν-e scattering
- in fact $q^2 \ll \Lambda_{QCD}^2$: evaluate at $q^2 = 0$ (plus controlled corrections)
- in contrast to HVP in $(g-2)_\mu$, need mixed I_3-Q correlator

$$\frac{\alpha}{\pi}\left(\hat{\Pi}^{(3)}_{3\gamma}(0) - 2\sin^2\theta_W \hat{\Pi}^{(3)}_{\gamma\gamma}(0)\right)$$

$$(q^\mu q^\nu - q^2 g^{\mu\nu})\Pi_{\gamma\gamma}(q^2) = 4i\pi^2 \int \mathrm{d}^d x\, e^{iq\cdot x}\langle 0|T\{J^\mu_\gamma(x)\, J^\nu_\gamma(0)\}|0\rangle$$

$$(q^\mu q^\nu - q^2 g^{\mu\nu})\Pi_{3\gamma}(q^2) = 4i\pi^2 \int \mathrm{d}^d x\, e^{iq\cdot x}\langle 0|T\{J^\mu_3(x)\, J^\nu_\gamma(0)\}|0\rangle$$

Virtual corrections

$$\hat{\Pi}^{(3)}_{\gamma\gamma} = \sum_{i,j} Q_i Q_j \Pi^{ij} = \frac{4}{9}\Pi^{uu} + \frac{1}{9}\Pi^{dd} + \frac{1}{9}\Pi^{ss} - \frac{4}{9}\Pi^{ud} - \frac{4}{9}\Pi^{us} + \frac{2}{9}\Pi^{ds},$$

$$\hat{\Pi}^{(3)}_{3\gamma} = \sum_{i,j} T^3_i Q_j \Pi^{ij} = \frac{1}{2}\left(\frac{2}{3}\Pi^{uu} + \frac{1}{3}\Pi^{dd} + \frac{1}{3}\Pi^{ss} - \Pi^{ud} - \Pi^{us} + \frac{2}{3}\Pi^{ds}\right)$$

SU(3)$_f$: $\Pi^{uu} = \Pi^{dd} = \Pi^{ss}$ and $\Pi^{ud} = \Pi^{us} = \Pi^{ds}$

$$\Longrightarrow \quad \hat{\Pi}^{(3)}_{3\gamma}(0) \approx \hat{\Pi}^{(3)}_{\gamma\gamma}(0)$$

SU(2)$_f$ + OZI: $\Pi^{uu} = \Pi^{dd}, \; \Pi^{ss} = 0 \quad \Pi^{ud} = \Pi^{us} = \Pi^{ds} = 0.$

$$\Longrightarrow \quad \hat{\Pi}^{(3)}_{3\gamma} = 9\hat{\Pi}^{(3)}_{\gamma\gamma}/10.$$

For numerical analysis:

$$\hat{\Pi}^{(3)}_{\gamma\gamma}(0)\big|_{\mu=2\,\mathrm{GeV}} = 3.597(21)$$

$$\hat{\Pi}^{(3)}_{3\gamma}(0) = (1 \pm 0.2)\,\hat{\Pi}^{(3)}_{\gamma\gamma}(0)$$

Real radiation

Phase space integration

$$\mathrm{d}\sigma_{\mathrm{LO}}^{\nu_\ell e\to\nu_\ell e\gamma} = \frac{\alpha}{4\pi}\frac{m\omega}{\pi^3}\left[\left(c_{\mathrm{L}}^{\nu_\ell e}\right)^2\tilde{\mathrm{I}}_{\mathrm{L}} + c_{\mathrm{R}}^2\tilde{\mathrm{I}}_{\mathrm{R}} + c_{\mathrm{L}}^{\nu_\ell e}c_{\mathrm{R}}\tilde{\mathrm{I}}_{\mathrm{R}}^{\mathrm{L}}\right]$$

$$\tilde{\mathrm{I}}_i = \int \frac{R_i}{m^2\omega^2}\delta^4(k+p-k_\gamma-k'-p')\frac{\mathrm{d}^3\vec{k}_\gamma}{2k_\gamma}\frac{\mathrm{d}^3\vec{k}'}{2\omega'}\frac{\mathrm{d}^3\vec{p}\,'}{2E'}$$

$$R_{\mathrm{L}} = -\mathrm{I}_{\mathrm{L}}\left[\frac{p^\mu}{(p\cdot k_\gamma)} - \frac{p'^\mu}{(p'\cdot k_\gamma)}\right]^2 m^2\omega^2 + \frac{(k\cdot p')\,(k'\cdot p')}{(k_\gamma\cdot p')} - \frac{(k\cdot p)\,(k'\cdot p)}{(k_\gamma\cdot p)} + \frac{(k\cdot p)\,(k'\cdot p')}{(k_\gamma\cdot p')} - \frac{(k\cdot p)\,(k'\cdot p')}{(k_\gamma\cdot p)}$$

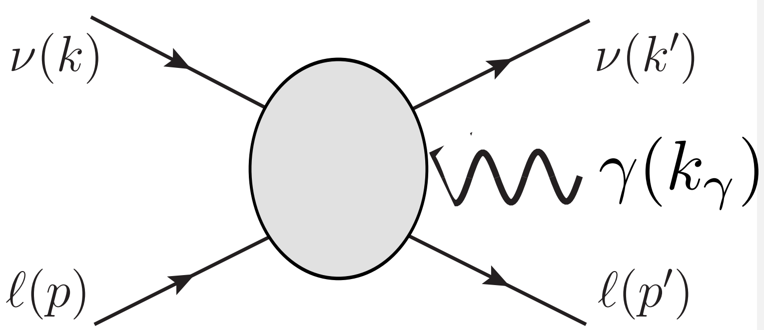
$$+ \frac{(k'\cdot p')\,(k\cdot k_\gamma)}{(k_\gamma\cdot p)}\left(1+\frac{m^2}{(k_\gamma\cdot p)} - \frac{(p\cdot p')}{(k_\gamma\cdot p')}\right) + \frac{(k\cdot p)\,(k'\cdot k_\gamma)}{(k_\gamma\cdot p')}\left(1-\frac{m'^2}{(k_\gamma\cdot p')} + \frac{(p\cdot p')}{(k_\gamma\cdot p)}\right)$$

Regions decomposition

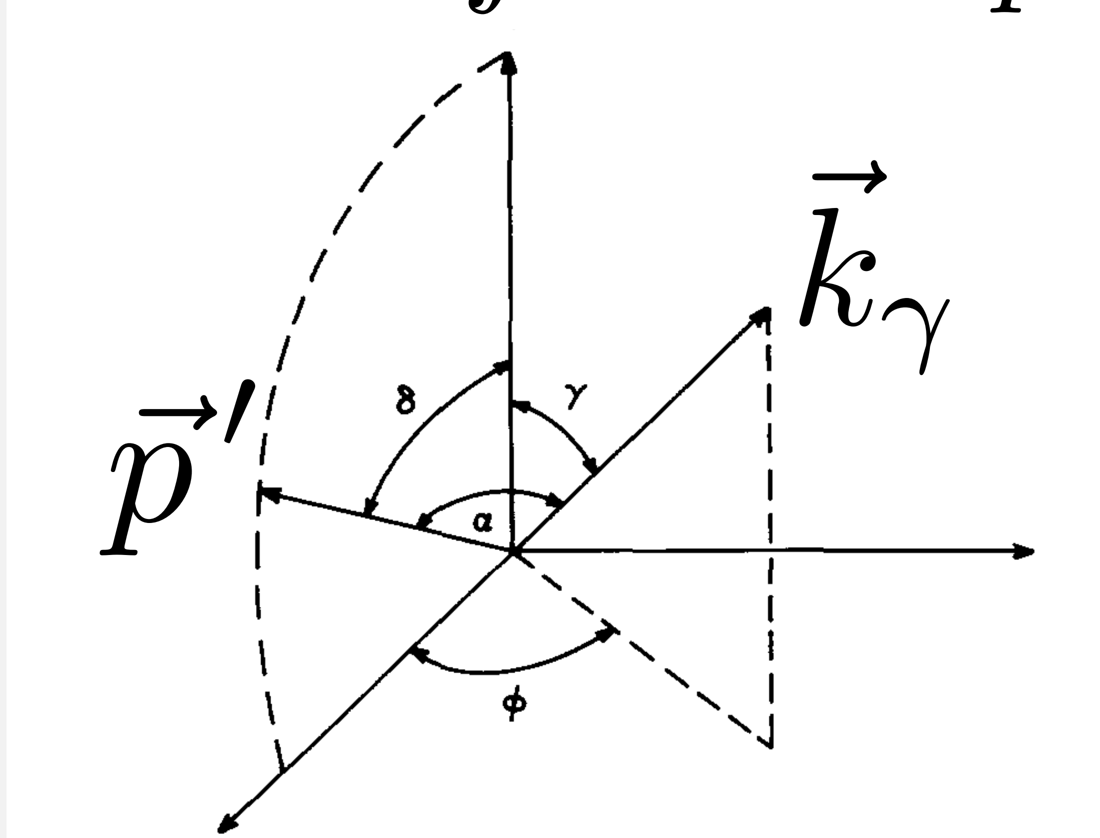
$$\mathrm{d}\sigma_{\mathrm{LO}}^{\nu_\ell e\to\nu_\ell e\gamma}+\mathrm{d}\sigma_{\mathrm{NLO}}^{\nu_\ell e\to\nu_\ell e}=\left[1+\frac{\alpha}{\pi}\left(\delta_v+\delta_s+\delta_{\mathrm{I}}+\delta_{\mathrm{II}}\right)\right]\mathrm{d}\sigma_{\mathrm{LO}}^{\nu_\ell e\to\nu_\ell e}$$

$$+\mathrm{d}\sigma_{v}^{\nu_\ell e\to\nu_\ell e}+\mathrm{d}\sigma_{\mathrm{dyn}}^{\nu_\ell e\to\nu_\ell e}+\mathrm{d}\sigma_{\mathrm{NF}}^{\nu_\ell e\to\nu_\ell e\gamma}$$

Real radiation



$$\vec{f}=\vec{k}+\vec{p}-\vec{p}'$$



Ram, PR 155, 1539 (1967)

Sarantakos, Sirlin, Marciano NPB 217, 84 (1983)

Real radiation

Consistency checks

- cancellation of Sudakov logarithms in electron energy spectrum

$$\delta_v \underset{\beta\to 1}{\sim} -\frac{1}{8}\ln^2(1-\beta)\,, \qquad \delta_s \underset{\beta\to 1}{\sim} -\frac{1}{4}\ln^2(1-\beta)\,, \qquad \delta_{\mathrm{II}} \underset{\beta\to 1}{\sim} \frac{3}{8}\ln^2(1-\beta)$$

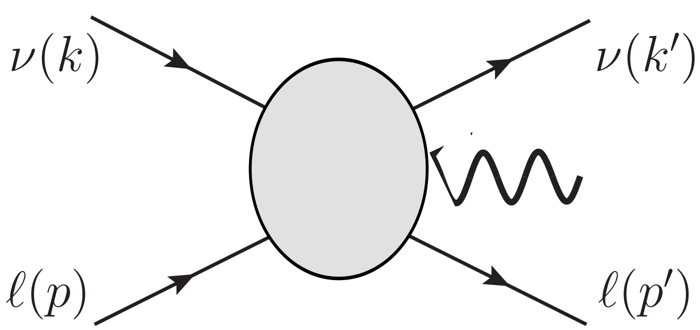
- soft photon log near electron threshold

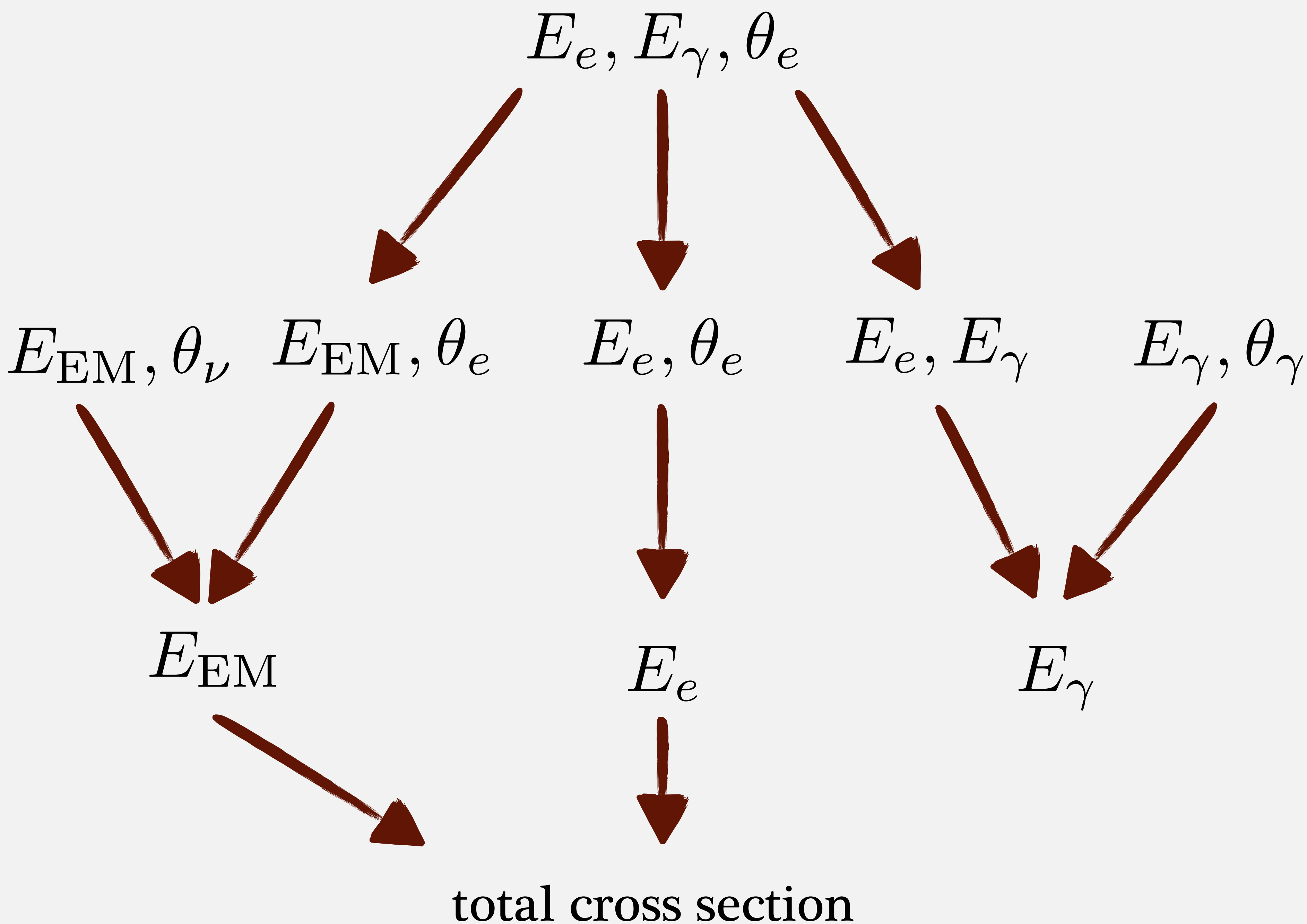
$$\frac{\mathrm{d}\sigma_{\mathrm{LO}}^{\nu_\ell e\to\nu_\ell e\gamma} + \mathrm{d}\sigma_{\mathrm{NLO}}^{\nu_\ell e\to\nu_\ell e}}{\mathrm{d}\sigma_{\mathrm{LO}}^{\nu_\ell e\to\nu_\ell e}} \approx -\frac{\alpha}{\pi}\frac{2}{\beta}\left(\beta - \frac{1}{2}\ln\frac{1+\beta}{1-\beta}\right)\ln\frac{E_0'-E'}{m}$$

- alternate orders of integration (next slide)

triple, double, single differentials and total cross section

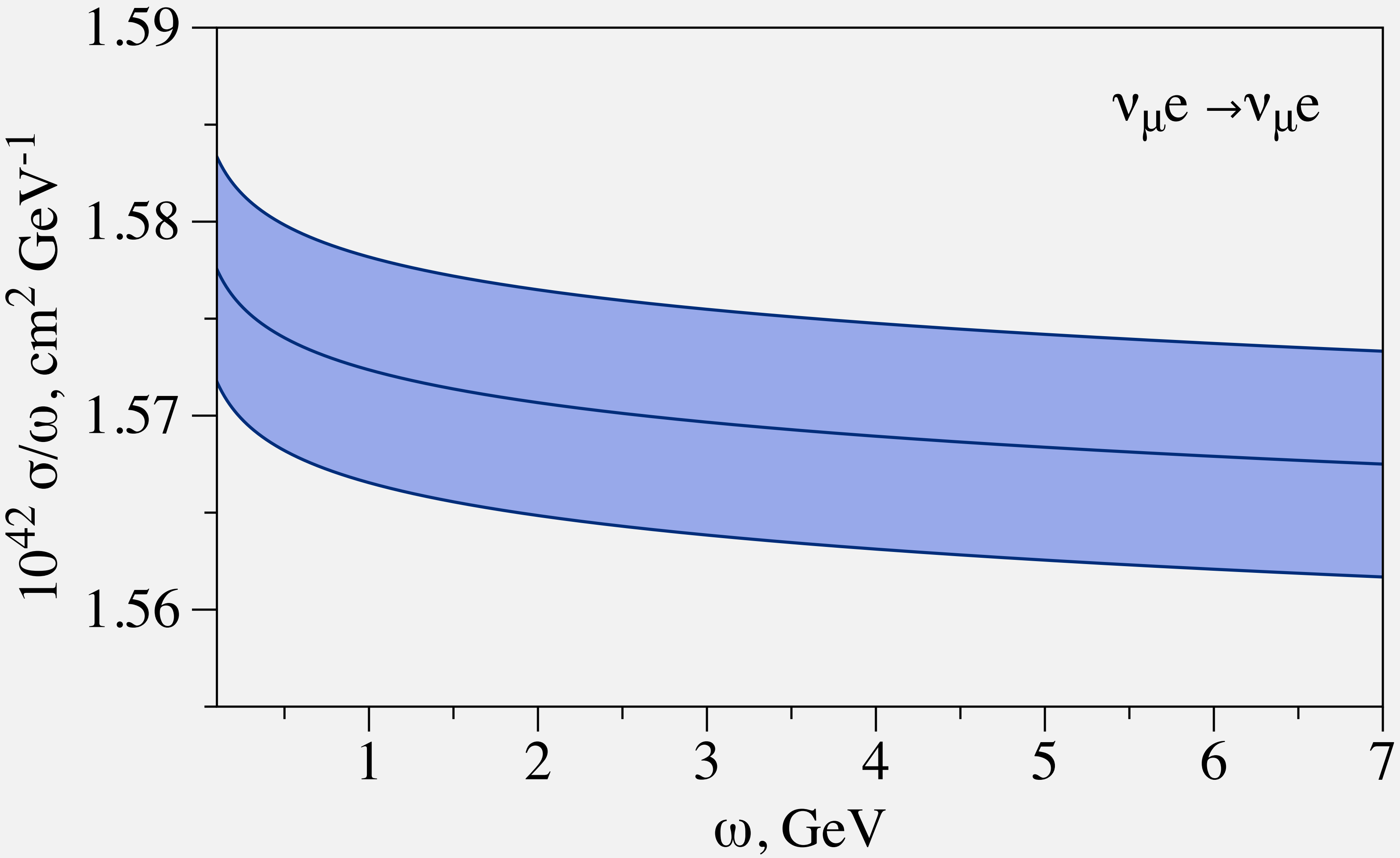
Distributions





Results

- total cross sections (and uncertainties!)

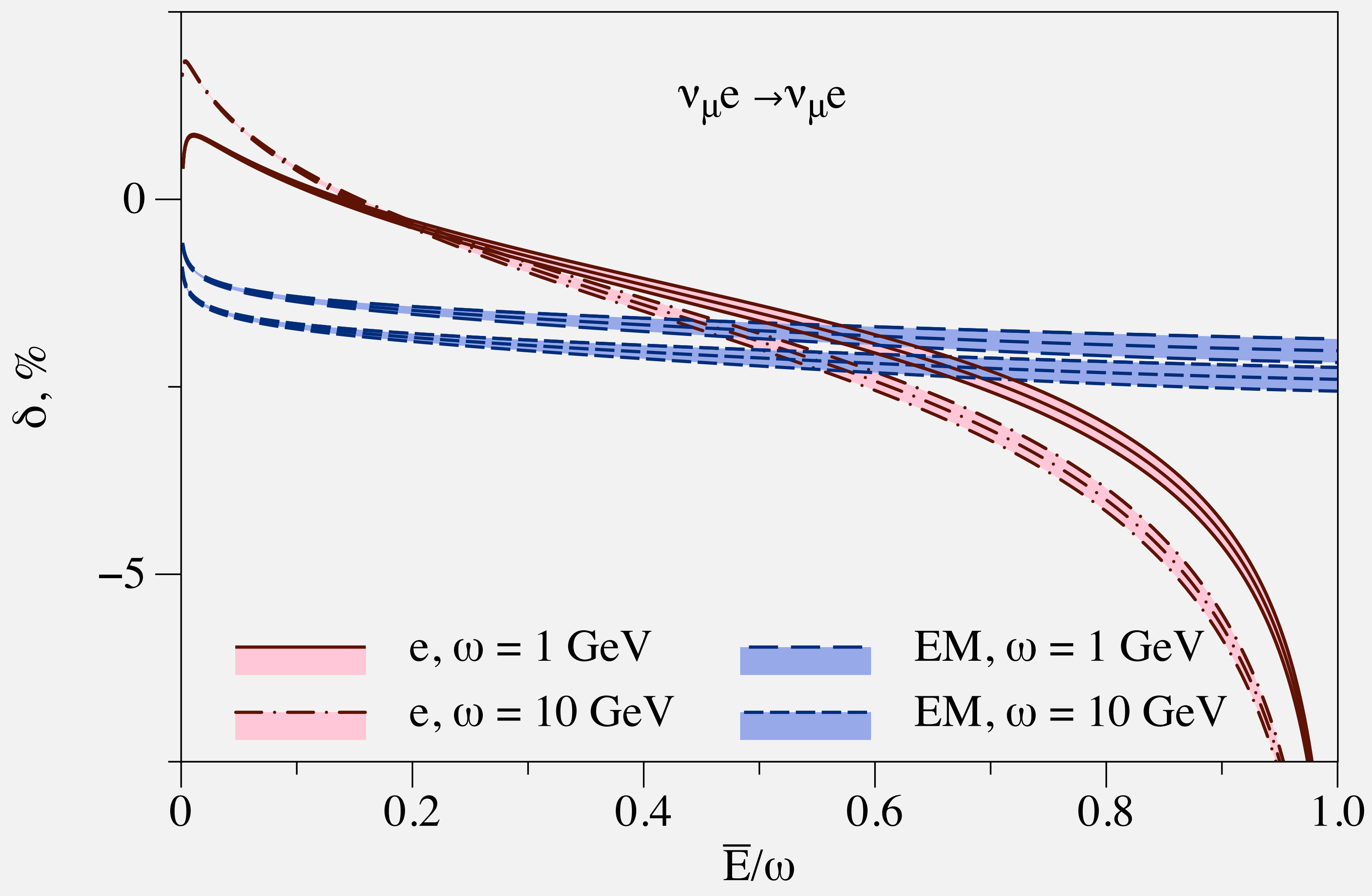


$$\sigma[\nu_\mu e \to \nu_\mu e(\gamma)] = \left[1.5724 \times 10^{-42}\,\mathrm{cm}^2\right] \times \left[1 \pm 0.0037_{\mathrm{had}} \pm 0.0003_{\mathrm{EW}} \pm 0.00007_{\mathrm{pert}}\right]$$

Results

- electron and e.m. energy spectrum

$$\delta = \frac{d\sigma_{\text{LO}}^{\nu_\ell e \to \nu_\ell e \gamma} + d\sigma_{\text{NLO}}^{\nu_\ell e \to \nu_\ell e} - d\sigma_{\text{LO}}^{\nu_\ell e \to \nu_\ell e}}{d\sigma_{\text{LO}}^{\nu_\ell e \to \nu_\ell e}}$$



Results

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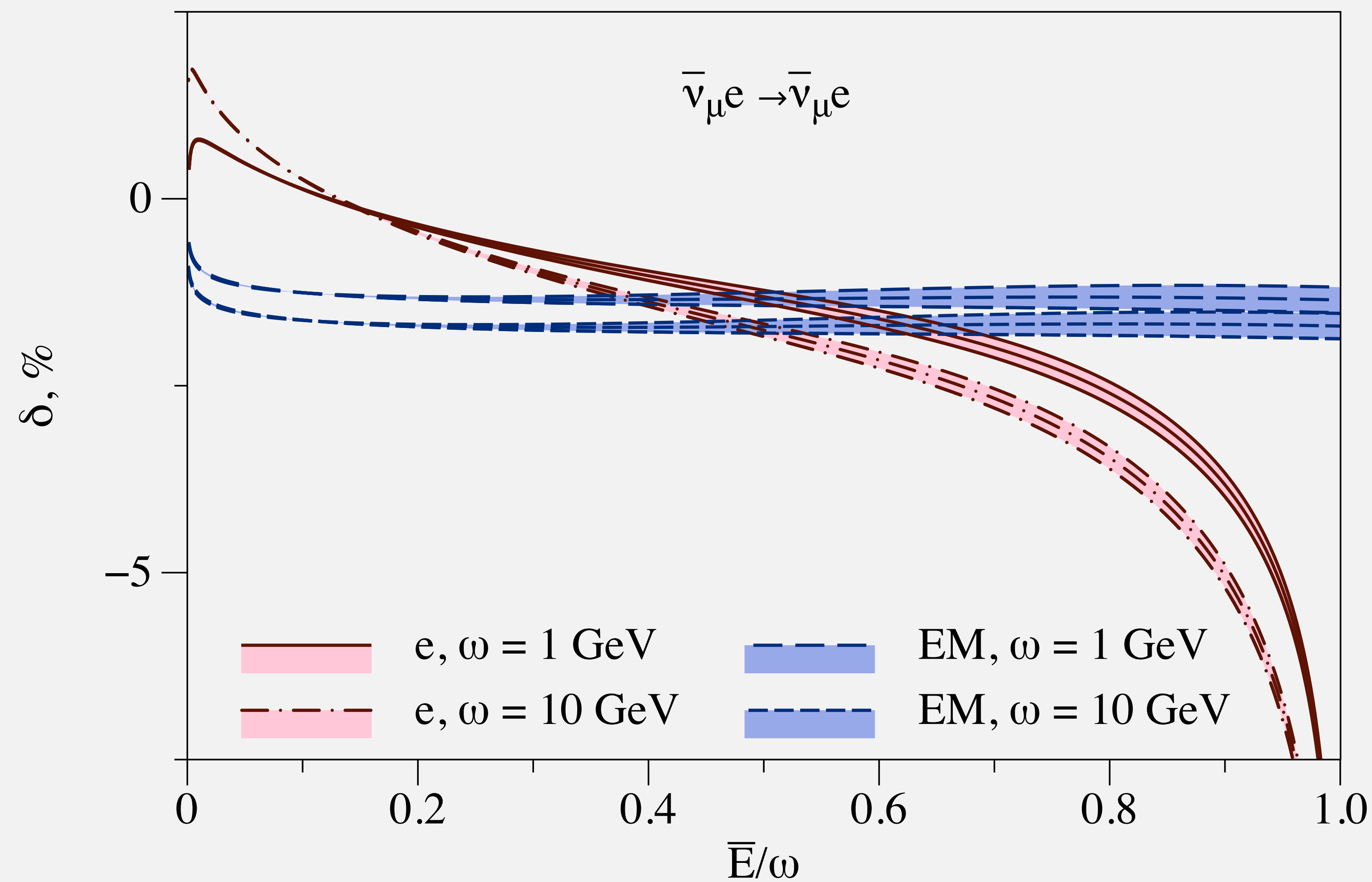
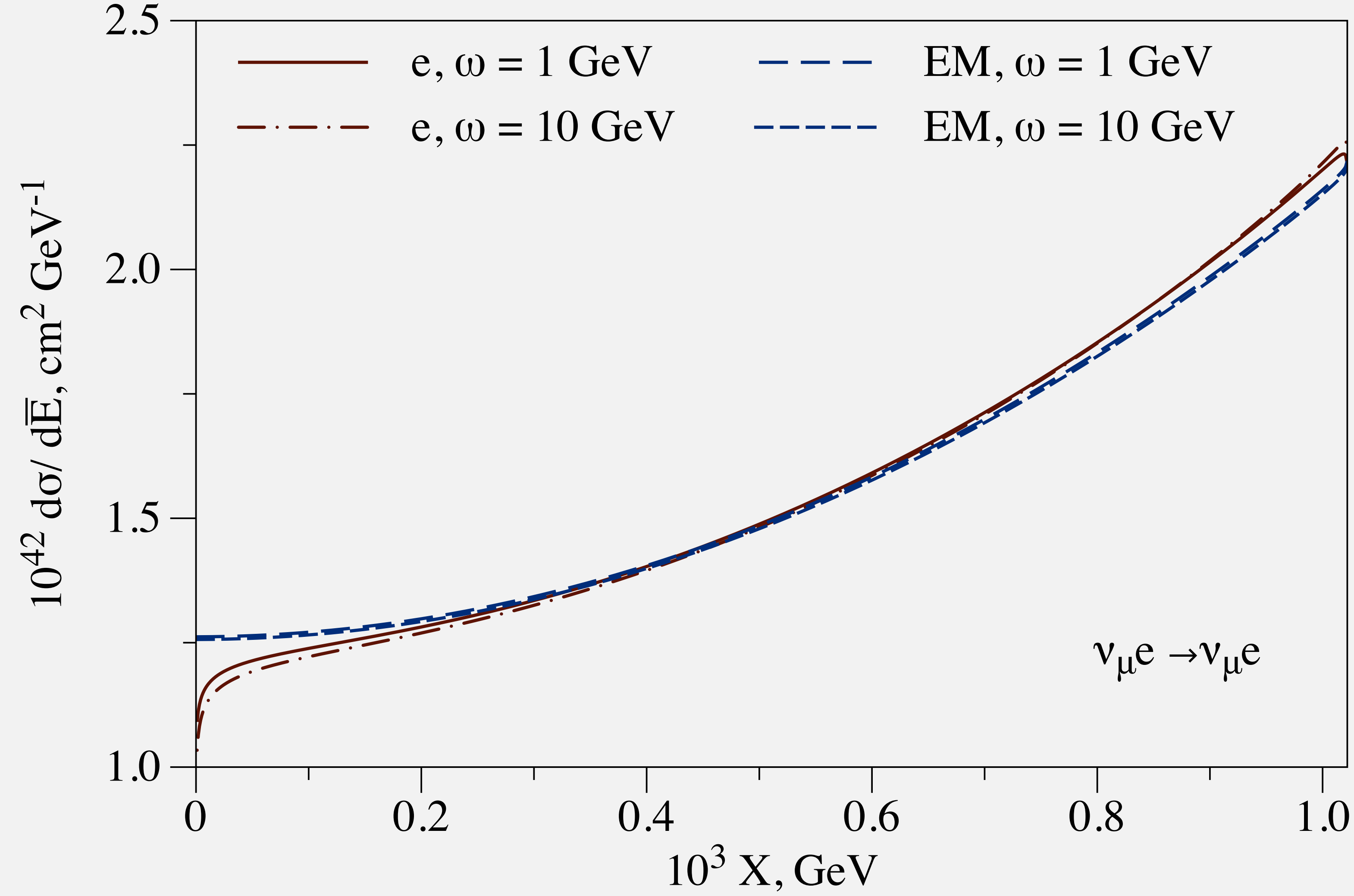


Results

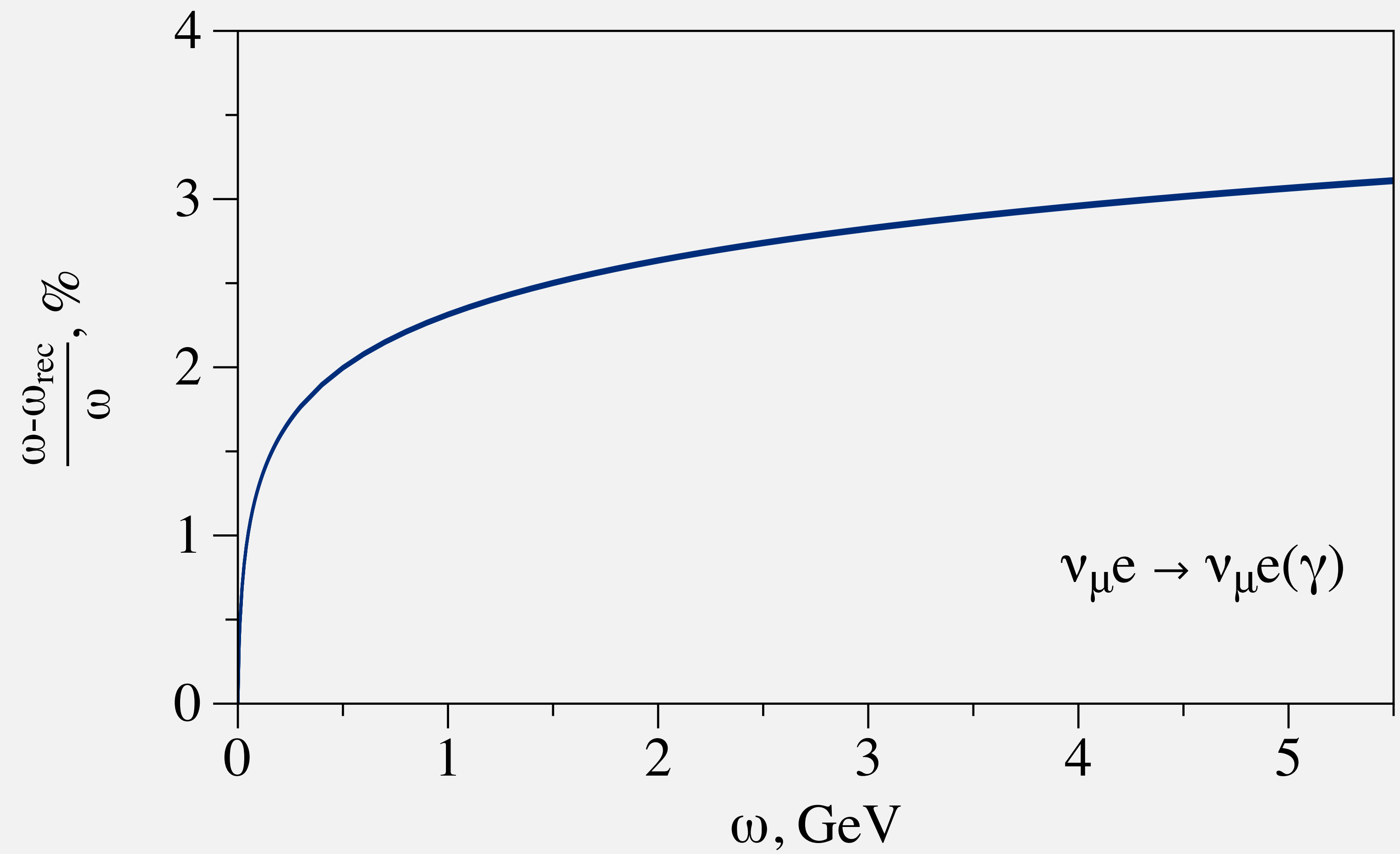
- cut dependence of flux constraints



$$X = 2m\left(1 - \frac{\bar{E}}{\omega}\right) \approx E_e \theta_e^2$$

Results

- neutrino energy reconstruction



$$\omega_{\rm rec} = \frac{m|\vec{p}_e|}{(E_e + m)\cos\theta_e - |\vec{p}_e|}$$

Summary and Outlook

Neutrino-electron scattering and radiative corrections

- corrections large compared to anticipated experimental precision

- theoretical uncertainty dominated by I_3-Q HVP, target for lattice QCD

- milliradian angular resolution at DUNE should provide capability for neutrino energy reconstruction: smearing by radiative corrections must be included *(cf. 1910.10996)*

- extension to scattering on nucleons and nuclei: factorization into hard, soft, collinear, with hard functions parameterized and measured