Volume Reflection Simulations using Yazynin's Channeling Code and Application to Possible e- Experiment

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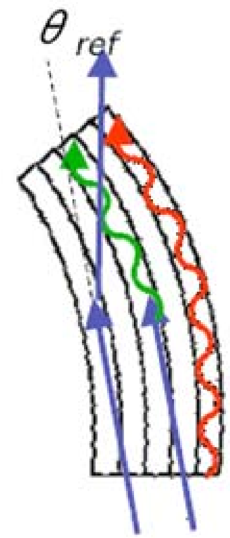
Special thanks to Igor Yazynin (IHEP) for kindly responding to questions and generously sharing his code. Thanks to Walter Scandale (CERN) for information on volume reflection and capture.

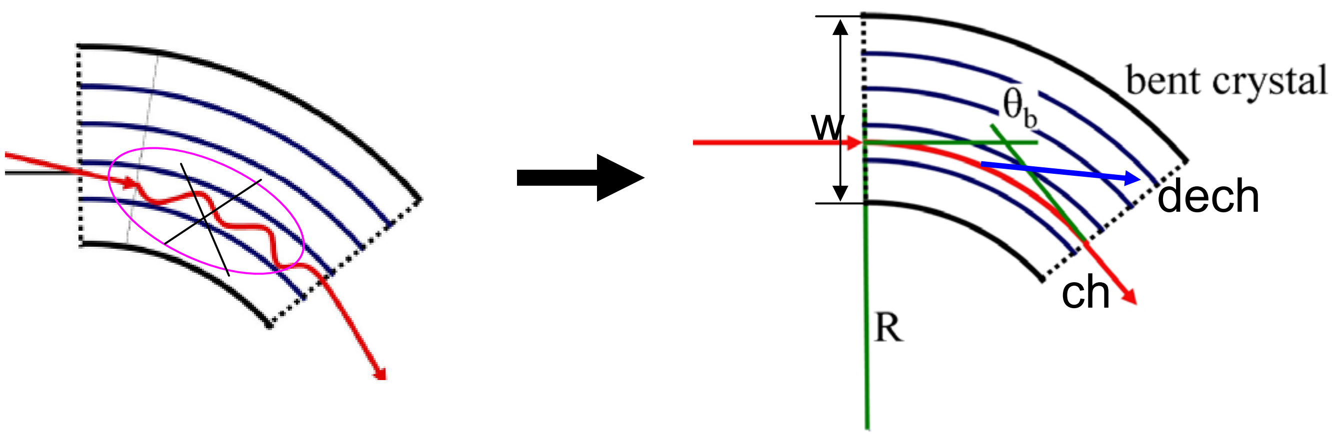
This is a "decision-tree" code, not full Monte Carlo.

Yazynin Code includes processes:

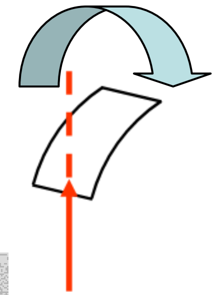
- multiple scattering
- channeling
- volume capture
- de-channeling
- *volume reflection*



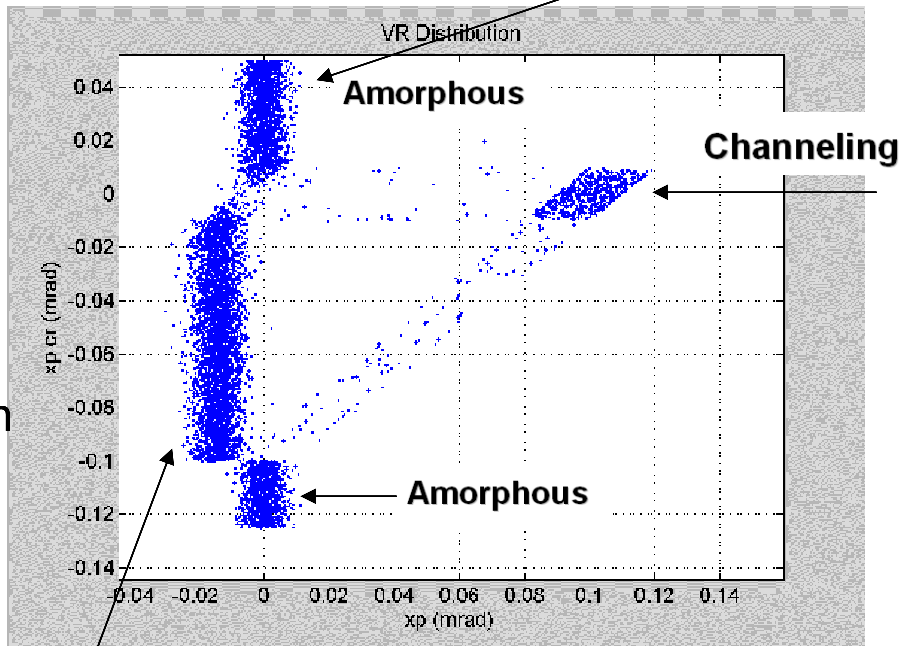
Basic approx: Code replaces details of particle orbits with Monte Carlo fits based on distribution fcns and analytic formulas for trajectories over long distances (not on scale of betatron motion in bent crystal). It applies probabilities to dechanneling, volume capture, volume reflection, amorphous transport, Coulomb and nucl scattering angles, energy loss, etc. Both proton and electron versions of code exist.

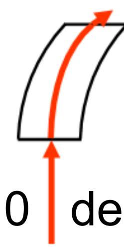


Example: 400 GeV protons, Si(110), crystal R = 10 m, length=1mm

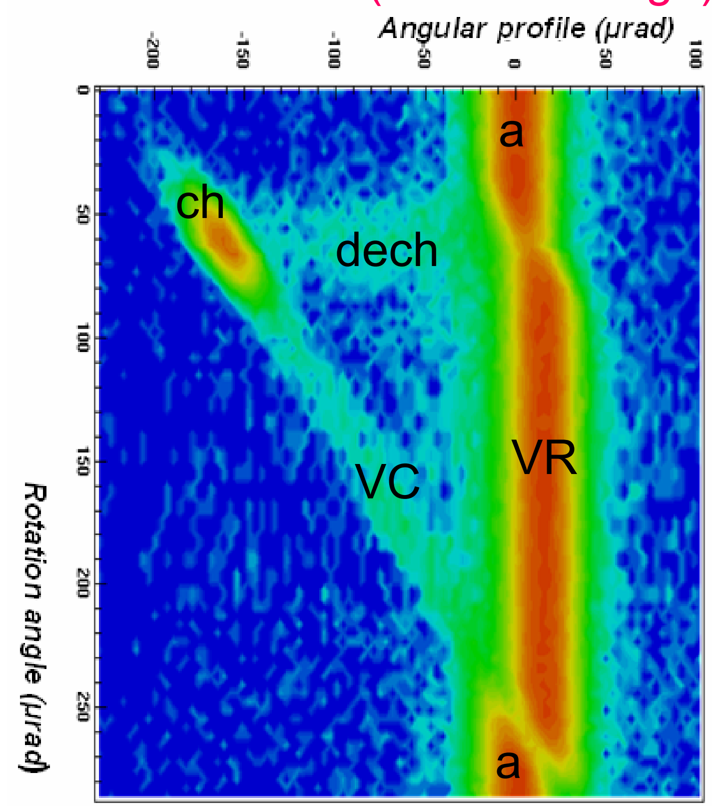


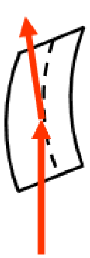
"Rotation Angle"- crystal orientation rel. to beam input direction





Usual expt plot (other than sign)



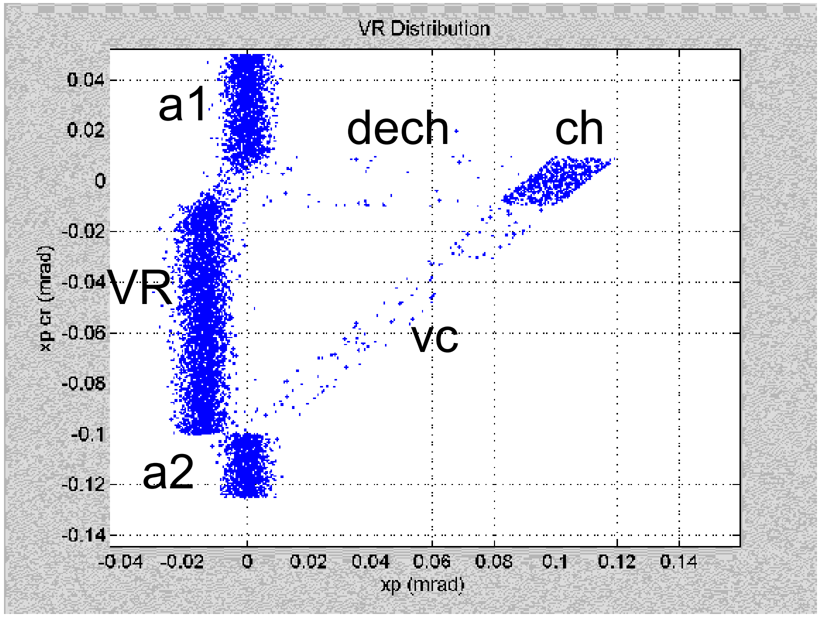


"Angular Profile"
- change in particle angle relative to input angle which is induced by crystal

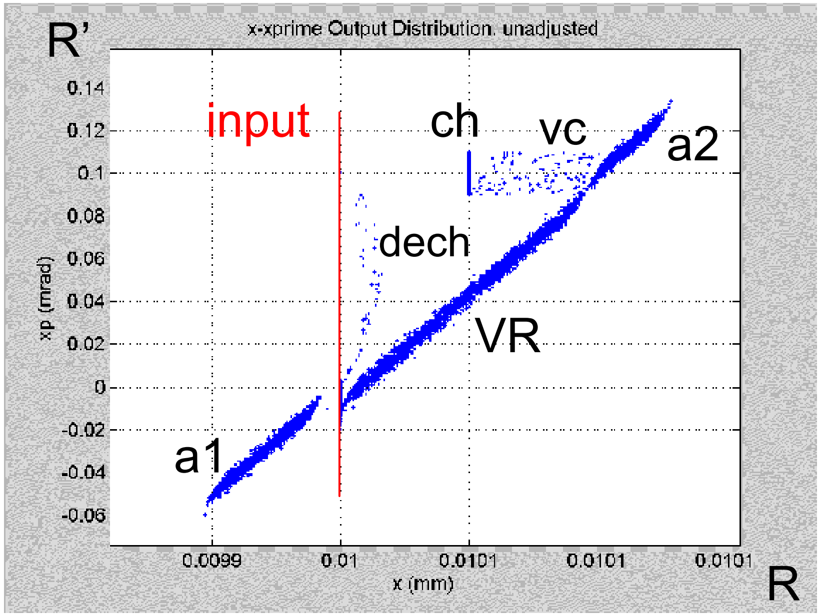
Max rot angle= xtal thickness/R=0.1 mrad

Example: Code runs 7000 protons (400 GeV), as pencil beam at R (horiz coord, not a radius) =0.01 mm, Z (vert) =1 mm with Z'=0, and R' uniformly spread over [-0.05, +0.125] mrad. Transports each proton through 1 mm long, curved (10m) Si crystal.

VR Angular Profile Plot



Output phase space R'-R



We exercised the p and e- codes over a large range of energies, curvature radii, crystal thickness (primarily Si) to understand limitations and compare to other work and data where available. Code was originally written for use with multi-hundred GeV - TeV protons, so some approximations are not expected to be valid in all regimes.

What we found and modified in our test version of the code:

1. VC particles channeled orders of magnitude deeper than physically reasonable. Changed the VC dechanneling formula to same as for normally channeled particles.

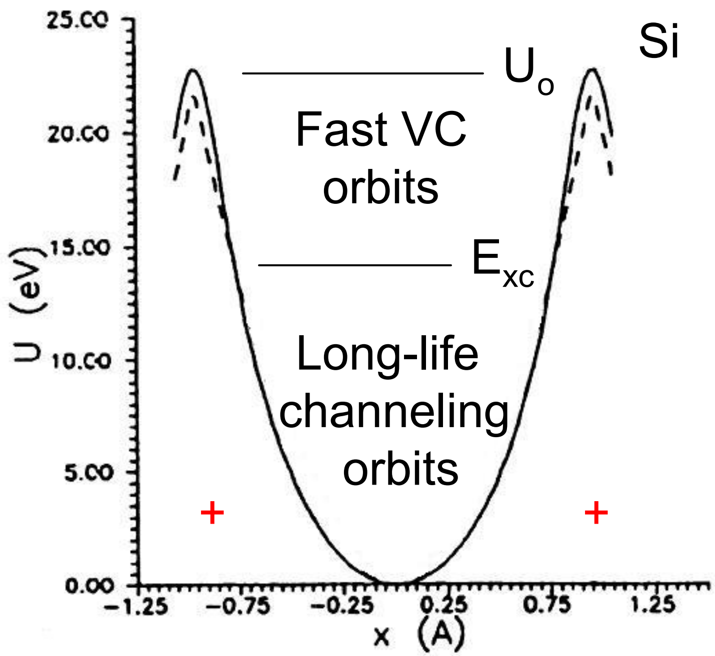
2. Added a bremsstrahlung energy loss term ~ E to the electron code.

3. Modified the multiple scattering formula to include $\log(z/Z_{rad})$ term so MS is correctly reduced for thin crystals (still need to add plural scatt. for ultra-thin).

4. At low energy and large $x=R_{crys}/R_{crit}$, code's VC probability > 1 (intended for proton E>100 GeV, x<30). Modified VC probability following Taratin and Scandale's potential well capture probability ($\sim E^{-1.5}$), and used an exponential form to keep P<1.

5. Prior to VC at a plane deep in the crystal, code applied no MS, resulting in too narrow of angle spread for these particles. We added MS, and when angle increase is too large, we set capture probability to zero.

Taratin and Scandale's potential-well capture probability, NIM B262 (2007) 340 (modified from an argument of Biryukov & Chesnokov)



Volume capture (VC) results from MS-induced energy transitions of order $U_0 - E_{xc}$ from above barrier to top-levels of potential.

$$KE = ½ (E_0 /c^2) v^2 = ½ E_0 \theta^2$$

$$d (KE)/ dz = ½ E_0 \, d\theta_{Nms}{}^2/dz$$

Capture length $L_N \approx (U_0 - E_{xc}) / d (KE)/ dz$

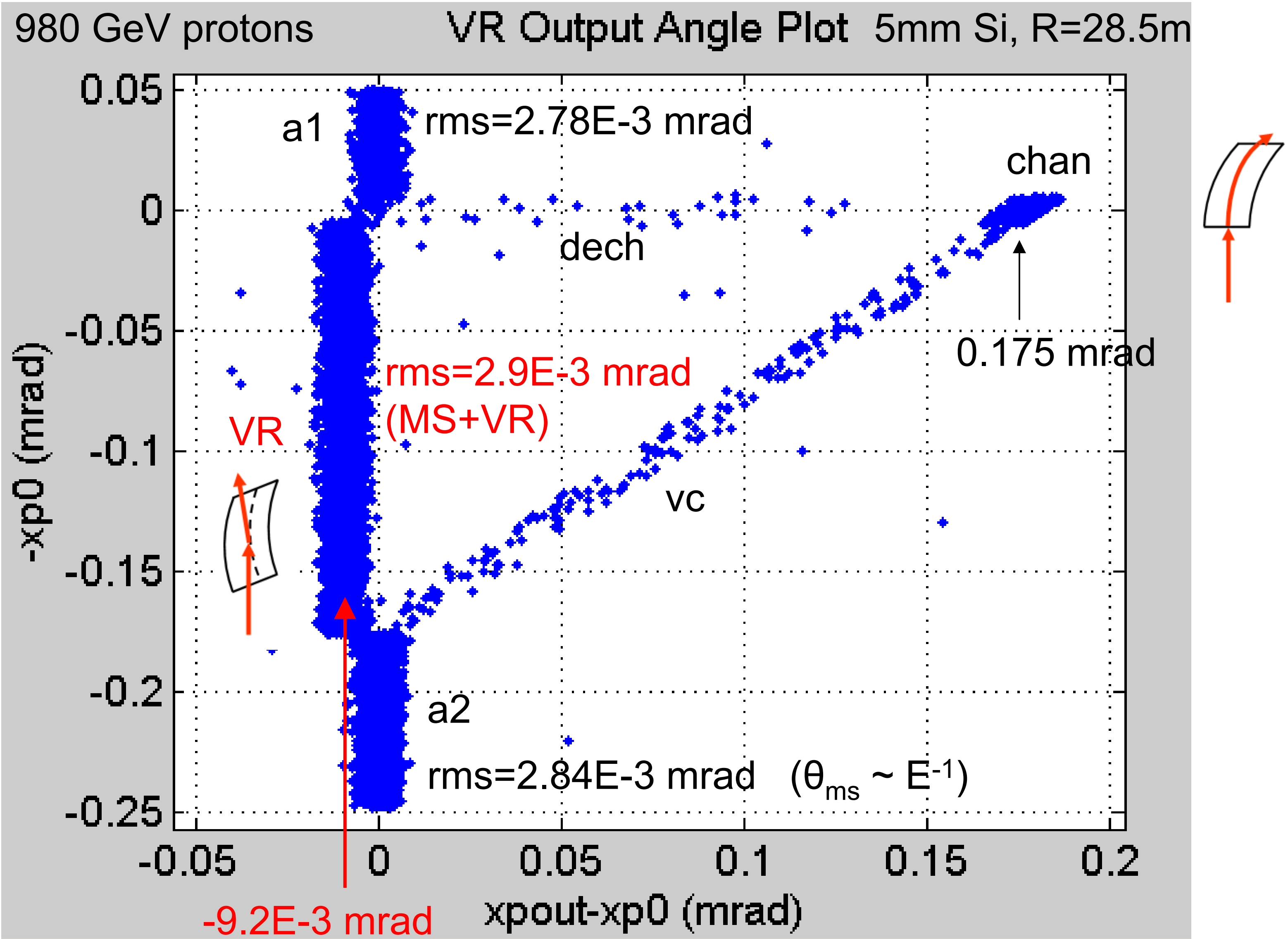
VC probability in curved crystal ≈
Distance that particle angle is within $\theta_{c\,0}$ of plane / Capture length

$$P \approx R\theta_{c\,0} / L_N \quad (P<<1)$$

To insure P<1 for all regimes in code, we replace this by the usual decay rate form

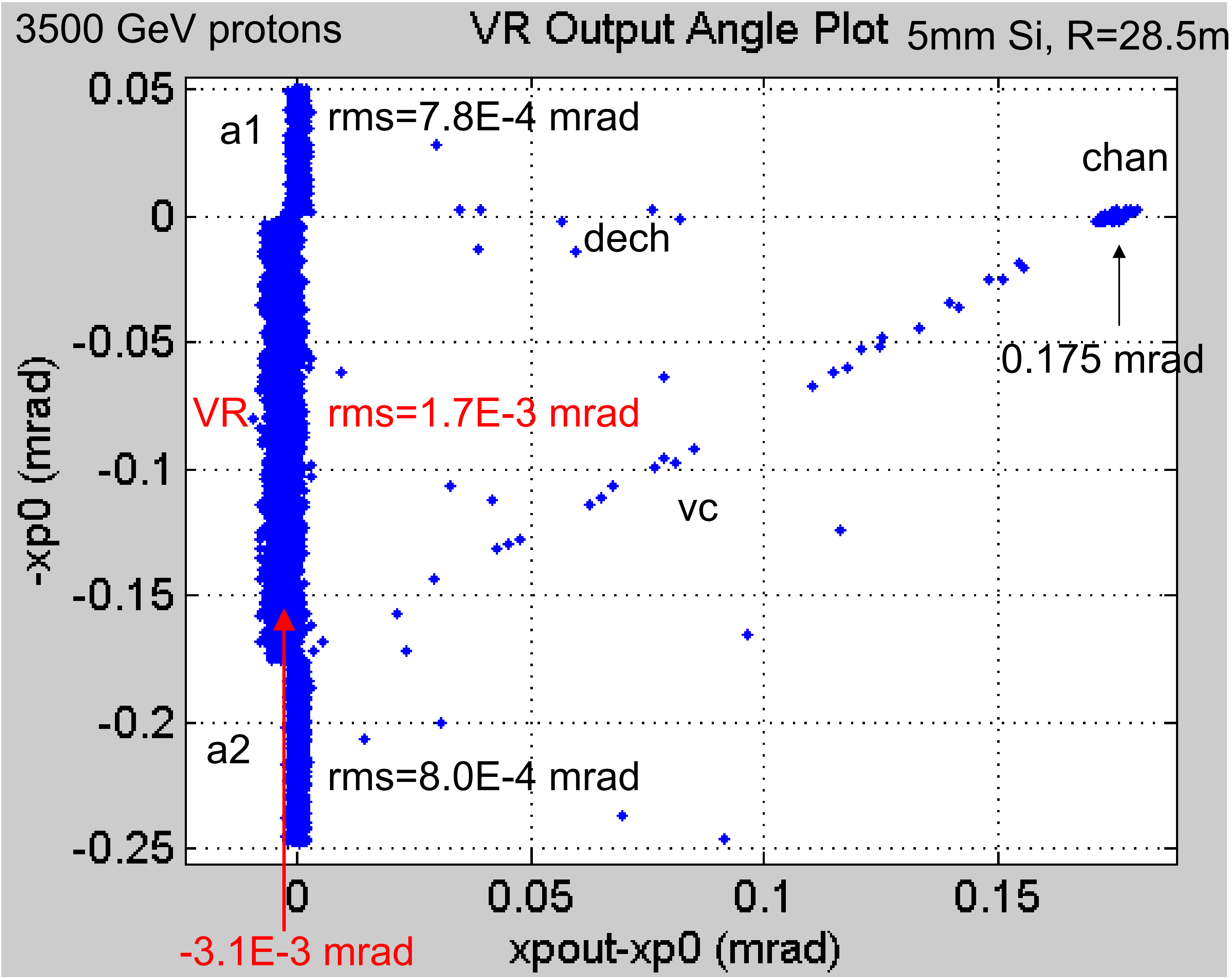
$$P = 1- \exp(- R\theta_{c0} / L_N)$$

* For thin crystal where $s < R\theta_{c\,0}$, include correction $s / (s+ R\theta_{c\,0})$



Code VR: $\theta_{refl} = -1.5\,\theta_{c\,0}\,(1 - 1.67\,(R_{crit}/R)) \sim E^{-1/2}$ $\theta_{c\,0} \sim E^{-1/2}$ $R_{crit} \sim E$ $L_{dech} \sim E$

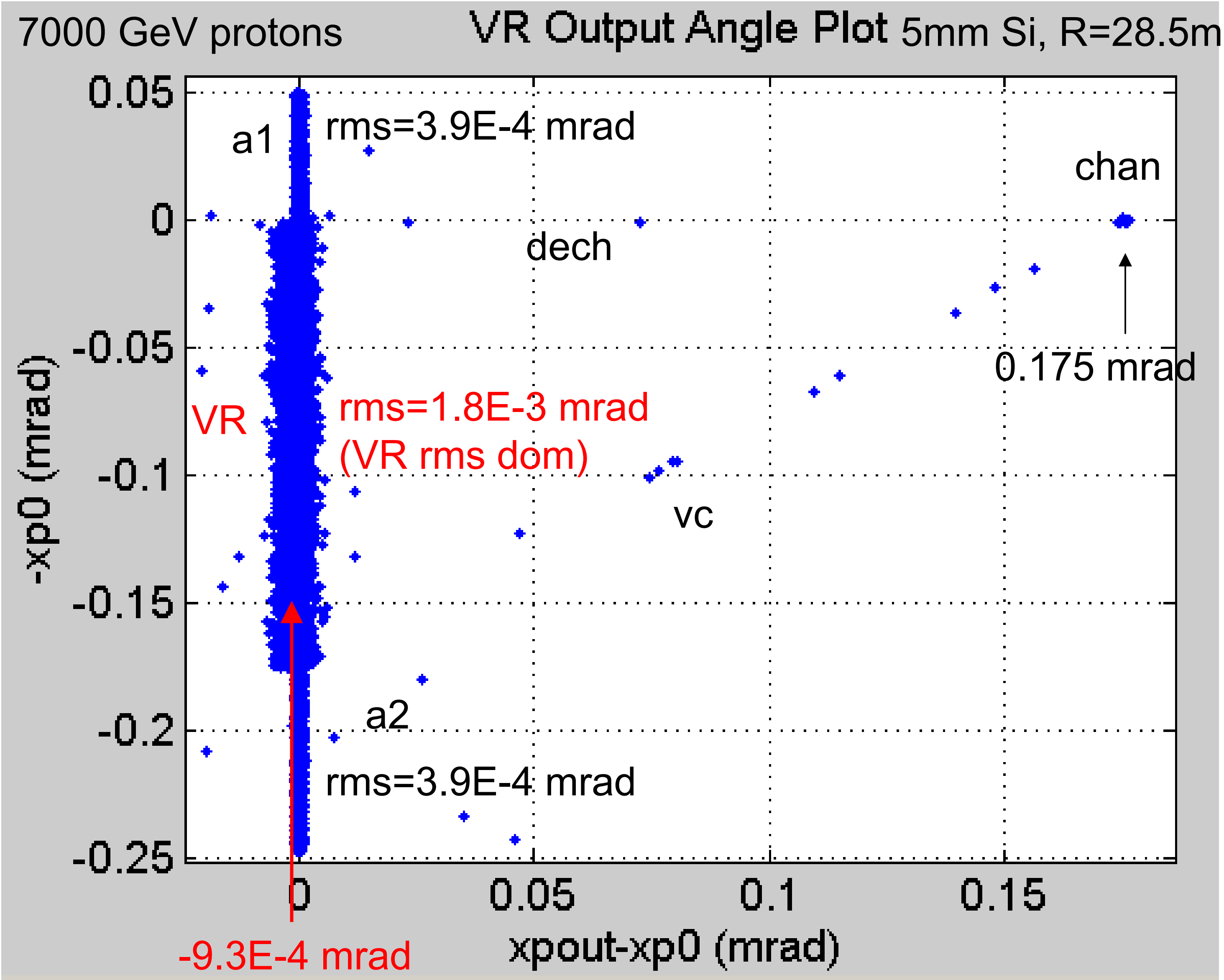
VR rms = $1.7\,\theta_{c\,0}\,R_{crit}/R \sim E^{1/2}$ (MS ~ 1/E) (6.4E-3 mrad) (2 m) (0.49m)



Code VR: $\theta_{refl} = -1.5\ \theta_{c\,0}\,(1 - 1.67\ (R_{crit} / R))$

VR rms = $1.7\ \theta_{c\,0}\, R_{crit} / R$

$\theta_{c\,0} \sim E^{-1/2}$ (3.4E-3 mrad) $R_{crit} \sim E$ (7.3 m) $L_{dech} \sim E$ (1.75m)

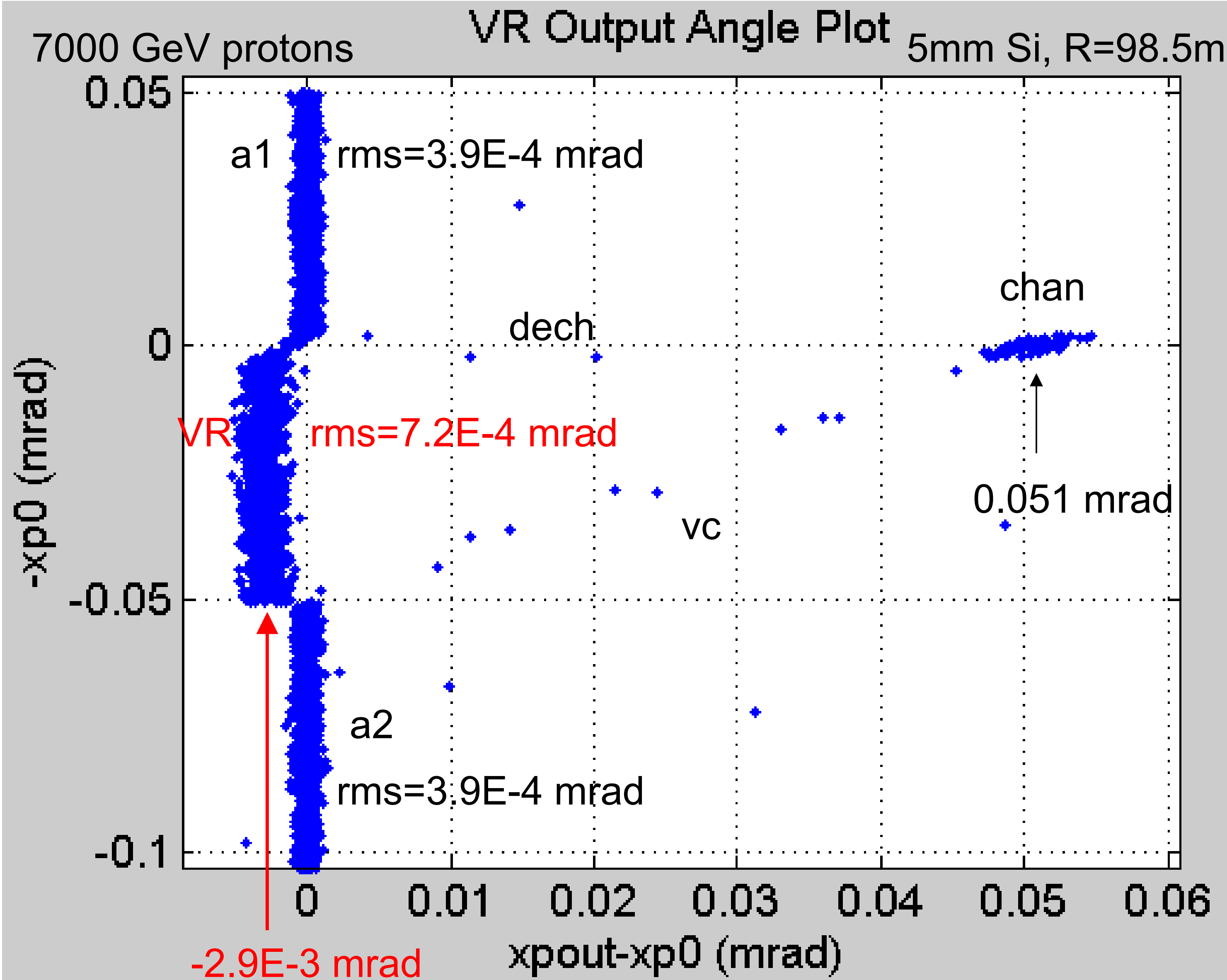


$\theta_{c\,0} \sim E^{-1/2}$ $R_{crit} \sim E$ $L_{dech} \sim E$

(2.4E-3 mrad) (14.6 m) (3.5 m)

Code VR: $\theta_{refl} = -1.5\ \theta_{c\,0}\,(0.1972\ R/R_{crit} - 0.1472)$

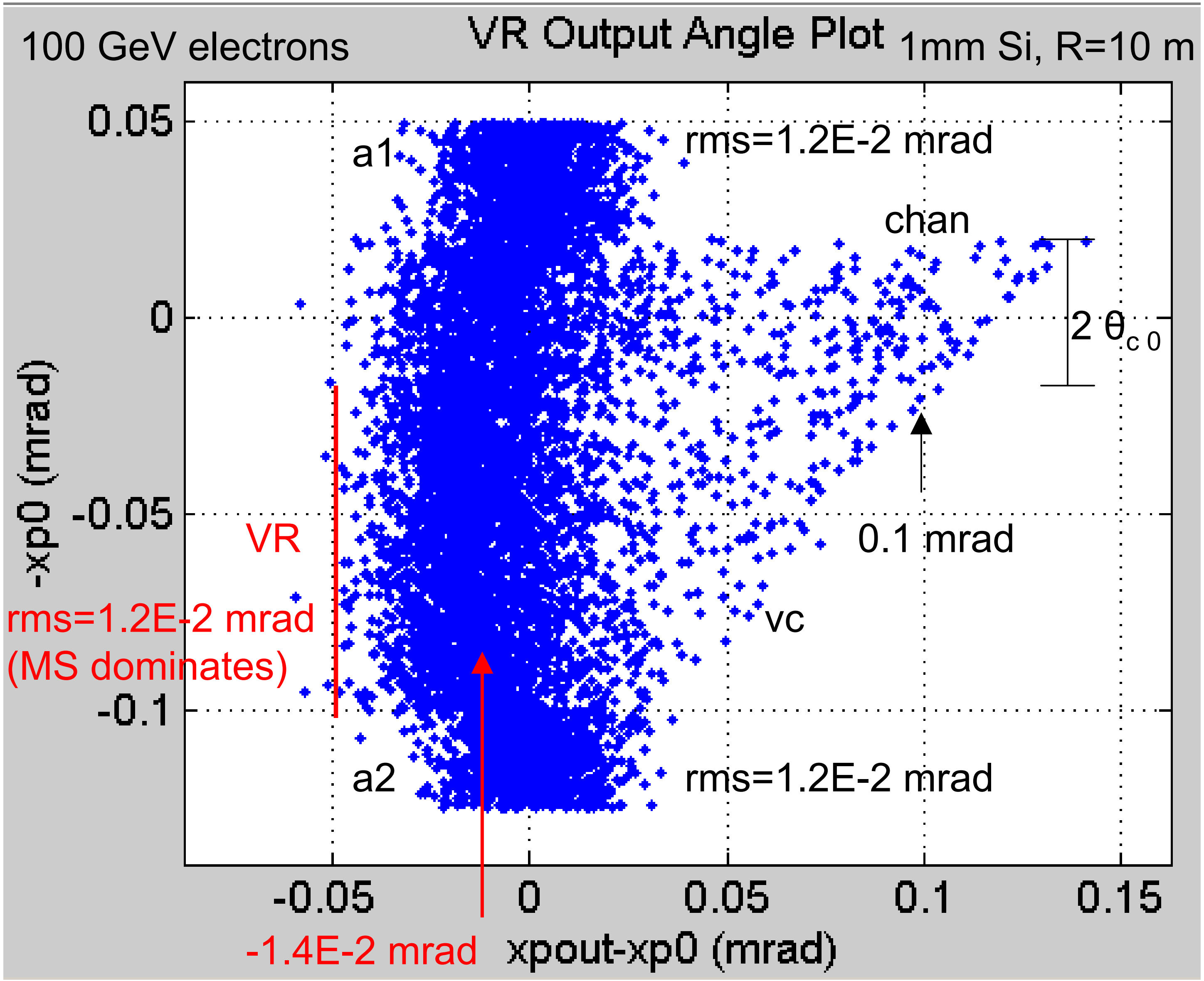
VR rms = $0.714\ \theta_{c\,0}\sin(0.4713\ R/R_{crit} + 0.85)$



Code VR: $\theta_{refl} = -1.5\ \theta_{c\,0} (1 - 1.67\ (R_{crit} / R))$

VR rms = $1.7\ \theta_{c\,0} R_{crit} / R$

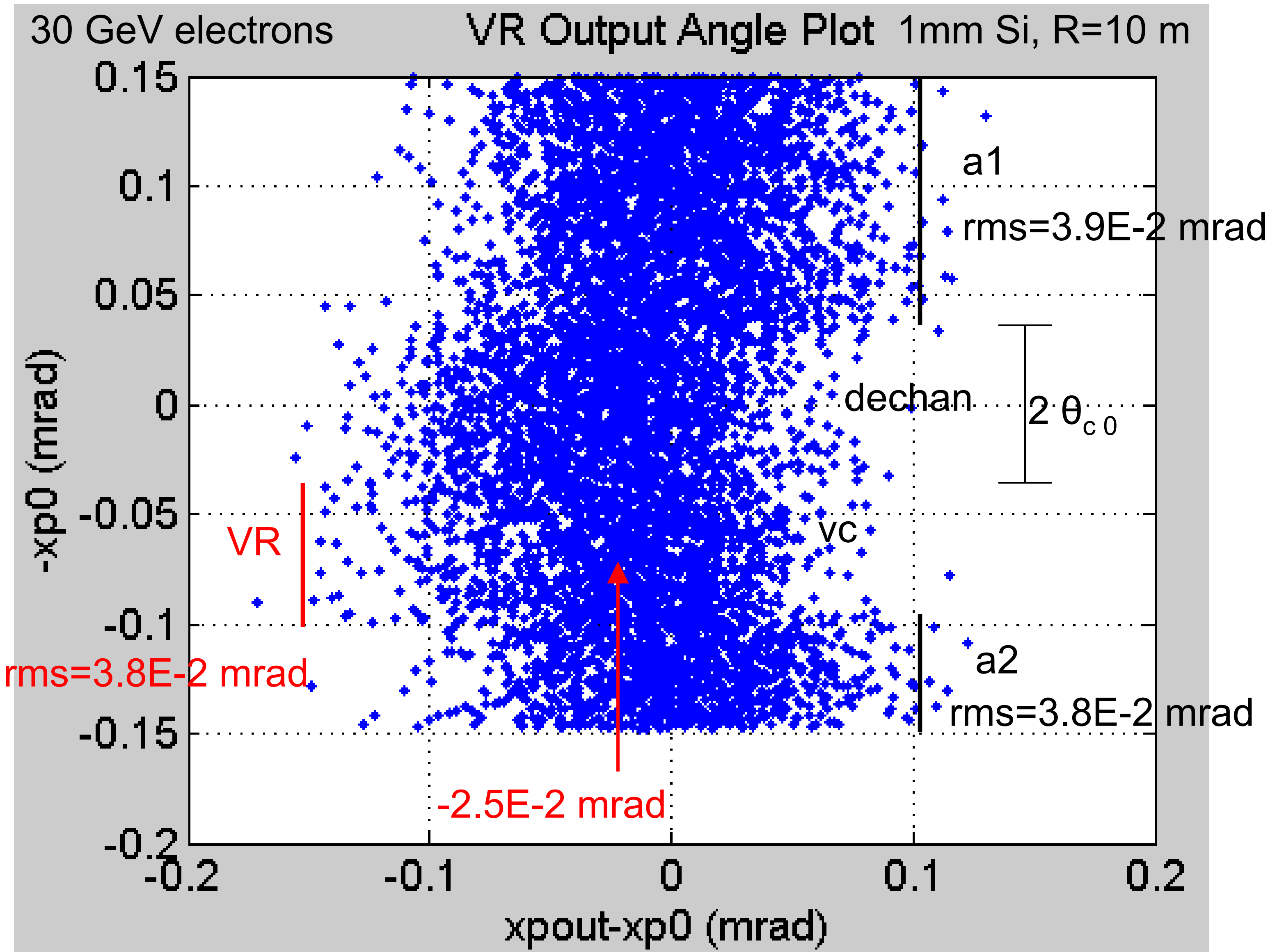
$\theta_{c\,0} \sim E^{-1/2}$ (2.4E-3 mrad) $R_{crit} \sim E$ (14.6 m) $L_{dech} \sim E$ (3.5 m)



Code VR: $\theta_{refl} = -0.8\ \theta_{c\,0}\,(1 - 2.55\,(R_{crit} / R))$

VR rms $= 1.8\ \theta_{c\,0}\,R_{crit} / R$ (negligible)

$\theta_{c\,0} \sim E^{-1/2}$ (0.02 mrad) $R_{crit} \sim E$ (0.21 m) $L_{dech} \sim E$ (3 mm)



$\theta_{c\,0} \sim E^{-1/2}$ $\quad R_{crit} \sim E$ $\quad L_{dech} \sim E$

(0.037 mrad) (0.063 m) (0.9 mm)

Code VR: $\theta_{refl} = -0.8\ \theta_{c\,0}\,(1 - 2.55\ (R_{crit}\,/\,R))$

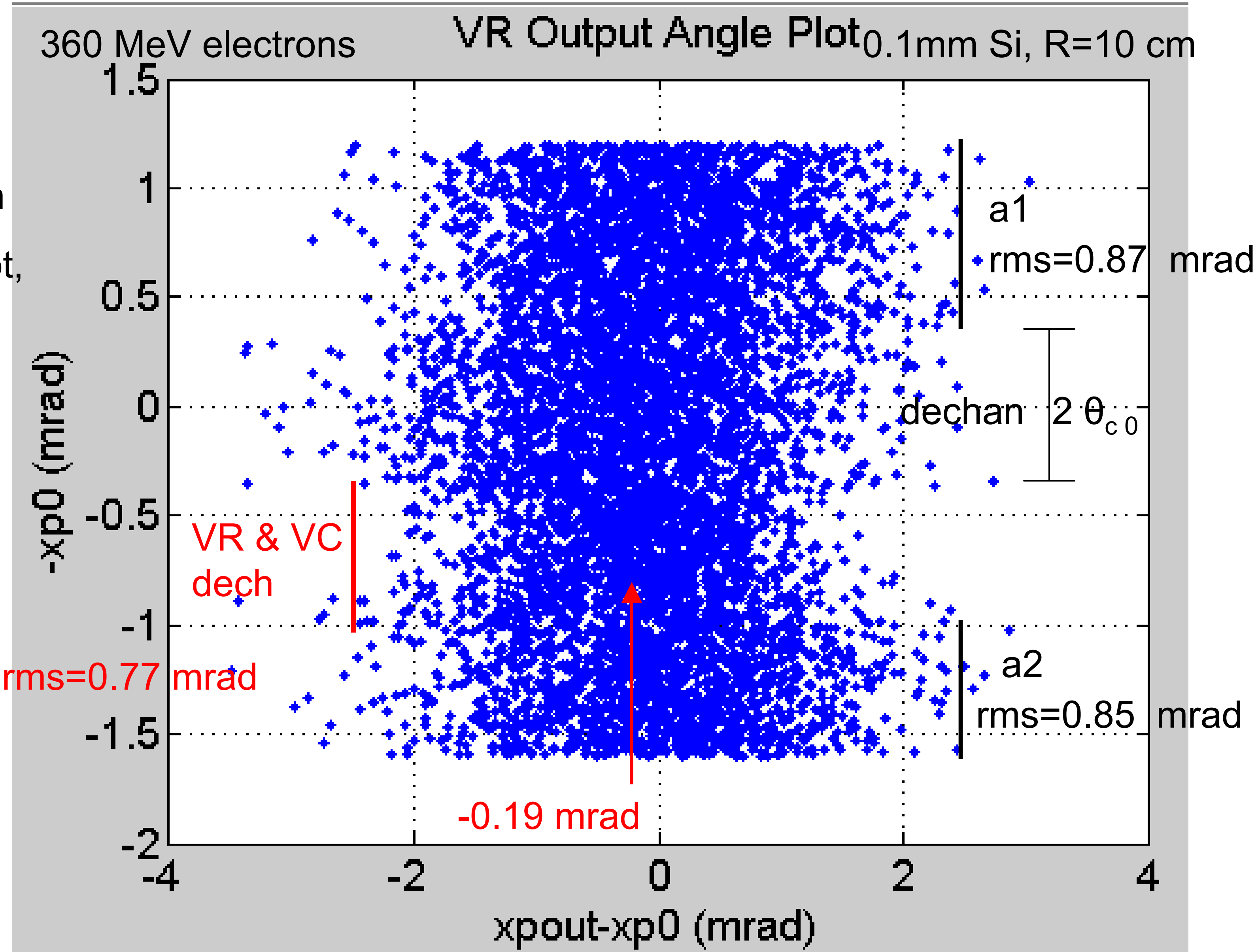
VR rms = $1.8\ \theta_{c\,0}\, R_{crit}\,/\,R$ (negligible)

SLAC
NLCTA

$\varepsilon_n \sim 10\ \mu m$

10 μm spot,

1 mrad



Code VR: $\theta_{refl} = -0.8\ \theta_{c\,0}(1 - 2.55\ (R_{crit}/R))$

VR rms = $1.8\ \theta_{c\,0} R_{crit}/R$ (negligible)

$\theta_{c\,0} \sim E^{-1/2}$	$R_{crit} \sim E$	$L_{dech} \sim E$
(0.34 mrad)	(0.76 mm)	(0.01 mm)

Summary

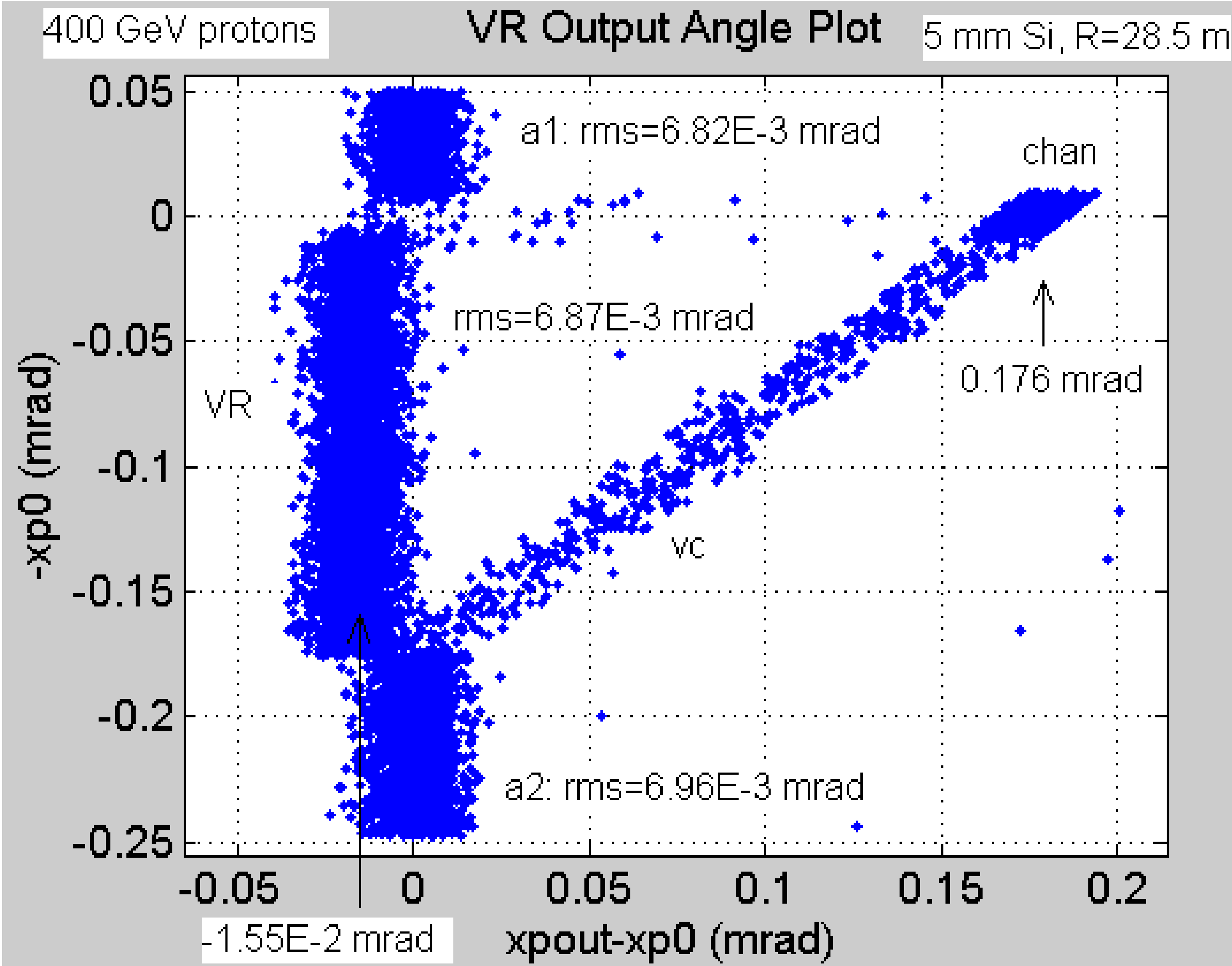
1. We made several improvements to Yazynin's code for VR/VC probabilities, dechanneling of VC particles, multiple scattering, and e- energy loss, which extend applicable range of R, E, crystal thickness for both p and e.

2. The coded formulas for VR angle and rms values still need to be confirmed and generalized (current version from Monte Carlo fits over restricted energy range). Basically the code gives VR ~ $\theta_{c\,0}$ (with R_{crit} / R correction) , VR rms ~ $\theta_{c\,0}\, R_{crit} / R$, θ_{MS}~1/E.

3. At LHC energies ($\theta_{c\,0}$ ~2 μrad), crystal curvature radius needs to be many tens of meters to reduce VR rms spread < μrad and obtain good VR angular separation (~ 2-3 μrad). This reduces VR angular acceptance (~thickness/R), but it is still tens of micro-radians.

4. For an e- experiment at 360 MeV, MS (~ 1/E) dominates the rms spreads, and the VR deflection tends to get lost in this spread, even for 100 micron thick Si. De-channeled particles tend to escape in same direction as VR deflection, adding to the apparent population of "deflected" particles. VR deflection ~ 0.2 mrad which is slightly less than $\theta_{c\,0}$, and rms spread is nearly 4 times this.

Extra slides



Code VR: $\theta_{refl} = 1.5\ \theta_{c0}\,(1 - 1.67\,(R_{crit}/R))$

$\theta_{c0} \sim E^{-1/2}$ (0.01 mrad) $R_{crit} \sim E$ (0.835m) $L_{dech} \sim E$ (0.2m)

Profile plots

Phase space plots