Dark matter direct detection from 1 to ∞ phonons

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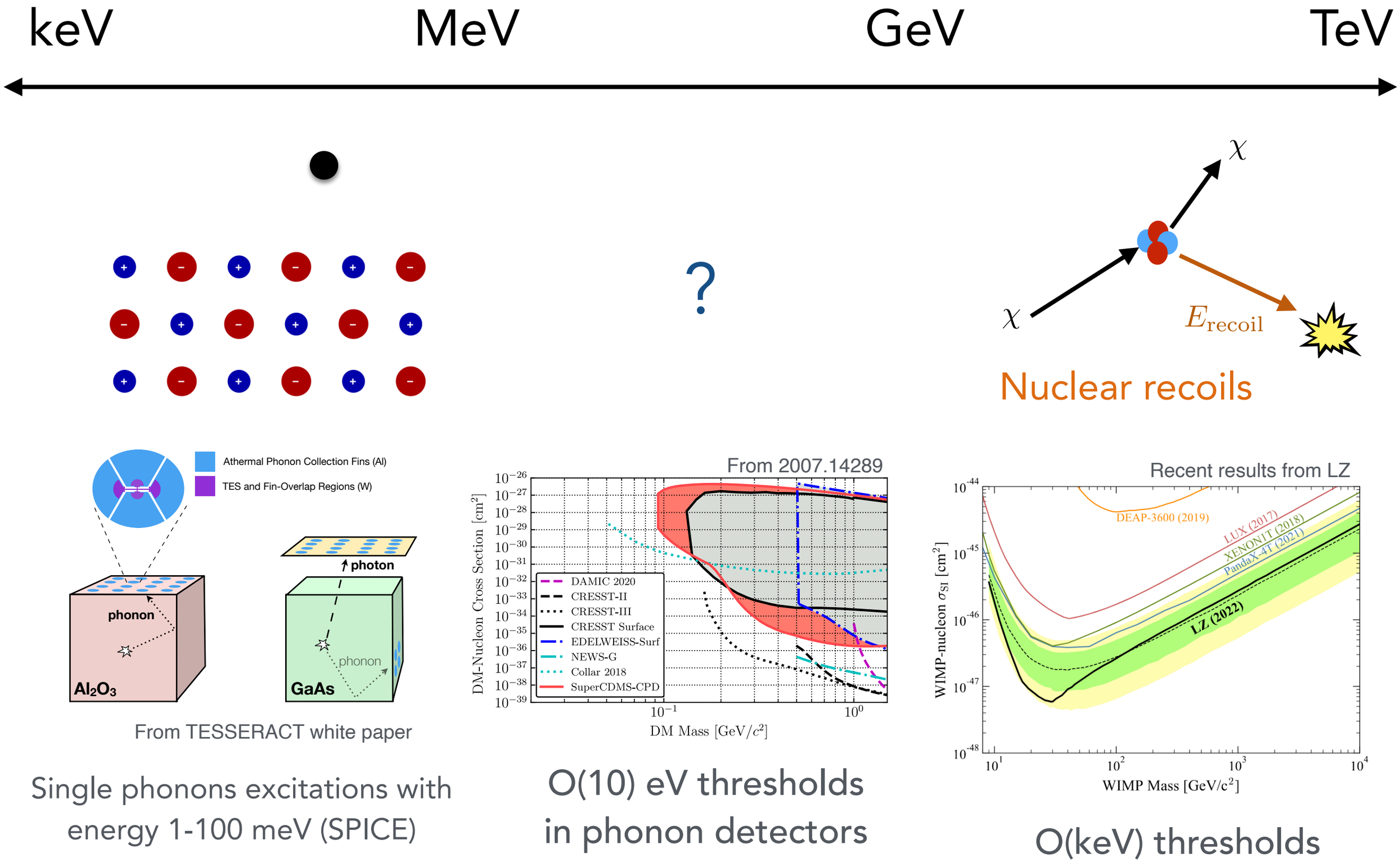
"Summiting the Unknown" workshop

Dark matter mass

keV MeV GeV TeV



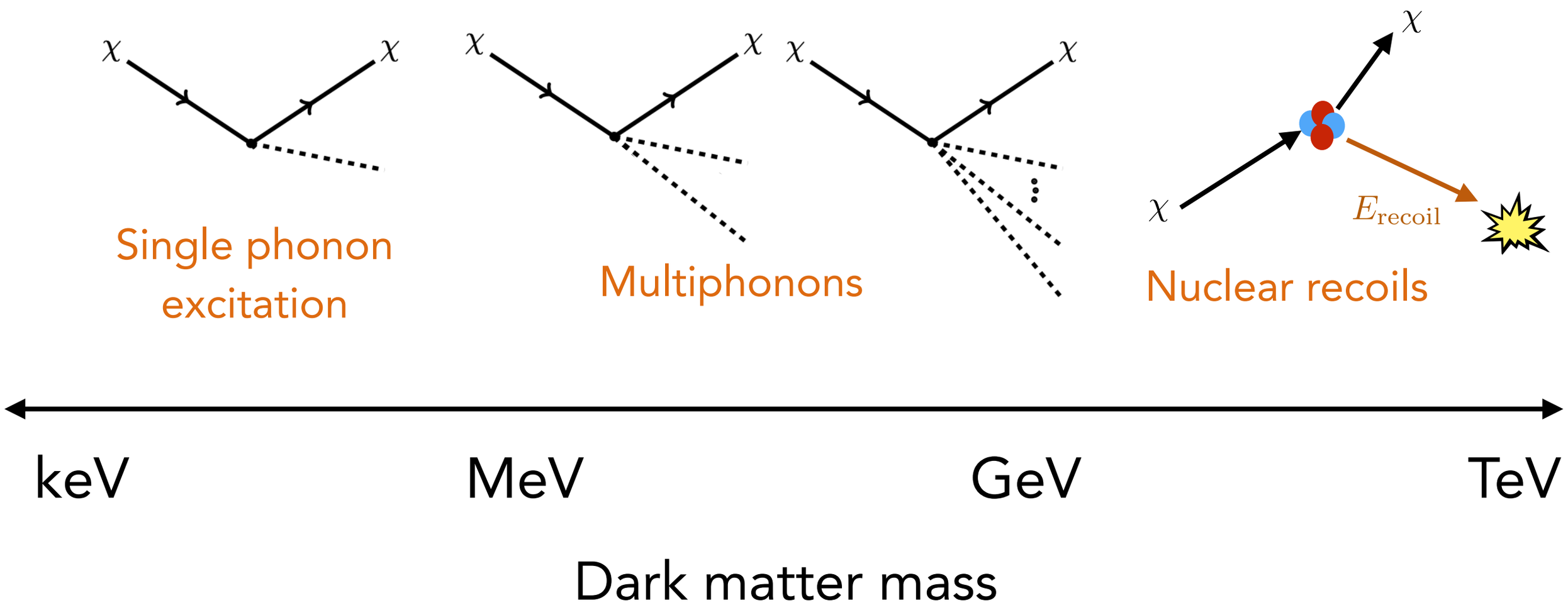
Single phonons excitations with energy 1-100 meV (SPICE)

O(10) eV thresholds in phonon detectors

O(keV) thresholds

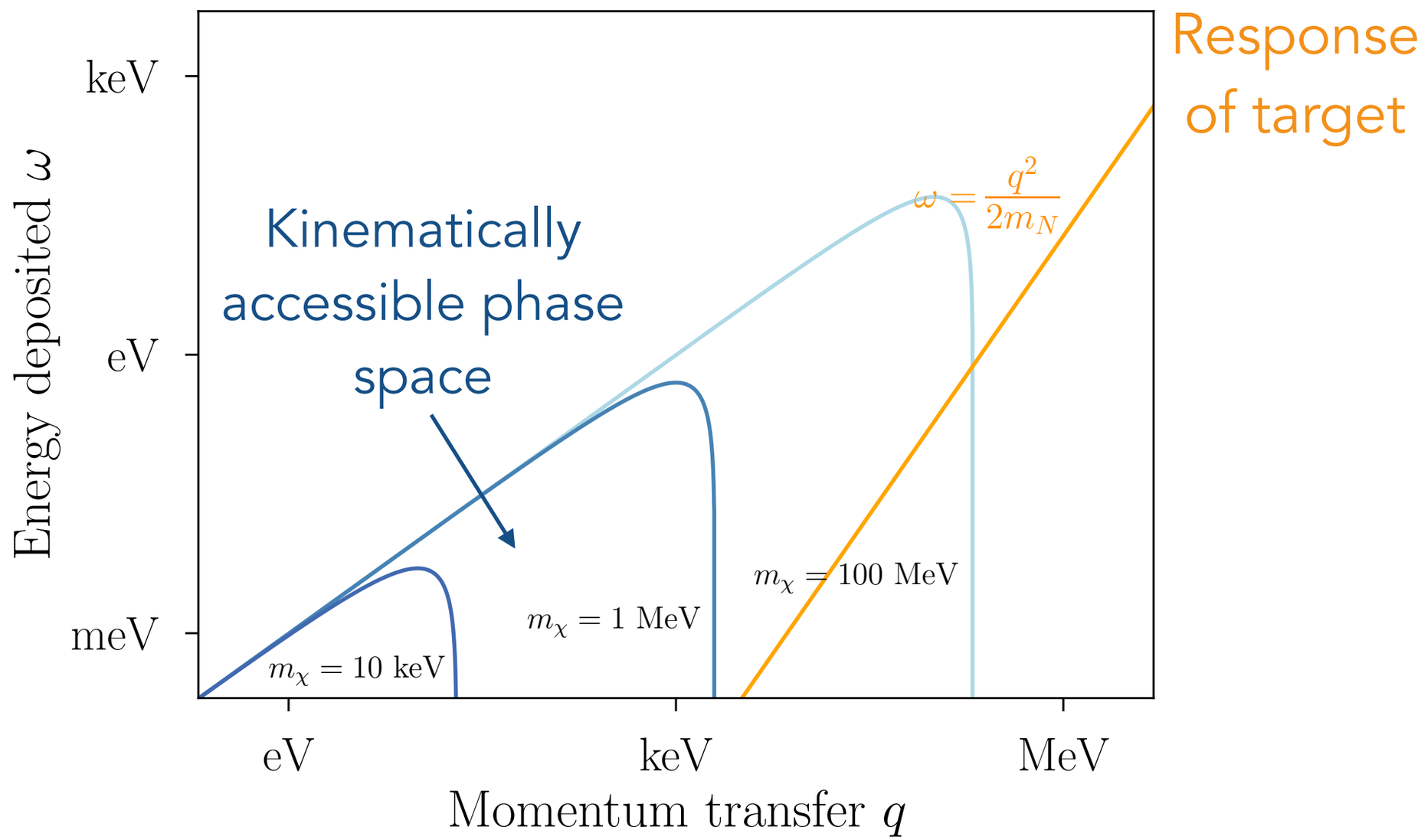
DM-nucleus scattering in crystals

(A similar picture is expected for superfluid He)



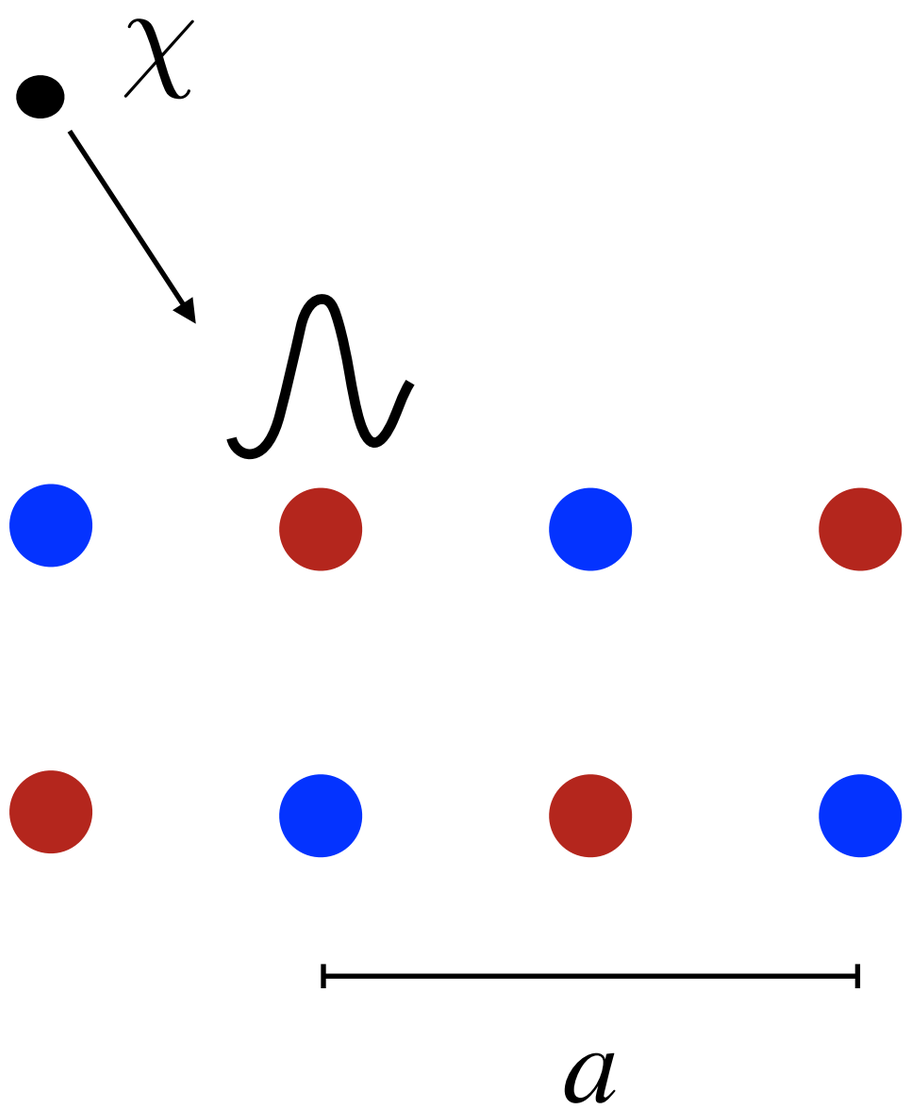
Brian Campbell-Deem, Knapen, TL, **Ethan Villarama** 2205.02250
Campbell-Deem, Cox, Knapen, TL, Melia 1911.03482
Knapen, Kozaczuk, TL 2011.09496

DM scattering off free nuclei



Free nucleus approximation is no longer valid at low q, ω

What does DM-nucleus scattering look like in a crystal?



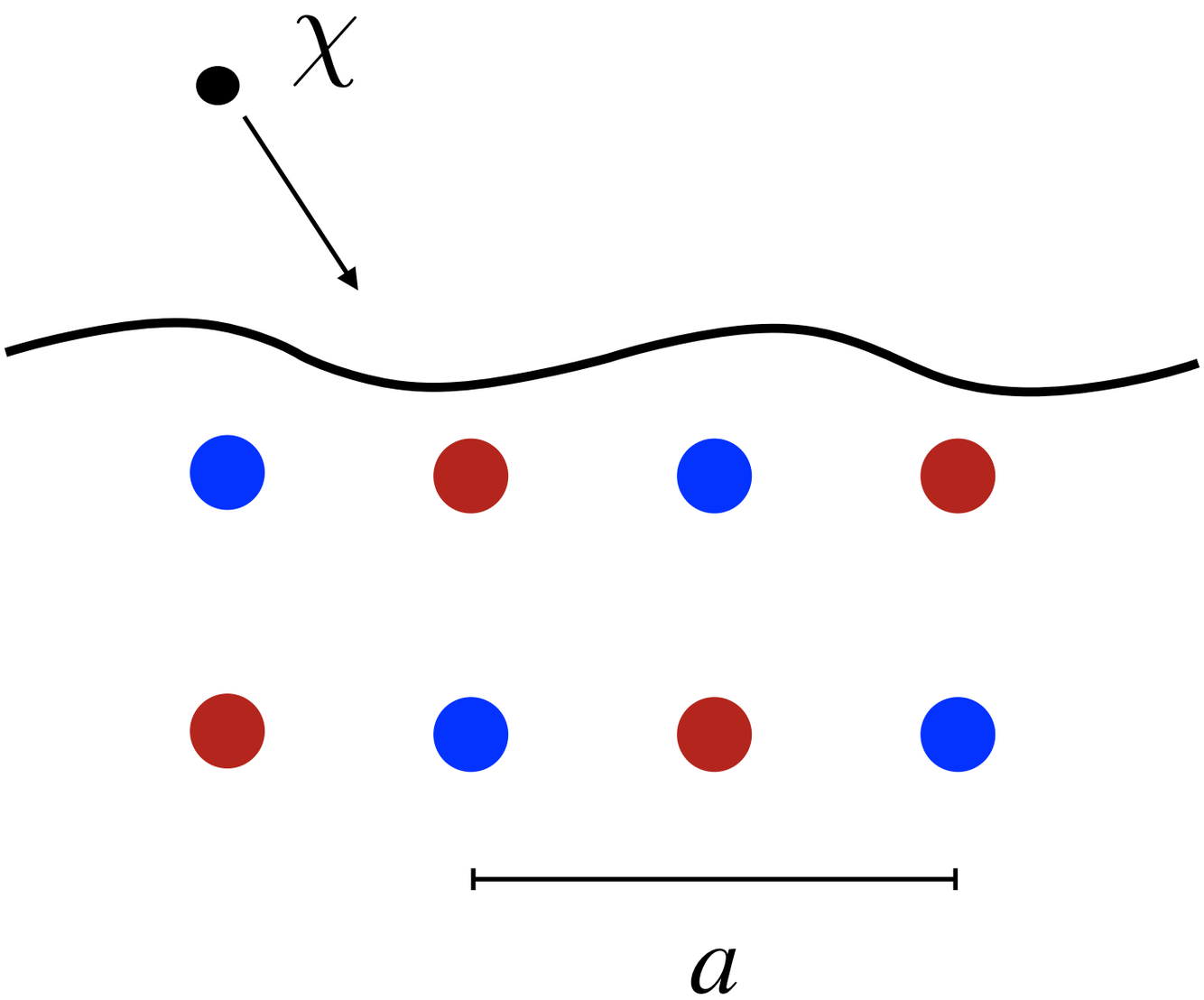
When momentum transfer

$$q \gg q_{\rm BZ} = \frac{2\pi}{a} \sim \text{few keV}$$

and $\omega \gg \bar{\omega}_{\rm phonon} \sim$ 10-100 meV

DM scatters off an individual nucleus

What does DM-nucleus scattering look like in a crystal?



When momentum transfer

$$q \ll q_{\rm BZ} = \frac{2\pi}{a}$$

and $\omega \sim \bar{\omega}_{\rm phonon}$

DM excites collective

excitations = phonons

DM scattering rate

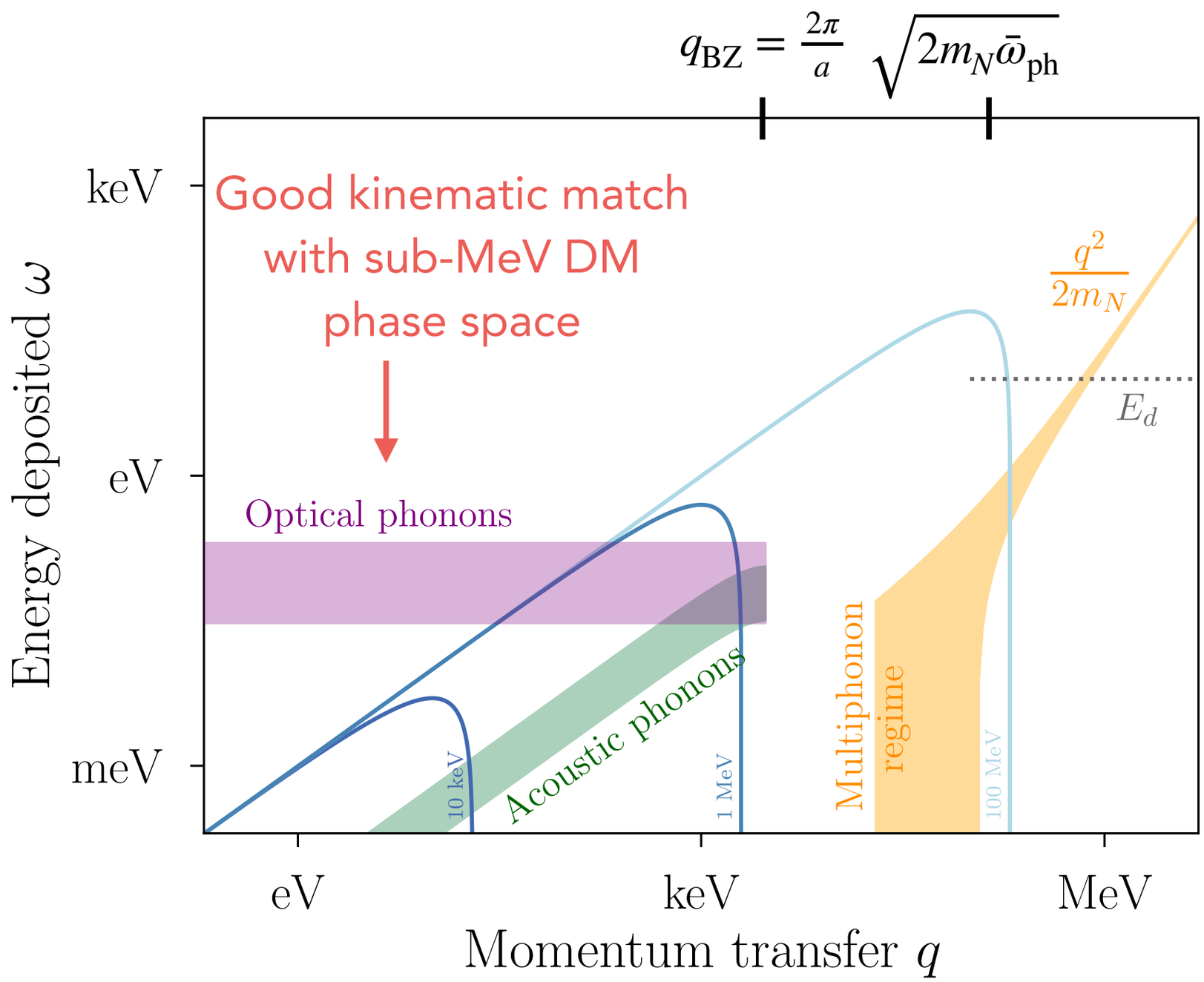
$$\frac{d\sigma}{d^3\mathbf{q}\,d\omega} \propto \sigma_{\chi p}\,\overbrace{|\tilde{F}_{\rm med}(q)|^2}^{\text{DM-mediator form factor}}\,\underbrace{S(\mathbf{q},\omega)}_{\text{Dynamic structure factor captures response of target}}\,\delta(\omega - \mathbf{q}\cdot\mathbf{v} + \frac{q^2}{2m_\chi})$$

For free nuclei and spin-independent interactions:

$$S(\mathbf{q},\omega) \propto A_N^2\,\delta\left(\omega - \frac{q^2}{2m_N}\right)$$

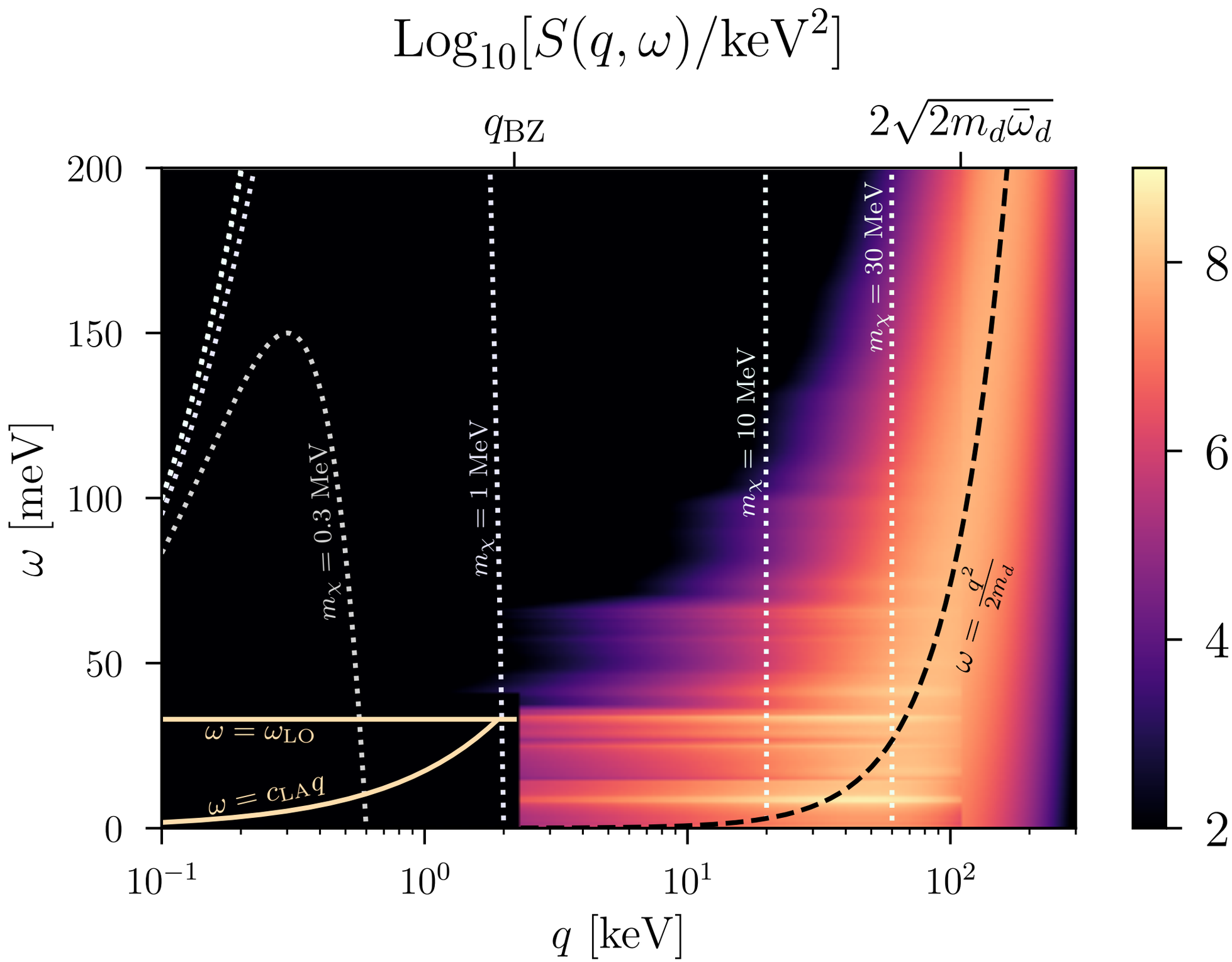
Goal: understand $S(\mathbf{q},\omega)$ from the single phonon to the nuclear recoil regime

DM-nucleus scattering in a crystal



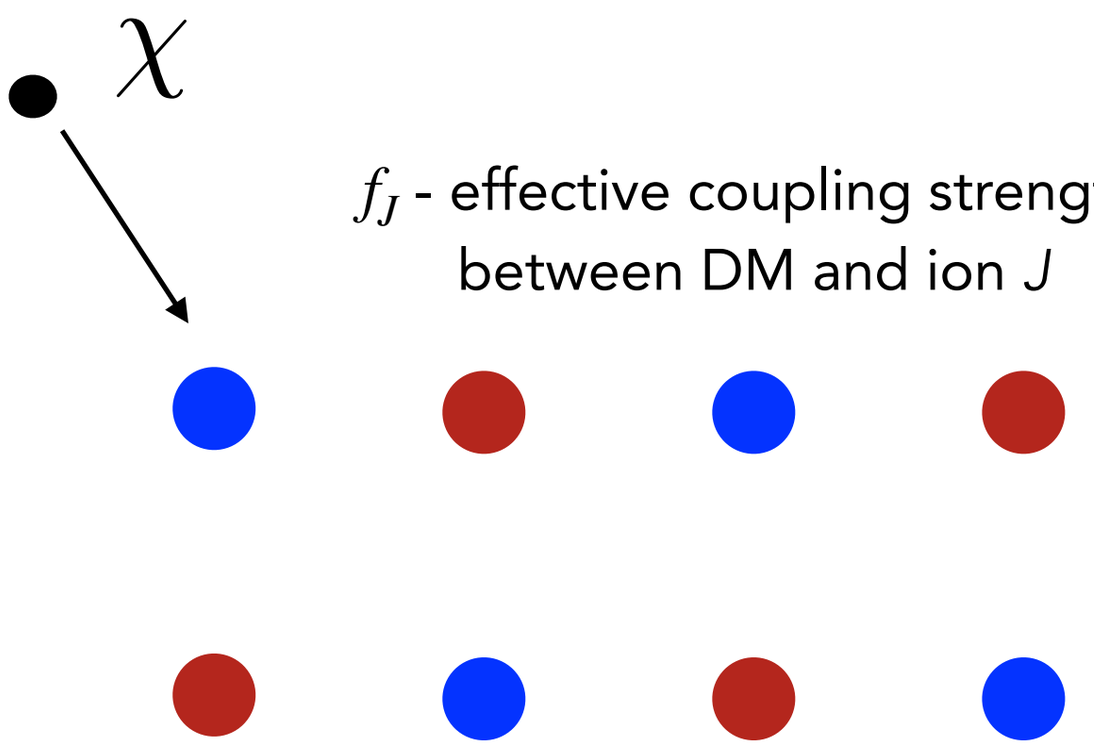
Kahn, TL 2108.03239
Trickle, Zhang, Zurek, Inzani, Griffin 1910.08092

Structure factor for GaAs



Campbell-Deem, Knapen, TL, Villarama 2205.02250

DM-nucleus interaction



f_J - effective coupling strength between DM and ion J

Short range SI interaction

$$\sigma_{\chi p} = 4\pi b_p^2$$

Scattering potential in Fourier space

$$V(\mathbf{q}) \propto b_p \sum_J f_J \, e^{i\mathbf{q}\cdot\mathbf{r}_J}$$

$$S(\mathbf{q},\omega) \equiv \frac{2\pi}{V} \sum_f \left| \sum_J \langle \Phi_f | f_J e^{i\mathbf{q}\cdot\mathbf{r}_J} | 0 \rangle \right|^2 \delta\left(E_f - \omega\right)$$

$$= \frac{1}{V} \sum_{J,\,J'}^{N} f_J f_{J'}^* \int_{-\infty}^{\infty} dt \, \langle e^{-i\mathbf{q}\cdot\mathbf{r}_{J'}(0)} e^{i\mathbf{q}\cdot\mathbf{r}_J(t)} \rangle e^{-i\omega t}$$

Contains interference terms between different atoms → single phonon excitations

Theory of neutron scattering: Squires 1996, Schober 2014

Dynamic structure factor

Phonon comes into play through positions of ions:

$$\mathbf{r}_J(t) = \mathbf{r}_J^0 + \mathbf{u}_J(t)$$

Quantized displacement field $\mathbf{u}_J(t) \sim \sum_{\mathbf{q}} \frac{1}{\sqrt{2m_N\omega_{\mathbf{q}}}} \left(\hat{a}_{\mathbf{q}}^\dagger \mathbf{e}_{\mathbf{q}} e^{-i\mathbf{q}\cdot\mathbf{r}_J + i\omega_{\mathbf{q}}t} + \text{h.c.}\right)$

Phonon dispersions $\omega_{\mathbf{q}}$ and eigenvectors $\mathbf{e}_{\mathbf{q}}$ calculated by first-principles approaches (density functional theory)

Single phonon contribution has been studied extensively in literature

$$S^{n=1}(\mathbf{q},\omega) \sim \sum_{J,J'} f_J f_{J'} \int dt \, \langle \mathbf{q}\cdot\mathbf{u}_J(0) \; \mathbf{q}\cdot\mathbf{u}_{J'}(t)\rangle e^{-i\omega t}$$

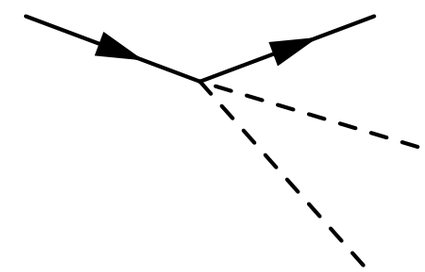
Griffin, Knapen, TL, Zurek 1807.10291; Griffin, Inzani, Trickle, Zhang, Zurek 1910.10716
Griffin, Hochberg, Inzani, Kurinsky, TL, Yu 2020; Coskuner, Tickle, Zhang, Zurek 2102.09567

Dynamic structure factor

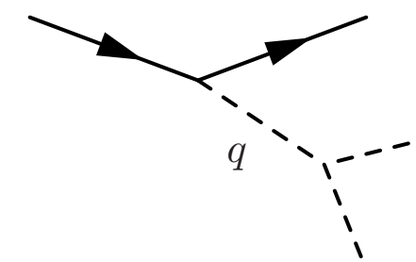
Expansion in $q^2/(M_N\omega)$ (and anharmonic interactions):

$$S(\mathbf{q},\omega) = \ (\text{0-phonon}) + (\text{1-phonon}) + (\text{2-phonon}) + \cdots$$

Harmonic



Anharmonic



Quickly becomes more complicated to evaluate for more than 1 phonon

Our approach: use harmonic & incoherent approximations

Incoherent approximation for $q > q_{\mathrm{BZ}}$ or $n > 1$ phonons

Neglect interference terms entirely:

$$S(\mathbf{q},\omega) \approx \frac{1}{V}\sum_{J}^{N}(f_J)^2 \int_{-\infty}^{\infty} dt\, \langle e^{-i\mathbf{q}\cdot\mathbf{u}_J(0)} e^{i\mathbf{q}\cdot\mathbf{u}_J(t)}\rangle e^{-i\omega t}$$

Given in terms of auto-correlation function

Motivation for $q > q_{\mathrm{BZ}}$: scatter off individual nuclei at large q

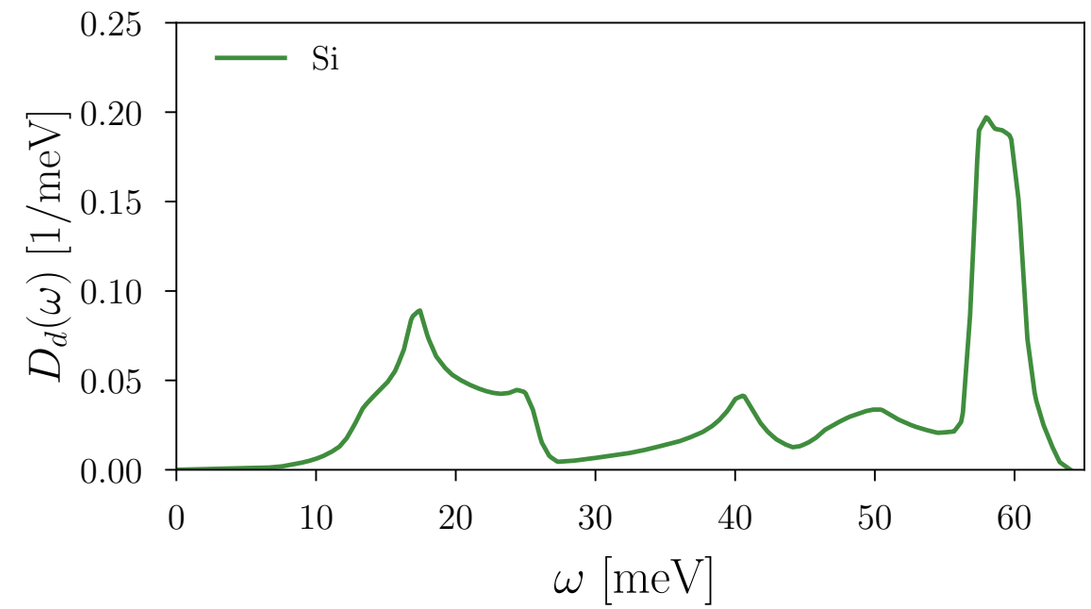
Motivation for $n > 1$: momentum gets distributed over multiple phonons, and the motions of individual atoms will be less correlated.

Auto-correlation can be approximated using the phonon density of states

$$\mathbf{u}_J(t) \sim \sum_{\mathbf{q}} \frac{1}{\sqrt{2m_N\omega_{\mathbf{q}}}} \left(\hat{a}^\dagger_{\mathbf{q}}\mathbf{e}_{\mathbf{q}} e^{-i\mathbf{q}\cdot\mathbf{r}_J + i\omega_{\mathbf{q}}t} + \text{h.c.}\right)$$

$$\langle \mathbf{q}\cdot\mathbf{u}_J(0)\mathbf{q}\cdot\mathbf{u}_J(t)\rangle \approx \frac{q^2}{2m_N}\int d\omega' \frac{D(\omega')}{\omega'} e^{i\omega' t}$$

In the harmonic, isotropic limit

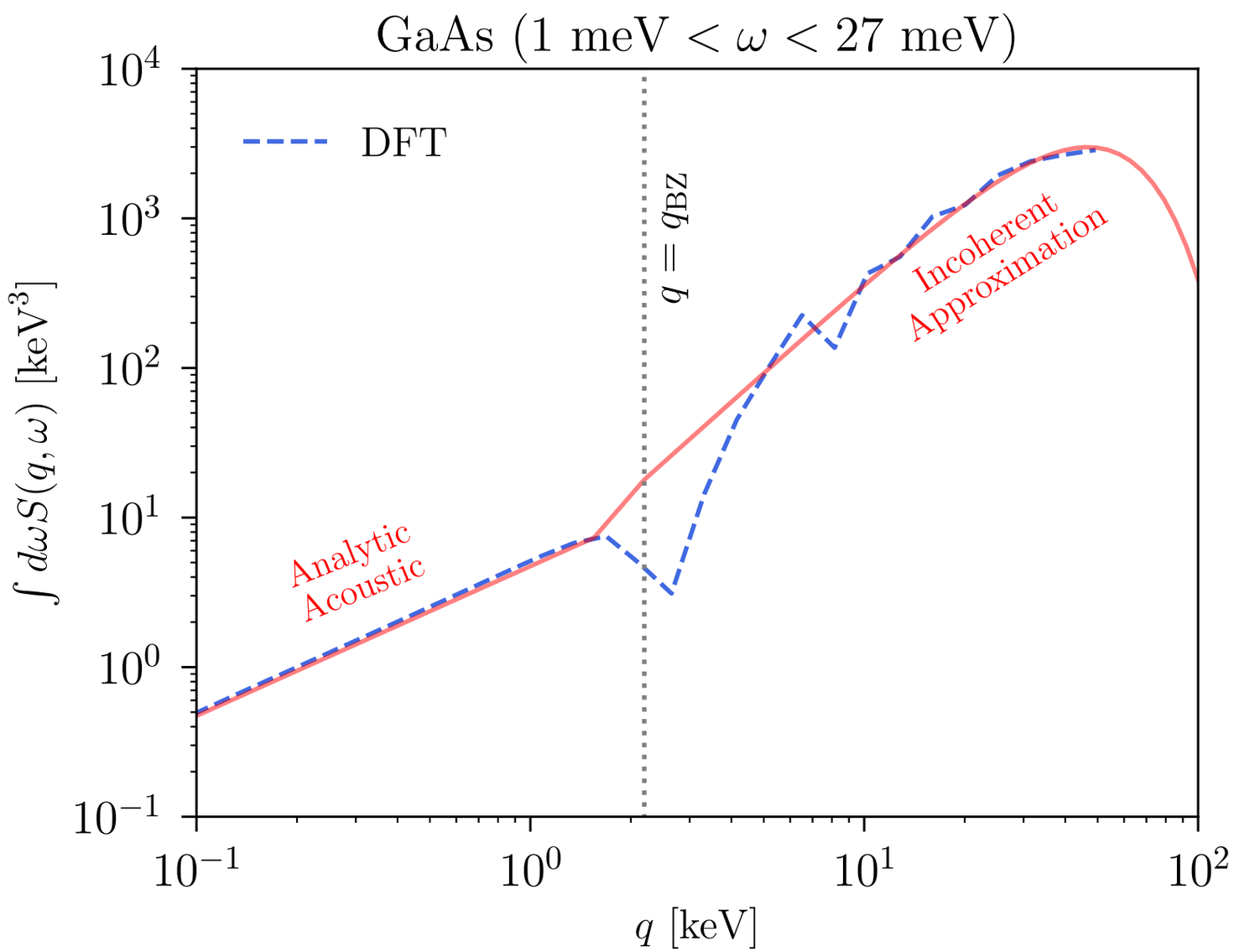


Dynamic structure factor with incoherent approximation:

$$S(q,\omega) \propto \sum_J e^{-2W_J(q)}(f_J)^2 \sum_n \frac{1}{n!}\underbrace{\left(\frac{q^2}{2m_N}\right)^n \left(\prod_{i=1}^n \int d\omega_i \frac{D(\omega_i)}{\omega_i}\right)}_{\sim \left(\frac{q^2}{2m_N\bar{\omega}_{\text{ph}}}\right)^n} \delta\left(\sum_j \omega_j - \omega\right)$$

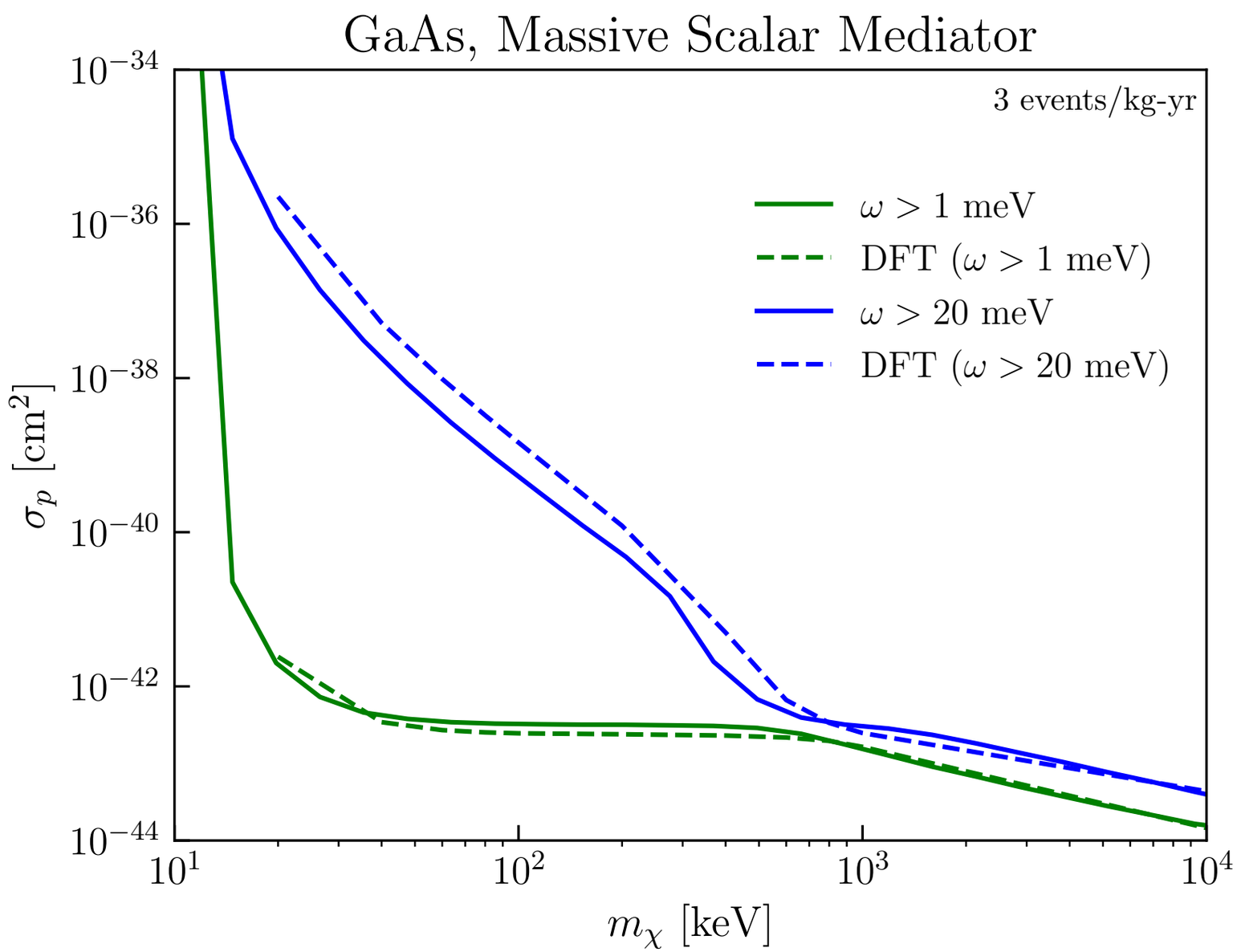
$q \approx \sqrt{2m_N\bar{\omega}_{\text{ph}}}$ for many phonons to contribute

Comparison with full (DFT) calculation for n=1 phonon

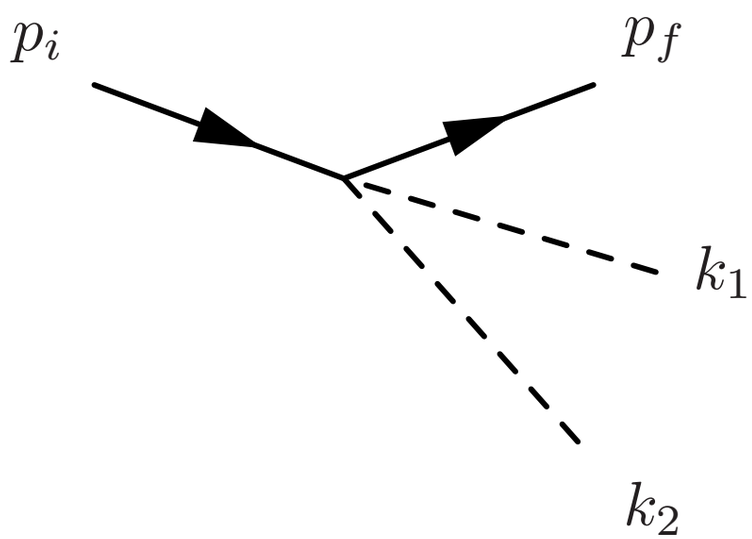


Incoherent approximation captures integrated structure factor

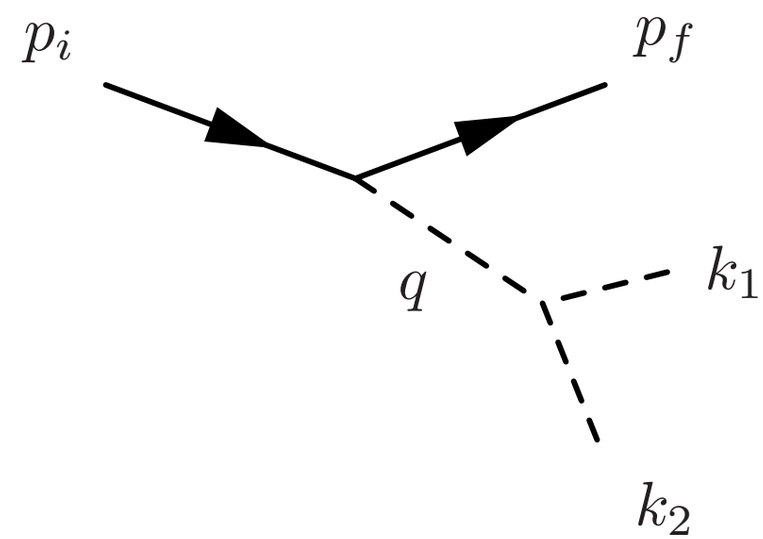
Comparison with full (DFT) calculation for n=1 phonon



2 phonons



Harmonic



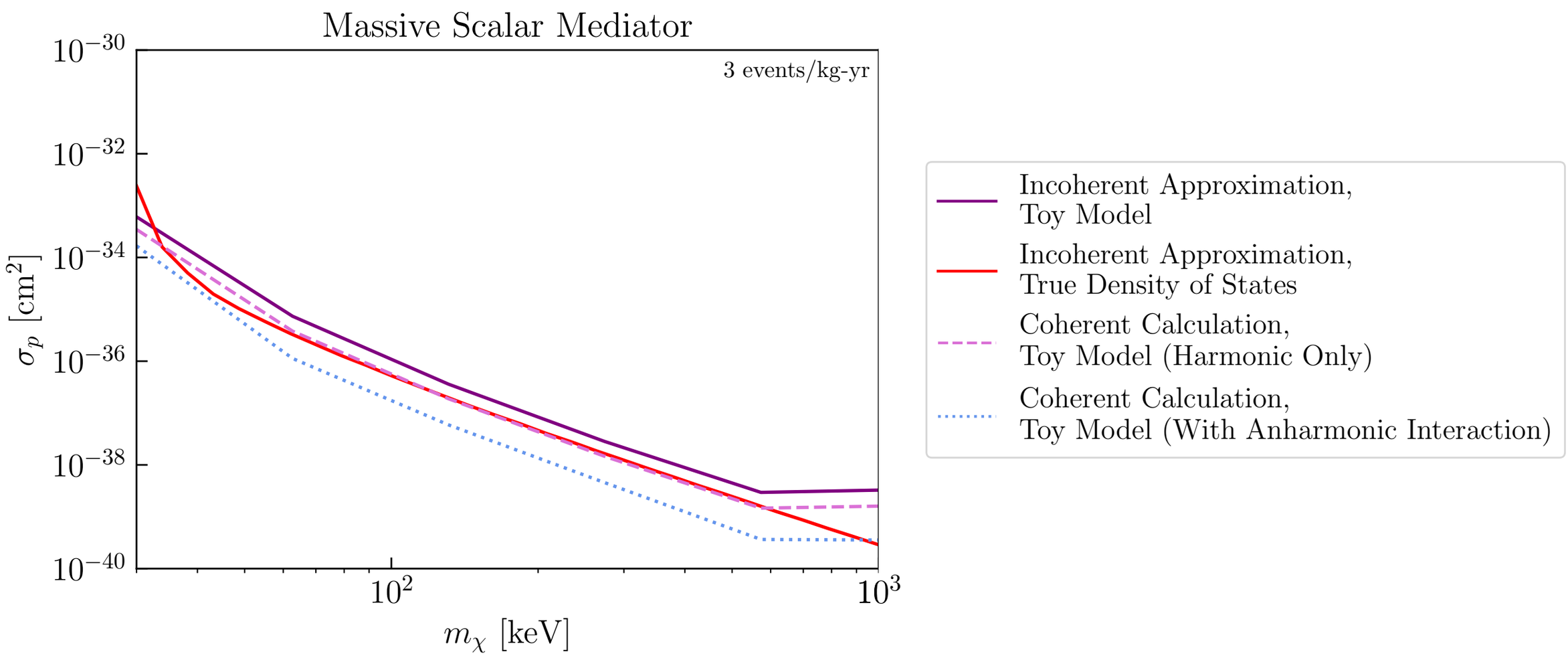
Anharmonic

Calculated in long-wavelength ($q \ll q_{\rm BZ}$) limit in crystals

Campbell-Deem, Cox, Knapen, TL, Melia 1911.03482

Calculated in superfluid He:
Schutz and Zurek 1604.08206
Knapen, TL, Zurek 1611.06228
Acanfora, Esposito, Polosa 1902.02361

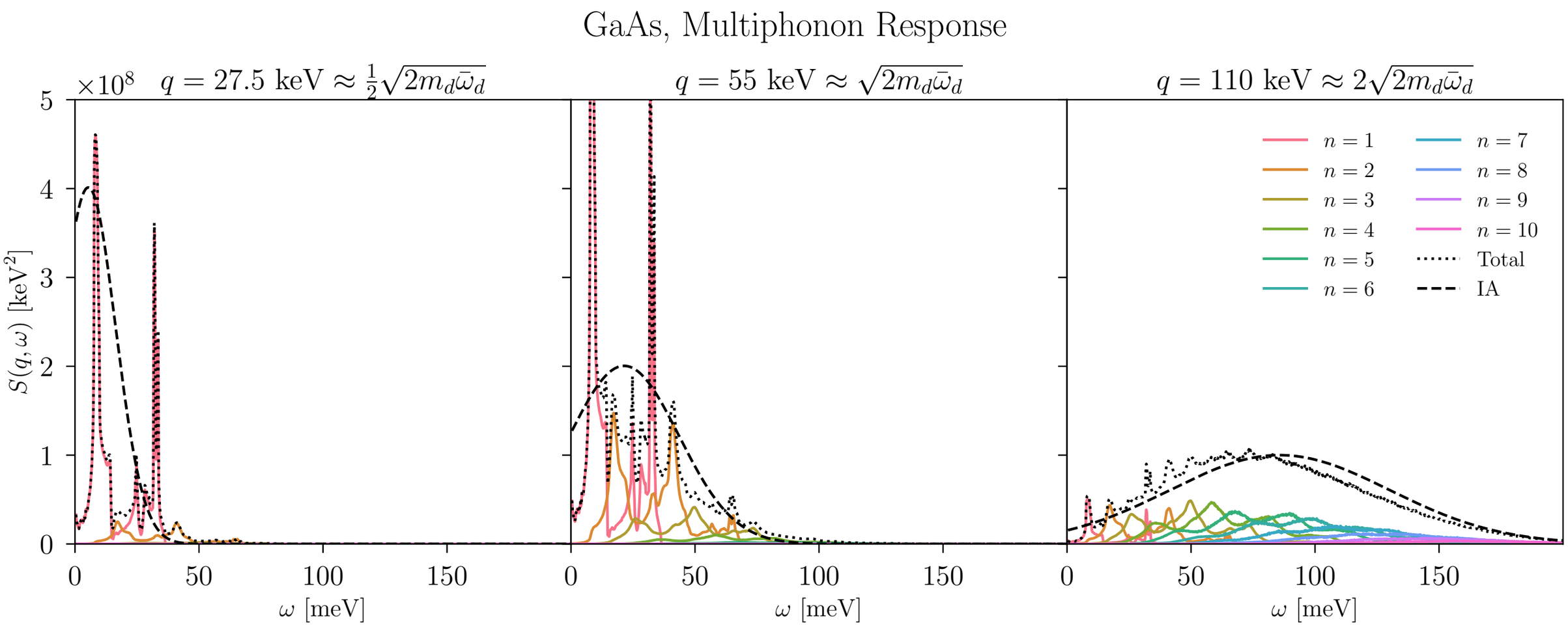
GaAs 2-phonon, ($\omega > 40$ meV)



Incoherent approximation works to within a factor of few for $q < q_{\mathrm{BZ}}$, comparing to harmonic result. Anharmonic interactions give another factor of few correction.

This should work better with higher q and n.

Multiphonons become important around $q = \sqrt{2m_N\bar{\omega}_{\rm ph}}$



$q = \frac{1}{2}\sqrt{2m_N\bar{\omega}_{\rm ph}}$:
dominated by
n=1 phonon

$q = \sqrt{2m_N\bar{\omega}_{\rm ph}}$:
contributions from
n=1,2,3,4…

$q = 2\sqrt{2m_N\bar{\omega}_{\rm ph}}$:
can be approximated by
Gaussian envelope
(Impulse Approximation)

Impulse approximation

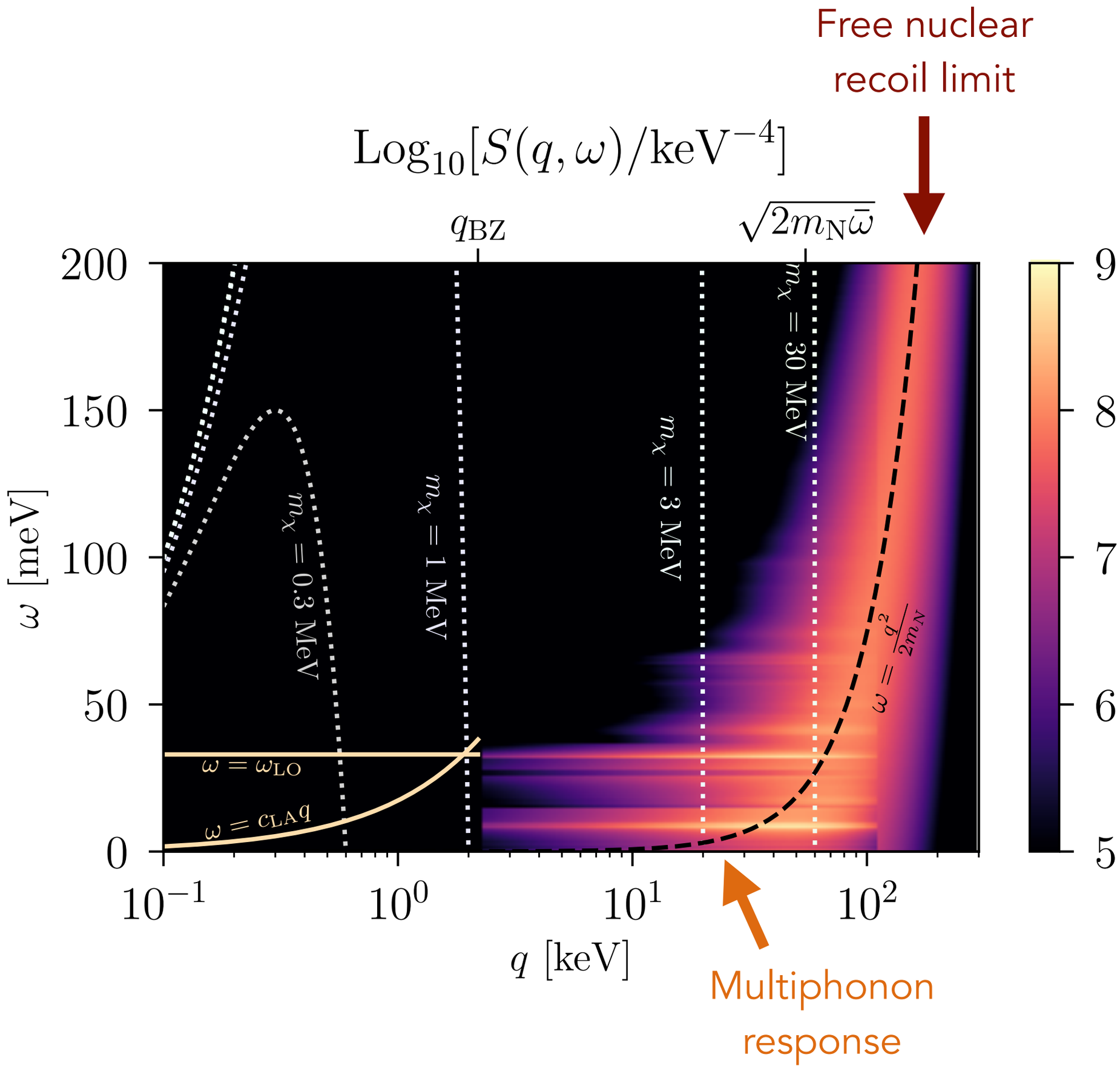
When $q \gg \sqrt{2m_N\bar{\omega}_{\rm ph}}$, "re-sum" the n-phonon contributions and directly evaluate by saddle-point approximation:

$$S^{\rm IA}(q,\omega) \propto \sum_J f_J^2 \sqrt{\frac{2\pi}{\Delta^2}} \exp\left(-\frac{(\omega - \frac{q^2}{2m_N})^2}{2\Delta^2}\right), \quad \Delta^2 = \frac{q^2\bar{\omega}_{\rm ph}}{2m_N}$$

As $\omega \gg \bar{\omega}_{\rm ph}$, $\Delta/\omega \to 0$, take narrow-width limit:

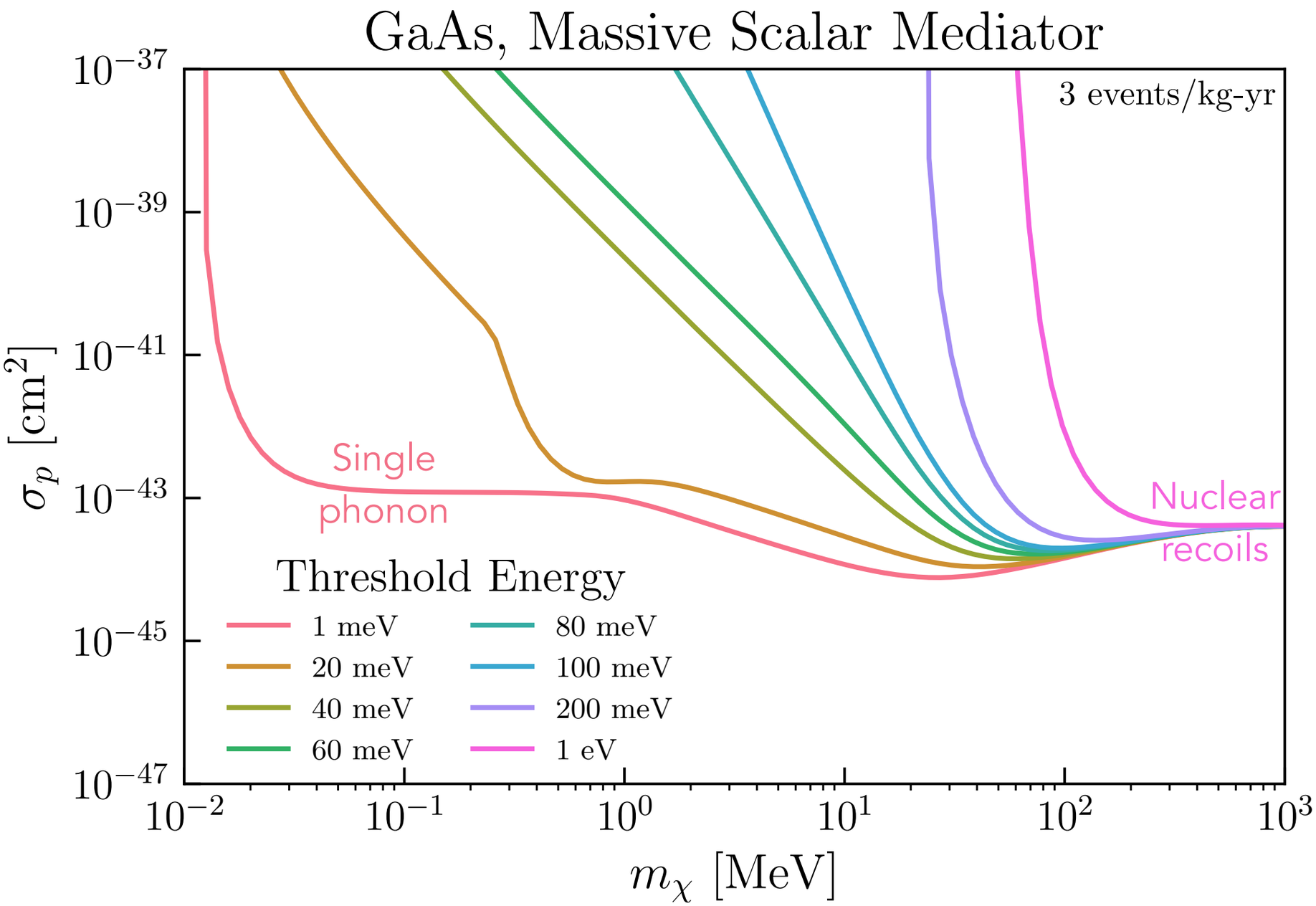
$$S(q,\omega) \propto \sum_J f_J^2 \; \delta\left(\omega - \frac{q^2}{2m_N}\right)$$

reproducing free nuclear recoils



Campbell-Deem, Knapen, TL, Villarama 2205.02250

DM scattering rate



Campbell-Deem, Knapen, TL, Villarama 2205.02250
See also Kahn, Krnjaic, Mandava 2011.09477

DM scattering in crystals

First steps towards describing DM-nucleus scattering into multiphonons.