Quantum Entanglement In Neutrino Oscillations (ID 44)

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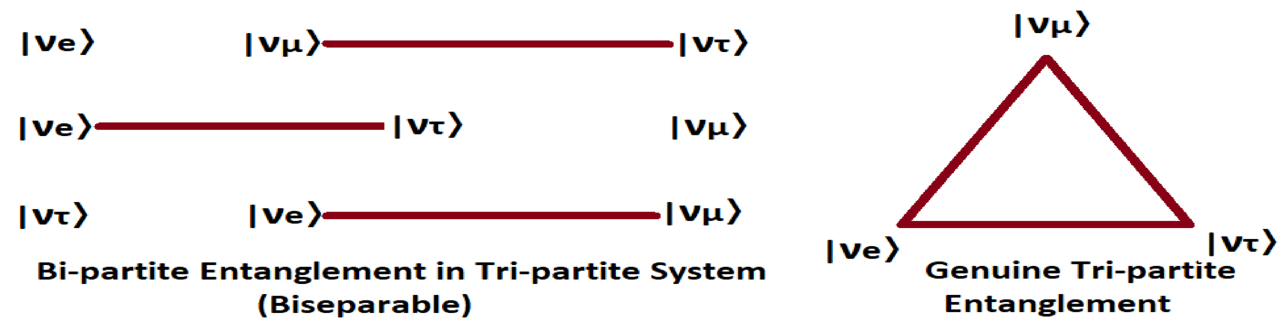
Abstract

We quantify bipartite and tripartite entanglement for two and three flavor neutrino oscillations in terms of two and three-qubit states (known as W states) used in quantum information theory. We calculate the concurrence, negativity and three tangle and show genuine tripartite entanglement in terms of a residual entanglement that satisfies a monogamy inequality. We use this analogy to outline the simulation of a neutrino oscillation on a quantum computer. We suggest the implementation of entanglement in neutrino systems on a IBM quantum processor.

Introduction

- The neutrino flavor states $|\nu_\alpha\rangle$ are linear superposition of mass eigenstates ($|\nu_j\rangle$), $|\nu_\alpha\rangle = \Sigma_j U_{\alpha j}|\nu_j\rangle$. The time evolution follows $|\nu_\alpha(t)\rangle = \Sigma_j e^{-iE_jt}U_{\alpha j}|\nu_j\rangle$, where $U_{\alpha j}$ are the elements of the PMNS (Pontecorvo-Maki-Nakagawa-Sakita) matrix and E_j is the energy associated with the mass eigenstates $|\nu_j\rangle$. This is a superposition state.
- In the three mode basis, we identify each flavor state ($\alpha = e, \mu, \tau$) at t=0 as: $|\nu_e\rangle = |1\rangle_e \otimes |0\rangle_\mu \otimes |0\rangle_\tau \equiv |100\rangle_e$; $|\nu_\mu\rangle = |0\rangle_e \otimes |1\rangle_\mu \otimes |0\rangle_\tau \equiv |010\rangle_\mu$; and $|\nu_\tau\rangle = |0\rangle_e \otimes |0\rangle_\mu \otimes |1\rangle_\tau \equiv |001\rangle_\tau$
- Different possible ways of visualising three mode state entanglement are :



Bi-Partite Entanglement in Two-Flavor Neutrino Oscillations

- In two-flavor ($\nu_\alpha \to \nu_\beta$) mixing, $|\nu_e(t)\rangle = \tilde{U}_{ee}(t)|10\rangle_e + \tilde{U}_{e\mu}(t)|01\rangle_\mu$, where $|10\rangle$ and $|01\rangle$ are two-qubit states, the density matrix is $\rho^{e\mu}(t) = |\nu_e(t)\rangle\langle\nu_e(t)| = (\tilde{U}_{ee}(t)|10\rangle + \tilde{U}_{e\mu}(t)|01\rangle)(\tilde{U}^*_{ee}(t)\langle 10| + \tilde{U}^*_{e\mu}(t)\langle 01|)$
- Positive Partial Transpose (PPT) criterion is a condition for determining entanglement in bi-partite system. It states that if the partial transposition $\rho^{T_e}_{pq,rs}(t) = \rho^{e\mu}_{rq,ps}(t)$ or $\rho^{T_\mu}_{pq,rs}(t) = \rho^{e\mu}_{ps,rq}(t)$ of a density matrix is a positive operator with all positive eigenvalues then the system is unentangled. If the system has even one negative eigenvalues then it is entangled.
- Various measures of bi-partite entanglement are :

Entanglement Measures	Results obtained from $\rho^{T_\mu}(t)$
1.PPT Criterion for an entanglement	Eigenvalues of $\rho^{T_\mu}(t)$ are $\lambda_1 = P_d$, $\lambda_2 = P_a$, $\lambda_3 = \sqrt{P_dP_a}$, $\lambda_4 = -\sqrt{P_dP_a}$
2.Negativity $N = \|\|\rho^{T_\mu}\|\| - 1$	$N_{e\mu} = 2\sqrt{P_aP_d}$

- Using the 'Spin-flipped' density matrix, $\tilde{\rho}^{e\mu}(t) = (\sigma_y \otimes \sigma_y)\rho^{*e\mu}(t)(\sigma_y \otimes \sigma_y)$ where σ_x and σ_y are Pauli matrices, we calculate concurrence: $C(\rho^{e\mu}(t)) \equiv [max(\mu_1 - \mu_2 - \mu_3 - \mu_4, 0)]$, in which $\mu_1, ..., \mu_4$ are the eigenvalues of the matrix $\rho^{e\mu}(t)\tilde{\rho}^{e\mu}(t)$.
- Tangle: $\tau(\rho^{e\mu}) \equiv [max(\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4, 0)]^2$ and $S(\rho^{e\mu}) = 1 - Tr(\rho^{e\mu})^2$.

Entanglement Measures	Results obtained from $\rho^{e\mu}(t)$
1.Concurrence $C(\rho^{e\mu}(t))$	Only one eigenvalue(μ) is non zero: $2\sqrt{P_aP_d}$ thus $C_{e\mu} = 2\sqrt{P_aP_d}$
2.Tangle $\tau(\rho^{e\mu})$	$\tau_{e\mu} = 4P_aP_d$
5. Linear Entropy $S(\rho^{e\mu})$	$S_{e\mu} = 4P_aP_d = \tau_{e\mu} = C^2_{e\mu}$

- The figure below shows the time evolution of the various measures of entanglement compared to the oscillation probabilities in a typical reactor experiment.

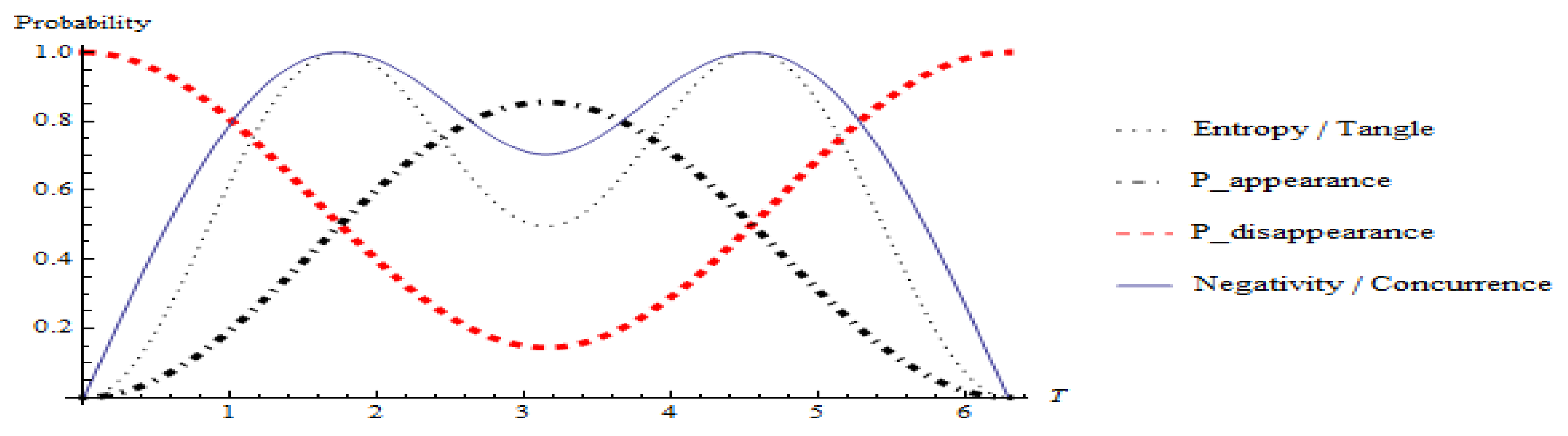


Figure 1:$\tau_{e\mu}$, $S_{e\mu}$, $N_{e\mu}$, $C_{e\mu}$, P_d, and P_a as functions of the scaled time $T \equiv \Delta m^2 t/2E$ for $|\nu_e(t)\rangle$, considering the experimental value $\sin^2\theta = 0.310$, where θ= mixing angle.

- We see that when $P_a = P_d = 0.5$, all measures of entanglement tend to 1 i.e, $N_{e\mu} = \tau_{e\mu} = C_{e\mu} = S_{e\mu} = 1$, which corresponds to maximally entangled state.

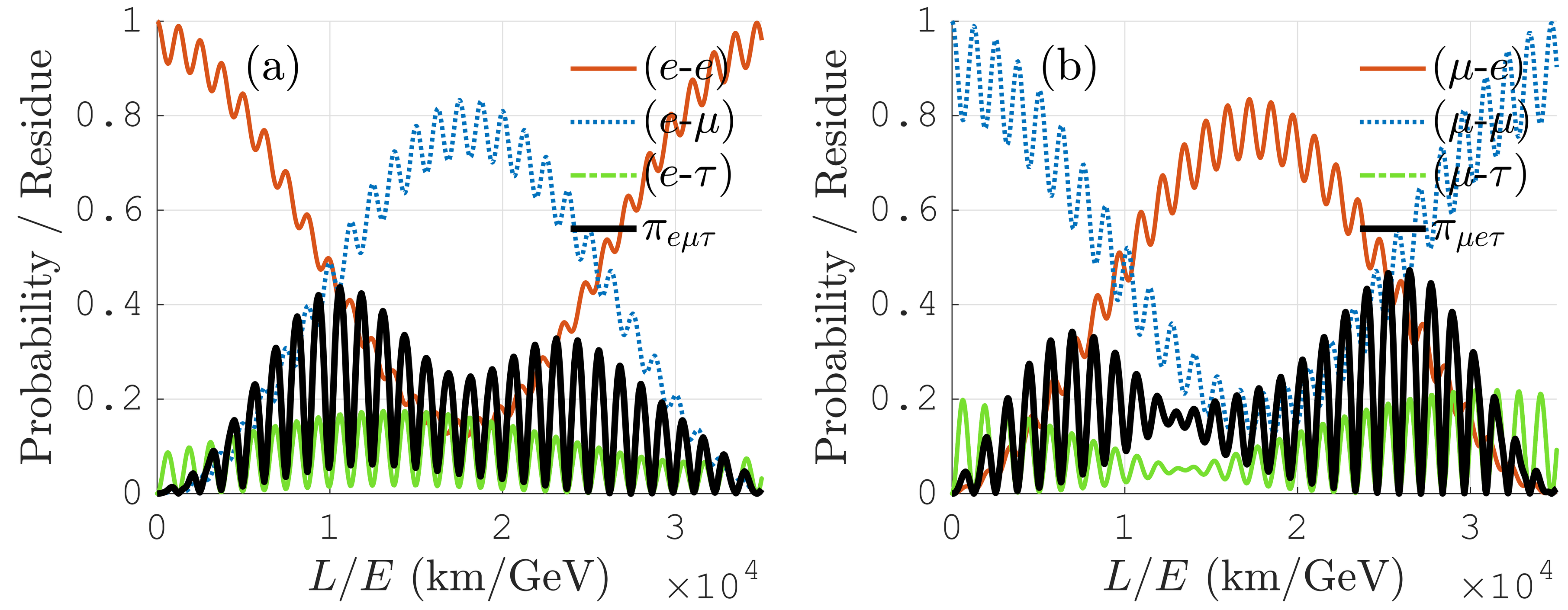
Tri-Partite Entanglement in Three-Flavor Neutrino Oscillations

- Biseparable states are formed in a three particle system, by considering two out of three modes state as a single state.
- The density matrix of the time evolved electron neutrino flavor state is $\rho^{e\mu\tau}(t) = |\tilde{U}_{ee}(t)|^2 |100\rangle\langle 100| + |\tilde{U}_{e\mu}(t)|^2 |010\rangle\langle 010| + |\tilde{U}_{e\tau}(t)|^2 |001\rangle\langle 001| + \tilde{U}_{ee}(t)\tilde{U}^*_{e\mu}(t)|100\rangle\langle 010| + \tilde{U}_{ee}(t)\tilde{U}^*_{e\tau}(t)|100\rangle\langle 001| + \tilde{U}_{e\mu}(t)\tilde{U}^*_{e\tau}(t)|010\rangle\langle 001| + |\tilde{U}_{e\tau}(t)\tilde{U}^*_{ee}(t)|001\rangle\langle 100| + \tilde{U}_{e\tau}(t)\tilde{U}^*_{e\mu}(t)|001\rangle\langle 010|$.
- The pairwise measures of entanglement are negativity ($N^2_{e(\mu\tau)}$), concurrence ($C^2_{e(\mu\tau)}$), tangle ($\tau_{e(\mu\tau)}$) and linear entropy ($S_{e(\mu\tau)}$). Because we have effectively reduced the three ν system to bi-partite system, the measures remain the same. $N^2_{e(\mu\tau)} = C^2_{e(\mu\tau)} = \tau_{e(\mu\tau)} = S_{e(\mu\tau)} = 4P_aP_d$.
- For tri-partite entanglement, criterion is known as Coffman-Kundu-Wooters (CKW) inequality. It states that the sum of quantum correlations between e and μ, and between e and τ, is either less than or equal to the quantum correlations between e and $\mu\tau$ (treating it as a single object) : $C^2_{e\mu} + C^2_{e\tau} \leq C^2_{e(\mu\tau)}$, $\tau_{e\mu} + \tau_{e\tau} \leq \tau_{e(\mu\tau)}$ and $N^2_{e\mu} + N^2_{e\tau} \leq N^2_{e(\mu\tau)}$.

Bi-separable entanglement measures	Results from $\rho^{e\mu\tau}(t)$
1. Concurrence equality	$C^2_{e\mu} + C^2_{e\tau} = C^2_{e(\mu\tau)}$
2. Tangle equality	$\tau_{e\mu} + \tau_{e\tau} = \tau_{e(\mu\tau)}$
3. Negativity inequality	$N^2_{e\mu} + N^2_{e\tau} < N^2_{e(\mu\tau)}$

- There is one more genuine measures of tri-partite entanglement quantified by three-tangle and three-π negativity known as residual entanglement.
- The residual entanglement three-π for electron neutrino flavor state $|\nu_e(t)\rangle$ is, (see Fig.2)

$$\pi_{e\mu\tau} = \frac{4}{3}[|\tilde{U}_{ee}(t)|^2\sqrt{|\tilde{U}_{ee}(t)|^4 + 4|\tilde{U}_{ee}(t)|^2|\tilde{U}_{e\tau}(t)|^2} + |\tilde{U}_{e\mu}(t)|^2\sqrt{|\tilde{U}_{e\mu}(t)|^4 + 4|\tilde{U}_{ee}(t)|^2|\tilde{U}_{e\tau}(t)|^2} + |\tilde{U}_{e\tau}(t)|^2\sqrt{|\tilde{U}_{e\tau}(t)|^4 + 4|\tilde{U}_{ee}(t)|^2|\tilde{U}_{e\mu}(t)|^2} - |\tilde{U}_{ee}(t)|^4 - |\tilde{U}_{e\mu}(t)|^4 - |\tilde{U}_{e\tau}(t)|^4] > 0.$$



Figure(2):(a) Time evolved electron neutrino flavor state $|\nu_e(t)\rangle$ (relevant to reactor experiment) and (b) a muon flavor state $|\nu_\mu(t)\rangle$ (relevant to accelerator experiment) vs scale of distance per energy unit $\frac{L}{E}$. At $\frac{L}{E} > 0$ entanglement among three-flavor modes occurs i.e, the black curve $\pi_{e\mu\tau} > 0$ or $\pi_{e\mu\tau} > 0$, and exhibits a typical oscillatory behavior. $\pi_{e\mu\tau}$ reaches the maximum value 0.436629 (see Fig.2(a)) when transition probabilities are $P_{\nu_{e\to e}} = 0.39602$, $P_{\nu_{e\to\mu}} = 0.435899$, and $P_{\nu_{e\to\tau}} = 0.168081$. Similarly, for $|\nu_\mu(t)\rangle$, $\pi_{\mu e\tau}$ reaches the maximum value 0.472629 (see Fig.2(b)).

- The residual entanglement tri-partite results are :

Residual Entanglement	Tri-Partite results
Three-tangle $\tau_{e\mu\tau} = C^2_{e(\mu\tau)} - C^2_{e\mu} + C^2_{e\tau}$	$\tau_{e\mu\tau} = 0$
Three-π $\pi_{e\mu\tau} = \frac{1}{3}(N^2_{e(\mu\tau)} + N^2_{\mu(e\tau)} + N^2_{\tau(e\mu)} - 2N^2_{e\mu} - 2N^2_{e\tau} - 2N^2_{\mu\tau})$	$\pi_{e\mu\tau} > 0$

Quantum Simulation On IBMQ Computer

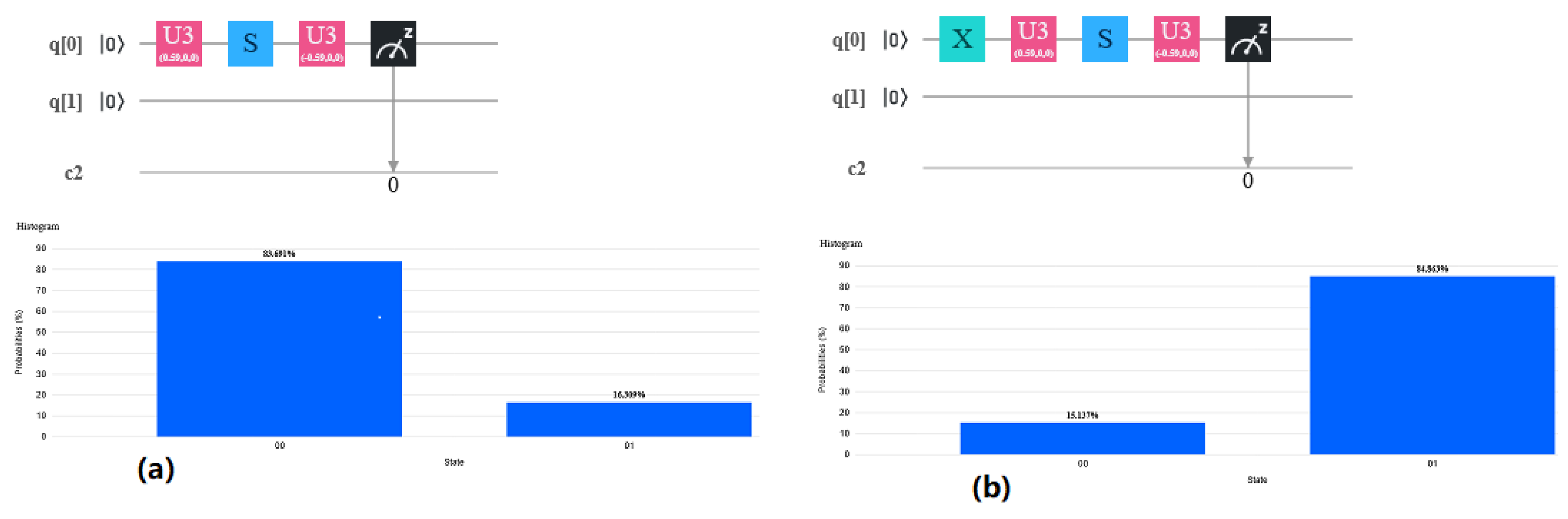


Figure 2:Quantum circuits embodying two-flavor neutrino oscillation and histogram plot (probabilities) for (a) $\nu_e \to \nu_e$ and (b) $\nu_e \to \nu_\mu$.

- $$\begin{pmatrix}|\nu_e(t)\rangle \\ |\nu_e(t)\rangle\end{pmatrix} = \begin{pmatrix}cos\theta & sin\theta \\ -sin\theta & cos\theta\end{pmatrix}\begin{pmatrix}1 & 0 \\ 0 & e^{i\phi}\end{pmatrix}\begin{pmatrix}cos\theta & -sin\theta \\ sin\theta & cos\theta\end{pmatrix}\begin{pmatrix}\nu_e(0) \\ \nu_\mu(0)\end{pmatrix}$$ $$= \mathrm{U3}(2\theta, 0, 0)U1(t)U3(-2\theta, 0, 0)\begin{pmatrix}\nu_e(0) \\ \nu_\mu(0)\end{pmatrix}, \quad \mathrm{U3}(\Phi, \phi, \lambda) = \begin{pmatrix}cos\frac{\Phi}{2} & -sin\frac{\Phi}{2}e^{i\lambda} \\ sin\frac{\Phi}{2}e^{i\phi} & cos\frac{\Phi}{2}e^{i(\lambda+\phi)}\end{pmatrix}$$ is an IBMQ U3 Universal gate, $U1(t) = S(\phi)$ is a time-evolution operation gate, where $\phi = \frac{\Delta m^2 t}{2E}$ and X is the Pauli-X gate.
- In quantum optics, the action of quantum mechanical beam splitter interferometer is given by $SU(2)$ matrix, which performs exactly the same transformation on photons as the neutrino mixing matrix does. Thus, the entanglement in a two flavor neutrino mixing is akin to entanglement via mode swapping due to beam splitter. Quantum simulation of such system on quantum computer is in progress.

Result

The tri-partite result $\pi_{e\mu\tau} > \tau_{e\mu\tau} = 0$ or $\pi_{\mu e\tau} > \tau_{\mu e\tau} = 0$ imply that the three-neutrino state shows the remarkable property of having a genuine form of three way entanglement similar to the W-state in quantum information processing.