We quantify the tripartite and tripartite entanglement for two and three flavor neutrino oscillations in terms of two and three-qubit states (known as W states) used in quantum information theory. We calculate the concurrence, negativity and three and two qubit and show genuine tripartite entanglement in terms of a residual entanglement that satisfies a monogamy inequality. We use this analogy to outline the simulation of a neutrino oscillation on a quantum computer. We suggest the implementation of entanglement in neutrino systems on an IBM quantum processor.

Quantum Simulation On IBMQ Computer

Abstract

We quantify the tripartite and tripartite entanglement for two and three flavor neutrino oscillations in terms of two and three-qubit states (known as W states) used in quantum information theory. We calculate the concurrence, negativity and three and two qubit and show genuine tripartite entanglement in terms of a residual entanglement that satisfies a monogamy inequality. We use this analogy to outline the simulation of a neutrino oscillation on a quantum computer. We suggest the implementation of entanglement in neutrino systems on an IBM quantum processor.

Introduction

The neutrino flavor states $|\nu_i\rangle$ are linear superpositions of mass eigenstates $|\nu_e\rangle$, $|\nu_\mu\rangle$, $|\nu_\tau\rangle$. The time evolution follows $|\psi(t)\rangle = \Sigma U_{ij} |\nu_i\rangle$, where $U_{ij}$ are the elements of the PMNS (Pontecorvo-Maki-Nakagawa-Sakata) matrix and $E$ is the energy associated with the mass eigenstates $|\nu_i\rangle$. This is a superposition state.

Different possible ways of visualising three mode state entanglement are:

- Bi-Partite Entanglement in Three-Flavor Neutrino Oscillations
- Tri-Partite Entanglement in Three-Flavor Neutrino Oscillations
- Tri-partite results

Tangle $\tau(\rho^{(3)})$ is defined as $\tau(\rho^{(3)}) = |\langle \psi | \rho^{(3)} | \psi \rangle|^2 = \frac{1}{1 + \langle \psi | \rho^{(3)} | \psi \rangle} - \frac{1}{2}$

Entanglement Measures

1. Concurrence $C(\rho^{(3)})$ obtained from $\rho^{(3)}$: $C(\rho^{(3)}) = \frac{1}{2} \sqrt{\lambda_1 - \lambda_2}$

2. Quadratic Entropy $S(\rho^{(3)})$ obtained from $\rho^{(3)}$: $S(\rho^{(3)}) = 1 - Tr(\rho^{(3)})^2$

3. Linear Entropy $S(\rho^{(3)})$ obtained from $\rho^{(3)}$: $S(\rho^{(3)}) = 1 - Tr(\rho^{(3)})^2$

4. Negativity $\mathcal{N}(\rho^{(3)})$ obtained from $\rho^{(3)}$: $\mathcal{N}(\rho^{(3)}) = \max(0, Tr(\rho^{(3)}))$

The figure below shows the time evolution of the various measures of entanglement compared to the oscillation probabilities in a typical reactor experiment.

Figure 1: A three-partite state is created and the probability of finding the state in each mode is plotted over time. The concurrence, negativity, and quadratic entropy are shown with their respective values.

Figure 2: (a) Time-evolved electron neutrino flavor state $|\nu_e(0)\rangle$ (relevant to reactor experiment) and (b) a mass flavor state $|\nu_\mu(0)\rangle$ (relevant to accelerator experiment) to scale of distance per energy $\frac{1}{E}$ for $|\nu_e(0)\rangle$.

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Figure 3: Quantum circuits encoding two-flavor neutrino oscillation and histogram plot (probabilities for $|\psi\rangle \rightarrow \phi$ and $\lambda|\phi\rangle \rightarrow \phi$).

Figure 4: (a) The residual entanglement tri-partite results are:

- Residual Entanglement
- Tri-partite results

- Tangle $\tau_{\rho(3)} = C_{\rho(3)} - \frac{1}{2} = 0$
- $3\pi$-angle $\pi_{\rho(3)} = \frac{1}{3}(N_{\rho(3)}^{\rho(3)} + N_{\rho(3)}^{\rho(3)} + N_{\rho(3)}^{\rho(3)}) = 2N_{\rho(3)}^{\rho(3)}$

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Figure 5: Quantum circuits encoding three-flavor neutrino oscillation and histogram plot (probabilities for $|\psi\rangle \rightarrow \phi$ and $\lambda|\phi\rangle \rightarrow \phi$).

Figure 6: (a) The residual entanglement tri-partite results are:

- Residual Entanglement
- Tri-partite results

- Tangle $\tau_{\rho(3)} = C_{\rho(3)} - \frac{1}{2} = 0$
- $3\pi$-angle $\pi_{\rho(3)} = \frac{1}{3}(N_{\rho(3)}^{\rho(3)} + N_{\rho(3)}^{\rho(3)} + N_{\rho(3)}^{\rho(3)}) = 2N_{\rho(3)}^{\rho(3)}$

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Figure 7: Quantum circuits encoding three-flavor neutrino oscillation and histogram plot (probabilities for $|\psi\rangle \rightarrow \phi$ and $\lambda|\phi\rangle \rightarrow \phi$).