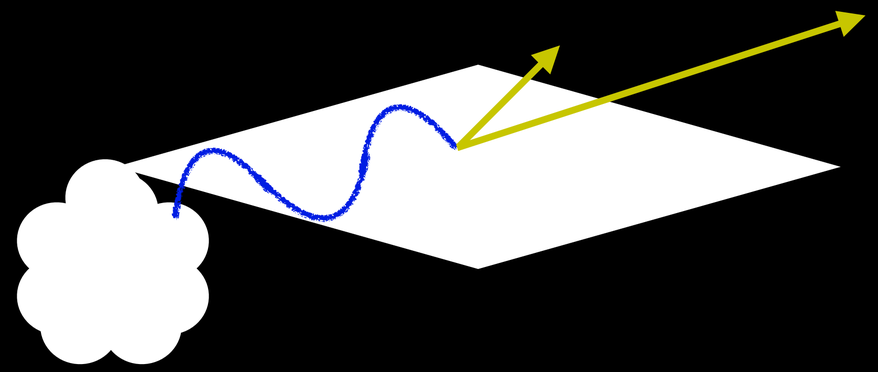
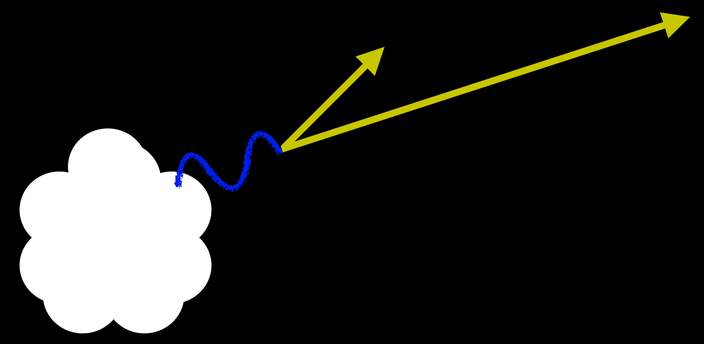




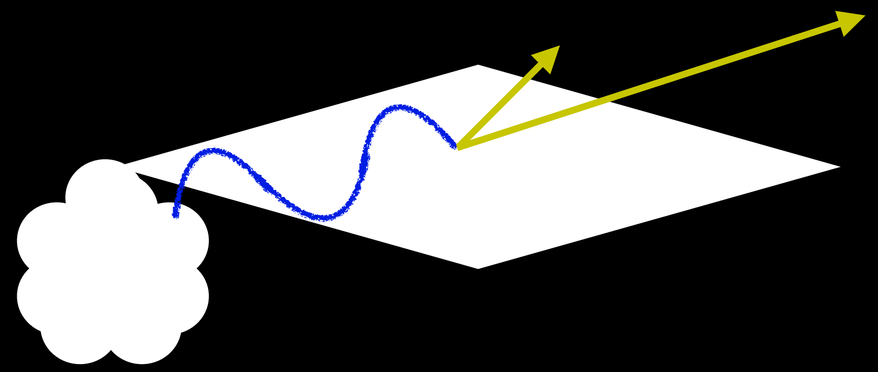
Universality near the RMC endpoint

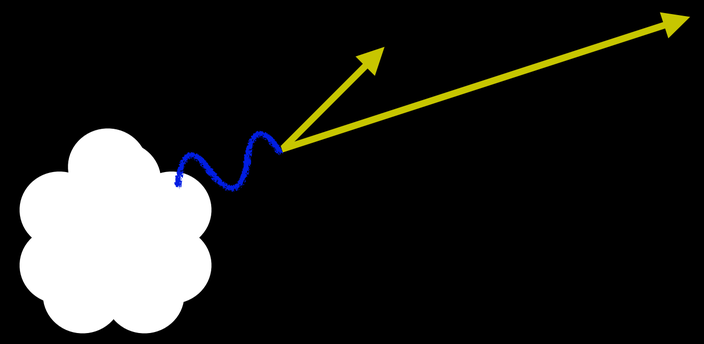
(Internal*) conversion as external conversion

$$\mu^- \, \mathrm{Al} \to \nu_\mu \, \gamma^{(*)} \, \mathrm{Mg} \qquad \mu^- \, \mathrm{Al} \to e^+ \, \mathrm{Na}$$

Ryan Plestid | July 21st 2020 | New Perspectives 2020

Intensity frontier fellow at FNAL | Postdoctoral Scholar at UKY





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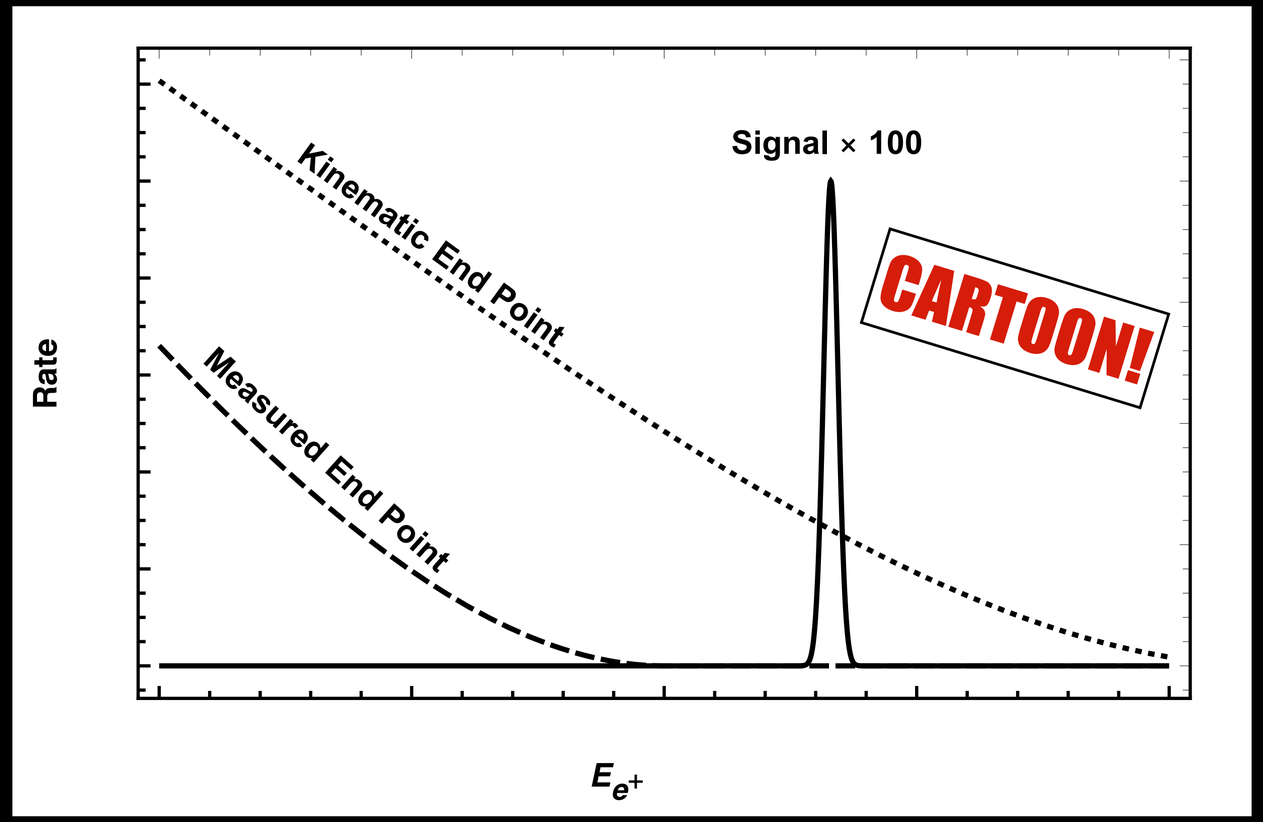
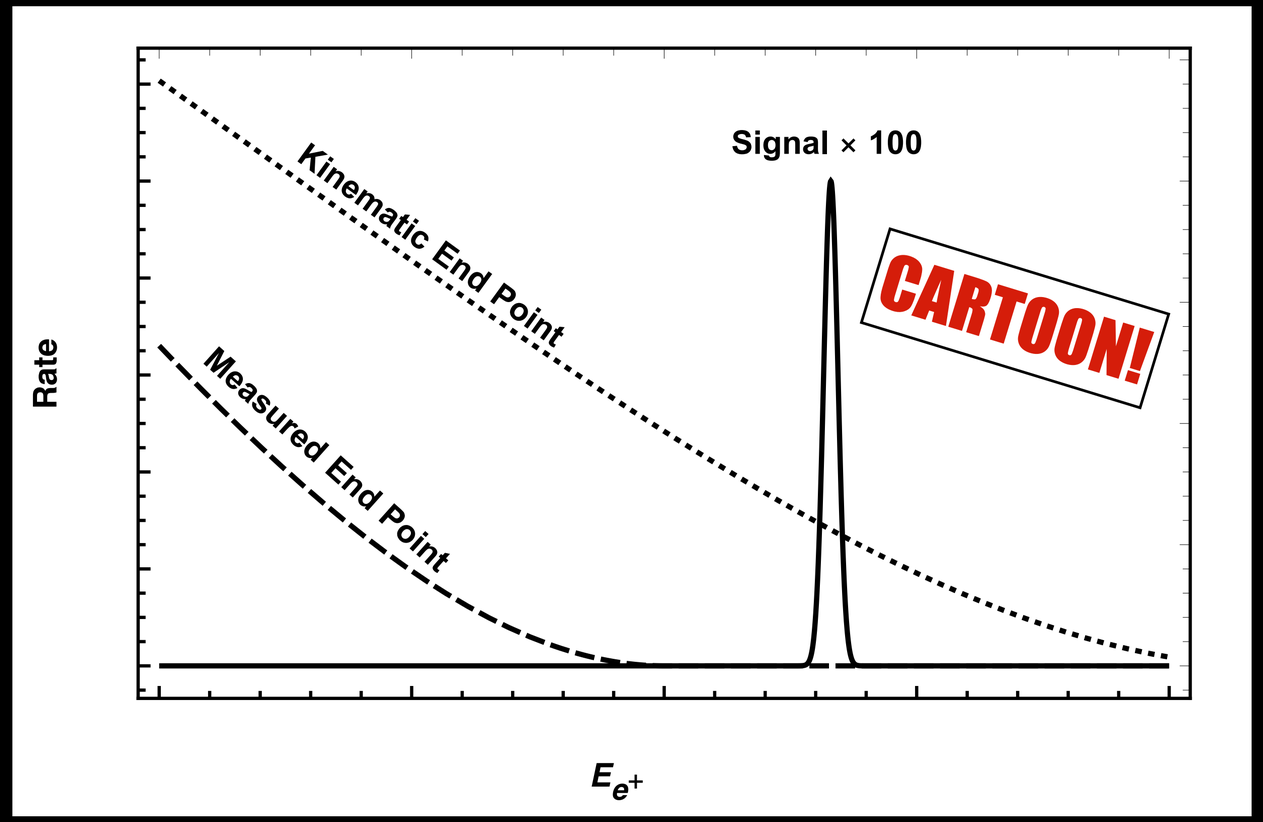


Signal: Positron from muon capture on ^{27}Al

$$\mu^- \, \mathrm{Al} \rightarrow e^+ \, \mathrm{Na}$$

Motivation



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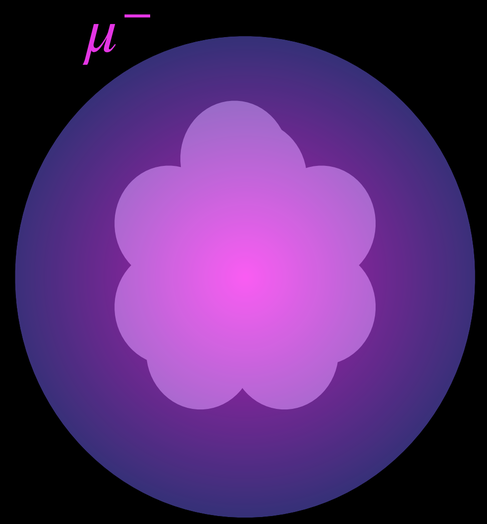

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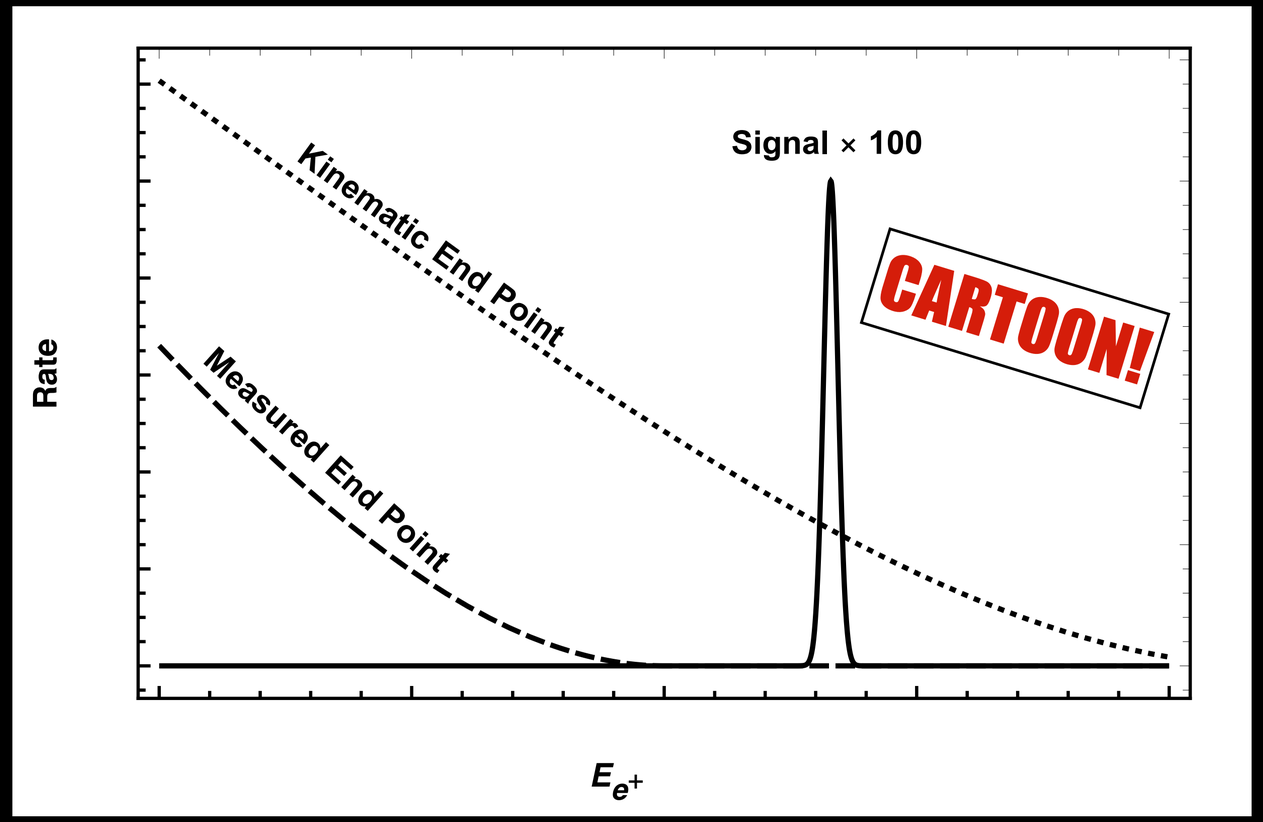

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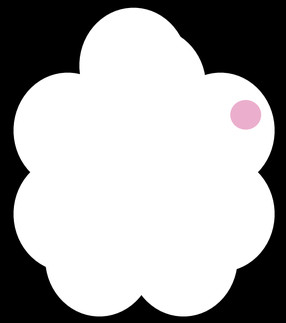

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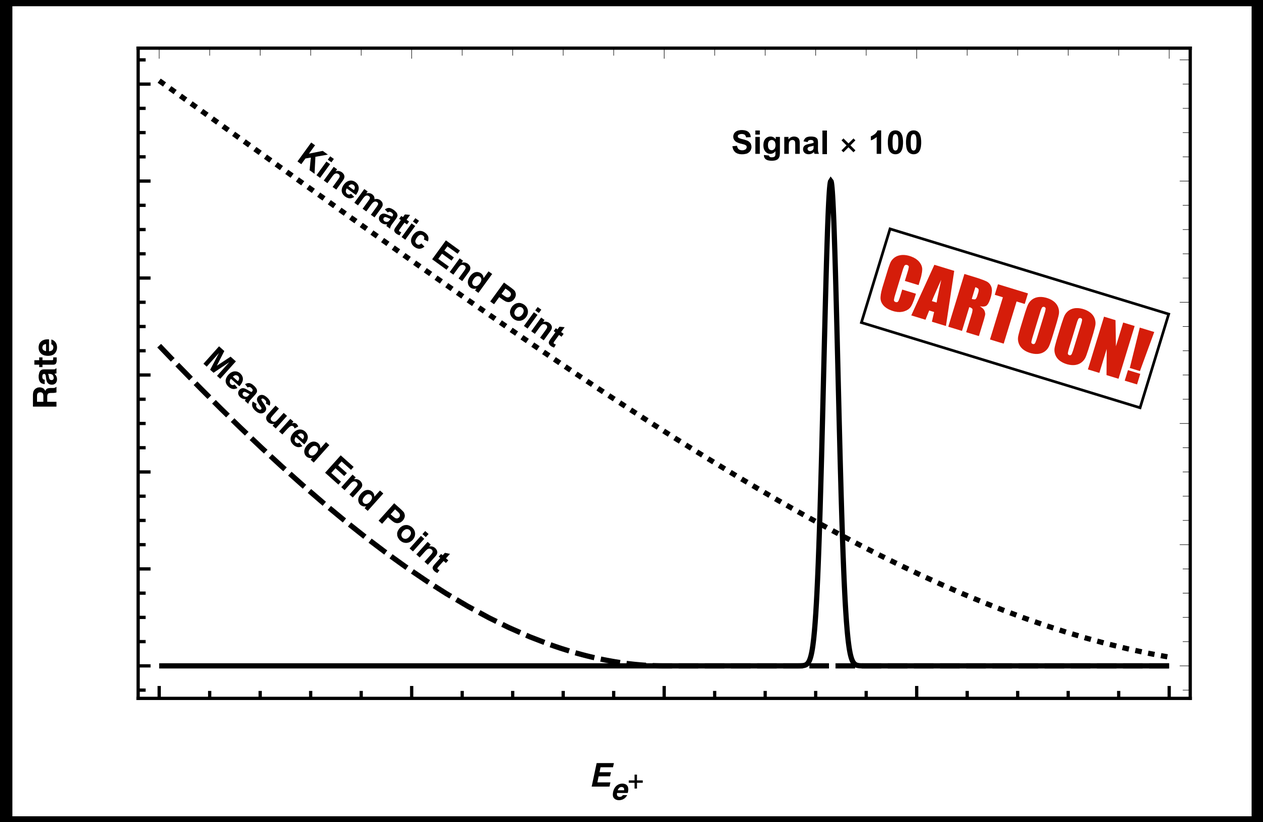

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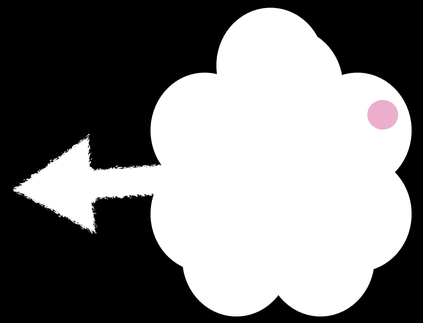

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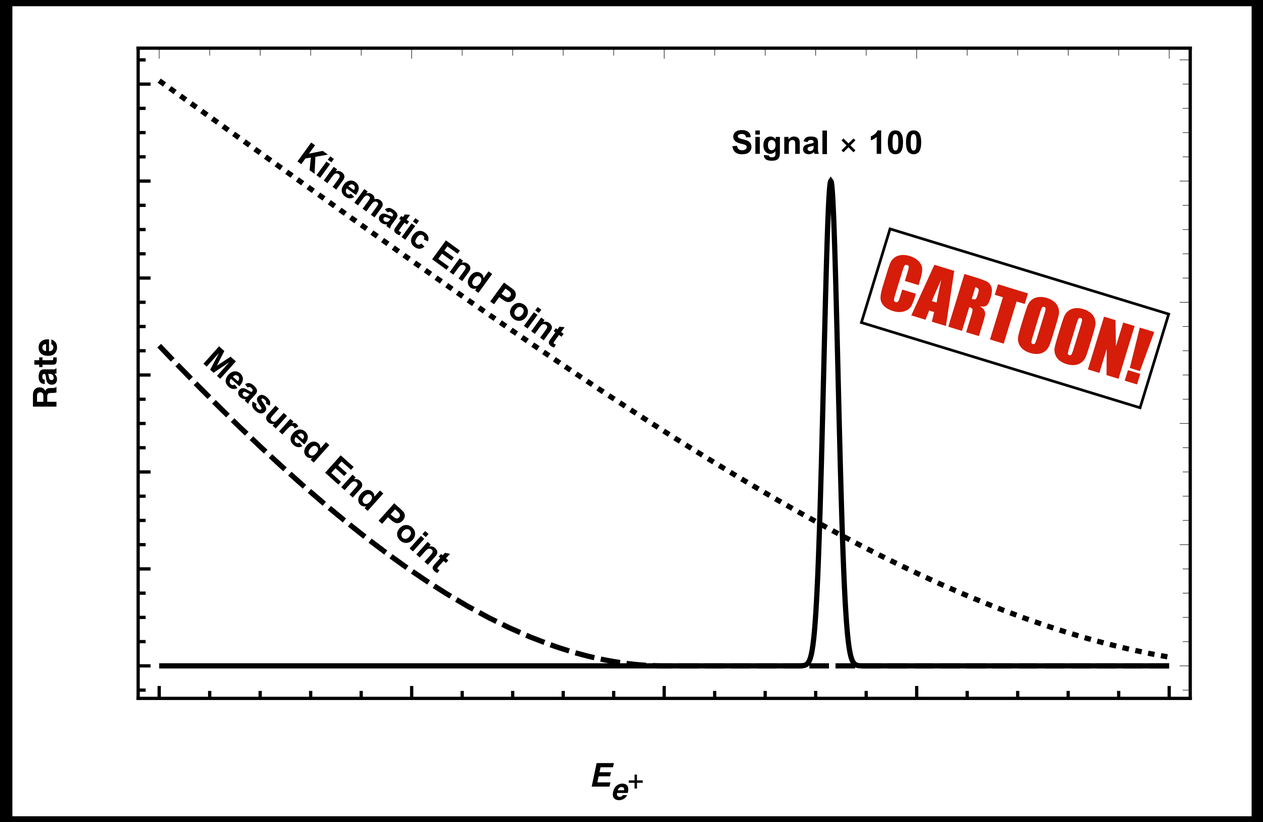

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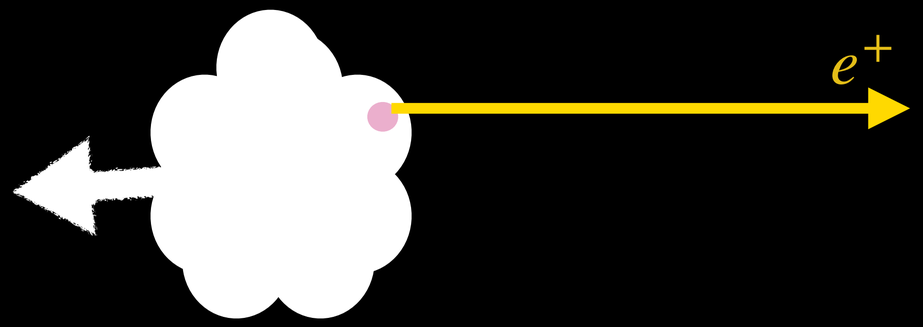

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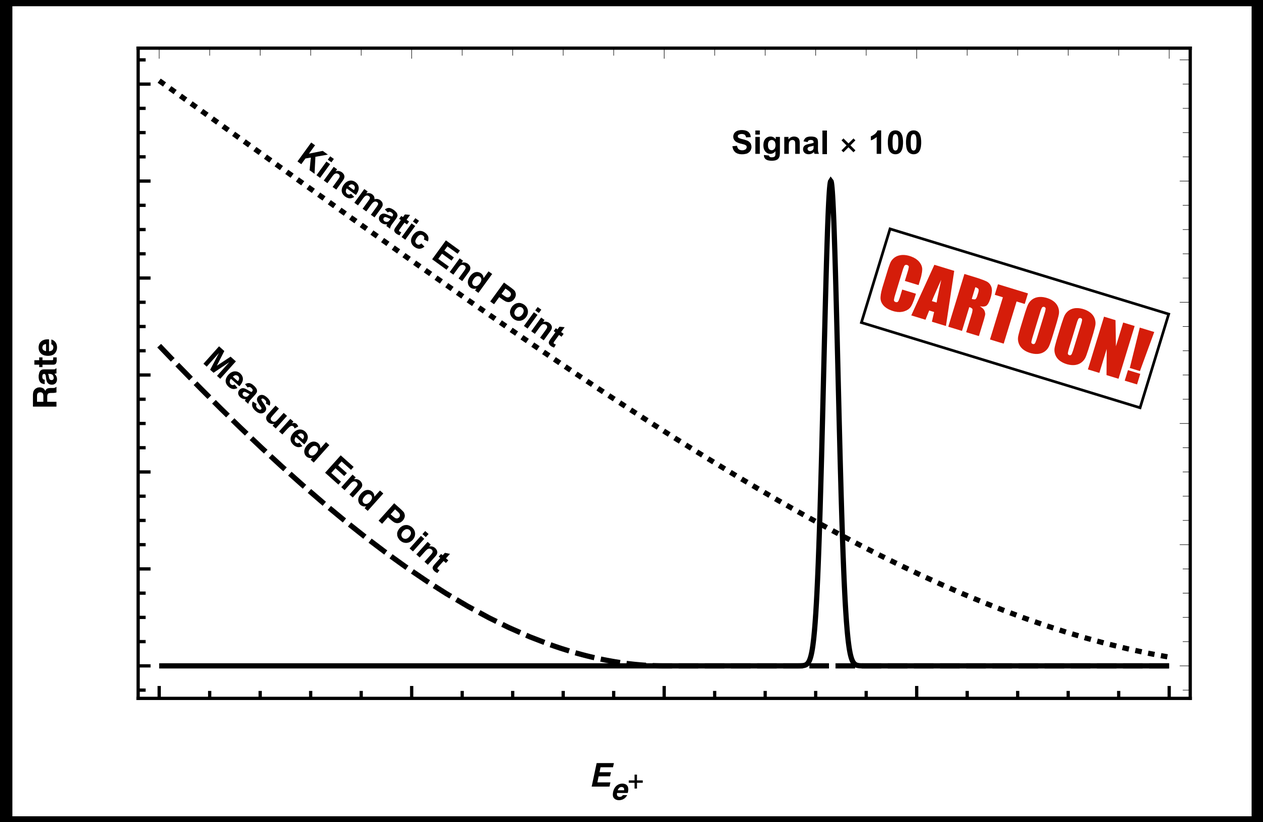

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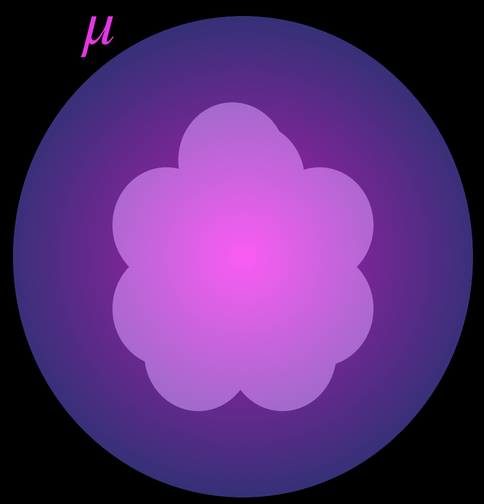

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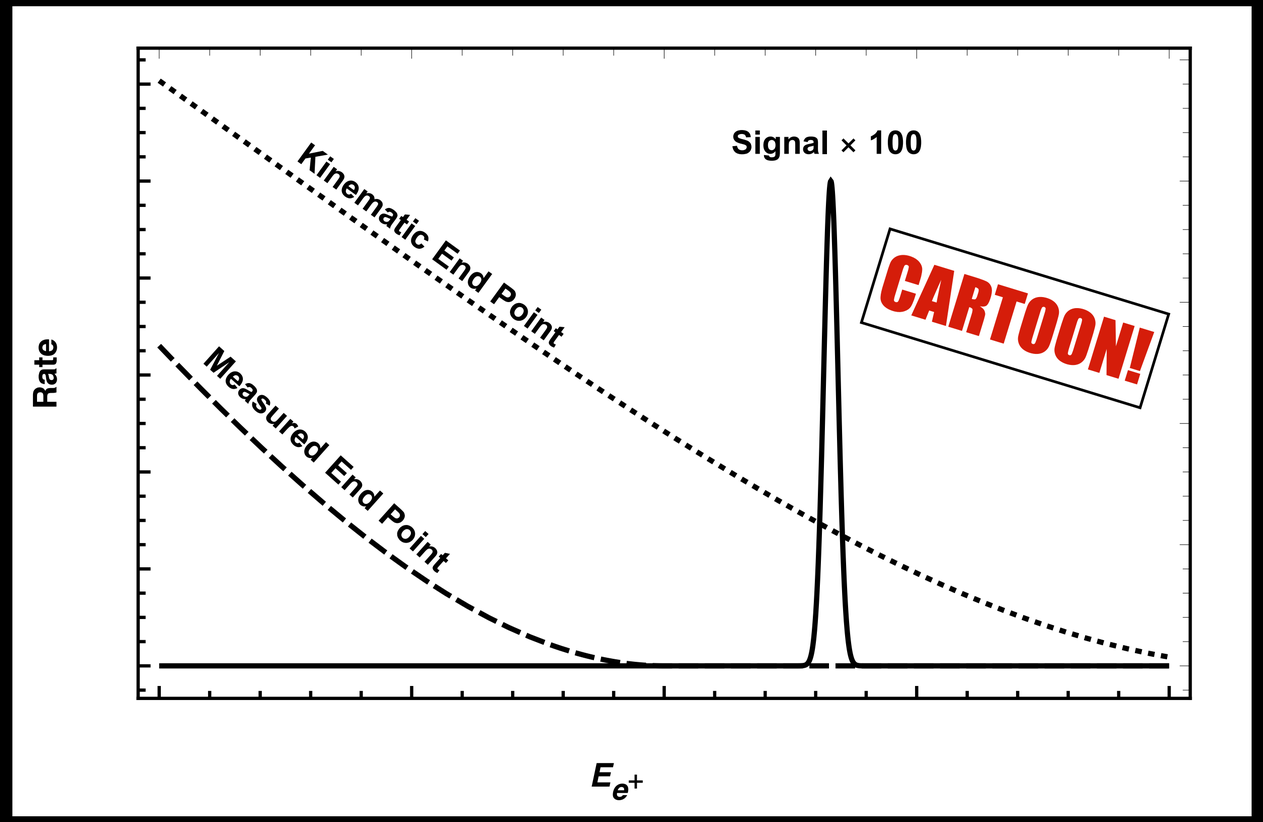

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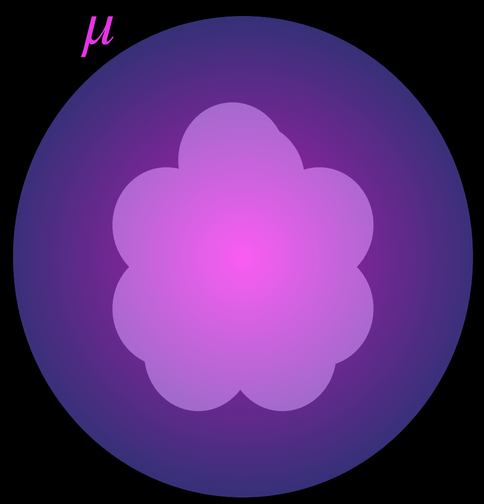

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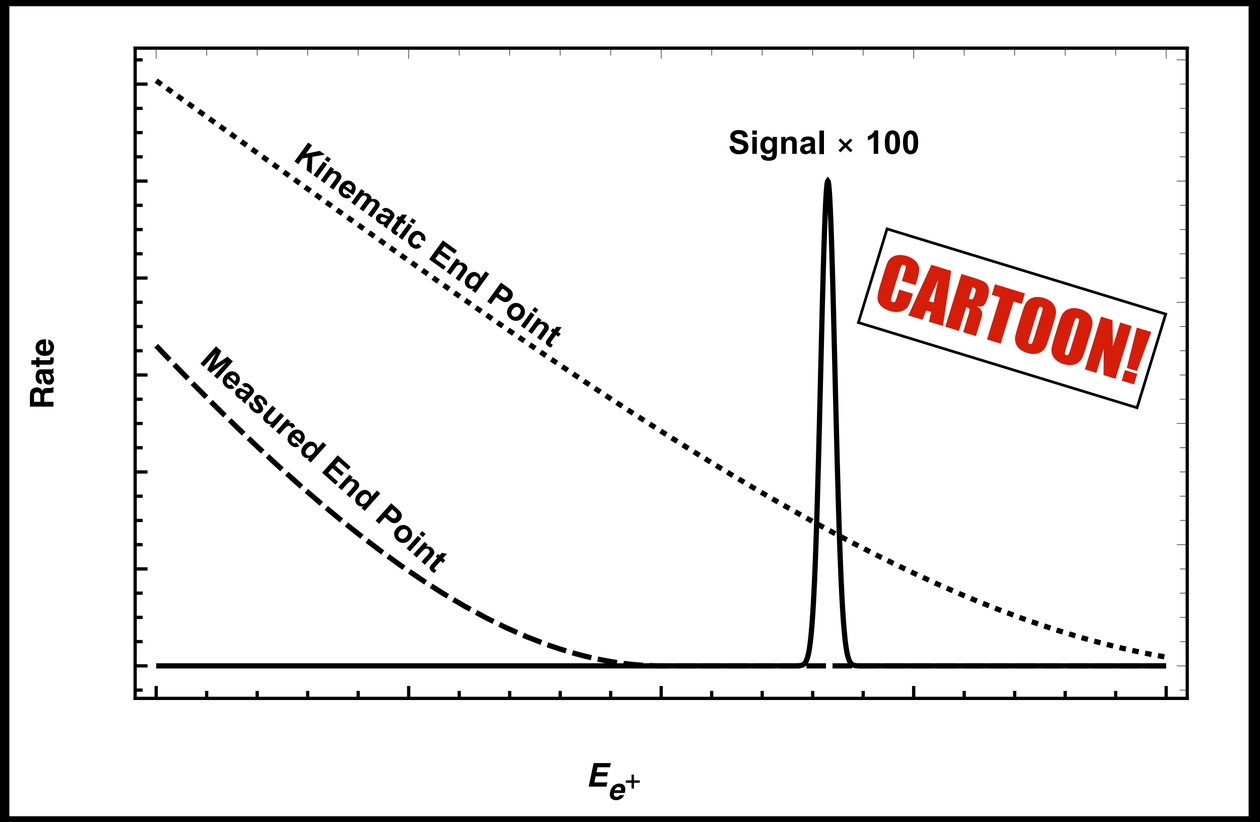

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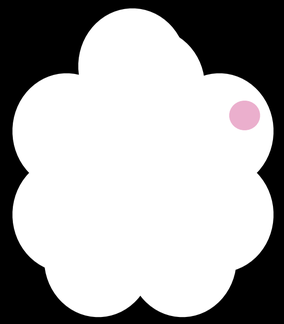

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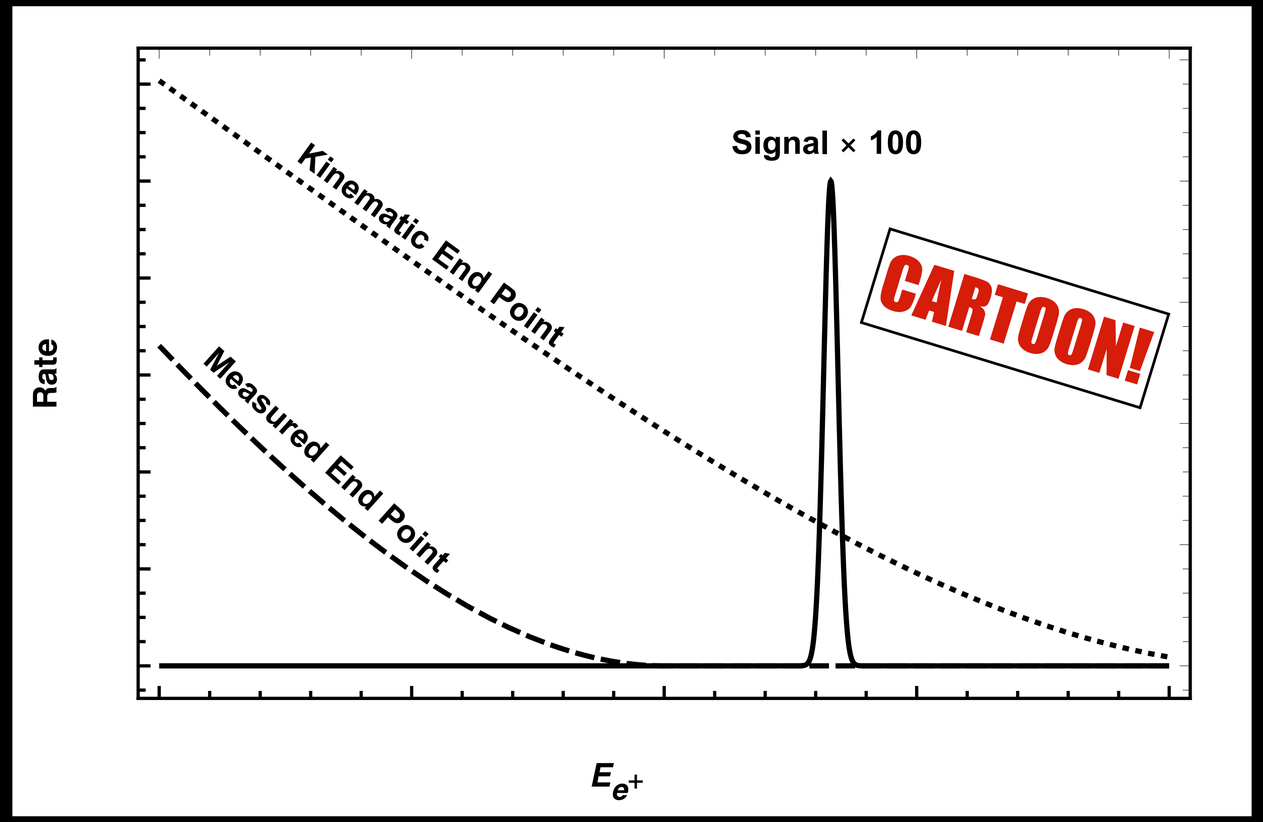

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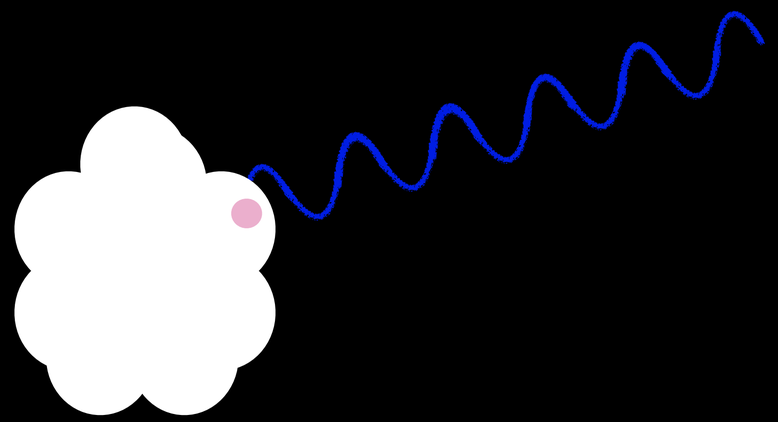

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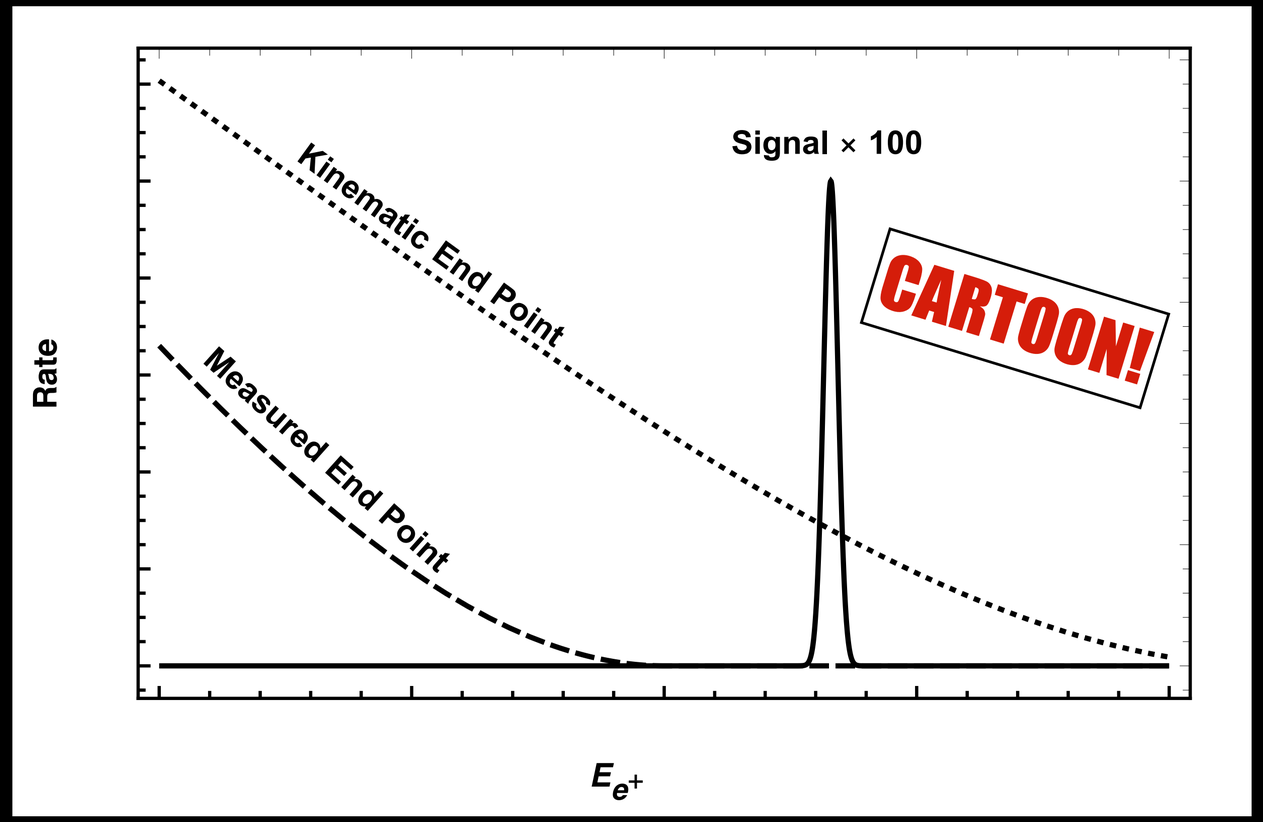

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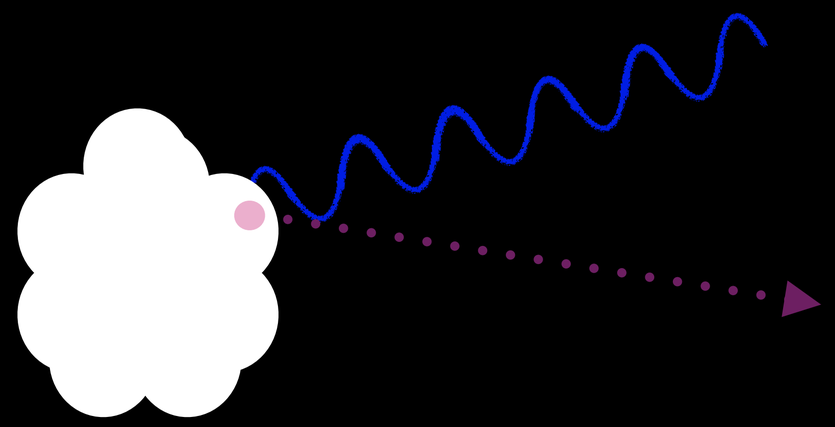

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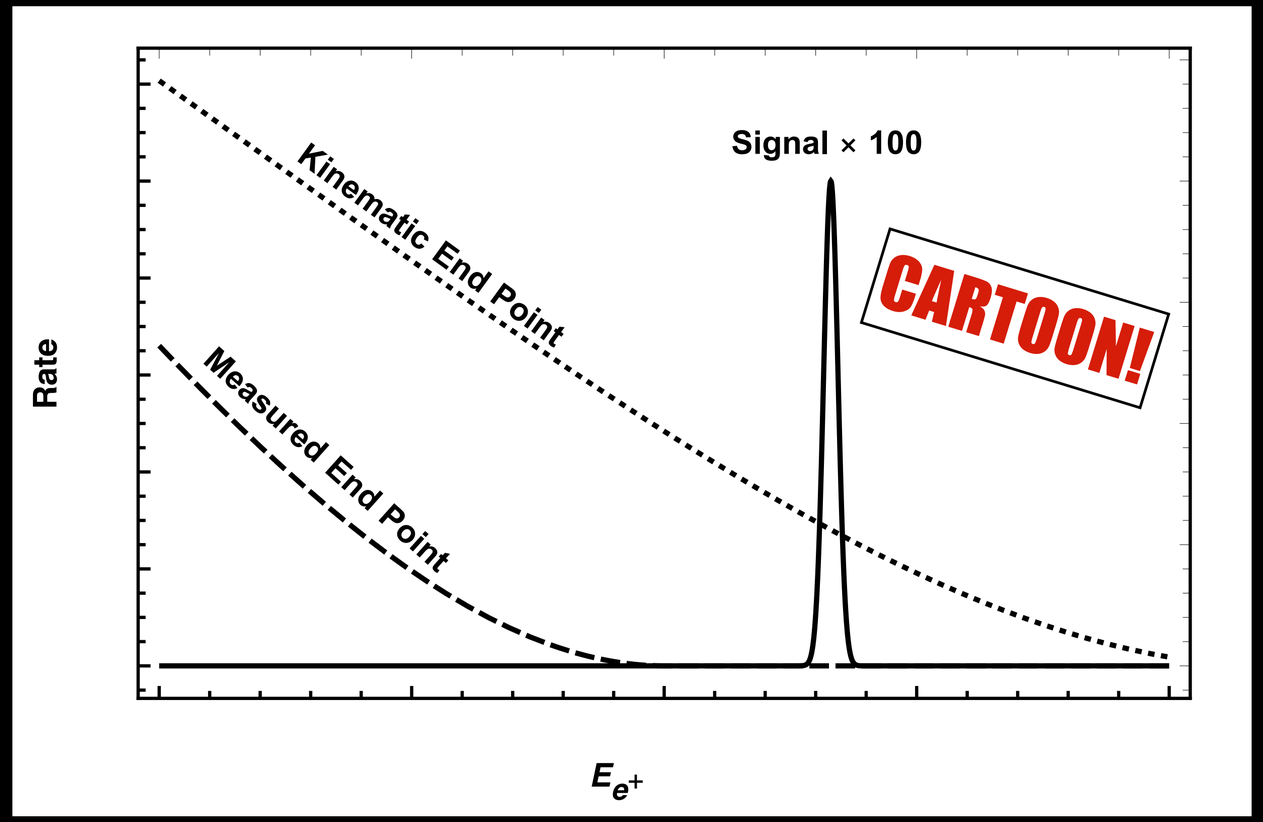

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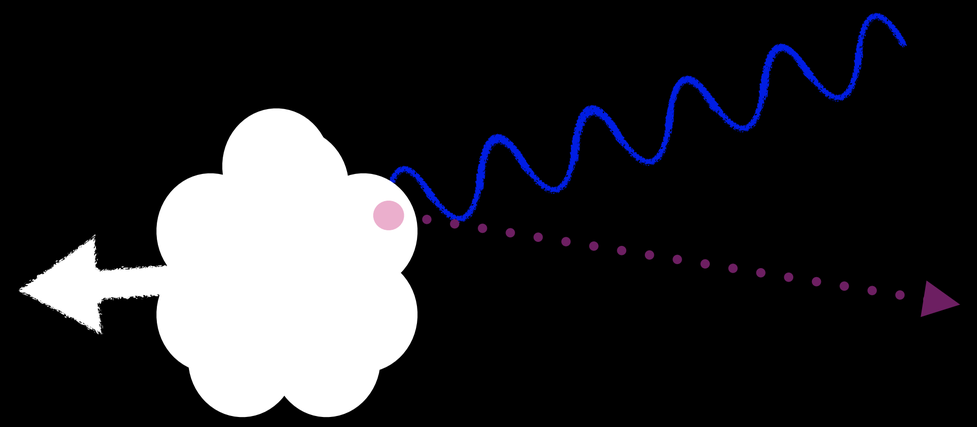

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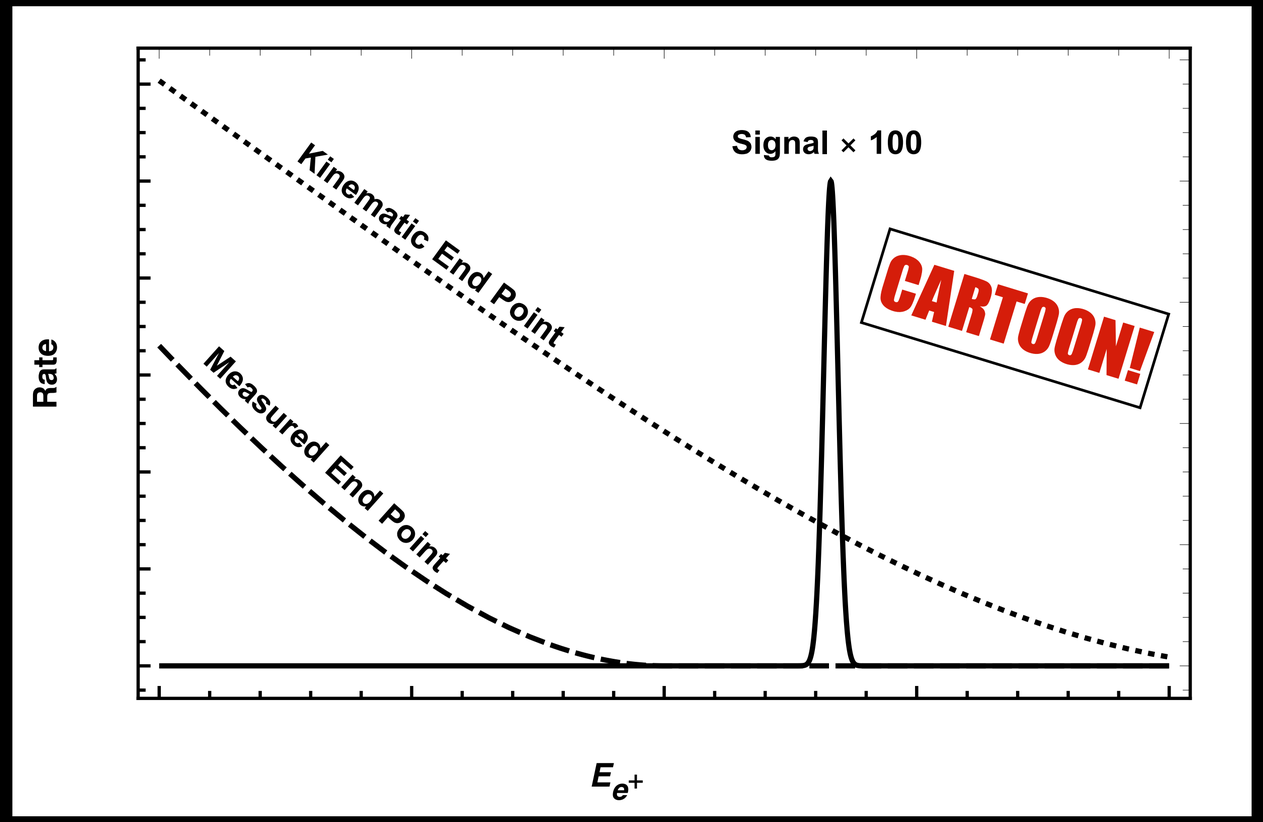

Q: What is spectrum of real photons?

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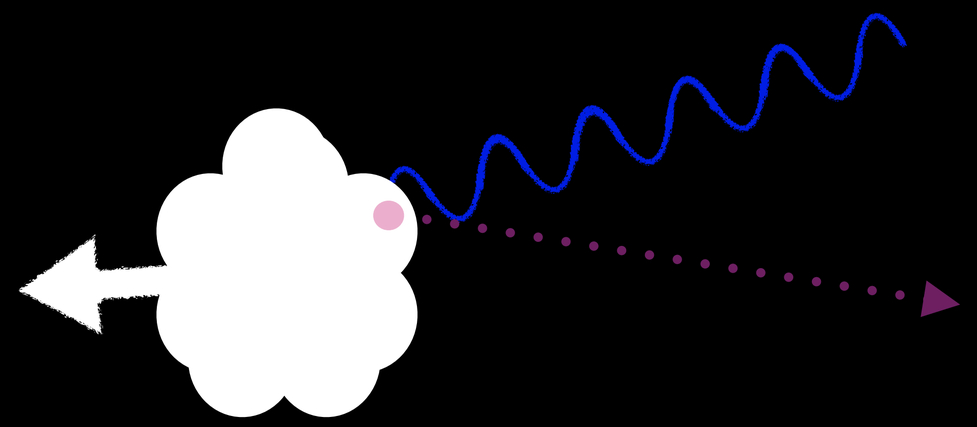

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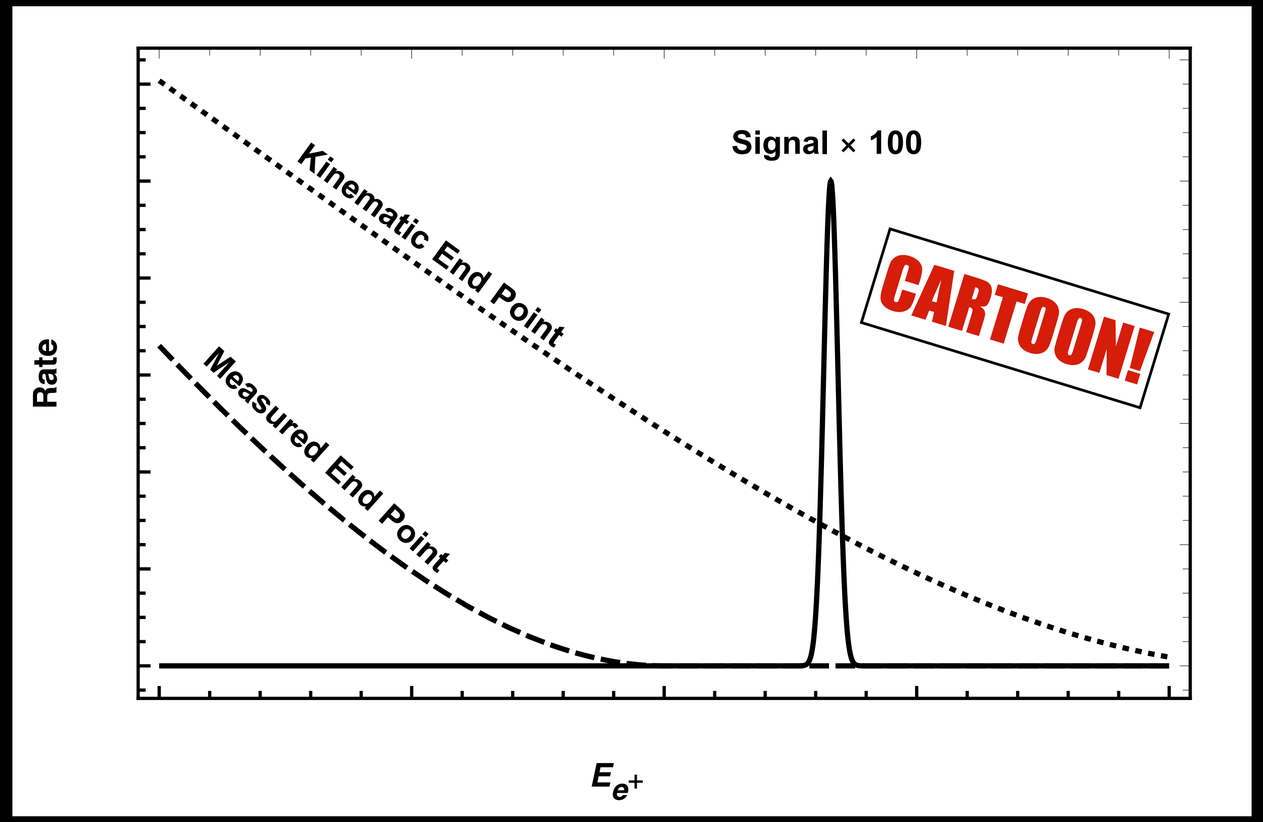

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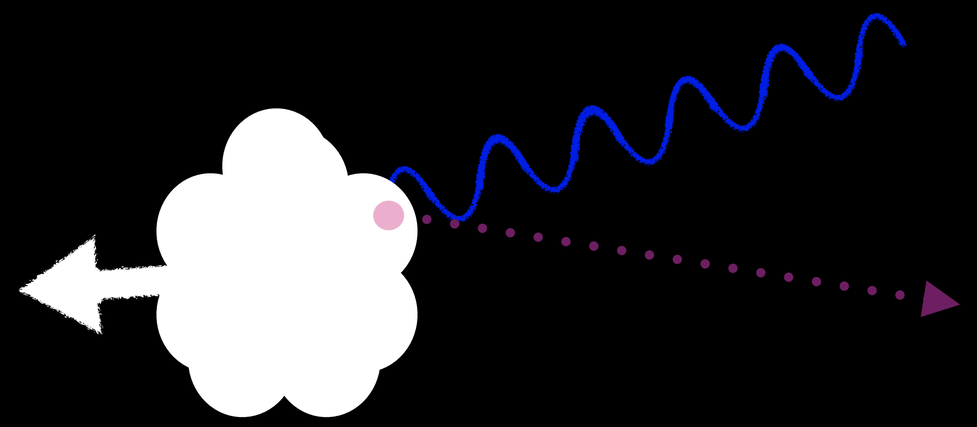
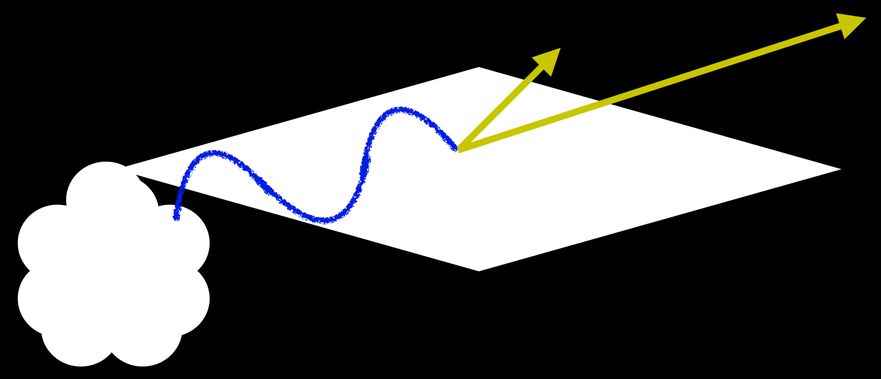


Q: What is spectrum of real photons?

A: For this talk we assume this will be *measured*

External

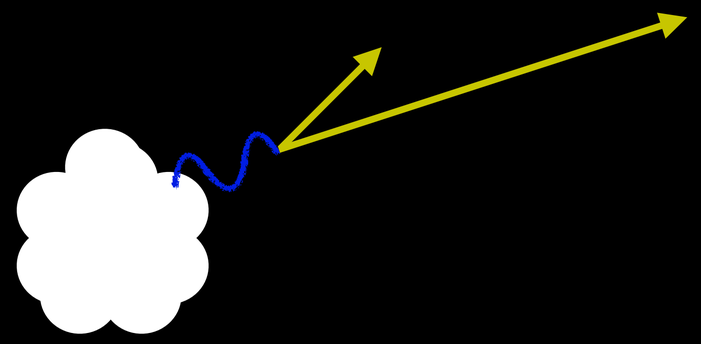


Compton scattering

In-medium pair production

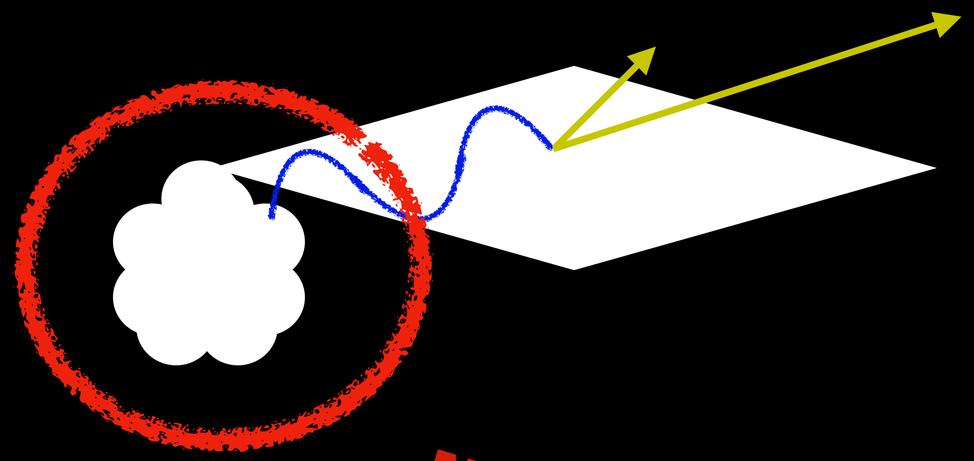
Detector/target dependent

Internal

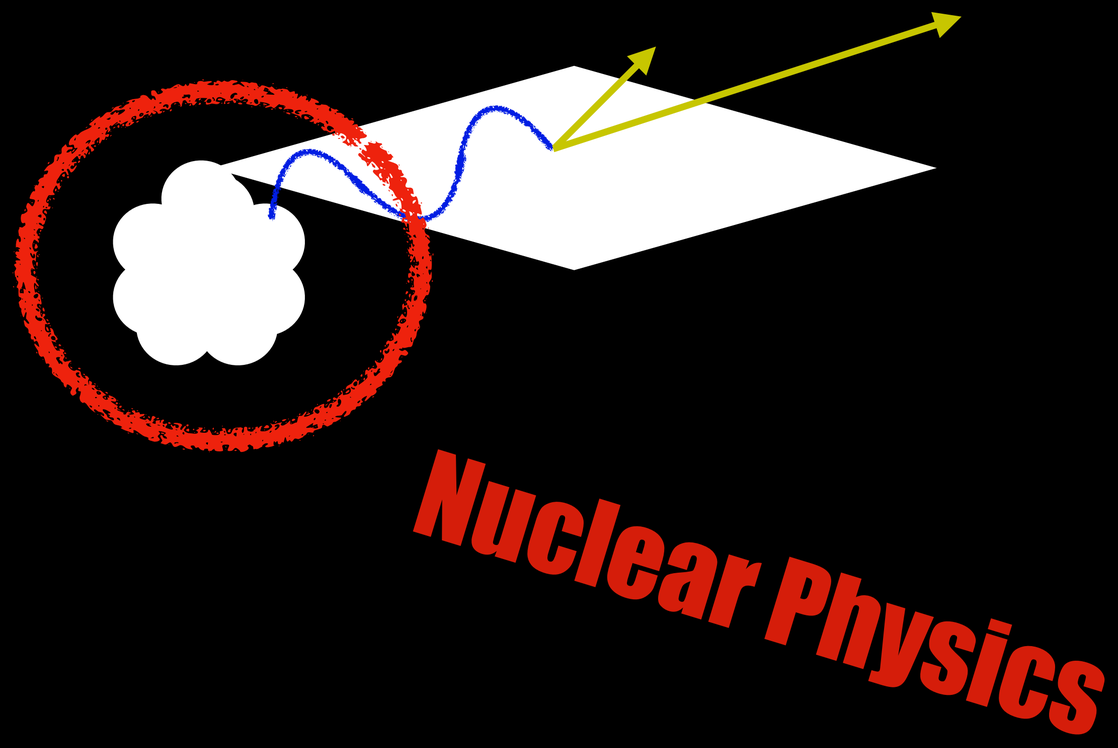


Virtual photon to $e^+ e^-$

Detector independent

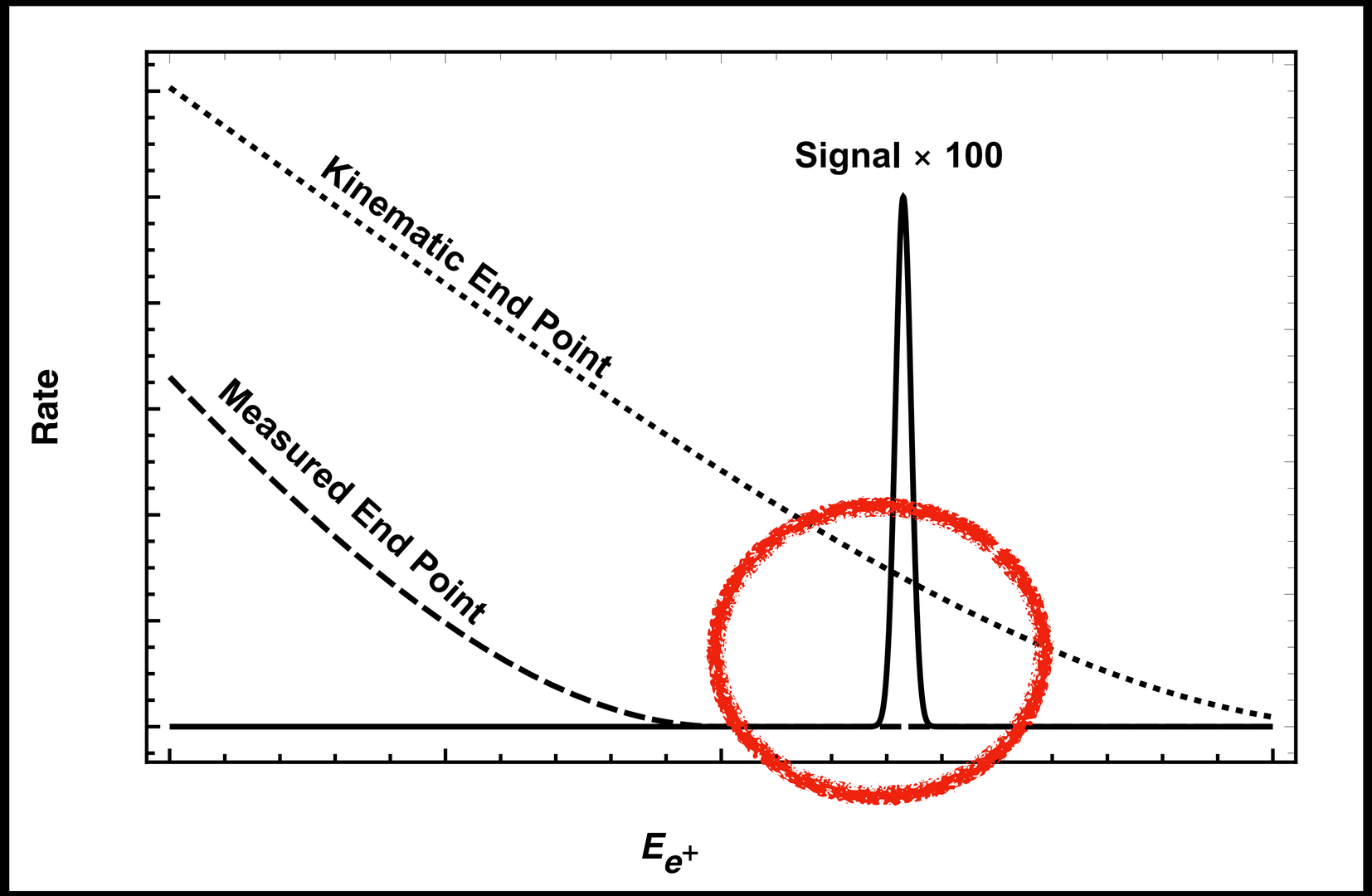


Nuclear Physics



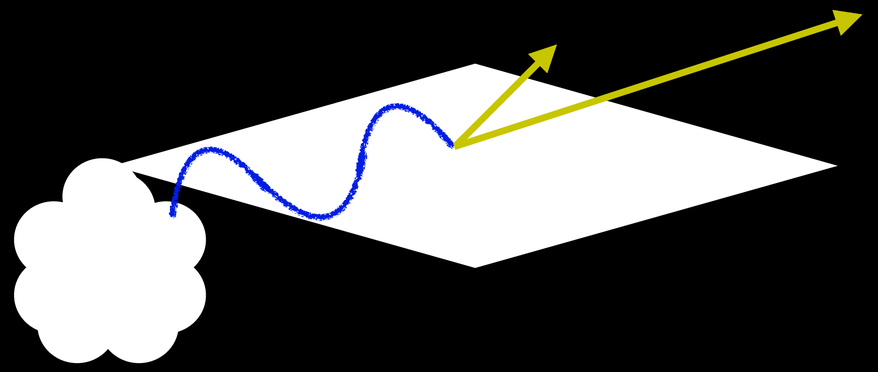
Can we absorb all the nuclear physics into measurable quantities?

Yes, but only near the end point



Thats okay!

External conversion



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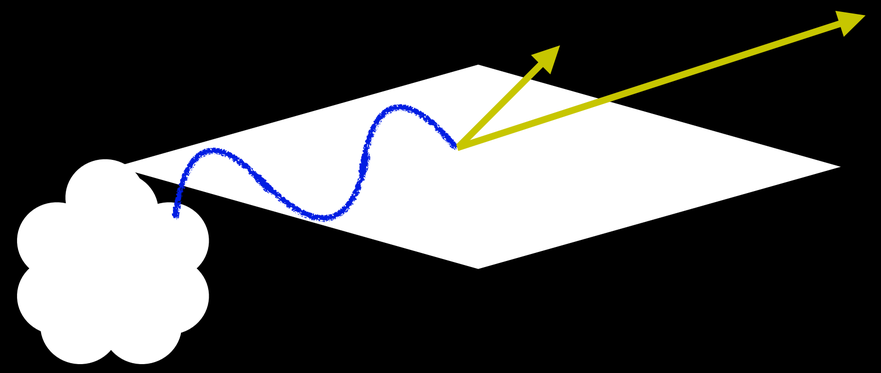
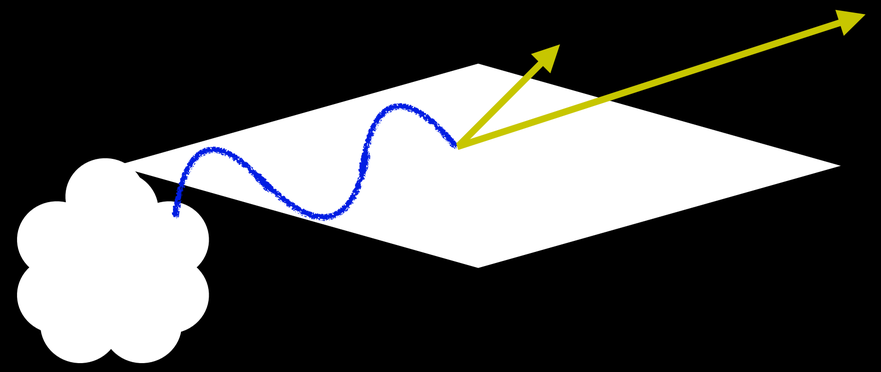


$$\frac{\mathrm{d}\Gamma}{\mathrm{d}E_+} = \int \mathrm{d}E_\gamma P(E_+ \,|\, E_\gamma) \times \frac{\mathrm{d}\Gamma_{\mathrm{RMC}}}{\mathrm{d}E_\gamma}$$

External conversion



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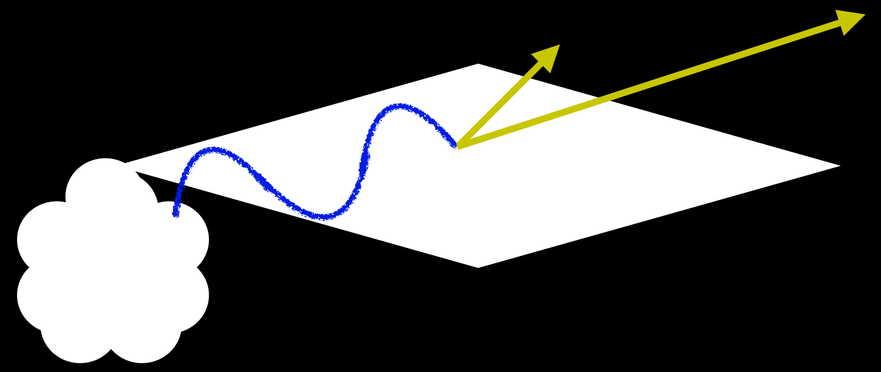
$$P(E_+ \,|\, E_\gamma)$$

External conversion



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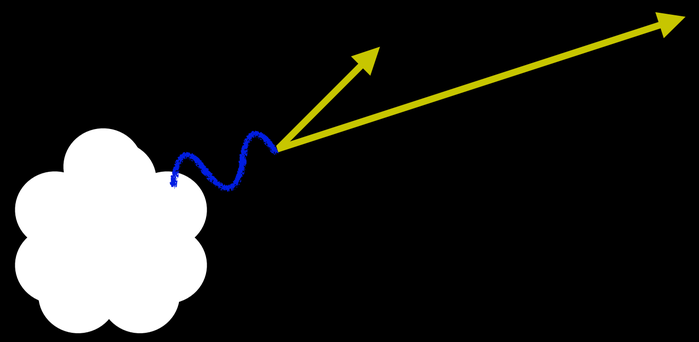
$$P(E_+ \,|\, E_\gamma)$$

$$\langle |\mathscr{M}_0|^2 \rangle_{\mathrm{Nuclear}}$$

How can we relate internal and external conversion?

Internal conversion



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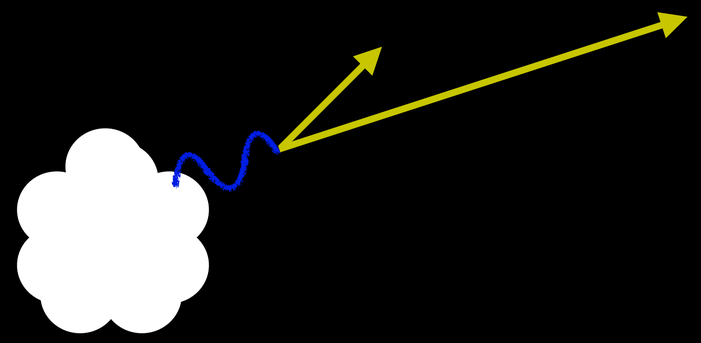
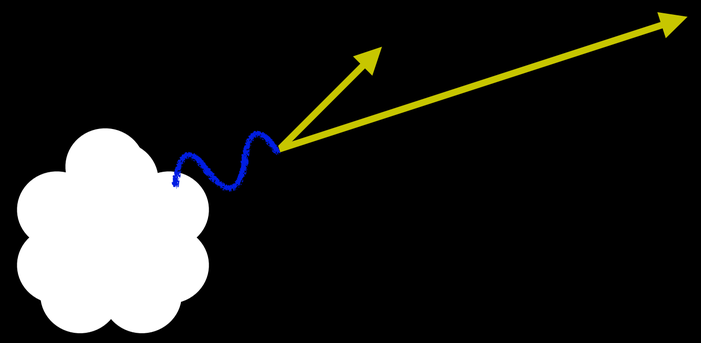


$$\mathrm{d}\Gamma = \mathrm{d}\Phi_4 \mathscr{M}_*^{\mu\nu} L_{\mu\nu} \times \frac{\sqrt{4\pi\alpha}}{m_*^4}$$

Internal conversion



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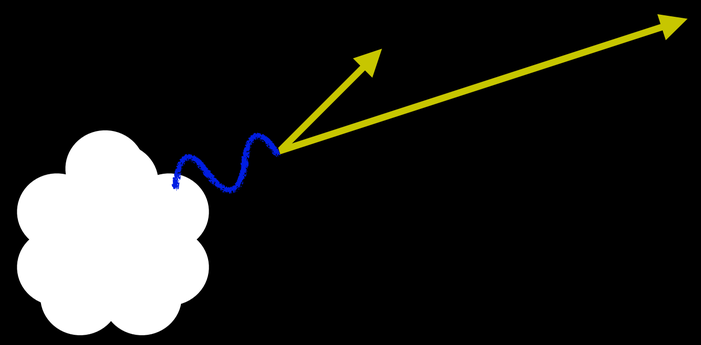

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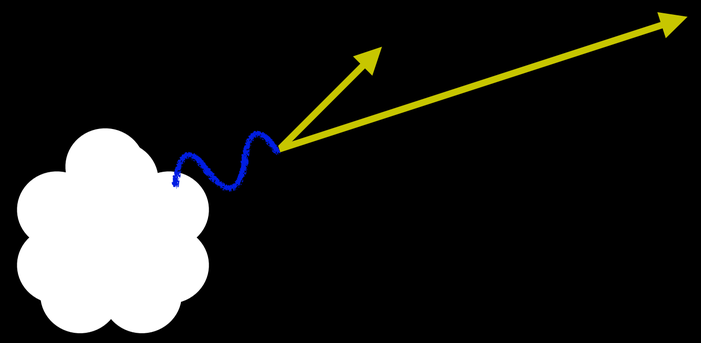
Off-shell

$$\langle |\mathscr{M}_*|^2 \rangle_{\text{Nuclear}}$$

Internal conversion



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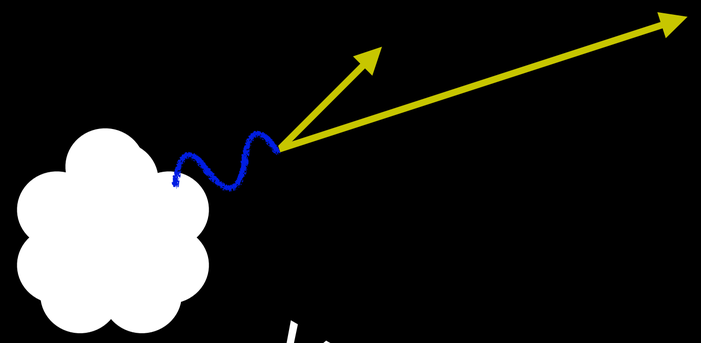
4-body phase space

Off-shell

$$\langle |\mathscr{M}_*|^2 \rangle_{\text{Nuclear}}$$

Internal conversion



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Virtual photon mass

Off-shell

4-body phase space

$$\langle |\mathscr{M}_*|^2 \rangle_{\mathrm{Nuclear}}$$

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``off-shell RMC" phase space

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``off-shell RMC" phase space

Virtual photon mass

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``off-shell RMC" phase space **Virtual photon mass** **Electron phase space**

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``off-shell RMC" phase space

Virtual photon mass

Electron phase space

Do these integrals

$$\mathrm{d}\Gamma = \mathrm{d}\Phi_4 \mathscr{M}_*^{\mu\nu} L_{\mu\nu} \times \frac{\sqrt{4\pi\alpha}}{m_*^4}$$

$$\mathrm{d}\Phi_4 = \mathrm{d}\Phi_3(\gamma^*, \nu, \mathrm{Mg}) \times \frac{\mathrm{d}m_*^2}{2\pi} \times \mathrm{d}\Phi_2(\gamma^* \to e^+e^-)$$

``off-shell RMC" phase space

Virtual photon mass

Electron phase space

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NUCLEAR PHYSICS

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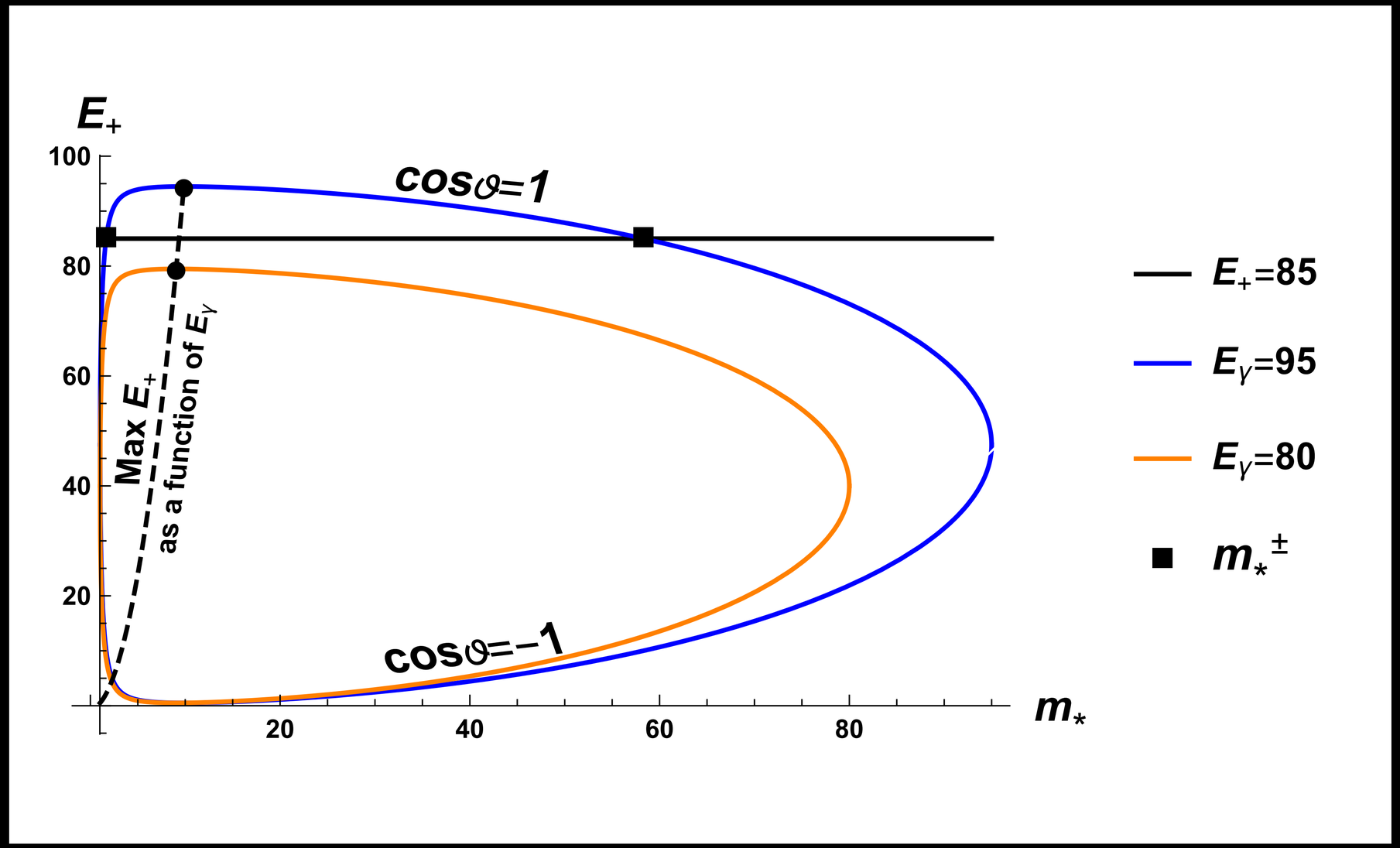
Phase space to integrate over



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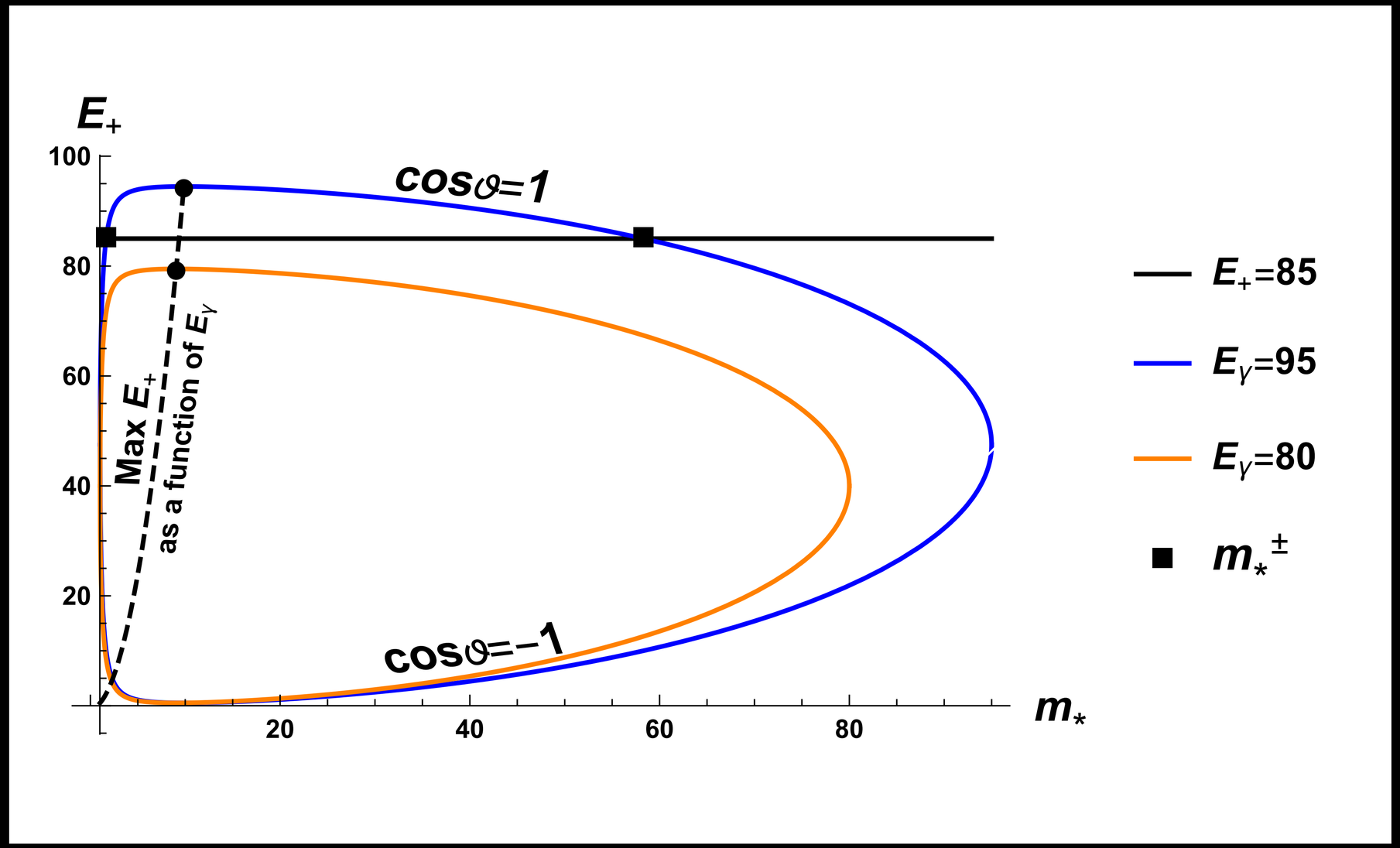
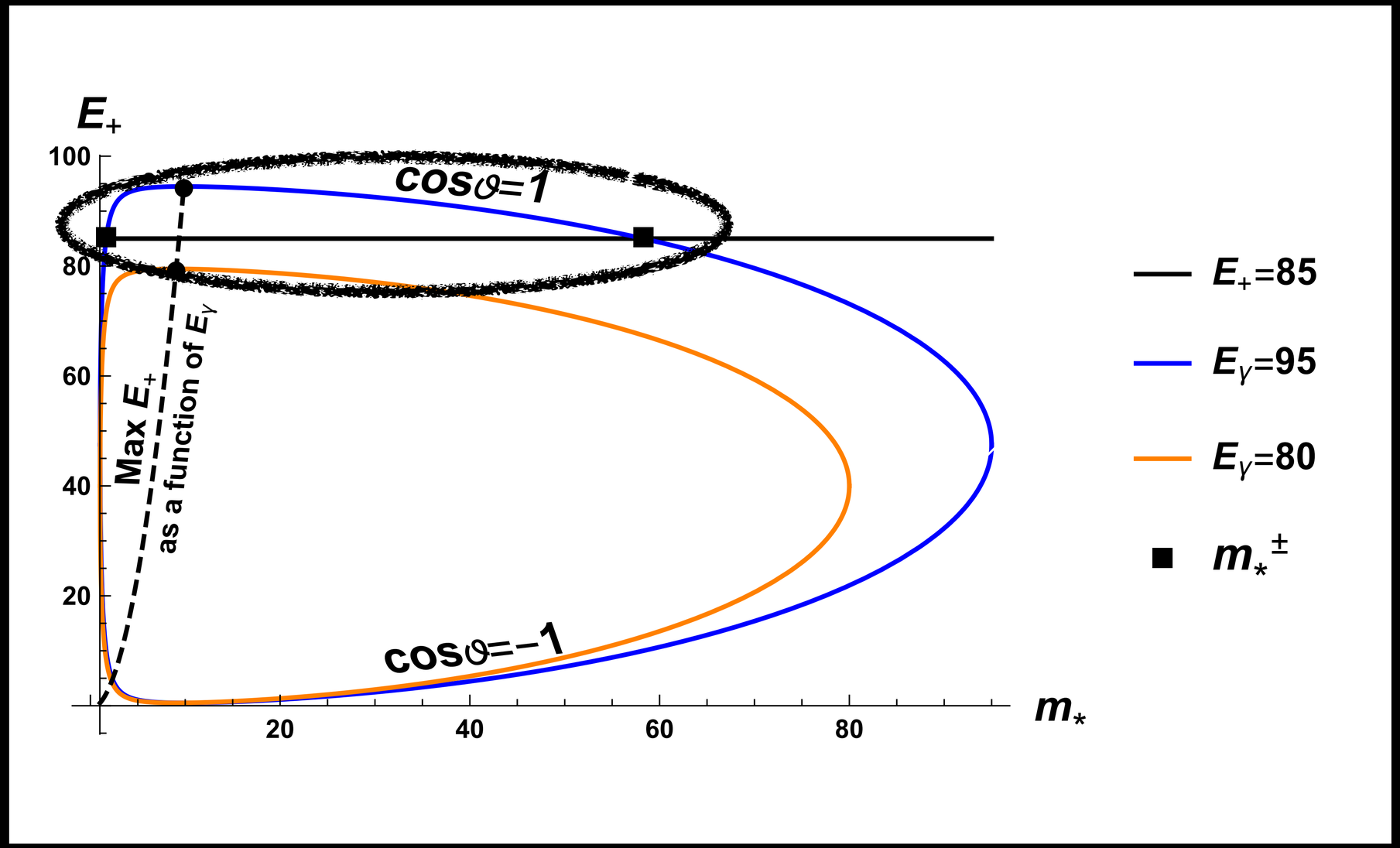


Lets zoom in

Phase space to integrate over



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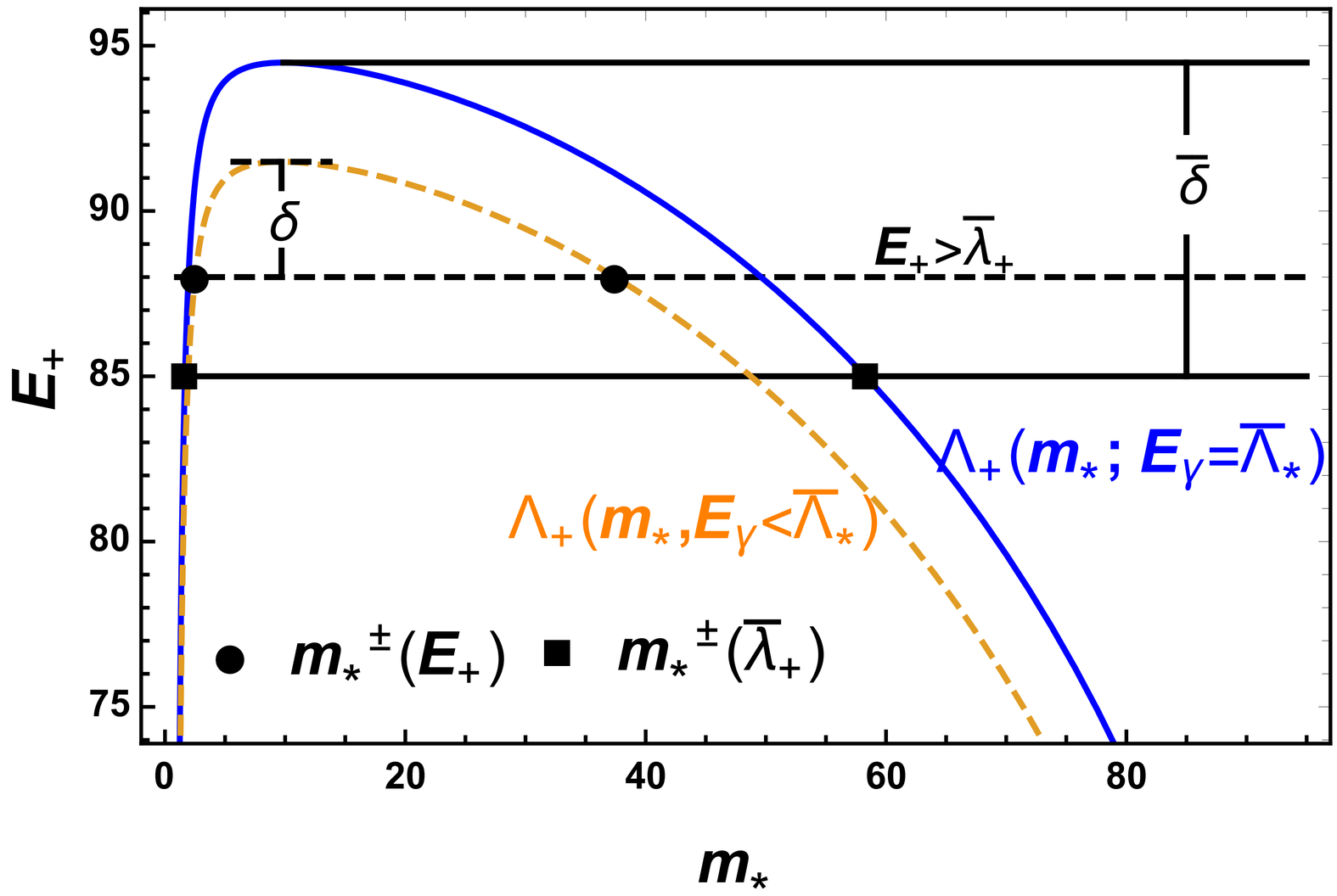

$$\delta = \frac{E_\gamma - m_e - E_+}{E_\gamma}$$

$$m_* \lesssim 2\sqrt{\delta}\, m_\mu$$

$$m_*^2 \sim \delta$$

$$1 - \cos\vartheta \sim \delta$$

$$\mathrm{d}\Gamma = \underbrace{\mathrm{d}\Phi_3 \langle |\mathscr{M}_0|^2 \rangle}_{\mathrm{d}\Gamma_{\mathrm{RMC}}} \times \underbrace{\int \frac{\mathrm{d}m_*^2}{2\pi} \times \mathrm{d}\Phi_2 \text{ (stuff)}}_{P_{\mathrm{int}}(E_+|E_\gamma) + O(\delta)}$$

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More precise statements in progress

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More precise statements in progress

Work ongoing with Richard Hill

If you measure the real photon spectrum you can predict the positron spectrum near the end point

or

The nuclear physics is in the photons

Many thanks to

Pavel Murat, Michael Mackenzie, & Stefano Di Falco

That's all Folks!

Coulomb-induced asymmetry

Near end point:

- Positrons have high energy ~90 MeV
- Electrons have low energy ~ few MeV
- Photon has a virtuality of ~ 10s MeV i.e. range of ~10 fm

We have *low energy* electrons

They are within a short-range of the nucleus.

This is very similar to beta-decay

Coulomb-induced asymmetry

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}E_+} = \int \mathrm{d}E_\gamma \frac{\mathrm{d}\Gamma}{\mathrm{d}E_\gamma} \times P_{\mathrm{int}}(E_+ \,|\, E_\gamma)$$

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(as in beta decay)

Slow electrons are more likely
to be produced in a strong field

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Enhances rate near end point!

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