FUNDAMENTALS OF RADIO ASTRONOMY II

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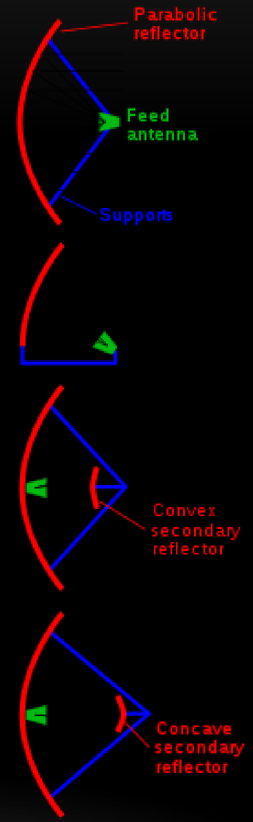
OPTICAL DESIGN OF RADIO TELESCOPES

Prime focus

Off-axis (or offset feed)

Cassegrain

Gregorian



TYPICAL RADIO TELESCOPE ANTENNAS

- **Parabolic**: The primary mirror is a parabola
- **Steerable**: The antenna can move in two angular directions to track sources across the sky.
- **Alt-Az Mount**: The antenna tracks in altitude (elevation) and azimuth
- **Cassegrain focus**: A secondary mirror (subreflector) is placed in front of the prime focus of the primary reflector and focuses the radio waves to a receiver located behind the main reflector.
- **On-axis**: The antenna axis is the same as the optical axis, resulting in a symmetric antenna.

A TYPICAL RADIO TELESCOPE: ATCA

EXCEPTIONS: OFF-AXIS DESIGNS

In on-axis designs, the secondary and the feed-legs block radiation and cause side lobes due to diffraction. Off-axis designs are more efficient. An excellent example is the Green Bank Telescope (GBT)

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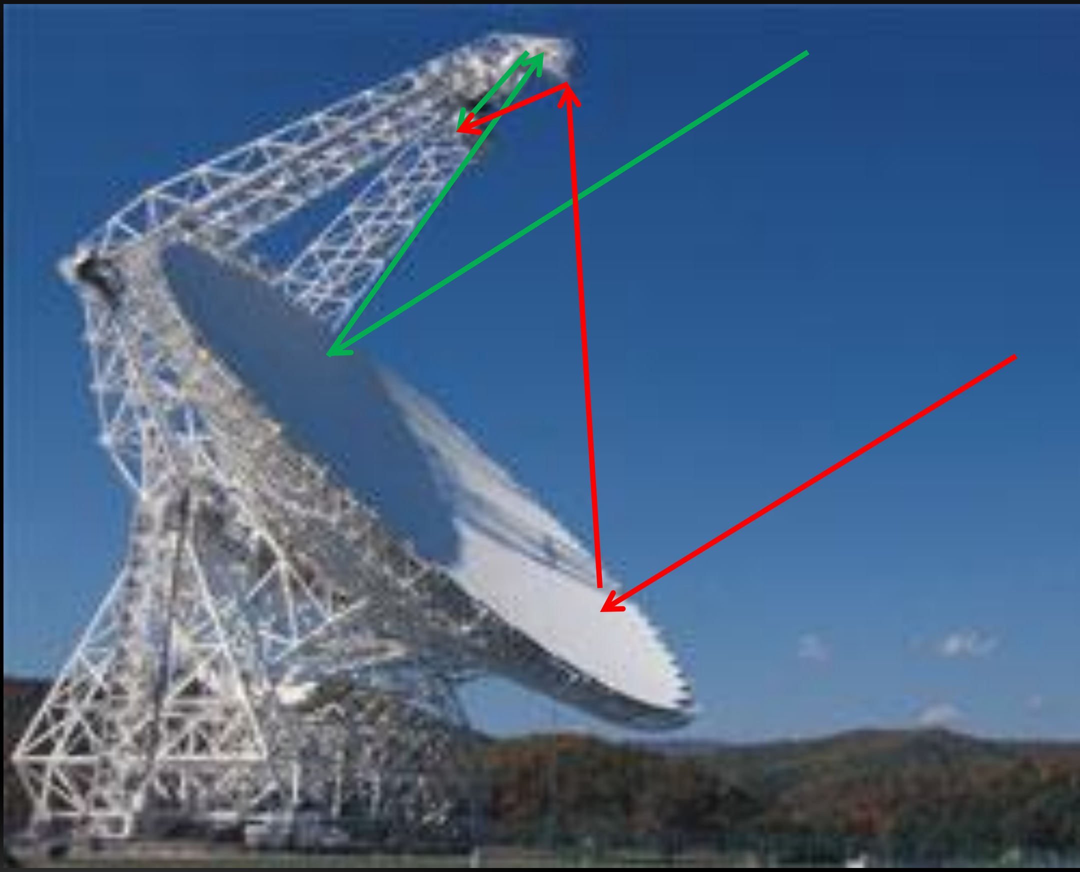


EXCEPTIONS: GREGORIAN FOCUS

For some applications, particularly off-axis designs, it is sometimes more practical to place the secondary behind the prime focus in a Gregorian-foucs scheme rather than in front of the prime focus (Cassegrain-focus)

EXCEPTIONS: ARECIBO: SPHERICAL AND NON-STEERABLE (TRANSIT)

For huge telescopes it may be impractical or expensive to build a steerable surface. The 300 m Arecibo telescope has a fixed spherical surface that can only observe sources passing nearly overhead. It focuses to a line rather than a point.

ANTENNA BEAM PATTERNS: DIRECTIVITY AND GAIN

Directivity: If P_{TR} is the total power radiated by an antenna, and $p(\theta,\varphi)$ is the power radiated per unit solid angle in the direction (θ,φ) , then the *directivity function* $D(\theta,\phi)$ is defined as

$$D(\theta,\phi) = 4\pi \frac{p(\theta,\phi)}{P_{TR}}$$

Gain: If P_T is the total power supplied to an antenna, and $p(\theta,\phi)$ is the power radiated per unit solid angle in the direction (θ,ϕ) , then the *power gain function* $G(\theta,\phi)$ is defined as

$$G(\theta,\phi) = 4\pi \frac{p(\theta,\phi)}{P_T}$$

RADIATION EFFICIENCY

The antenna will not transmit all of the power supplied to it, since some power is lost due to ohmic losses. The **"radiation efficiency" η_R** is defined as

$$P_{TR} = \eta_R P_T$$

It follows that the Directivity is related to the Gain by

$$G(\theta, \phi) = \eta_R D(\theta, \phi)$$

EFFECTIVE AREA

Suppose a telescope with a **beam pattern** $f(\theta,\phi) = \frac{D(\theta,\phi)}{D_0} = \frac{G(\theta,\phi)}{G_0}$, where D_0 and G_0 are the maximum values of the Directivity and Gain functions, respectively, receives radio radiation from a source in some direction on the sky (α_0, δ_0). The source has a brightness distribution $S_\nu(\alpha,\delta)$. The total incremental power collected in the frequency range between ν and $\Delta\nu$, and delivered to the input of the receiver is

$$dP_\nu(\alpha_0,\delta_0) = \frac{1}{2}\eta_R\ \Delta\upsilon\ \ A_e(\alpha_0,\delta_0;\theta,\phi)\ \ dS_\upsilon(\alpha,\delta)$$

where ***A_e* is the "effective area"** (reception pattern) and the factor of ½ comes from the sensitivity to a single polarization. Integrating over solid angle yields

$$P_\nu(\alpha_0,\delta_0) = \frac{1}{2}\eta_R\Delta\upsilon\int_0^{4\pi} A_e\,(\alpha_0,\delta_0;\theta,\phi)\ \ I_\upsilon(\alpha,\delta)\ \ d\Omega$$

BRIGHTNESS TEMPERATURE AND ANTENNA TEMPERATURE

In the Rayleigh-Jeans limit, one may define the **Brightness Temperature T_B** as the temperature of the blackbody that would give the same intensity.

$$T_B = \frac{\lambda^2}{2k} I_\nu$$

In a similar way, one may define the **Antenna Temperature T_A** as the temperature of the blackbody that would lead to the equivalent power received by the antenna:

$$P_\nu(\alpha_0, \delta_0) = kT_A(\alpha_0, \delta_0)\Delta\upsilon$$

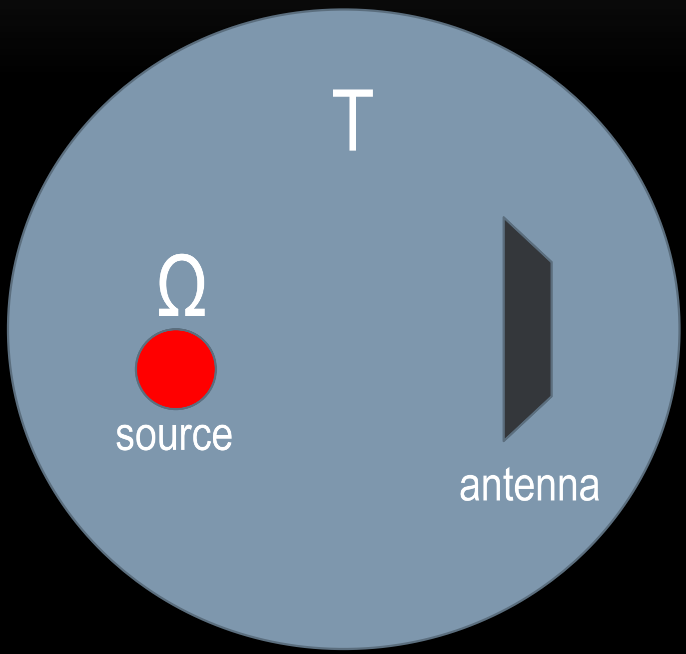
$$T_A(\alpha_0, \delta_0) = \frac{\eta_R}{\lambda^2}\int_0^{4\pi} A_e\,(\alpha_0, \delta_0; \theta, \phi) T_B(\alpha, \delta) d\Omega$$

RELATION BETWEEN GAIN AND EFFECTIVE AREA

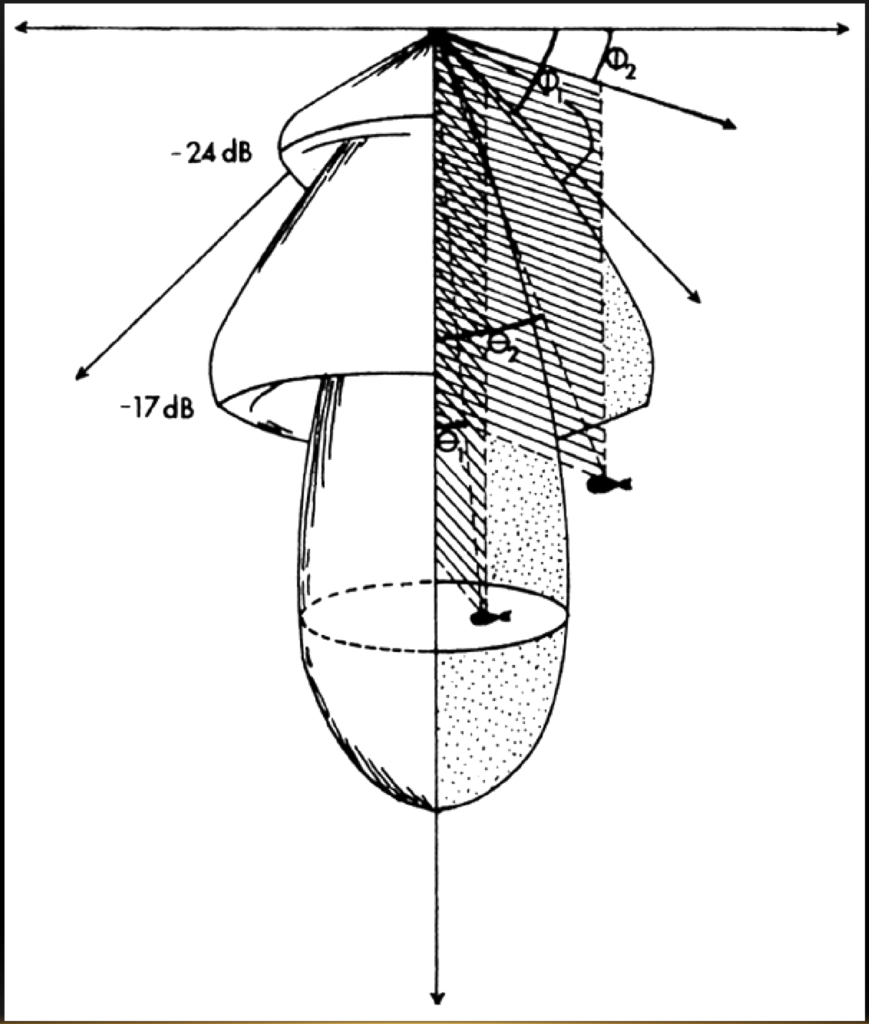
Suppose a small source and an antenna are inside an enclosure of temperature T in thermodynamic equilibrium. The power radiated to the object by the antenna, terminated in a resistor at temperature T, is just $kTG\Omega\Delta\nu/4\pi$. But the power from the object collected by the antenna is given by $\eta_R kTA_e\Omega\Delta\nu/\lambda^2$. Since the system is in thermodynamic equilibrium, the power radiated from the antenna to the object must equal the power radiated from the object to the antenna. Hence:

$$G(\theta,\phi) = \frac{4\pi}{\lambda^2}\eta_R A_e(\theta,\phi)$$

The transmission pattern is identical to the reception pattern!

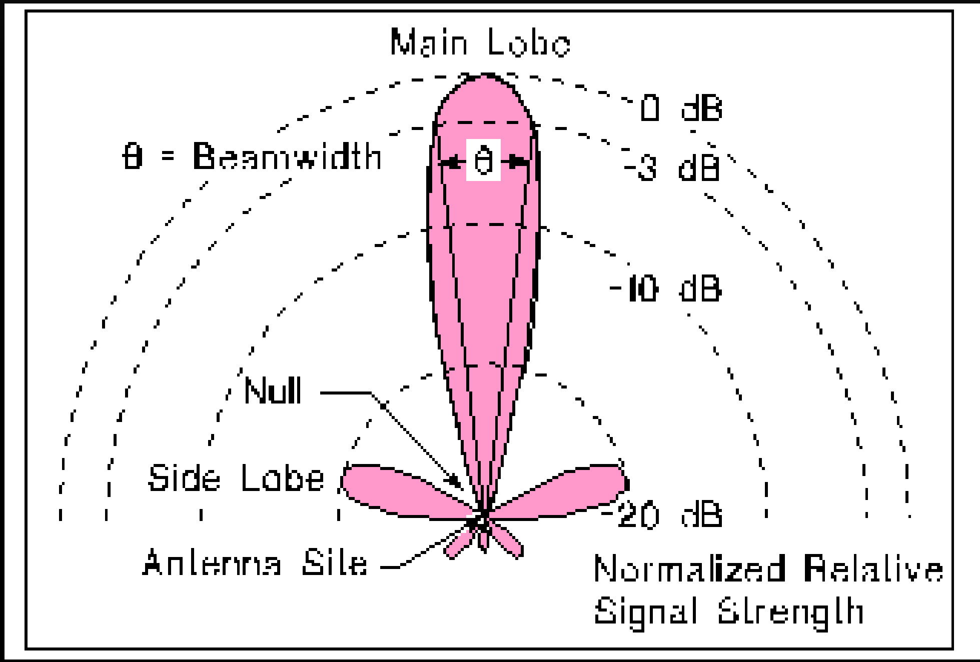


A TYPICAL BEAM PATTERN



SIDE LOBES

The beam pattern from a paraboloid typically has a sharp maximum in the forward direction, but also several smaller maxima in other directions. The solid angle subtended by the maximum down to the nulls is called the "main lobe". The secondary maxima are associated with "side lobes."

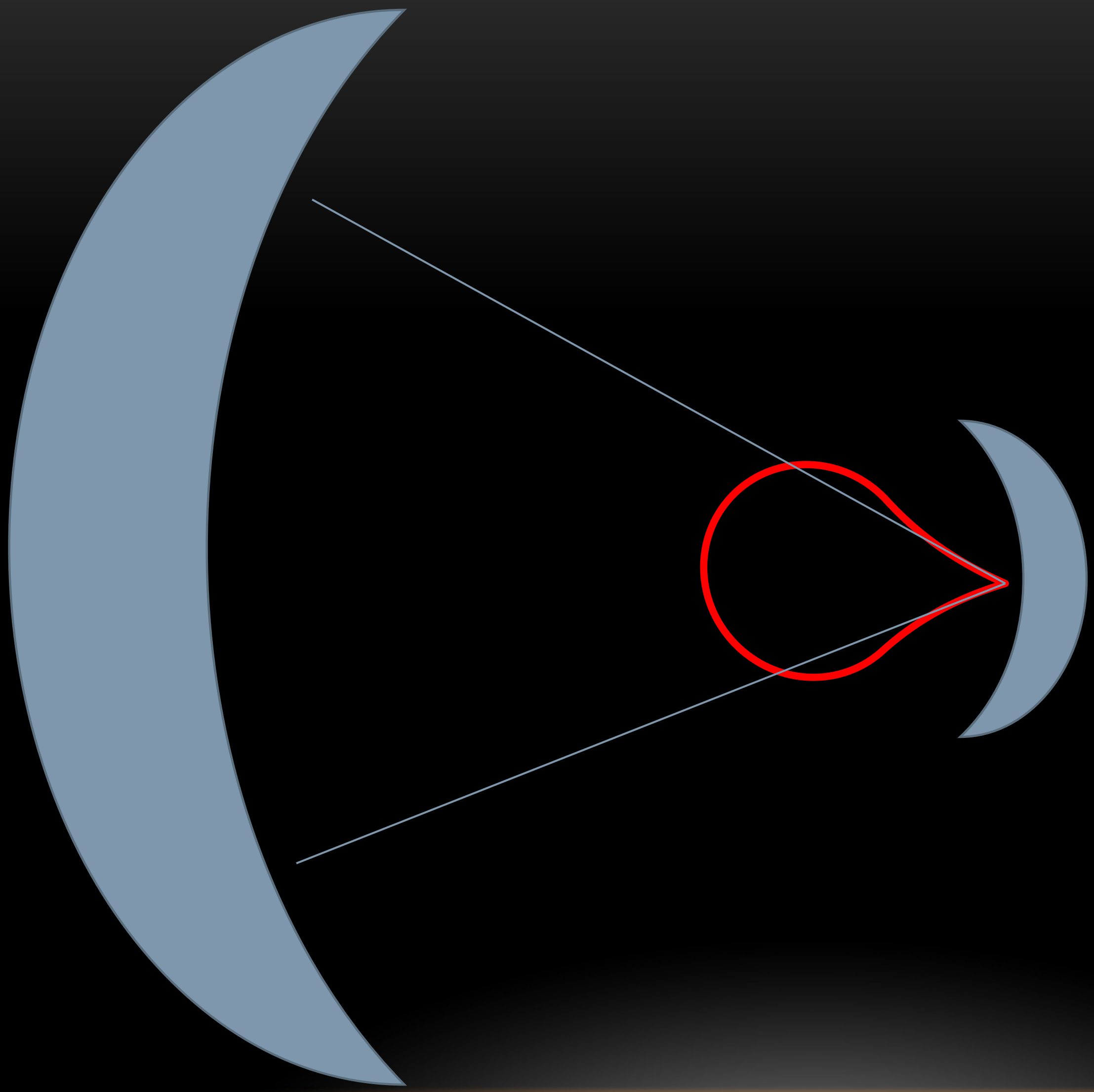


SOURCES IN SIDE LOBES

A source in the side lobe will contribute to the antenna temperature $T_A(\alpha_0, \delta_0)$ even though the source is far away from that position. There are two cases where this is problematic:

1. Bright point source : e.g., a maser.
2. Broad extended emission: e.g., 21 cm H I emission which is extended all over the sky and will always enter the side lobes ("stray radiation").

TAPERING



To minimize side lobes (caused by diffraction at sharp edges) the antenna pattern of the feed is often designed to "underilluminate" the main dish. This technique, called "tapering" minimizes sidelobes at the expense of a slight degradation in angular resolution.

EFFICIENCIES: DEGRADATION OF ANTENNA PERFORMANCE

- Radiation---due to ohmic losses
- Tapering---due to the underillumination of the main dish by the feed
- Scattering---due to scattering off the structures blocking the surface, e.g., feed legs and subreflector
- Spill-over—due to the beam pattern of the feed; not all of the radiation will in fact reflect off the subreflector, some will “spill over” onto the sky.
- Surface Irregularities---a rough surface will scatter radiation; the ideal is a perfectly smooth, reflective paraboloid.

The Total Efficiency is the product of all of these:

$$\eta = \eta_R \eta_T \eta_S \eta_{SO} \eta_{SC} \eta_P$$

SURFACE IRREGULARITIES

Surface roughness affects the antenna efficiency η_P, which is a strong function of frequency.

Suppose an antenna has a surface with an rms roughness (deviation from a perfect paraboloid) of ε. The efficiency due to surface roughness η_P is then given by

$$\eta_P = \exp[-B(\frac{4\pi\varepsilon}{\lambda})^2]$$

Here B is a factor between 0 and 1 that depends on the radius of curvature, increasing as the radius of curvature decreases. For a focal ratio f/D = 0.7, B ~ 0.9. For a surface roughness $\varepsilon=\lambda/30$, the efficiency $\eta_P = 0.85$. Note the very fast deterioration at shorter wavelengths ($\eta_P = 0.24$ for $\varepsilon=\lambda/10$).

Antennas rapidly lose performance if pushed to higher frequencies.

MAIN BEAM TEMPERATURE

For extended sources, especially for sources larger than the main lobe but smaller than the side lobe pattern, it is often desirable to account for the efficiency of the antenna integrated only over the main beam. This is called the "main-beam efficiency"

$$\eta_{MB} = \frac{1}{G_0} \int_0^{MB} G(\theta, \phi) d\Omega$$

The corresponding **Main-Beam Temperature *T_{MB}*** is defined as folllows:

$$T_A = \eta_{MB} T_{MB}$$

The main-beam temperature is the brightness temperature of a uniform disk that fills (only) the main beam and which produces the observed antenna temperature. It is the beam-averaged brightness temperature of an extended source over the solid angle of the main beam.

CONVERTING FLUXES TO MAIN-BEAM TEMPERATURES

Since the main beam temperature is a **surface brightness**, it is a unit of **intensity**. However for compact sources it is often useful to convert from fluxes to main beam temperatures. The formula is

$$T_{MB} = \frac{\lambda^2 F_\nu}{2k\Omega} = 13.6\ K\ (\frac{\lambda mm}{\theta"})^2 (\frac{F_\nu}{Jy})$$

For example, at 3mm wavelength, a source of 1 Jy will produce T_{mb} of 0.1 K in the θ=35” Mopra beam.

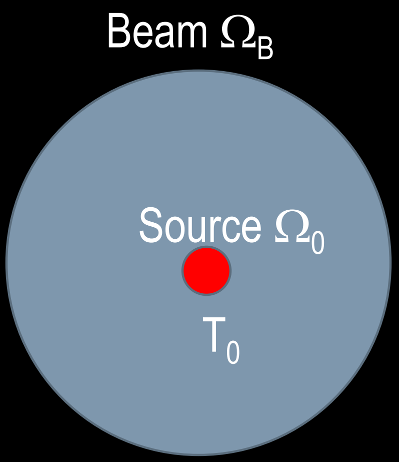
BEAM DILUTION

Consider a **"point source,"** defined as a source that subtends a solid angle Ω_S so small that the beam pattern $f(\theta,\phi)$ does not vary appreciably across the source.

Let us approximate the source as a disk with a brightness temperature T_0 over the solid angle Ω_S and zero anywhere outside this range.

The observed antenna temperature then becomes

$$T_A = \frac{G_0}{4\pi} \int T_B\,(\alpha,\delta)\; d\Omega = \frac{G_0}{4\pi} T_0 \Omega_S$$



Defining the **beam solid angle**

$$\Omega_B = \frac{4\pi}{D_0} = \int_0^{4\pi} f(\theta,\phi) d\Omega$$

We find $$T_A = \eta_R T_0 \frac{\Omega_S}{\Omega_B}$$

The antenna temperature is the average brightness temperature of the source, "diluted" by the ratio of the source solid angle to the beam solid angle.

This ratio is sometimes called the filling factor $\phi = \frac{\Omega_S}{\Omega_B}$

THE RADIO MAP IS A CONVOLUTION OF SOURCE BRIGHTNESS WITH ANTENNA PATTERN

The effect of the finite resolution of the antenna is to “smear” the true brightness distribution on the sky.

For small angles, maps are gridded onto a rectangular grid using the transformation

$$\xi = \alpha \cos \delta_0 \qquad \text{and} \qquad \eta = \delta - \delta_0$$

The element of solid angle in these coordinates is $d\Omega = d\xi\ d\eta$

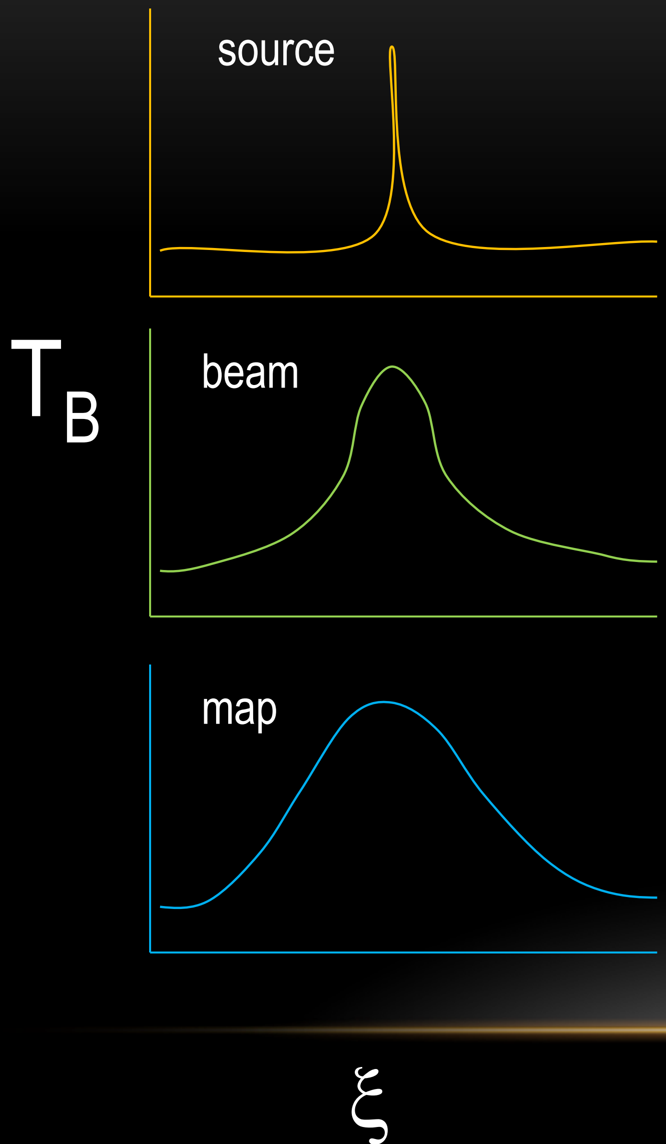
Using this substitution we can rewrite the equation for antenna temperature

$$T_A(\xi_0, \eta_0) = \int G\,(\xi - \xi_0, \eta - \eta_0)\, T_B(\xi, \eta)\, d\xi\ d\eta$$

This is a convolution equation (or an unnormalized cross-correlation integral).

The observed antenna temperature distribution is the convolution of the TRUE brightness distribution with the antenna beam pattern.

CONVOLUTION WITH BEAM



A map of a point source will yield exactly the beam pattern.

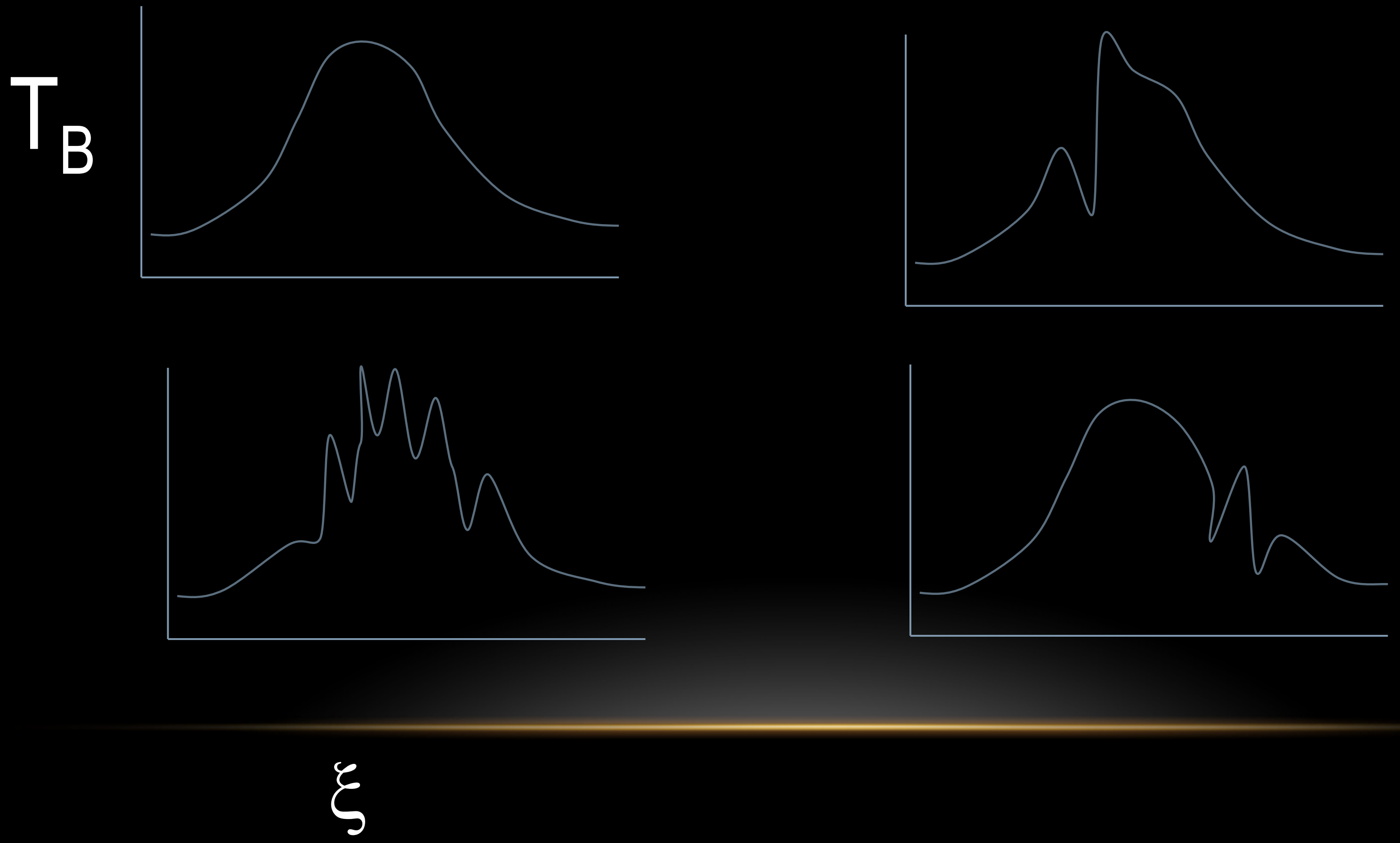
A map of an extended source will be broadened by the beam. Suppose the beam and the source distributions are approximately Gaussian. If the angular size, say FWHM, of the beam is θ_B and that of the source is θ_S, then the observed angular size in the map θ_{obs} will be:

$$\theta_{obs}{}^2 = \theta_S{}^2 + \theta_B{}^2$$

ANTENNA SMOOTHING

Because the antenna pattern is only sensitive to certain spatial frequencies (of order the size of the main beam), any radio antenna is insensitive to small scale structures.

Each of these true distributions will yield the *same map* for a large main beam.



EFFECTS OF ATMOSPHERE: LOSSY LINE

The atmosphere can be regarded as a “lossy” line at temperature T. Consider a transmission line terminated by resistors and in thermal equilbrium at temperature T. The line will be characterized by a power absorption coefficient κ_ν such that the power absorbed per unit distance is $\frac{dP}{dz} = -\kappa_\nu P$

The power flowing from one end of the line down the resister is $kT\Delta\nu$. Since each point of the line can be considered as being terminated by a resistor from the rest of the line, this must be the power flowing in one direction at all points. The line, however, is lossy and absorbs a power $\kappa_\nu P = \kappa_\nu kT\Delta\nu$.

These two statements can be reconciled only if the line also GENERATES a power $\kappa_\nu kT\Delta\nu$ per unit length . If one then transmits a power P down the line of length L, then

$$\frac{dP}{dz} = \kappa_\upsilon kT\ \Delta\upsilon\ - \kappa_\nu P$$

Solving and writing in terms of brightness temperatures:

$$T_{out} = T_{in} e^{-\kappa_\nu L} + T(1 - e^{-\kappa_\nu L})$$

ATMOSPHERIC SELF-EMISSION

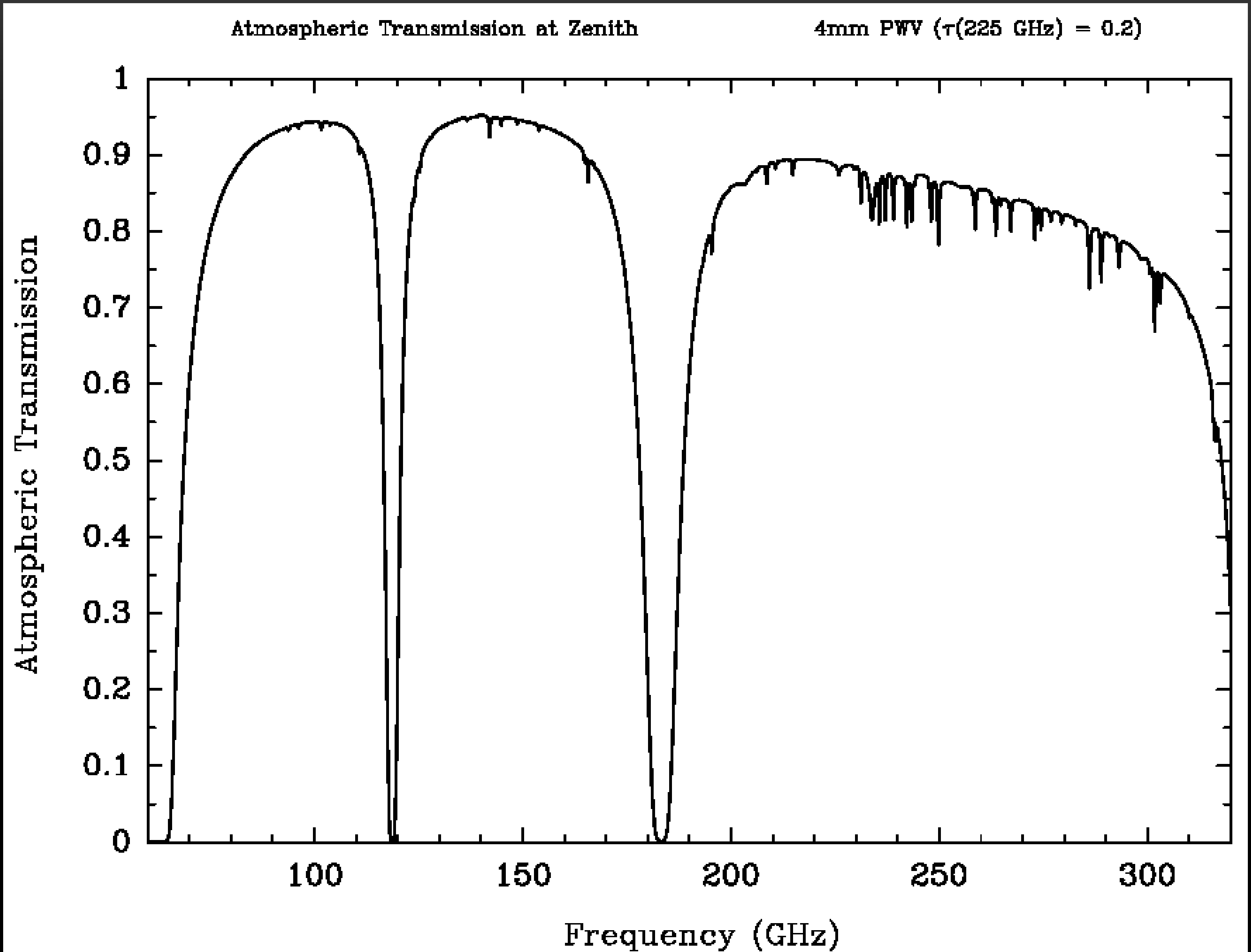
Radiation traveling through the atmosphere has the same equation as a lossy transmission line. The measured antenna temperature T_A will depend on the original, unattenuated source temperature $T_{A,0}$, the temperature of the atmosphere T_{atm}, the absorption coefficient κ_ν, and the path length through the atmosphere L

$$T_A = T_{A,0} e^{-\kappa_\nu L} + T_{atm}(1 - e^{-\kappa_\nu L})$$

Not only is the signal attenuated (first term) but the atmosphere also emits. The second term can often dominate over the first, and it becomes very difficult to subtract off the large, fluctuating atmospheric signal to arrive at the small astronomical signal $T_{A,0}$.

WHEN YOU SHOULD BE CONCERNED BY THE ATMOSPHERE

- The atmosphere is quite transparent at most radio frequencies EXCEPT
  - Long wavelengths (λ > 1 meter): Ionospheric effects dominate
  - Short wavelengths (λ < 2 cm): Atmospheric molecular lines, mostly H_2O vapor, but also O_2 and liquid H_2O
- At these wavelengths, you can minimize the effects of atmosphere by
  - Observing at sources at their highest elevation
  - Observing from excellent, dry sites (Chile, Antarctica, outer space)
  - Selecting relatively transparent windows between molecular bands
  - Observing in good weather, and stable conditions
- By careful calibration the atmospheric effects can be removed to recover $T_{A,0}$. The corrected antenna temperature is usually called T_A^*



Atmospheric Transmission at Zenith
4mm PWV (τ(225 GHz) = 0.2)
Atmospheric Transmission
Frequency (GHz)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
100
150
200
250
300

RADIO SPECTROSCOPY

- ATOMIC LINES
  - Fine Structure
  - Hyperfine structure
  - Recombination
- MOLECULAR LINES
  - Rotational transitions
  - Vibrational transitions
  - Electronic transitions
  - Inversion transitions

EXCITATION TEMPERATURE

A spectral line will emit with a certain brightness temperature above that of the background or continuum temperature. It is related to the beam filling factor ϕ and the **excitation temperature *T_{ex}*** by

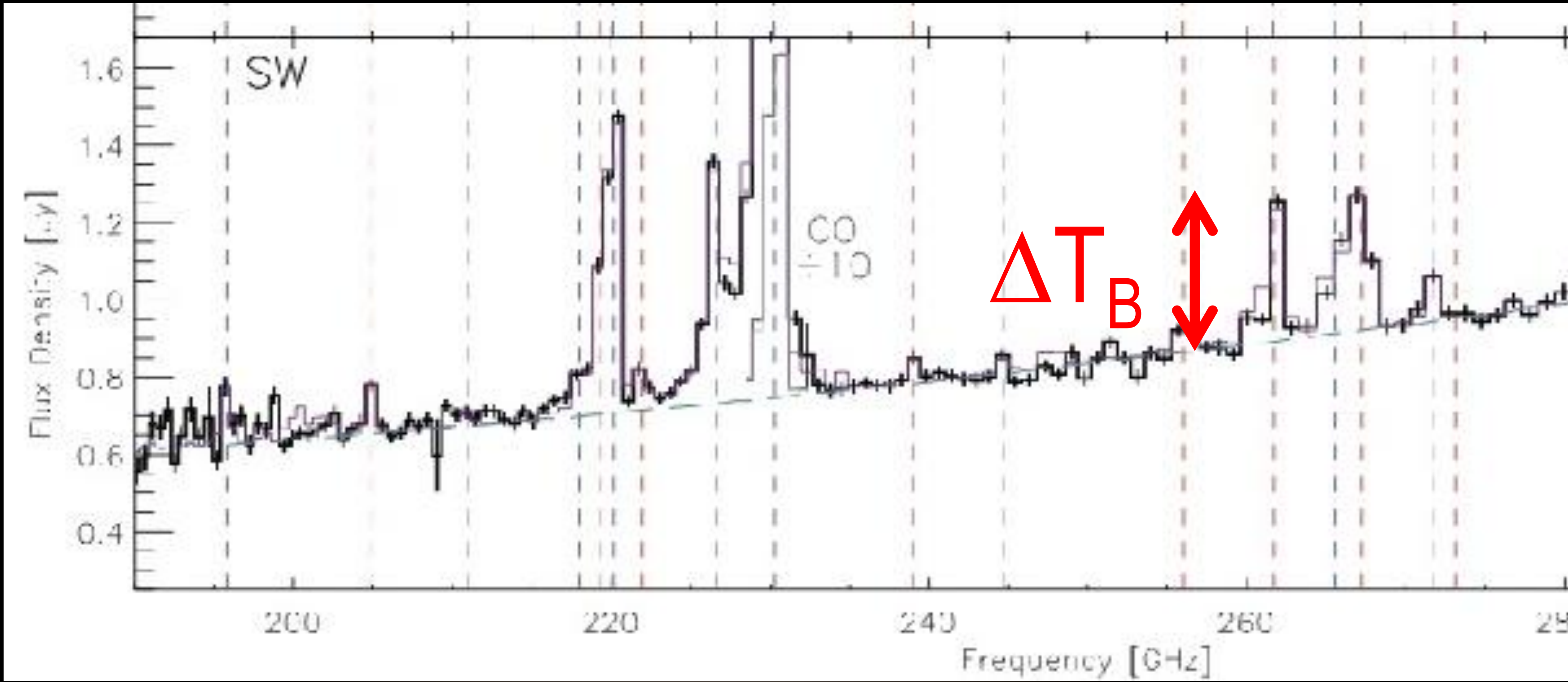
$$\Delta T_{MB} = \phi(T_{ex} - T_{BG})(1 - e^{-\tau})$$

Here T_{ex} is defined as the equivalent temperature at which a Maxwell-Boltzmann distribution at gives rise to the same population levels:

$$\frac{n_u}{n_l} \stackrel{\text{def}}{=} \frac{g_u}{g_l} e^{\frac{-(E_u - E_l)}{kT_{ex}}}$$

Spectral lines emit like a blackbody at temperature *T_{ex}*.

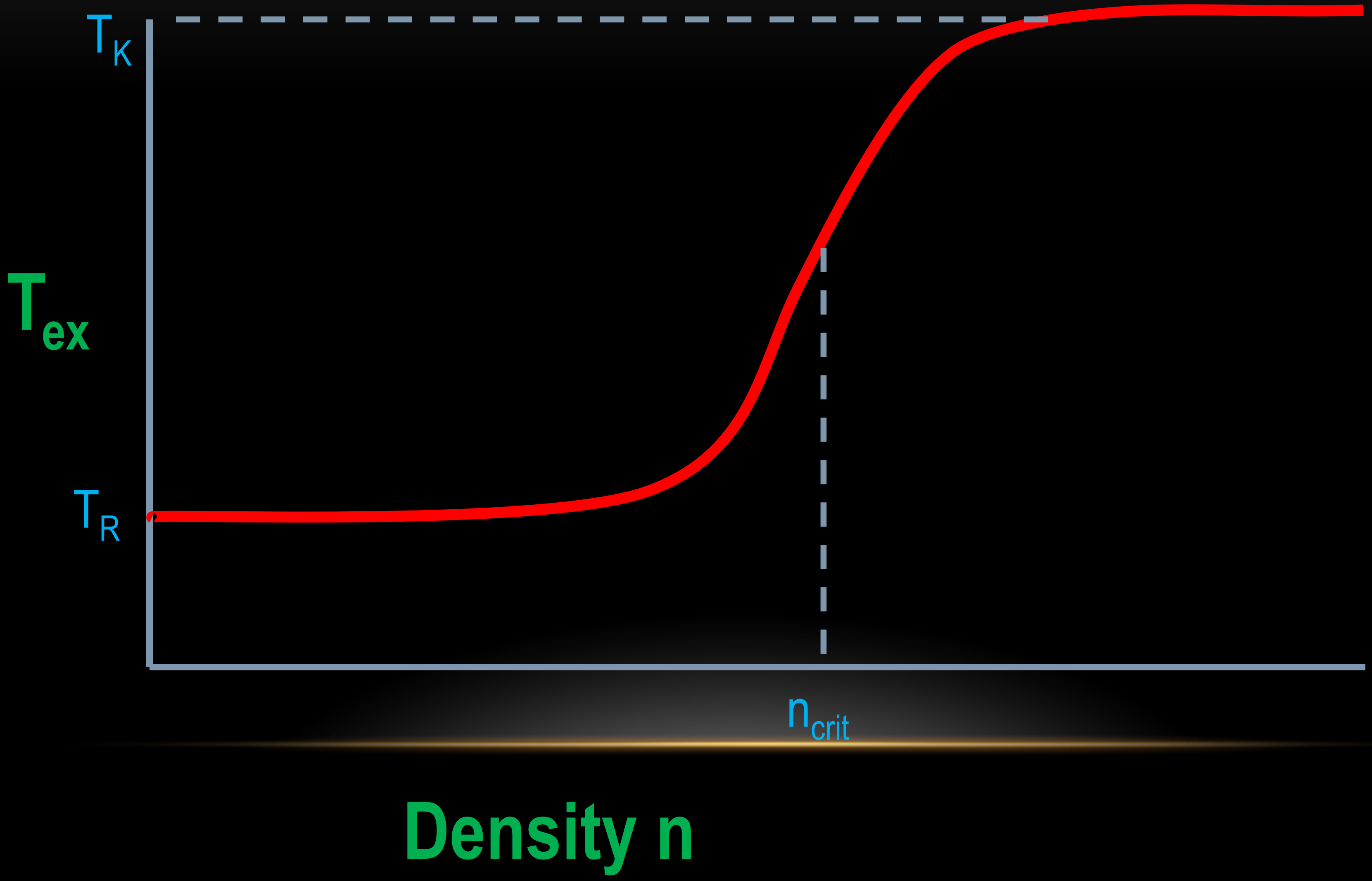
$\Delta T_B = T_{LINE} - T_{CONTINUUM}$



EXCITATION AS A FUNCTION OF DENSITY

- The exact level populations depend on the competition between radiative and collisional excitation.
- The radiative rate is the Einstein A coefficient A_{ul}.
- The collisional rate is the collisional rate coefficient $C_{ul} = n_u \gamma_{ul}(T)$. Classically the rate of collisions is just $n\sigma v$, so $\gamma_{ul} = \sigma v$. In practice this is a good approximation, but quantum effects can also become important.
- In the high-density limit, collisions dominate and the level populations come into equilibrium at the gas kinetic temperature T_K.
- In the low-density limit, radiation dominates and the level populations come into equilibrium at the radiation temperature T_R (the equivalent blackbody temperature to produce the radiation field at the frequency of interest).
- For radio observations T_R is typically smaller than the gas kintetic temperature (usually $T_R \sim 2.7$ K, and $T_K > 10$ K).

THE CRITICAL DENSITY



BRIGHTNESS AND DENSITY

In the radio, spectral lines "turn on" and become bright only when the density equals or exceeds the critical density n_{crit}.

$$n_{crit} = \frac{A}{\gamma}$$

Since the Einstein A coefficient depends on the atom's (or molecule's) electric dipole moment μ and the transition frequency ν as

$$A \sim \mu^2 \nu^3$$

Spectral lines with higher dipole moments and higher frequencies require much higher densities for excitation. Thus, one can choose a line of interest to probe particular density regimes, from very diffuse gas up to very dense gas.

TAKE HOME MESSAGES

- Antennas are designed with many competing considerations; but typically use steerable paraboloids with an alt-az mount and a Cassegrain focus.
- The beam pattern characterizes both the transmission pattern and the reception pattern.
- The observed source brightness is a convolution of the true brightness with the beam pattern. The result is a broader distribution and the lack of knowledge of fine-scale structure.
- At wavelengths where there is significant opacity, the atmosphere both attenuates the astronomical signal and emits its own signal.
- Spectral lines emit like a black body at with a temperature Tex.
- In the radio, spectral lines “turn on” at the critical density.