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Models of Neutrino Masses and Mixings

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Plan of the lectures

- Reminder of basic notions and facts

- Model building ideas and tricks

- Is Tri-Bimaximal mixing a coincidence or a hint?

  Models of TB mixing - Discrete flavor symmetries A4, S4...

- Is “quark lepton complementarity” a coincidence or a hint?

  Bimaximal mixing with corrections from the ch. lepton sector - a model based on S4

In the last decade data on ν oscillations have added some (badly needed) fresh experimental input to particle physics

ν masses are not all vanishing but they are very small

This suggests that ν's are Majorana particles and L is not conserved

ν mixing angles follow a different pattern from quark mixings

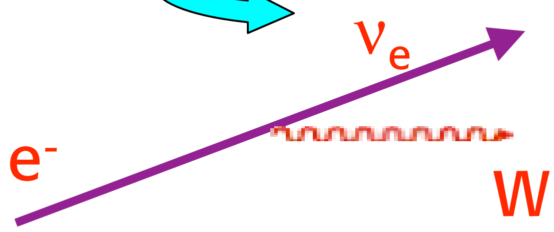
For ν masses and mixings we do not have so far a "Standard Model": many possibilities are still open.

In fact, this is also the case for quarks and charged leptons: we do not have a theory of flavour that explains the observed spectrum, mixings and CP violation.

Thus ν's are interesting because they can provide new clues on this important problem

ν Oscillations Imply Different ν Masses

ν_e: same weak isospin doublet as e^-



flavour mass

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U^+ \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

U: mixing matrix

$U = U_{PMNS}$

Pontecorvo
Maki, Nakagawa, Sakata

e.g 2 flav.

$$\nu_e = \cos\theta\,\nu_1 + \sin\theta\,\nu_2$$
$$\nu_\mu = -\sin\theta\,\nu_1 + \cos\theta\,\nu_2$$

Stationary source:

Stodolsky

$\nu_{1,2}$: different mass, different x-dep:

$$\nu_a(x) = e^{ip_a x}\,\nu_a \qquad p_a^2 = E^2 - m_a^2$$

$$P(\nu_e <-> \nu_\mu) = |< \nu_\mu(L)|\,\nu_e>|^2 = \sin^2(2\theta)\cdot\sin^2(\Delta m^2 L/4E)$$

At a distance L, ν_μ from μ^- decay can produce e^- via charged weak interact's

Solid evidence for solar and atmosph. ν oscillations

Δm^2 values fixed:

$\Delta m^2_{atm} \sim 2.5\ 10^{-3}\ eV^2$,

$\Delta m^2_{sol} \sim 8\ 10^{-5}\ eV^2$

Miniboone has not confirmed LSND

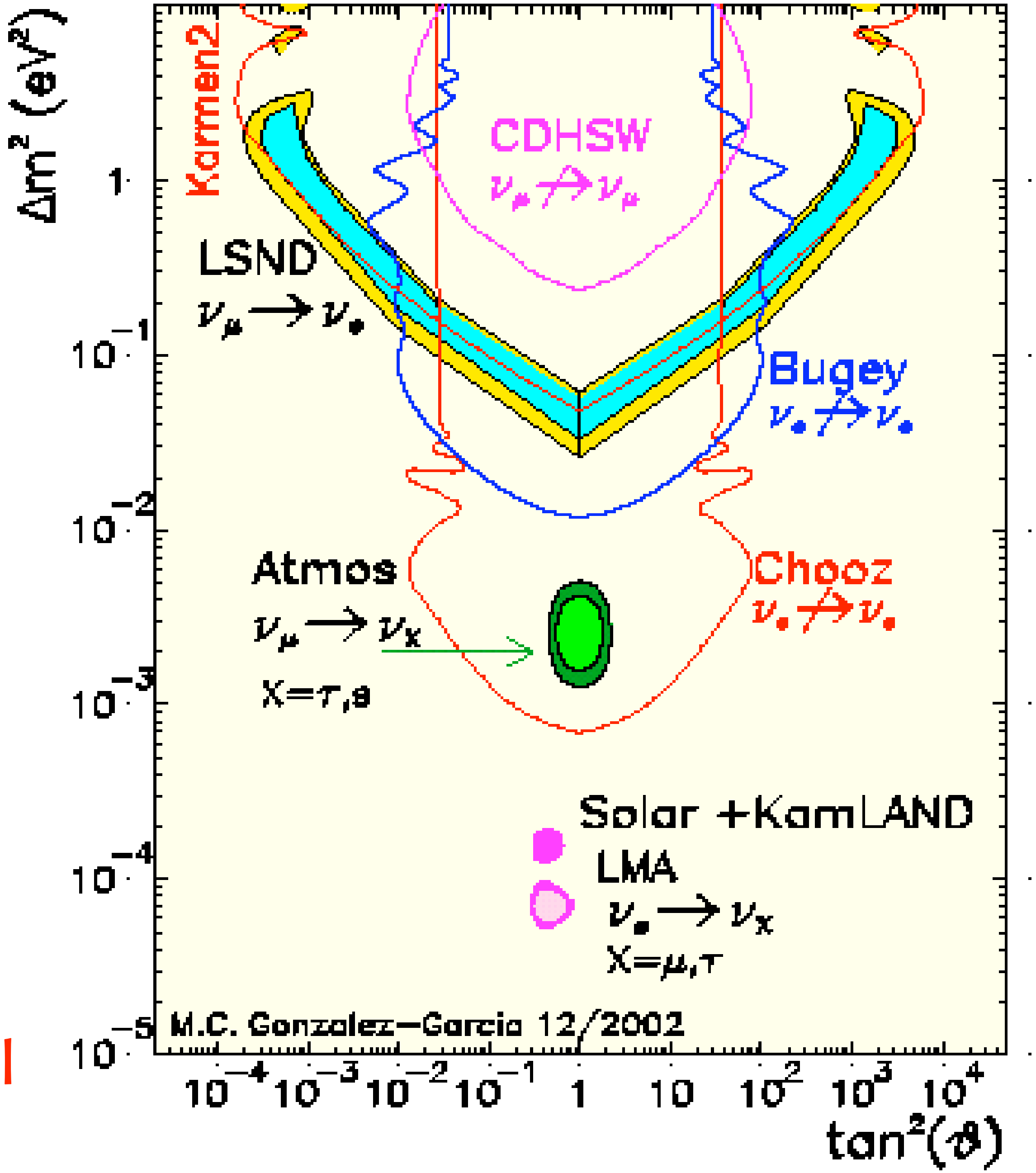
3 ν's are enough!

mixing angles:

θ_{12} (solar) large

θ_{23} (atm) large,~ maximal

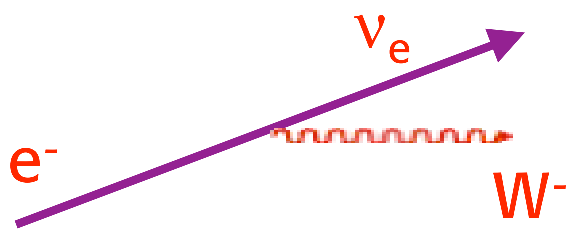
θ_{13} (CHOOZ) small



3-ν Models

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U^+ \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

flavour mass



$U = U_{PMNS}$

Pontecorvo
Maki, Nakagawa, Sakata

In basis where e^-, μ^-, τ^- are diagonal:

δ: CP violation

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \sim$$

s = solar: large

CHOOZ: $|s_{13}| <\sim 0.2$

$$\sim \begin{bmatrix} c_{13}\,c_{12} & c_{13}\,s_{12} & s_{13}e^{-i\delta} \\ \cdots & \cdots & c_{13}\,s_{23} \\ \cdots & \cdots & c_{13}\,c_{23} \end{bmatrix}$$

atm.: ~ max

(some signs are conventional)

In general: $U = U^+_e U_\nu$

$$m_\nu \sim U^* \begin{pmatrix} e^{i\alpha_1}m_1 & 0 & 0 \\ 0 & e^{i\alpha_2}m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} U^+$$

$L^T m_\nu L$

In general 9 parameters:
3 masses, 3 angles,
3 phases

The extra phases appear because the Majorana mass is $L^T L$ and not L^{bar} L.

Note:
- m_ν is symmetric
- phases can be included in m_i

Relation between masses and frequencies:

$P(\nu_e \leftrightarrow \nu_\mu) = P(\nu_e \leftrightarrow \nu_\tau) = 1/2 \sin^2 2\theta_{12} \cdot \sin^2 \Delta_{sun}$

$P(\nu_\mu \leftrightarrow \nu_\tau) = \sin^2 \Delta_{atm} - 1/4 \sin^2 2\theta_{12} \cdot \sin^2 \Delta_{sun}$

$$\Delta_{sun} = \frac{m_2^2 - m_1^2}{4E} L \quad ; \quad \Delta_{atm} = \frac{m_3^2 - m_{1,2}^2}{4E} L$$

here by m^2 we mean $|m^2|$

In our def.: $\Delta_{sun} > 0$, $\Delta_{atm} >$ or < 0

Defining:

$$\Delta m^2_{atm} = m_3^2 - m_2^2 > \text{or} < 0$$

$$\Delta m^2_{sol} = m_2^2 - m_1^2 > 0$$

one has:

$$m_3^2 = \overline{m^2} + \frac{2}{3}\Delta m^2_{atm} + \frac{1}{3}\Delta m^2_{sol}$$

$$m_2^2 = \overline{m^2} - \frac{1}{3}\Delta m^2_{atm} + \frac{1}{3}\Delta m^2_{sol}$$

$$m_1^2 = \overline{m^2} - \frac{1}{3}\Delta m^2_{atm} - \frac{2}{3}\Delta m^2_{sol}$$

and

$$\overline{m^2} >> \left|\Delta m^2_{atm}\right| > \Delta m^2_{sol}$$ degenerate

$$\Delta m^2_{atm} < 0$$ inverse hierarchy

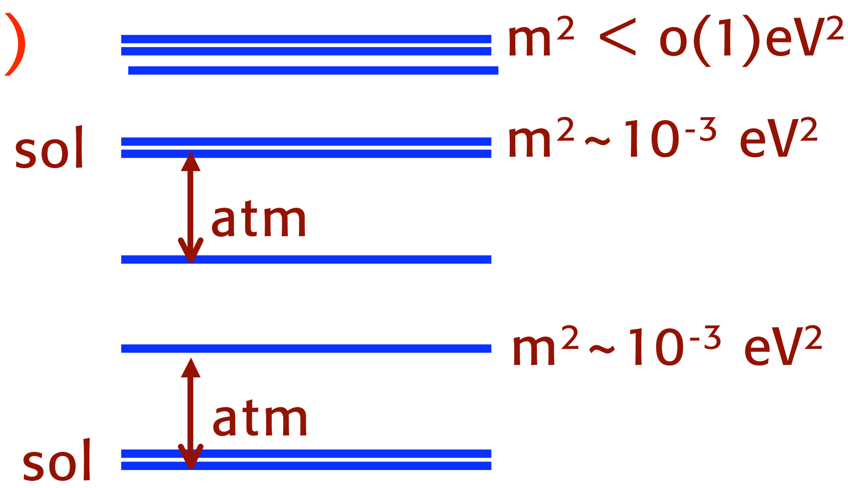
$$\Delta m^2_{atm} > 0$$ normal hierarchy

The current experimental situation on ν masses and mixings has much improved but is still incomplete

- what is the absolute scale of ν masses?
- value of θ_{13}......
- pattern of spectrum (sign of Δm^2_{atm})
- no detection of $0\nu\beta\beta$ (i.e. no proof that ν's are Majorana)

3 light ν's are OK (MiniBoone)

- Degenerate ($m^2 >> \Delta m^2$)
- Inverse hierarchy
- Normal hierarchy



Different classes of models are still possible

Neutrino oscillation parameters

- 2 distinct frequencies
- 2 large angles, 1 small

parameter	best fit	2σ	3σ
$\Delta m^2_{21}\ [10^{-5}\mathrm{eV}^2]$	$7.65^{+0.23}_{-0.20}$	7.25–8.11	7.05–8.34
$\lvert\Delta m^2_{31}\rvert\ [10^{-3}\mathrm{eV}^2]$	$2.40^{+0.12}_{-0.11}$	2.18–2.64	2.07–2.75
$\sin^2\theta_{12}$	$0.304^{+0.022}_{-0.016}$	0.27–0.35	0.25–0.37
$\sin^2\theta_{23}$	$0.50^{+0.07}_{-0.06}$	0.39–0.63	0.36–0.67
$\sin^2\theta_{13}$	$0.01^{+0.016}_{-0.011}$	$\lesssim 0.040$	$\lesssim 0.056$

Schwetz et al '08

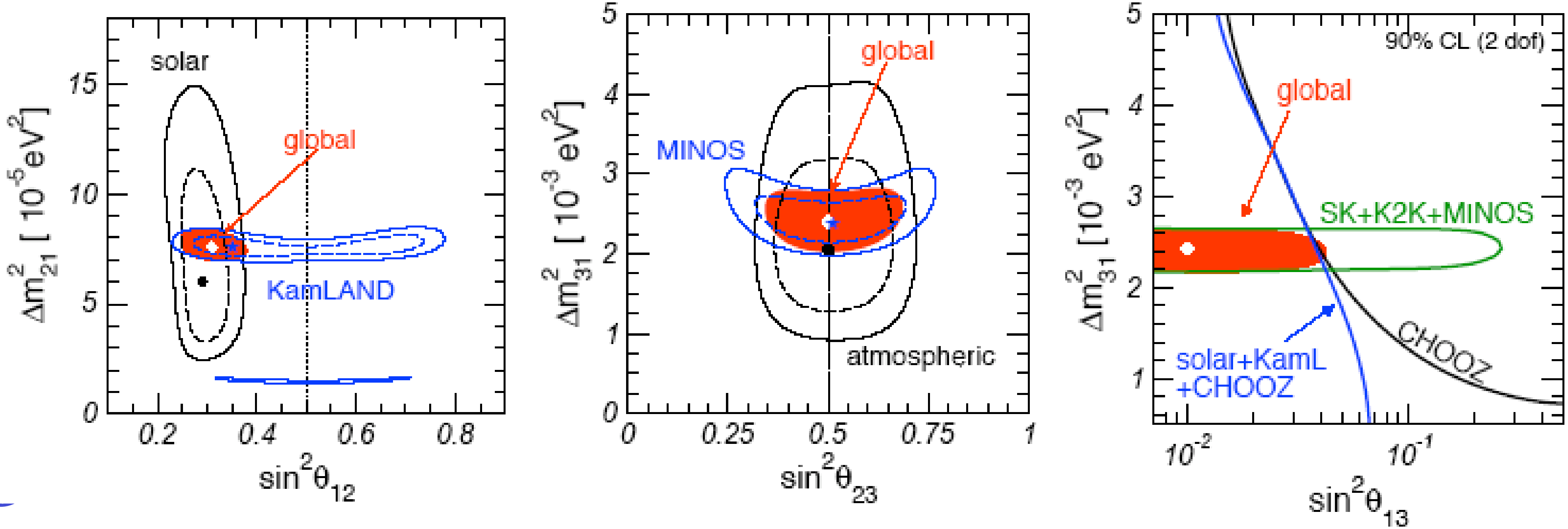
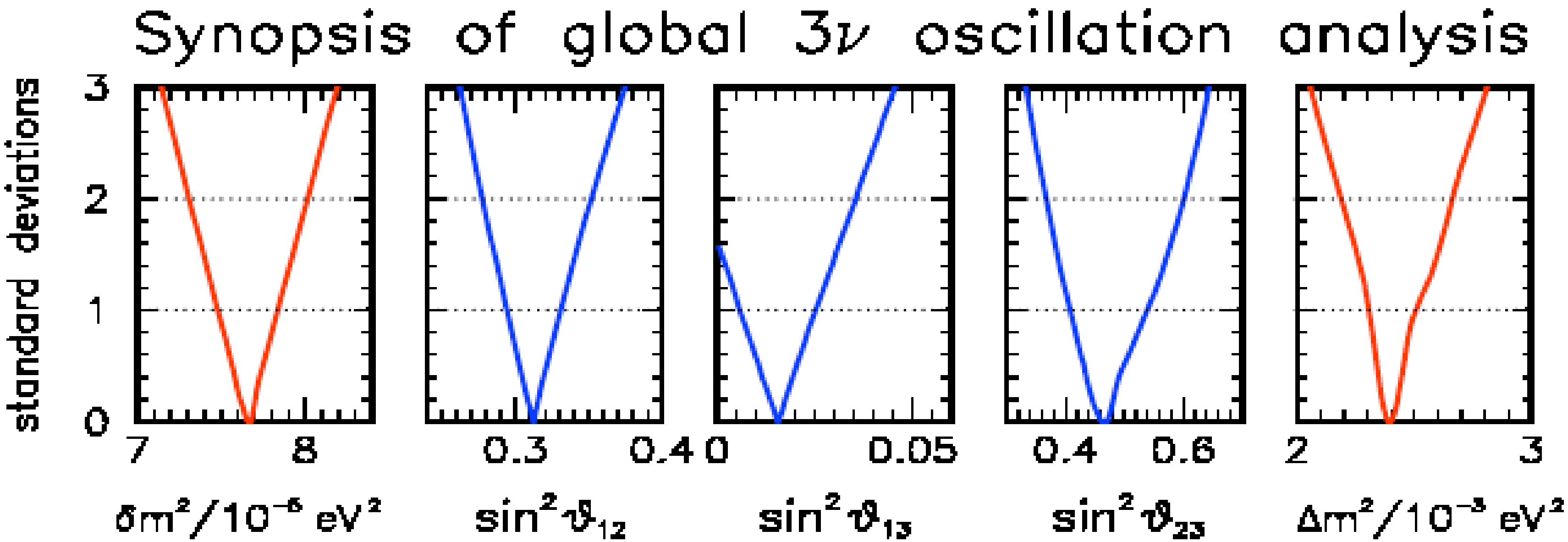


Table 1: Global 3ν oscillation analysis (2008): best-fit values and allowed n_σ ranges, from Ref. [4].

Parameter	$\delta m^2/10^{-5}$ eV^2	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$	$\Delta m^2/10^{-3}$ eV^2
Best fit	7.67	0.312	0.016	0.466	2.39
1σ range	7.48 – 7.83	0.294 – 0.331	0.006 – 0.026	0.408 – 0.539	2.31 – 2.50
2σ range	7.31 – 8.01	0.278 – 0.352	< 0.036	0.366 – 0.602	2.19 – 2.66
3σ range	7.14 – 8.19	0.263 – 0.375	< 0.046	0.331 – 0.644	2.06 – 2.81

Fogli et al '08

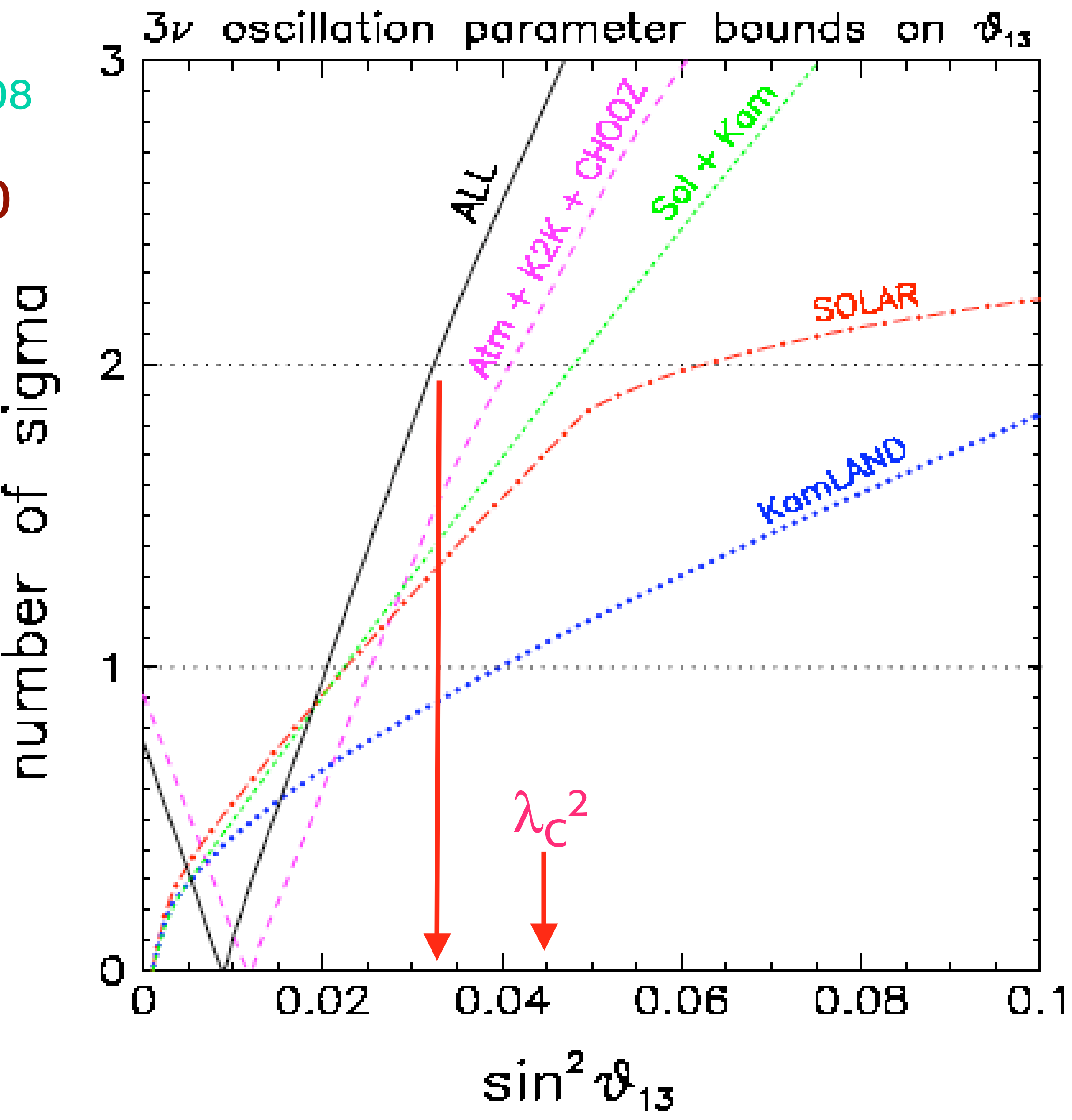


θ_{13} bounds

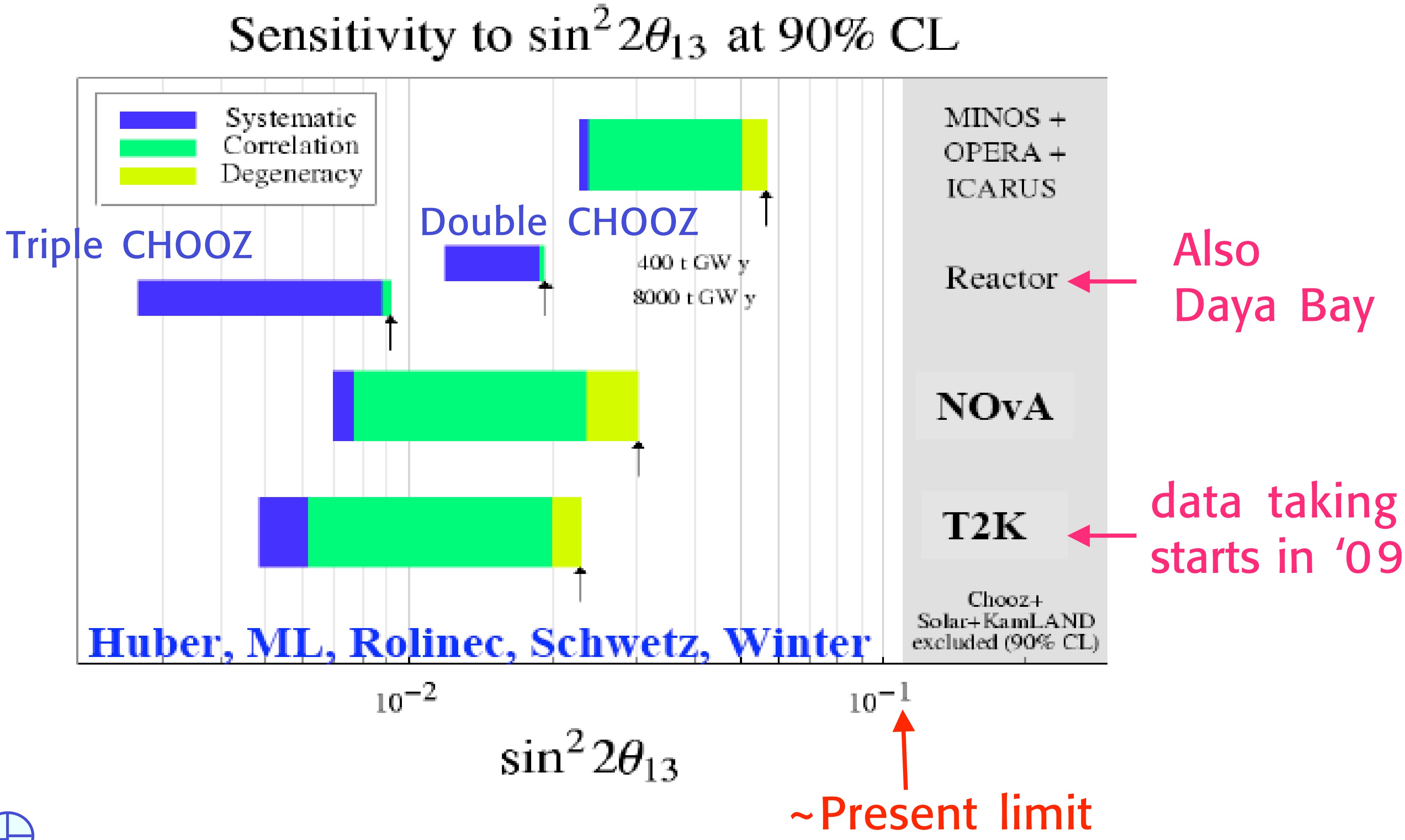
Fogli et al '08

$\sin^2\theta_{13}=0.016\pm0.010$

The 95% upper bound on $\sin\theta_{13}$ is close to $\lambda_C = \sin\theta_C$



Measuring θ_{13} is crucial for future ν-oscill's experiments (eg CP violation)



ν oscillations measure Δm^2. What is m^2?

$\Delta m^2_{atm} \sim 2.5\ 10^{-3}\ eV^2 = (0.05\ eV)^2$; $\Delta m^2_{sun} \sim 8\ 10^{-5}\ eV^2 = (0.009\ eV)^2$

- Direct limits

$m_{"\nu e"} < 2.2$ eV
$m_{"\nu\mu"} < 170$ KeV
$m_{"\nu\tau"} < 18.2$ MeV

End-point tritium β decay (Mainz, Troitsk)
Future: Katrin
0.2 eV sensitivity
(Karsruhe)

$m_{ee} = |\Sigma\, U_{ei}^2\, m_i|$

- $0\nu\beta\beta$ $\quad m_{ee} < 0.2 - 0.7 - ?$ eV (nucl. matrix elmnts)

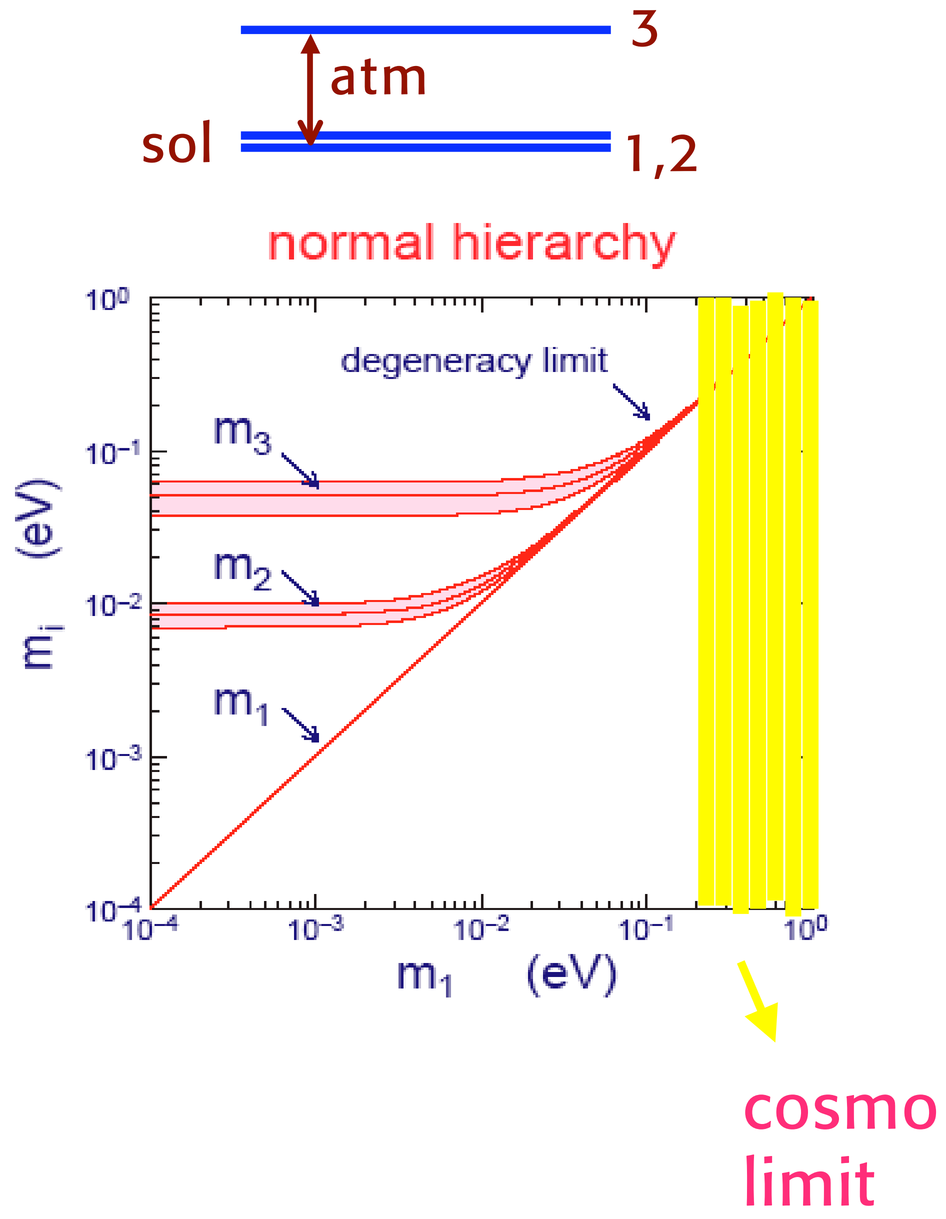
Evidence of signal? Klapdor-Kleingrothaus

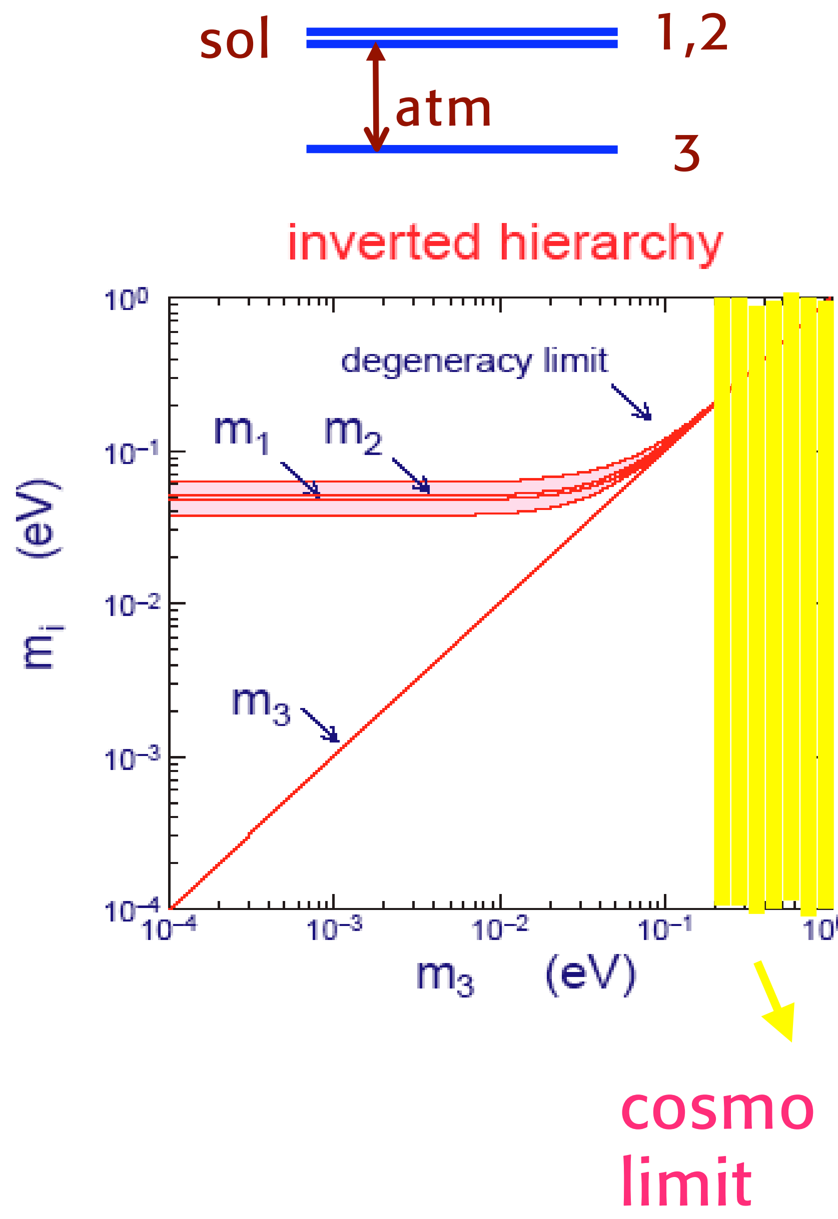
- Cosmology $\quad \Omega_\nu h^2 \sim \Sigma_i m_i\ /94\text{eV}$ $\quad (h^2 \sim 1/2)$

$\Sigma_i m_i < 0.2\text{-}0.7$ eV (dep. on data&priors)

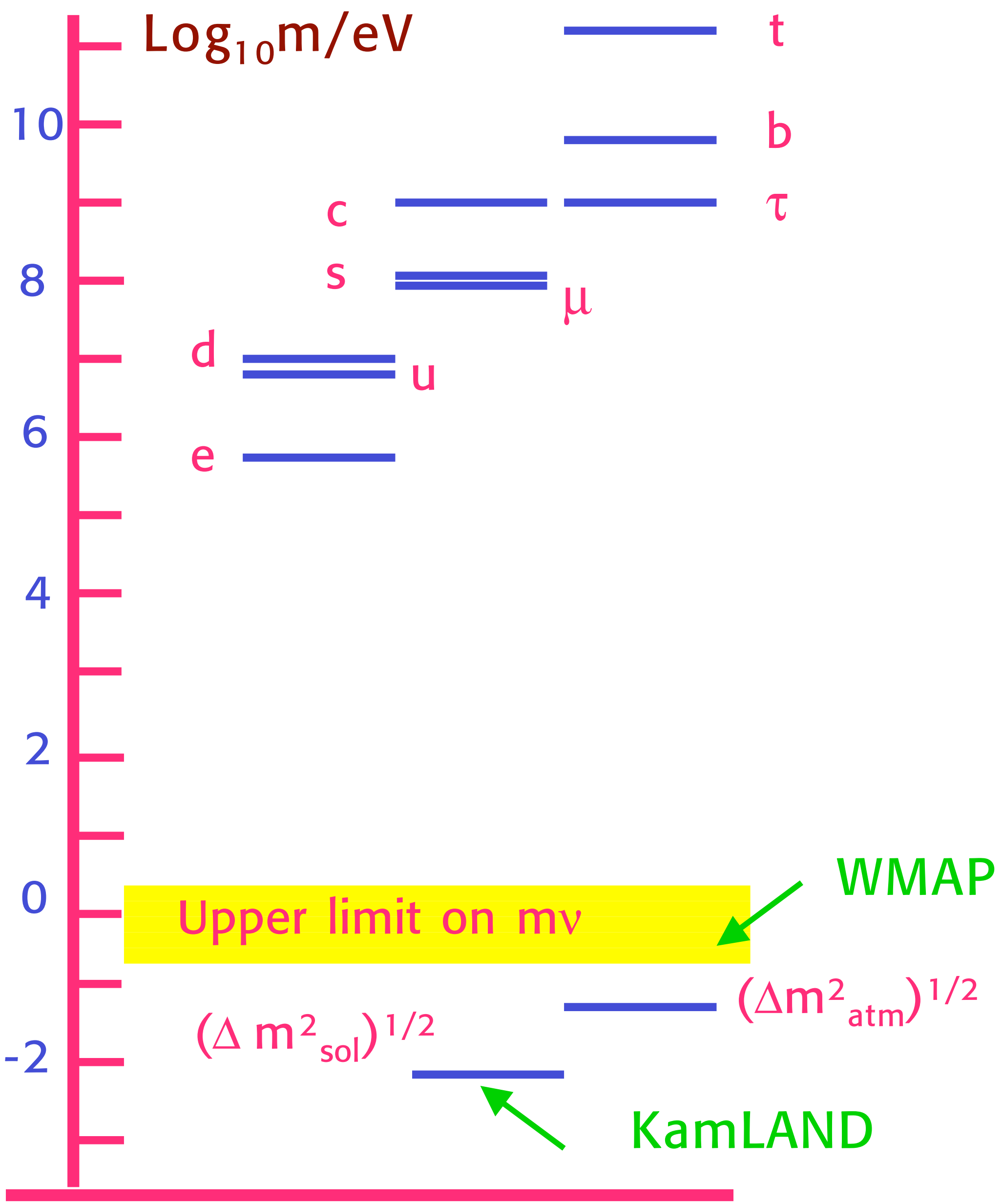
WMAP, SDSS, 2dFGRS, Ly-α

Any ν mass $< 0.06 - 0.23 - 2.2$ eV





Only moderate degeneracy allowed



Neutrino masses are really special!

$m_t/(\Delta m^2_{atm})^{1/2} \sim 10^{12}$

Massless ν's?

- no ν_R
- L conserved

Small ν masses?

- ν_R very heavy
- L not conserved

See-Saw Mechanism

Minkowski;
Yanagida; Gell-Mann, Ramond , Slansky;
Glashow; Mohapatra, Senjanovic…..

$M\nu^T{}_R\nu_R$ allowed by SU(2)xU(1)

Large Majorana mass M (as large as the cut-off)

$m_D\bar{\nu}_L\nu_R$ Dirac mass m from Higgs doublet(s)

$$\begin{array}{cc} & \begin{array}{cc}\nu_L & \nu_R\end{array} \\ \begin{array}{c}\nu_L \\ \nu_R\end{array} & \begin{bmatrix} 0 & m_D \\ m_D & M \end{bmatrix}\end{array}$$

$M \gg m_D$

Eigenvalues

$$|\nu_{light}| = \frac{m_D{}^2}{M} \quad , \quad \nu_{heavy} = M$$

In general ν mass terms are:

$$L_\nu = \bar{\nu}_L y \nu_R H + h.c. + \nu_R{}^T M_R \nu_R + \nu_L{}^T \frac{\lambda}{M_L} \nu_L HH$$

$$O_5 = \ell^T \frac{\lambda}{M_L} \ell HH$$

Dirac
$m_D = yv$
$v = \langle 0|H|0 \rangle$

Majorana

$$m = \frac{\lambda v^2}{M_L}$$

More general see-saw mechanism:

$$\begin{array}{cc} & \begin{array}{cc} \nu_L & \nu_R \end{array} \\ \begin{array}{c} \nu_L \\ \nu_R \end{array} & \begin{bmatrix} \lambda v^2/M_L & m_D \\ m_D & M_R \end{bmatrix} \end{array}$$

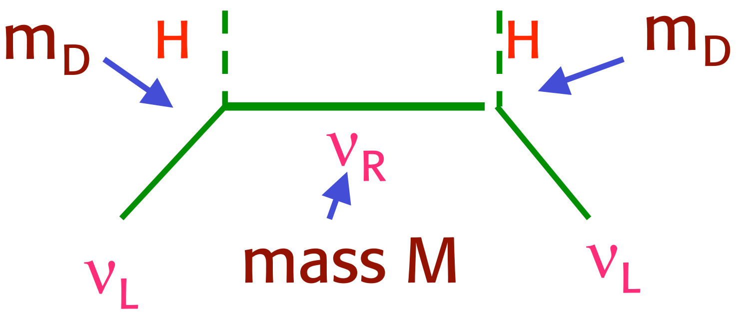
$m_{light} \sim \dfrac{m_D{}^2}{M_R}$ and/or $\dfrac{\lambda v^2}{M_L}$

$m_{heavy} \sim M_R$

$$m_{eff} = \nu^T{}_L m_{light} \nu_L$$

See-saw diagrams

Type 1



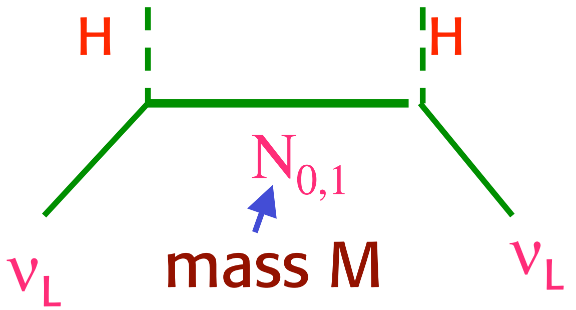
$\nu_L^T m_\nu \nu_L$

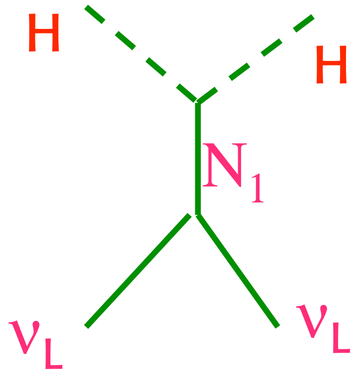
$m_\nu = m_D^T M^{-1} m_D$

More in general: non ren. O_5 operator

$$O_5 = \ell^T \frac{\lambda}{M_L} \ell H H$$

e.g from
Type 2





Type 3

$N_{0,1}$: new particle $I_W=0,1$

Whatever the underlying dynamics O_5 is a general effective description of light Majorana neutrino masses

ν oscillations point to very large values of M

A very natural and appealing explanation:

ν's are nearly massless because they are Majorana particles and get masses through L non conserving interactions suppressed by a large scale $M \sim M_{GUT}$

$$m_\nu \sim \frac{m^2}{M}$$

$m: \leq m_t \sim v \sim 200$ GeV

M: scale of L non cons.

Note:

$m_\nu \sim (\Delta m^2_{atm})^{1/2} \sim 0.05$ eV

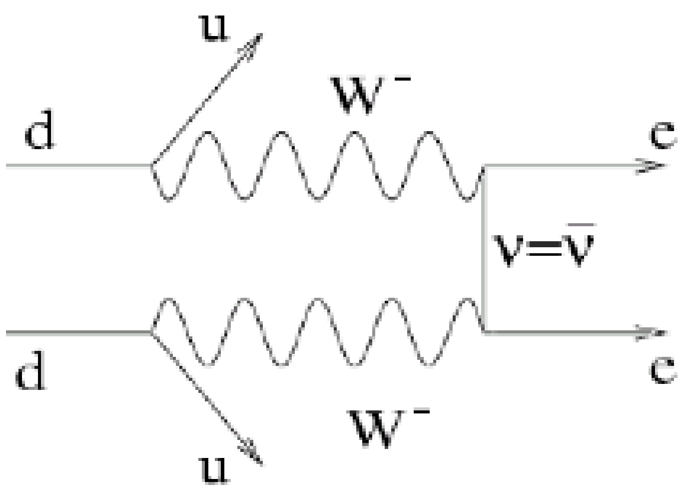
$m \sim v \sim 200$ GeV

$M \sim 10^{15}$ GeV

Neutrino masses are a probe of physics at M_{GUT} !

All we know from experiment on ν masses strongly indicates that ν's are Majorana particles and that L is not conserved (but a direct proof still does not exist).

Detection of $0\nu\beta\beta$ would be a proof of L non conservation. Thus a big effort is devoted to improving present limits and possibly to find a signal.



Heidelberg-Moscow
IGEX
Cuoricino
Nemo
Sokotvina
DAMA
•••••

$0\nu\beta\beta$ = dd -> $uue^{-}e^{-}$

$0\nu\beta\beta$ would prove that L is not conserved and ν's are Majorana
Also can tell degenerate, inverted or normal hierarchy

$$|m_{ee}|=c_{13}^2\,[m_1 c_{12}^2+e^{i\alpha}m_2 s_{12}^2]+m_3 e^{i\beta}s_{13}^2$$

Degenerate: $\sim|m||c_{12}^2+e^{i\alpha}s_{12}^2|\sim|m|(0.3\text{-}1)$

$|m_{ee}|\sim |m|\ (0.3\ \text{-}1)\leq 0.23\text{-}1$ eV

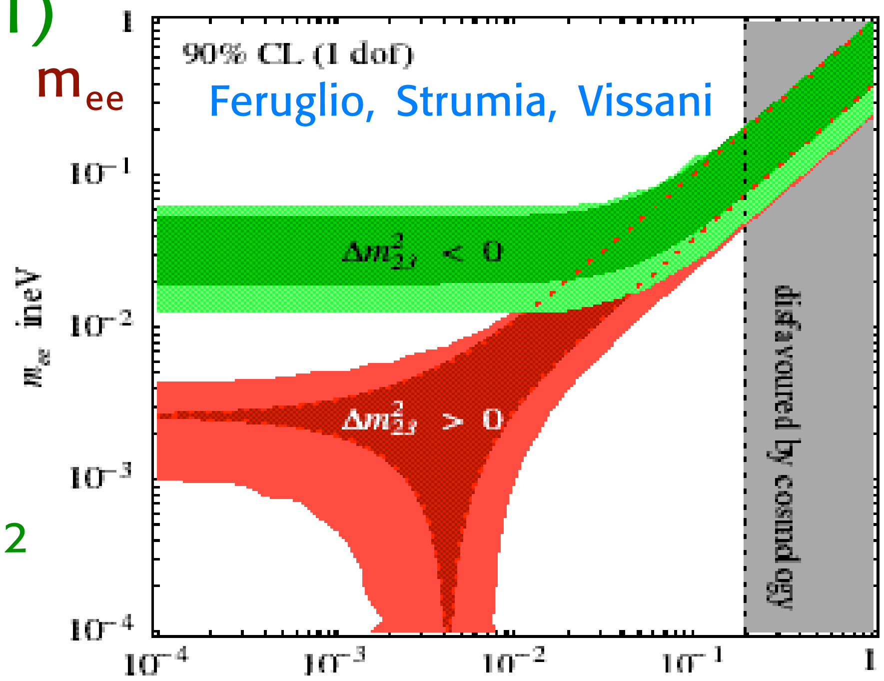
IH: $\sim(\Delta m^2_{atm})^{1/2}|c_{12}^2+e^{i\alpha}s_{12}^2|$

$|m_{ee}|\sim (1.6\text{-}5)\ 10^{-2}$ eV

NH: $\sim(\Delta m^2_{sol})^{1/2}s_{12}^2+(\Delta m^2_{atm})^{1/2}e^{i\beta}s_{13}^2$

$|m_{ee}|\sim$ (few) 10^{-3} eV

Full dependence on min m_ν



Present exp. limit: $m_{ee}< 0.3\text{-}0.5$ eV
(and a hint of signal????? Klapdor Kleingrothaus)

Neutrinos and GUT's

In each family 16 fermions $\begin{pmatrix} u \\ d \end{pmatrix}_L, u^c, d^c, \begin{pmatrix} \nu \\ e \end{pmatrix}_L, e^c, \nu^c$

SU(5): $16 = \bar{5} + 10 + 1$ ν_R is an optional in SU(5)!

In SO(10) all 16 fermions in one single irreducible repr.'n!

The Majorana mass $M\nu^T_R\nu_R$ is invariant under SU(5) [M could be o(M_{Pl})]
but not under SO(10) [M expected at o(M_{GUT})]

Neutrinos support (SUSY) GUT's and SO(10)!

Models of ν masses and mixings

An interplay of different matrices:

$$m_\ell \to \bar{R} m_\ell L$$

$$m_\ell{}' = V_\ell{}^\dagger m_\ell U_\ell$$

$$m_\ell{}^{\dagger\prime} m_\ell{}' = U_\ell{}^\dagger m_\ell{}^\dagger m_\ell U_\ell$$

$$U_{PMNS} = U_\ell{}^\dagger U_\nu$$

neutrino diagonalisat'n

ch lepton diagonalisat'n

See-saw

$$m_\nu{}' = U_\nu{}^T m_\nu U_\nu$$

$$m_\nu = m_D{}^T M^{-1} m_D$$

neutrino Majorana mass

neutrino Dirac mass

For example, the large ν mixing vs the small q mixing can be due to the Majorana nature of ν's

General remarks

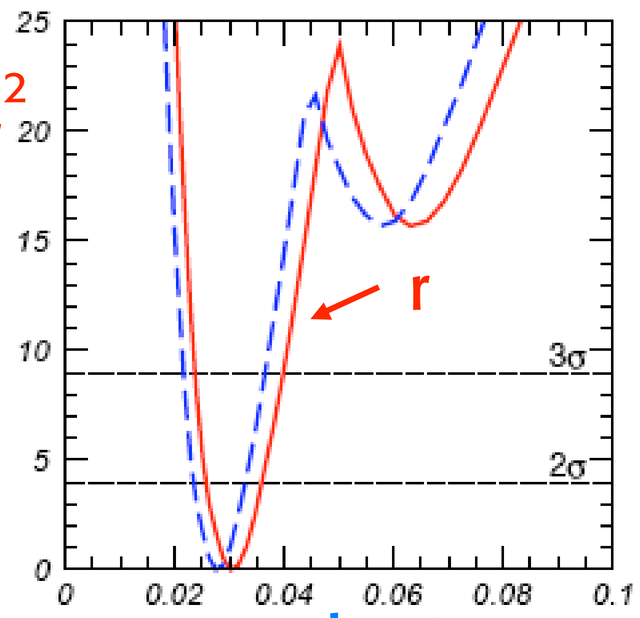
• After KamLAND, SNO and WMAP.... not too much hierarchy is found in ν masses:

$r \sim \Delta m^2_{sol} / \Delta m^2_{atm} \sim 1/30$

Only a few years ago could be as small as 10^{-8}!

Precisely at 3σ: $0.025 < r < 0.039$

Schwetz et al '08



or

$m_{heaviest} < 0.2$ - 0.7 eV
$m_{next} > \sim 8\ 10^{-3}$ eV

For a hierarchical spectrum: $\frac{m_2}{m_3} \approx \sqrt{r} \approx 0.2$

Comparable to $\lambda_C = \sin\theta_C$: $\lambda_C \approx 0.22$ or $\sqrt{\frac{m_\mu}{m_\tau}} \approx 0.24$

Suggests the same "hierarchy" parameters for q, l, ν
(small powers of λ_C) → e.g. θ_{13} not too small!

- Still large space for non maximal 23 mixing

  2-σ interval $0.37 < \sin^2\theta_{23} < 0.60$ Fogli et al '08

  Maximal θ_{23} theoretically hard

- θ_{13} not necessarily too small
  probably accessible to exp.

  Very small θ_{13} theoretically hard

- θ_{12} is at present the best measured angle

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Tri-Bimaximal mixing agrees with data at ~ 1σ

At 1σ: G.L.Fogli et al'08

$\sin^2\theta_{12} = 1/3$: 0.29-0.33

$\sin^2\theta_{23} = 1/2$: 0.41-0.54

$\sin^2\theta_{13} = 0$: $< \sim 0.02$

A coincidence or a hint?

There is an intriguing empirical relation:

$$\theta_{12} + \theta_C = (47.0 \pm 1.7)^\circ \sim \pi/4$$

Raidal'04

A coincidence or a hint?

In this lecture we take those as accidents, in next lectures as hints

I now review some ideas on model building

Old models are more generic and qualitative than present models. Some examples:

Anarchy
Semianarchy
Lopsided models
$U(1)_{FN}$
••••••

With better data the range for each mixing angle has narrowed and models have become more quantitative

e.g Tribimaximal mixing, A4, S4

Naively large mixing --> nearly degenerate masses

$$m_i^2 >> \Delta m^2_{ij}$$

Degenerate models are less favoured by now because of:

- No clear physical motivation: after all quark and charged lepton masses are very non degenerate

- Upper bounds on m^2 that limit $m^2/\Delta m^2_{atm}$
At present, no significant amount of hot dark matter is indicated by cosmology ---->
Typically: any ν mass < 0.23 eV
$\Delta m^2_{atm} \sim (0.05 \text{ eV})^2$
----> Only a moderate degeneracy is allowed

- Disfavoured by see-saw
- Possible renormalization group instability

It is difficult to marry degenerate models with see-saw

$$m_\nu \sim m_D^T M^{-1} m_D$$

(needs all degenerate or a sort of conspiracy between M and m_D)

So most degenerate models deny all relation to m_D and M and directly work with effective operators

$$O_5 = \ell^T \frac{\lambda}{M_L} \ell H H$$

Even if a symmetry guarantees degeneracy at the GUT scale it is difficult to protect it from corrections, e.g. from renormalisation group running

For degenerate models there can be large ren. group corrections to mixing angles and masses in the running from M_{GUT} dow to m_W

In fact the running rate is inv. prop. to mass differences

For a 2x2 case:

$$U^{Aa} = \begin{pmatrix} c_\vartheta & -s_\vartheta \\ s_\vartheta & c_\vartheta \end{pmatrix} \qquad t = \frac{1}{16\pi^2}\log\frac{m}{m_Z}$$

$$\frac{ds_\vartheta}{dt} = \kappa A_{21}(y_{e_2}^2 - y_{e_1}^2)s_\vartheta c_\vartheta^2 \quad \frac{dc_\vartheta}{dt} = -\kappa A_{21}(y_{e_2}^2 - y_{e_1}^2)s_\vartheta^2 c_\vartheta,$$

with $A_{21} = \frac{m_2 + m_1}{m_2 - m_1}$

$k = -3/2$ (SM), 1 (MSSM)

$y_e = m_e/v$ (SM), $m_e/v\cos\beta$ (MSSM)

RG corrections are generally negligible and can only be large for degenerate models especially at large $\tan\beta$

The observed mixings and splittings do not fit the typical result from pure evolution.

See, for example, Chankowski, Pokorski '01

In the class of degenerate models we can include anarchy

No order -> Anarchy

No symmetry, no dynamics assumed, only chance

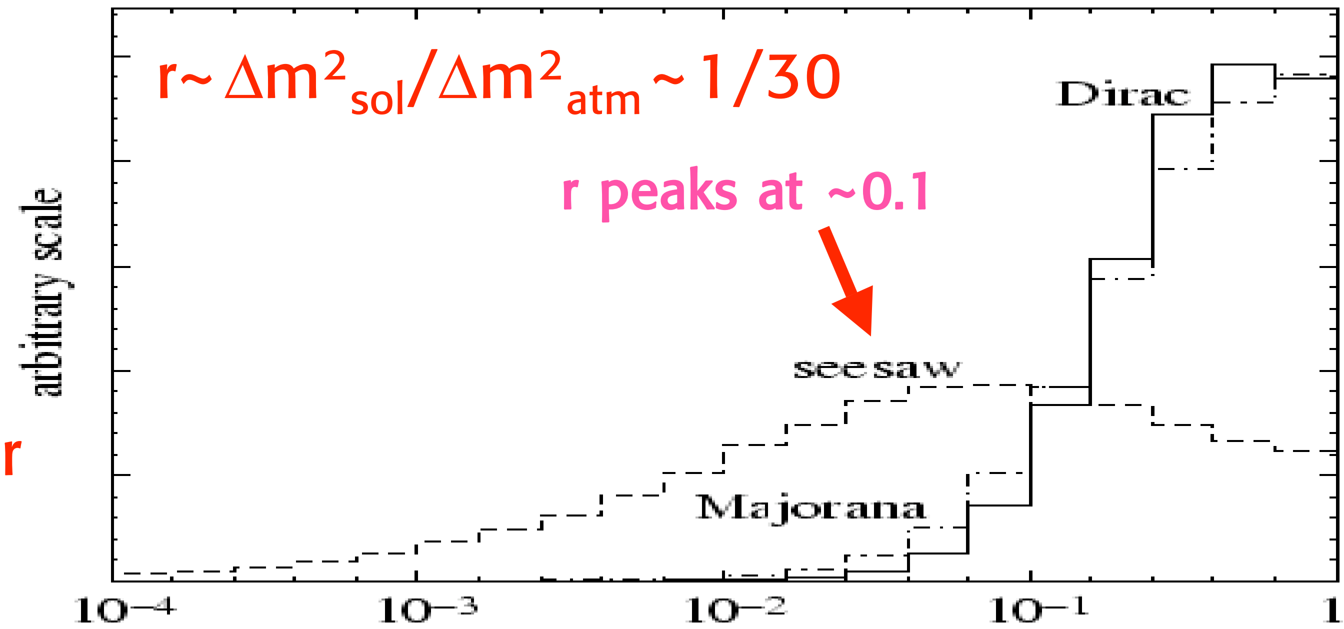
Anarchy (or accidental hierarchy):
No structure in the neutrino sector

A bit extreme!

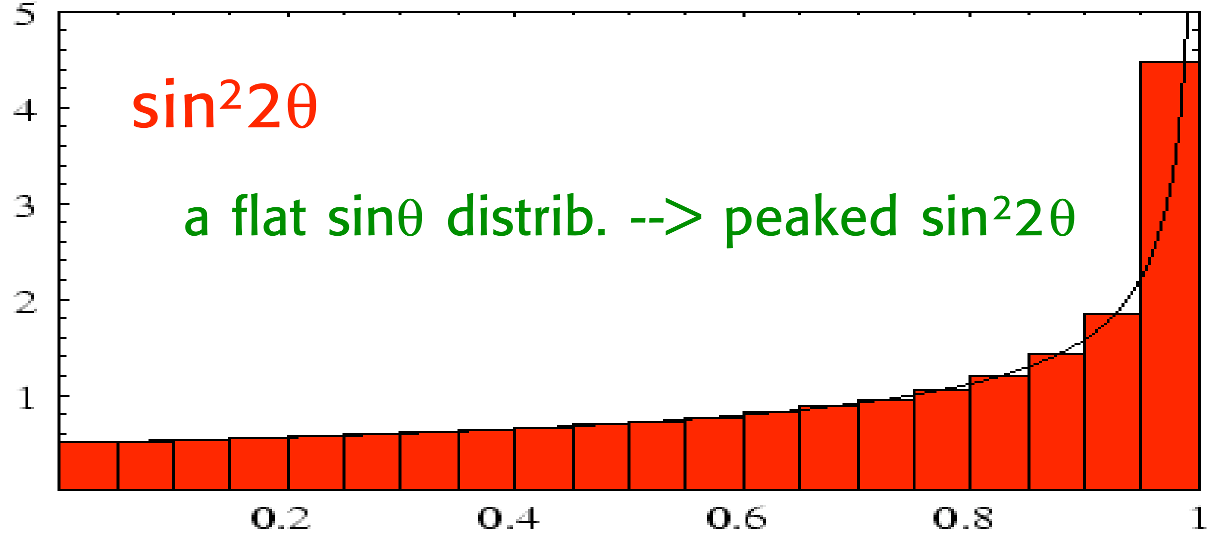
Hall, Murayama, Weiner

See-Saw:
$m_\nu \sim m^2/M$
produces hierarchy
from random m, M

could fit the data on r



But: all mixing angles
should be not too large,
not too small →



Predicts θ_{13} near bound
θ_{23} sizably non maximal

Anarchy can be realised in SU(5) by putting all the flavour structure in T ~ 10 and not in F^{bar} ~ 5^{bar}

$m_u \sim 10\,.10$ strong hierarchy $m_u : m_c : m_t$

$m_d \sim 5^{bar}\,.10 \sim m_e^T$ milder hierarchy $m_d : m_s : m_b$

or $m_e : m_\mu : m_\tau$

Experiment supports that d, e hierarchy is roughly the square root of u hierarchy

$m_\nu \sim 5^T\,.5$ or for see saw $(5.1)^T\,(1.1)\,(1.5)$

anarchy

For example, for the simplest flavour group, $U(1)_F$

1st fam. 2nd 3rd

$$\left\{\begin{array}{ll} T: & (3, 2, 0) \\ F^{bar}: & (0, 0, 0) \\ 1: & (0, 0, 0) \end{array}\right.$$

Hierarchy for masses and mixings via horizontal $U(1)_{FN}$ charges.

Froggatt, Nielsen '79

Principle:

A generic mass term

$$\bar{R}_1 m_{12} L_2 H$$

is forbidden by U(1) if $q_1+q_2+q_H$ not 0

q_1, q_2, q_H: U(1) charges of $\bar{R}_1, L_2, H$

U(1) broken by vev of "flavon" field θ with U(1) charge $q_\theta = -1$.

If vev $\theta = w$, and $w/M=\lambda$ we get for a generic interaction:

Δ_{charge}

$$\bar{R}_1 m_{12} L_2 H\ (\theta/M)^{q1+q2+qH} \qquad m_{12} \to m_{12}\ \lambda^{q1+q2+qH}$$

Hierarchy: More Δ_{charge} -> more suppression ($\lambda=\theta/M$ small)

One can have more flavons (λ, λ', ...)
with different charges (>0 or <0) etc -> many versions

A milder ansatz - Semianarchy: no structure in 23

Consider a matrix like $m_\nu \sim L^T L \sim \begin{bmatrix} \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & 1 & 1 \\ \lambda^2 & 1 & 1 \end{bmatrix}$ Note: $\theta_{13} \sim \lambda^2$

$\theta_{23} \sim 1$

$q(\bar{5}) \sim (2, 0, 0)$

with coeff.s of o(1) and det23~o(1)

["semianarchy", while $\lambda \sim 1$ corresponds to anarchy]

After 23 and 13 rotations $m_\nu \sim \begin{bmatrix} \lambda^4 & \lambda^2 & 0 \\ \lambda^2 & \eta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Normally two masses are of o(1) or r ~1 and $\theta_{12} \sim \lambda^2$

But if, accidentally, $\eta \sim \lambda^2$, then r is small and θ_{12} is large.

The advantage over anarchy is that θ_{13} is naturally small, but θ_{12} large and the hierarchy $m^2_3 >> m^2_2$ are accidental

Ramond et al, Buchmuller et al

⊕ With see-saw, one can do much better (see later)

Is normal hierarchy compatible with large ν mixings?

- In the 2-3 sector we need both
  large m_3-m_2 splitting and large mixing.

  $m_3 \sim (\Delta m^2_{atm})^{1/2} \sim 5\ 10^{-2}$ eV
  $m_2 \sim (\Delta m^2_{sol})^{1/2} \sim 9\ 10^{-3}$ eV

- The "theorem" that large Δm_{32} implies small mixing
  (pert. th.: $\theta_{ij} \sim 1/|E_i - E_j|$)
  is not true in general: all we need is (sub)det[23]~0

- Example: $m_{23} \sim \begin{bmatrix} x^2 & x \\ x & 1 \end{bmatrix}$

  Det = 0; Eigenvl's: 0, $1+x^2$
  Mixing: $\sin^2 2\theta = 4x^2/(1+x^2)^2$

So all we need are natural mechanisms for det[23]=0

For x~1
large splitting
and large mixing!

Examples of mechanisms for Det[23]~0

based on see-saw: $m_\nu \sim m^T_D M^{-1} m_D$

1) A ν_R is lightest and coupled to μ and τ

King; Allanach; Barbieri et al......

$$M \sim \begin{bmatrix} \varepsilon & 0 \\ 0 & 1 \end{bmatrix} \longrightarrow M^{-1} \sim \begin{bmatrix} 1/\varepsilon & 0 \\ 0 & 1 \end{bmatrix} \approx \begin{bmatrix} 1/\varepsilon & 0 \\ 0 & 0 \end{bmatrix}$$

$$m_\nu \sim \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1/\varepsilon & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a & c \\ b & d \end{bmatrix} \approx 1/\varepsilon \begin{bmatrix} a^2 & ac \\ ac & c^2 \end{bmatrix}$$

2) M generic but m_D "lopsided" $\qquad m_D \sim \begin{bmatrix} 0 & 0 \\ x & 1 \end{bmatrix}$

Albright, Barr; GA, Feruglio, ...

$$m_\nu \sim \begin{bmatrix} 0 & x \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ b & c \end{bmatrix} \begin{bmatrix} 0 & 0 \\ x & 1 \end{bmatrix} = c \begin{bmatrix} x^2 & x \\ x & 1 \end{bmatrix}$$

An important property of SU(5)

Left-handed quarks have small mixings (V_{CKM}),
but right-handed quarks can have large mixings (unknown).

In SU(5):
LH for d quarks $\Longleftrightarrow$ RH for l^- leptons

$\bar{5}$ 10

$m_d \sim \bar{d}_R d_L$

10 $\bar{5}$

$m_e \sim \bar{e}_R e_L$

$\bar{5}$: $(\underbrace{\bar{d},\bar{d},\bar{d},}_{R}\ \underbrace{\nu,e^-}_{L})$

$$\boxed{m_d = m_e^T}$$

cannot be exact, but approx.

Most "lopsided" models are based on this fact. In these models often large atmospheric mixing arises, at least in part, from the charged lepton sector.

- The correct pattern of masses and mixings, also including ν's, is obtained in simple models based on

$SU(5) x U(1)_{flavour}$

Ramond et al; GA, Feruglio+Masina; Buchmuller et al; King et al; Yanagida et al, Berezhiani et al; Lola et al.......

Offers a simple description of hierarchies, but it is not very predictive (large number of undetermined o(1) parameters)

Of course, SU(5) can also be coupled with non abelian flavour symmetries, eg $O(3)_F$, $SU(3)_F$, S3, A4, S4 (see next lectures) and become more predictive

- SO(10) models are more predictive but less flexible

Albright, Barr; Babu et al; Bajic et al; Barbieri et al; Buccella et al; King et al; Mohapatra et al; Raby et al; G. Ross et al

SU(5)xU(1)

1st fam. 2nd 3rd

Ψ_{10}: (5, 3, 0)
Ψ_5: (2, 0, 0)
Ψ_1: (1,-1, 0)

Equal 2,3 ch. for lopsided

Recall: $m_u \sim 10\ 10$
$m_d = m_e^T \sim 5^{bar}\ 10$
$m_{\nu D} \sim 5^{bar}\ 1$; $M_{RR} \sim 1\ 1$

No structure for leptons

No automatic det23 = 0

Automatic det23 = 0

Model	Ψ_{10}	$\Psi_{\bar{5}}$	Ψ_1	(H_u, H_d)
Anarchical (A)	(3,2,0)	(0,0,0)	(0,0,0)	(0,0)
Semi-Anarchical (SA)	(2,1,0)	(1,0,0)	(2,1,0)	(0,0)
Hierarchical (H_I)	(6,4,0)	(2,0,0)	(1,-1,0)	(0,0)
Hierarchical (H_{II})	(5,3,0)	(2,0,0)	(1,-1,0)	(0,0)
Inversely Hierarchical (IH_I)	(3,2,0)	(1,-1,-1)	(-1,+1,0)	(0,+1)
Inversely Hierarchical (IH_{II})	(6,4,0)	(1,-1,-1)	(-1,+1,0)	(0,+1)

all charges positive

not all charges positive

With suitable charge assignments all relevant patterns can be obtained

The optimised values of λ are of the order of λ_c or a bit larger (moderate hierarchy)

`model`	$\lambda(=\lambda')$
A_{SS}	0.2
SA_{SS}	0.25
$H_{(SS,II)}$	0.35
$H_{(SS,I)}$	0.45
$IH_{(SS,II)}$	0.45
$IH_{(SS,I)}$	0.25

Example: Normal Hierarchy

G.A., Feruglio, Masina'02

1st fam. 2nd 3rd

$q(10)$: $(5, 3, 0)$

$q(\bar{5})$: $(2, 0, 0)$

$q(1)$: $(1,-1, 0)$

Note: not all charges positive --> det23 suppression

$q(H) = 0$, $q(\bar{H}) = 0$

$q(\theta) = -1$, $q(\theta') = +1$

In first approx., with $\langle\theta\rangle/M \sim \lambda \sim \lambda' \sim 0.35 \sim o(\lambda_C)$

$10_i 10_j$

$$m_u \sim v_u \begin{bmatrix} \lambda^{10} & \lambda^8 & \lambda^5 \\ \lambda^8 & \lambda^6 & \lambda^3 \\ \lambda^5 & \lambda^3 & 1 \end{bmatrix},$$

$10_i \bar{5}_j$

$$m_d = m_e^T \sim v_d \begin{bmatrix} \lambda^7 & \lambda^5 & \lambda^5 \\ \lambda^5 & \lambda^3 & \lambda^3 \\ \lambda^2 & 1 & 1 \end{bmatrix}$$

"lopsided"

$\bar{5}_i 1_j$

$$m_{\nu D} \sim v_u \begin{bmatrix} \lambda^3 & \lambda & \lambda^2 \\ \lambda & \lambda' & 1 \\ \lambda & \lambda' & 1 \end{bmatrix},$$

$1_i 1_j$

$$M_{RR} \sim M \begin{bmatrix} \lambda^2 & 1 & \lambda \\ 1 & \lambda'^2 & \lambda' \\ \lambda & \lambda' & 1 \end{bmatrix}$$

Note: coeffs. 0(1) omitted, only orders of magnitude predicted with $\lambda \sim \lambda'$

$$\bar{5}_i 1_j \qquad\qquad 1_i 1_j$$

$$m_{\nu D} \sim v_u \begin{bmatrix} \lambda^3 & \lambda & \lambda^2 \\ \lambda & \lambda & 1 \\ \lambda & \lambda & 1 \end{bmatrix}, \qquad M_{RR} \sim M \begin{bmatrix} \lambda^2 & 1 & \lambda \\ 1 & \lambda^2 & \lambda \\ \lambda & \lambda & 1 \end{bmatrix}$$

see-saw $\quad m_\nu \sim m_{\nu D}{}^T M_{RR}{}^{-1} m_{\nu D}$

$$m_\nu \sim v_u{}^2/M \begin{bmatrix} \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & \boxed{\begin{matrix} 1 & 1 \\ 1 & 1 \end{matrix}} \end{bmatrix},$$

$$\det_{23} \sim \lambda^2$$

The 23 subdeterminant is automatically suppressed,
$\theta_{13} \sim \lambda^2$, θ_{12}, $\theta_{23} \sim 1$

This model works, in the sense that all small parameters are naturally due to various degrees of suppression.
But too many free parameters!!

Masses in SO(10) models

$16 \times 16 = 10 + 126 + 120$

If no non-ren mass terms are allowed a simplest model needs a 10 and a 126:

Bajc, Senjanovic, Vissani '02
Goh, Mohapatra, Ng '03

$$\mathcal{L}_Y = 10_H 16 y_{10} 16 + 126_H 16 y_{126} 16,$$

leading to

$$m_d = \alpha y_{10} + \beta y_{126}, \qquad m_e = \alpha y_{10} - 3\beta y_{126},$$

and $m_\nu \propto m_d - m_e \propto 126$

In the 23 sector, both m_d and m_e can be obtained as:

$$m_{d,e} \sim \begin{pmatrix} \lambda^2 & \lambda^2 \\ \lambda^2 & 1 \end{pmatrix}$$

Then b-τ unification forces a cancellation $1 \to \lambda^2$ in m_ν, which in turn makes a large 23 neutrino mixing.

One can arrange that θ_{12} is large, $r \sim \lambda^2$, θ_{13} near the bound

In other SO(10) models one avoids large Higgs represent'ns (120, 126) by relying on non ren. operators like $16_i\ 16_H\ 16_j\ 16'_H$ or $16_i\ 16_j\ 10_H 45_H$ (several of such terms are needed in order to reproduce all masses and mixings)

In the flavour-symmetric limit, the lowest dimension mass terms $16_3 16_3 10_H$ is only allowed for the 3rd family.

In particular, both lopsided and L-R symmetric models can be obtained in this way

Babu, Pati, Wilczek
Albright, Barr
Ji, Li, Mohapatra
Dermisek, Raby
....

GUT models often contain ad hoc ingredients and a lot of parameter fitting

Model	Hierarchy	$\sin^2 2\theta_{23}$	$\lvert U_{e3}\rvert^2$	$\sin^2\theta_{12}$
A [15]	NH	0.99	0.0025	0.31
AB [16]	NH	0.99	0.0020	0.28
BB [17]	NH	0.97	0.0021	0.29
BM [18]	NH	0.98	0.013	0.31
BO [19]	NH	0.99	0.0014	0.27
CM [20]	NH	1.00	0.013	0.27
CY [21]	NH	1.00	0.0029	0.29
DMM [22]	NH	1.00	0.0078	–
DR [23]	NH	0.98	0.0024	0.30
GK [24]	NH	1.00	0.00059	0.31
JLM [25]	NH	1.0	0.0189	0.29
VR [26]	NH	0.995	0.024	0.34
YW [27]	NH	0.96	0.04	0.29
S-B TBM	NH	$\gtrsim 0.94$	$\lesssim 10^{-3}$	–
S-B TBM	IH	$\gtrsim 0.91$	$\lesssim 10^{-2}$	–
S-B TBM	QD	–	–	–

Table 1: $SO(10)$ models and their predictions for the lepton mixing angles. If ranges are given we take the central value. Also given are the constraints, if any, on the mixing angles for the three possible mass orderings from the softly-broken tri-bimaximal mixing mass matrices.

Data have become more precise
Next lecture: models of Tri-Bimaximal mixing

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Comparison with experiment:

G.L.Fogli et al'08

At 1σ:

$\sin^2\theta_{12} = 1/3$: 0.29-0.33
$\sin^2\theta_{23} = 1/2$: 0.41-0.54
$\sin^2\theta_{13} = 0$: $< \sim 0.02$

The HPS mixing is clearly a very good approx. to the data!

Harrison, Perkins, Scott '02

Also called:
Tri-Bimaximal mixing

$$\nu_3 = \frac{1}{\sqrt{2}}(-\nu_\mu + \nu_\tau)$$

$$\nu_2 = \frac{1}{\sqrt{3}}(\nu_e + \nu_\mu + \nu_\tau)$$