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Models of Neutrino Masses and Mixings

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Hierarchy for masses and mixings via horizontal $U(1)_{FN}$ charges.

Froggatt, Nielsen '79

Principle: A generic mass term

$$\overline{R}_1 m_{12} L_2 H$$

is forbidden by U(1)
if $q_1+q_2+q_H$ not 0

q_1, q_2, q_H:
U(1) charges of
$\overline{R}_1, L_2, H$

U(1) broken by vev of "flavon" field θ with U(1) charge $q_\theta = -1$.
If vev $\theta = w$, and $w/M=\lambda$ we get for a generic interaction:

$$\overline{R}_1 m_{12} L_2 H \, (\theta/M)^{q1+q2+qH} \qquad m_{12} \to m_{12}\, \lambda^{q1+q2+qH}$$

($q1+q2+qH$: Δ_{charge})

Hierarchy: More Δ_{charge} -> more suppression ($\lambda=\theta/M$ small)

One can have more flavons (λ, λ', ...)
with different charges (>0 or <0) etc -> many versions

Anarchy can be realised in SU(5) by putting all the flavour structure in T ~ 10 and not in F^{bar} ~ 5^{bar}

$m_u \sim 10 . 10$ strong hierarchy $m_u : m_c : m_t$

$m_d \sim 5^{bar} . 10 \sim m_e^T$ milder hierarchy $m_d : m_s : m_b$

or $m_e : m_\mu : m_\tau$

Experiment supports that d, e hierarchy is roughly the square root of u hierarchy

$m_\nu \sim 5^T . 5$ or for see saw $(5.1)^T (1.1) (1.5)$

anarchy

For example, for the simplest flavour group, $U(1)_F$

1st fam. 2nd 3rd

$$\begin{cases} T : & (3, 2, 0) \\ F^{bar} : & (0, 0, 0) \\ 1 : & (0, 0, 0) \end{cases}$$

Is normal hierarchy compatible with large ν mixings?

- In the 2-3 sector we need both
  large m_3-m_2 splitting and large mixing.

  $m_3 \sim (\Delta m^2_{atm})^{1/2} \sim 5\ 10^{-2}$ eV
  $m_2 \sim (\Delta m^2_{sol})^{1/2} \sim 9\ 10^{-3}$ eV

- The "theorem" that large Δm_{32} implies small mixing
  (pert. th.: $\theta_{ij} \sim 1/|E_i - E_j|$)
  is not true in general: all we need is (sub)det[23]~0

- Example: $m_{23} \sim \begin{bmatrix} x^2 & x \\ x & 1 \end{bmatrix}$

  Det = 0; Eigenvl's: 0, $1+x^2$
  Mixing: $\sin^2 2\theta = 4x^2/(1+x^2)^2$

So all we need are natural mechanisms for det[23]=0

For $x \sim 1$
large splitting
and large mixing!

Examples of mechanisms for Det[23]~0

based on see-saw: $m_\nu \sim m^T_D M^{-1} m_D$

1) A ν_R is lightest and coupled to μ and τ

King; Allanach; Barbieri et al......

$$M \sim \begin{bmatrix} \varepsilon & 0 \\ 0 & 1 \end{bmatrix} \rightarrow M^{-1} \sim \begin{bmatrix} 1/\varepsilon & 0 \\ 0 & 1 \end{bmatrix} \approx \begin{bmatrix} 1/\varepsilon & 0 \\ 0 & 0 \end{bmatrix}$$

$$m_\nu \sim \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1/\varepsilon & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a & c \\ b & d \end{bmatrix} \approx 1/\varepsilon \begin{bmatrix} a^2 & ac \\ ac & c^2 \end{bmatrix}$$

2) M generic but m_D "lopsided" $\quad m_D \sim \begin{bmatrix} 0 & 0 \\ x & 1 \end{bmatrix}$

Albright, Barr; GA, Feruglio, ...

$$m_\nu \sim \begin{bmatrix} 0 & x \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ b & c \end{bmatrix} \begin{bmatrix} 0 & 0 \\ x & 1 \end{bmatrix} = c \begin{bmatrix} x^2 & x \\ x & 1 \end{bmatrix}$$

An important property of SU(5)

Left-handed quarks have small mixings (V_{CKM}),
but right-handed quarks can have large mixings (unknown).

In SU(5): LH for d quarks	⟷	RH for l^- leptons

$\bar{5}$ → $m_d \sim \bar{d}_R d_L$ ← 10

10 → $m_e \sim \bar{e}_R e_L$ ← $\bar{5}$

$\bar{5}$: $(\bar{d}, \bar{d}, \bar{d}, \nu, e^-)$, with $(\bar{d}, \bar{d}, \bar{d})$: R and (ν, e^-): L

$$m_d = m_e^T$$

cannot be exact, but approx.

Most "lopsided" models are based on this fact. In these models often large atmospheric mixing arises, at least in part, from the charged lepton sector.

- The correct pattern of masses and mixings, also including ν's, is obtained in simple models based on

SU(5)xU(1)$_{flavour}$

Ramond et al; GA, Feruglio+Masina; Buchmuller et al; King et al; Yanagida et al, Berezhiani et al; Lola et al.......

Offers a simple description of hierarchies, but it is not very predictive (large number of undetermined o(1) parameters)

Of course, SU(5) can also be coupled with non abelian flavour symmetries, eg O(3)$_F$, SU(3)$_F$, S3, A4, S4 (see next lectures) and become more predictive

- SO(10) models are more predictive but less flexible

Albright, Barr; Babu et al; Bajic et al; Barbieri et al; Buccella et al; King et al; Mohapatra et al; Raby et al; G. Ross et al

SU(5)xU(1)

1st fam. → 2nd → 3rd

Ψ_{10}: (5, 3, 0)
Ψ_{5}: (2, 0, 0) ← Equal 2,3 ch. for lopsided
Ψ_{1}: (1,-1, 0)

Recall: $m_u \sim$ 10 10
$m_d = m_e^T \sim$ 5^{bar} 10
$m_{\nu D} \sim$ 5^{bar} 1; $M_{RR} \sim$ 1 1

Model	Ψ_{10}	$\Psi_{\bar{5}}$	Ψ_1	(H_u, H_d)
Anarchical (A)	(3,2,0)	(0,0,0)	(0,0,0)	(0,0)
Semi-Anarchical (SA)	(2,1,0)	(1,0,0)	(2,1,0)	(0,0)
Hierarchical (H_I)	(6,4,0)	(2,0,0)	(1,-1,0)	(0,0)
Hierarchical (H_{II})	(5,3,0)	(2,0,0)	(1,-1,0)	(0,0)
Inversely Hierarchical (IH_I)	(3,2,0)	(1,-1,-1)	(-1,+1,0)	(0,+1)
Inversely Hierarchical (IH_{II})	(6,4,0)	(1,-1,-1)	(-1,+1,0)	(0,+1)

No structure for leptons → (Anarchical)

No automatic det23 = 0 → (Semi-Anarchical)

all charges positive

Automatic det23 = 0 → (Hierarchical)

not all charges positive

With suitable charge assignments all relevant patterns can be obtained

The optimised values of λ are of the order of λ_C or a bit larger (moderate hierarchy)

model	$\lambda(= \lambda')$
A_{SS}	0.2
SA_{SS}	0.25
$H_{(SS,II)}$	0.35
$H_{(SS,I)}$	0.45
$IH_{(SS,II)}$	0.45
$IH_{(SS,I)}$	0.25

Example: Normal Hierarchy

G.A., Feruglio, Masina'02

1st fam. → 2nd → 3rd

$q(10)$: (5, 3, 0)
$q(\bar{5})$: (2, 0, 0)
$q(1)$: (1,-1, 0)

Note: not all charges positive --> det23 suppression

$q(H) = 0,\ q(\bar{H}) = 0$
$q(\theta) = -1,\ q(\theta') = +1$

In first approx., with $<\theta>/M \sim \lambda \sim \lambda' \sim 0.35 \sim o(\lambda_C)$

$10_i 10_j$:

$$m_u \sim v_u \begin{bmatrix} \lambda^{10} & \lambda^8 & \lambda^5 \\ \lambda^8 & \lambda^6 & \lambda^3 \\ \lambda^5 & \lambda^3 & 1 \end{bmatrix},$$

$10_i \bar{5}_j$:

$$m_d = m_e^T \sim v_d \begin{bmatrix} \lambda^7 & \lambda^5 & \lambda^5 \\ \lambda^5 & \lambda^3 & \lambda^3 \\ \lambda^2 & 1 & 1 \end{bmatrix}$$

"lopsided"

$\bar{5}_i 1_j$:

$$m_{\nu D} \sim v_u \begin{bmatrix} \lambda^3 & \lambda & \lambda^2 \\ \lambda & \lambda' & 1 \\ \lambda & \lambda' & 1 \end{bmatrix},$$

$1_i 1_j$:

$$M_{RR} \sim M \begin{bmatrix} \lambda^2 & 1 & \lambda \\ 1 & \lambda'^2 & \lambda' \\ \lambda & \lambda' & 1 \end{bmatrix}$$

Note: coeffs. 0(1) omitted, only orders of magnitude predicted with $\lambda \sim \lambda'$

$\bar{5}_i 1_j$ $\quad$ $1_i 1_j$

$$m_{\nu D} \sim v_u \begin{bmatrix} \lambda^3 & \lambda & \lambda^2 \\ \lambda & \lambda & 1 \\ \lambda & \lambda & 1 \end{bmatrix}, \qquad M_{RR} \sim M \begin{bmatrix} \lambda^2 & 1 & \lambda \\ 1 & \lambda^2 & \lambda \\ \lambda & \lambda & 1 \end{bmatrix}$$

see-saw $\quad m_\nu \sim m_{\nu D}^T M_{RR}^{-1} m_{\nu D}$

$$m_\nu \sim v_u^2/M \begin{bmatrix} \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & 1 & 1 \\ \lambda^2 & 1 & 1 \end{bmatrix},$$

$\det_{23} \sim \lambda^2$

The 23 subdeterminant is automatically suppressed,
$\theta_{13} \sim \lambda^2$, θ_{12}, $\theta_{23} \sim 1$

This model works, in the sense that all small parameters are naturally due to various degrees of suppression.
But too many free parameters!!

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Tri-Bimaximal mixing agrees with data at ~ 1σ

At 1σ:

G.L.Fogli et al'08

$\sin^2\theta_{12} = 1/3$: 0.29-0.33

$\sin^2\theta_{23} = 1/2$: 0.41-0.54

$\sin^2\theta_{13} = 0$: $< \sim 0.02$

A coincidence or a hint?

There is an intriguing empirical relation:

$$\theta_{12} + \theta_C = (47.0 \pm 1.7)^\circ \sim \pi/4$$

Raidal'04

A coincidence or a hint?

For some time people considered limiting models with $\theta_{13} = 0$ and θ_{23} maximal and θ_{12} generic

$m_\nu = U \mathrm{diag}(m_1, m_2, m_3) U^T$

$$U = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

The most general mass matrix for $\theta_{13} = 0$ and θ_{23} maximal is given by (after ch. lepton diagonalization!!!):

$$m_\nu = \begin{bmatrix} x & y & y \\ y & z & w \\ y & w & z \end{bmatrix}$$

Neglecting Majorana phases it depends on 4 real parameters (3 mass eigenvalues and 1 mixing angle: θ_{12})

Inspired models based on $\mu-\tau$ symmetry

Grimus, Lavoura..., Ma,.... Mohapatra, Nasri, Hai-Bo Yu ....

Actually, at present, since KamLAND, the most accurately known angle is θ_{12}

G.L.Fogli et al'08

At $\sim 1\sigma$: $\sin^2\theta_{12} = 0.294\text{-}0.331$

By adding $\sin^2\theta_{12} \sim 1/3$ to $\theta_{13} \sim 0$, $\theta_{23} \sim \pi/4$:

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Harrison, Perkins, Scott '02

Some additional ingredient other than μ–τ symmetry needed!

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Comparison with experiment:

At 1σ: G.L.Fogli et al'08

$\sin^2\theta_{12} = 1/3$: 0.29-0.33

$\sin^2\theta_{23} = 1/2$: 0.41-0.54

$\sin^2\theta_{13} = 0$: $< \sim 0.02$

[Thanks to KamLAND, the most accurately known angle is θ_{12}]

Called:
Tri-Bimaximal mixing

Harrison, Perkins, Scott '02

$$\nu_3 = \frac{1}{\sqrt{2}}(-\nu_\mu + \nu_\tau)$$

$$\nu_2 = \frac{1}{\sqrt{3}}(\nu_e + \nu_\mu + \nu_\tau)$$

Tribimaximal Mixing

A simple mixing matrix compatible with all present data

$$U=\begin{bmatrix}\sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0\\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}}\\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}\end{bmatrix}$$

In the basis of diagonal ch. leptons:

$$m_\nu = U \mathrm{diag}(m_1, m_2, m_3) U^T$$

$$m_\nu = \frac{m_3}{2}\begin{bmatrix}0 & 0 & 0\\ 0 & 1 & -1\\ 0 & -1 & 1\end{bmatrix} + \frac{m_2}{3}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix} + \frac{m_1}{6}\begin{bmatrix}4 & -2 & -2\\ -2 & 1 & 1\\ -2 & 1 & 1\end{bmatrix}$$

Eigenvectors: $m_3 \rightarrow \frac{1}{\sqrt{2}}\begin{bmatrix}0\\ 1\\ -1\end{bmatrix}$ $\quad m_2 \rightarrow \frac{1}{\sqrt{3}}\begin{bmatrix}1\\ 1\\ 1\end{bmatrix}$ $\quad m_1 \rightarrow \frac{1}{\sqrt{6}}\begin{bmatrix}-2\\ 1\\ 1\end{bmatrix}$

Note: mixing angles independent of mass eigenvalues

Compare with quark mixings $\lambda_C \sim (m_d/m_s)^{1/2}$

The most general mass matrix for
$\sin^2\theta_{12} \sim 1/3,\ \theta_{13} \sim 0,\ \theta_{23} \sim \pi/4$

Tribimaximal Mixing

$$m_\nu = \begin{bmatrix} x & y & y \\ y & z & w \\ y & w & z \end{bmatrix} \longrightarrow m = \begin{pmatrix} x & y & y \\ y & x+v & y-v \\ y & y-v & x+v \end{pmatrix}$$

$$\sin^2 2\theta_{12} = \frac{8y^2}{(x-w-z)^2 + 8y^2}$$

$m_1 = x - y$
$m_2 = x + 2y$
$m_3 = x - y + 2v$

The 3 remaining parameters
are the mass eigenvalues

- For TB mixing matrix all mixing angles are fixed to particularly symmetric values

Sparked interest in constructing models that can naturally produce this highly ordered structure (very far from anarchy!)

Models based on the A4 discrete symmetry (even permutations of 1234) offer a minimal solution

Ma...;
GA, Feruglio, hep-ph/0504165, hep-ph/0512103
GA, Feruglio, Lin hep-ph/0610165
GA, Feruglio, Hagedorn, 0802.0090 [hep-ph]
Y. Lin, 0804.2867 [hep-ph]........

Larger finite groups: T′, $\Delta(27)$, S4

Feruglio et al;
Chen, Mahanthappa;
Frampton, Kephart; Lam;
Bazzocchi et al ....

Alternative models based on $SU(3)_F$ or $SO(3)_F$ or their finite subgroups

Verzielas, G. Ross
....

King ....

List of models with flavor symmetries

(incomplete, by symmetry):

S_3: Pakvasa et al. (1978) Derman (1979), Ma (2000), Kubo et al. (2003), Chen et al. (2004), Grimus et al. (2005), Dermisek et al. (2005), Mohapatra et al. (2006), ...

S4: Pakvasa et al. (1979), Derman et al. (1979), Lee et al. (1994), Mohapatra et al. (2004), Ma (2006), Hagedorn, ML and Mohapatra (2006), Caravaglios et al. (2006), ...

A_4: Wyler (1979), Ma et al. (2001), Babu et al. (2003), Altarelli et al. (2005,2006), He et al. (2006) ...

D_4: Seidl (2003), Grimus et al. (2003,2004), Kobayashi et al. (2005), ...

D_5: Ma (2004), Hagedorn et al. (2006).

D_n: Chen et al. (2005), Kajiyama et al. (2007), Frampton et al. (1995,1996,2000), Frigerio et al. (2005), Babu et al. (2005), Kubo (2005), ...

T': Frampton et al. (1994,2007), Aranda et al. (1999,2000), Feruglio et al. (2007), Chen and Mahanthappa (2007)

Δ_n: Kaplan et al. (1994), Chou et al. (1997), de Medeiros Varzielas et al. (2005), ...

T_7: Luhn et al.

A4

A4 is the discrete group of even perm's of 4 objects.
(the inv. group of a tetrahedron). It has $4!/2 = 12$ elements.

A4 transformations can be written in terms of S and T as:

$$1, T, S, ST, TS, T^2, TST, STS, ST^2, T^2S, T^2ST, TST^2$$

with: $S^2 = T^3 = (ST)^3 = 1$ [$(TS)^3 = 1$ also follows]

An element is abcd which means 1234 --> abcd

C_1: $1 = 1234$
C_2: $T = 2314$ $ST = 4132$ $TS = 3241$ $STS = 1423$
C_3: $T^2 = 3124$ $ST^2 = 4213$ $T^2S = 2431$ $TST = 1342$
C_4: $S = 4321$ $T^2ST = 3412$ $TST^2 = 2143$

C_1, C_2, C_3, C_4 are equivalence classes

x, x′ in same class if $[x' \sim gxg^{-1}]$ g: group element

A4 has 4 inequivalent irreducible representations:
a triplet and 3 different singlets

$3, 1, 1', 1''$ (promising for 3 generations!)

Note:

as many representations as equivalence classes

$\sum d_i^2 = 12$ $9+1+1+1=12$

S4 contains A4. S4 repr.ns: $3, 3', 2, 1, 1'$

$9+9+4+1+1=24$

Note: many models tried S3
S3 has no triplets but only $2, 1, 1'$
$4+1+1=6$
A4 is better in the lepton sector

Mohapatra, Nasri, Yu
Koide
Kubo et al
Kaneko et al
Caravaglios et al
Morisi
Picariello......

Three singlet inequivalent represent'ns:

Recall:
$S^2 = T^3 = (ST)^3 = 1$

$$\begin{cases} 1: & S=1,\ T=1 \\ 1': & S=1,\ T=\omega \\ 1'': & S=1,\ T=\omega^2 \end{cases}$$

$$\omega = \exp i\frac{2\pi}{3} = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$$
$$\omega^3 = 1$$
$$1 + \omega + \omega^2 = 0$$
$$\omega^2 = \omega^*$$

The only irreducible 3-dim represent'n is obtained by:

$$S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \qquad T = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

(S-diag basis)

An equivalent form:

$$VV^\dagger = V^\dagger V = 1$$

$$S' = \frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix} = VSV^\dagger \qquad T' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{bmatrix} = VTV^\dagger \qquad V = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{bmatrix}$$

(T-diag basis)

Cabibbo '78

A4 has only 4 irreducible inequivalent represt'ns: 1,1',1",3

Table of Multiplication:
1'x1'=1"; 1"x1"=1';1'x1"=1
3x3=1+1'+1"+3+3

A4 is well fit for 3 families!

Ch. leptons l ~ 3

e^c, μ^c, τ^c ~ 1, 1", 1'

In the S-diag basis consider 3: (a_1,a_2,a_3) $\xrightarrow{S}$ $(a_1,-a_2,-a_3)$

(a_1,a_2,a_3) $\xrightarrow{T}$ (a_2,a_3,a_1)

For $3_1=(a_1,a_2,a_3)$, $3_2=(b_1,b_2,b_3)$ we have in $3_1 x 3_2$:

$$1 = a_1 b_1 + a_2 b_2 + a_3 b_3$$

$$1' = a_1 b_1 + \omega^2 a_2 b_2 + \omega a_3 b_3$$

$$1" = a_1 b_1 + \omega a_2 b_2 + \omega^2 a_3 b_3$$

$$3 \sim (a_2 b_3, a_3 b_1, a_1 b_2)$$

$$3 \sim (a_3 b_2, a_1 b_3, a_2 b_1)$$

e.g. $1" = a_1b_1+\omega a_2b_2+\omega^2 a_3b_3 \xrightarrow{T} a_2b_2+\omega a_3b_3+\omega^2 a_1b_1 =$
$= \omega^2 [a_1b_1+\omega a_2b_2+\omega^2 a_3b_3]$

(under S, 1" is invariant)

In the T-diagonal basis we have:

$$VV^\dagger = V^\dagger V = 1$$

$$S' = \frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix} = VSV^\dagger \qquad T' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{bmatrix} = VTV^\dagger \qquad V = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{bmatrix}$$

Cabibbo '78

For $3_1=(a_1,a_2,a_3)$, $3_2=(b_1,b_2,b_3)$ we have in $3_1 x 3_2$:

$$1 = a_1b_1 + a_2b_3 + a_3b_2$$

$$1' = a_3b_3 + a_1b_2 + a_2b_1$$

$$1" = a_2b_2 + a_1b_3 + a_3b_3$$

We will see that in this basis the charged leptons are diagonal

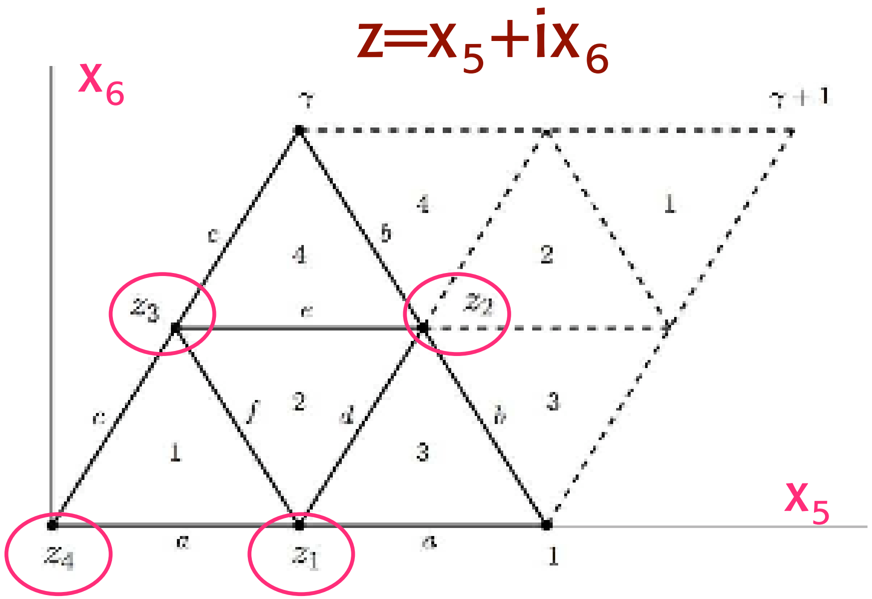
$$3_{symm} \sim \frac{1}{3}(2a_1b_1 - a_2b_3 - a_3b_2, 2a_3b_3 - a_1b_2 - a_2b_1, 2a_2b_2 - a_1b_3 - a_3b_1)$$

$$3_{antisymm} \sim \frac{1}{2}(a_2b_3 - a_3b_2, a_1b_2 - a_2b_1, a_1b_3 - a_3b_1)$$

What can be the origin of A4?

A4 (or some other discrete group) could arise from extra dimensions (by orbifolding with fixed points) as a remnant of 6-dim spacetime symmetry:

G.A.,F. Feruglio&Y. Lin, NP B775(2007)31
Adulpravitchai, Blum, Lindner '09



A torus with identified points:

$z \to z+1$

$z \to z+\gamma \quad \gamma = \exp(i\pi/3)$

and a parity $z \to -z$
leads to 4 fixed points
(equivalent to a tethraedron).

There are 4D branes at the fixed points where the SM fields live (additional gauge singlets are in the bulk)

⊕ A4 interchanges the fixed points

Under A4 the most common classification is:

lepton doublets $l \sim 3$, (in see-saw models $\nu^c \sim 3$)

$e^c, \mu^c, \tau^c \sim 1, 1'', 1'$ respectively

A4 breaking gauge singlet flavons $\phi_S, \phi_T, \xi \ldots \sim 3, 3, 1\ldots$

For SUSY version: driving fields $\phi_{0S}, \phi_{0T}, \xi_0 \ldots \sim 3, 3, 1\ldots$

with the alignment:

$$\langle \varphi_T \rangle = (v_T, 0, 0)$$
$$\langle \varphi_S \rangle = (v_S, v_S, v_S)$$
$$\langle \xi \rangle = u \quad , \quad \langle \tilde{\xi} \rangle = 0$$

!!! In a serious model the alignment must follow from the symmetries

In all versions there are additional symmetries:
e.g. a broken $U(1)_F$ symmetry and/or discrete symmetries Z_n to ensure hierarchy of charged lepton masses and to restrict allowed couplings

A baseline A4 model (a 4-dim SUSY version with see-saw)

$$w_l = y_e e^c(\varphi_T l) + y_\mu \mu^c(\varphi_T l)' + y_\tau \tau^c(\varphi_T l)'' + y(\nu^c l) +$$ ch. leptons

$$+(x_A\xi + \tilde{x}_A\tilde{\xi})(\nu^c\nu^c) + x_B(\varphi_S\nu^c\nu^c) + h.c. + ...$$ neutrinos

shorthand: Higg, U(1) flavon θ, and cut-off scale Λ omitted, e.g.:

$$y_e e^c(\varphi_T l) \sim y_e e^c(\varphi_T l) h_d \theta^4/\Lambda^5$$

Fields and their transformation properties

	l	e^c	μ^c	τ^c	ν^c	$h_{u,d}$	θ	φ_T	φ_S	ξ	φ_0^T	φ_0^S	ξ_0
A_4	3	1	1"	1'	3	1	1	3	3	1	3	3	1
Z_3	ω	ω^2	ω^2	ω^2	ω^2	1	1	1	ω^2	ω^2	1	ω^2	ω^2
$U(1)_{FN}$	0	4	2	0	0	0	-1	0	0	0	0	0	0
$U(1)_R$	1	1	1	1	1	0	0	0	0	0	2	2	2

In T-diag basis:
with this alignment:

!!!

$$\langle \varphi_T \rangle = (v_T, 0, 0)$$
$$\langle \varphi_S \rangle = (v_S, v_S, v_S)$$
$$\langle \xi \rangle = u \quad , \quad \langle \tilde{\xi} \rangle = 0$$

Ch. leptons are diagonal

$$m_l = v_T \frac{v_d}{\Lambda} \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}$$

$$y_\tau \approx O(1) \quad y_\mu \approx O(\lambda^2) \quad y_e \approx O(\lambda^4)$$

ν's are tri-bimaximal

$$m_\nu^D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} y v_u \qquad M = \begin{pmatrix} A + 2B/3 & -B/3 & -B/3 \\ -B/3 & 2B/3 & A - B/3 \\ -B/3 & A - B/3 & 2B/3 \end{pmatrix} u$$

$$A \equiv 2x_A \qquad B \equiv 2x_B \frac{v_S}{u}$$

after see-saw $m_\nu = (m_\nu^D)^T M^{-1} m_\nu^D$

recall:

$$m = \begin{pmatrix} x & y & y \\ y & x + v & y - v \\ y & y - v & x + v \end{pmatrix}$$

$$m_\nu = \frac{v_u^2}{\Lambda} \begin{pmatrix} a + 2b/3 & -b/3 & -b/3 \\ -b/3 & 2b/3 & a - b/3 \\ -b/3 & a - b/3 & 2b/3 \end{pmatrix}$$

Alignment

One more singlet is needed for vacuum alignment

The superpotential (at leading order):

This term defines ξ twiddle

$$
\begin{aligned}
w_d &= M(\varphi_0^T \varphi_T) + g(\varphi_0^T \varphi_T \varphi_T) \\
&+ g_1(\varphi_0^S \varphi_S \varphi_S) + g_2 \tilde{\xi}(\varphi_0^S \varphi_S) + g_3 \xi_0 (\varphi_S \varphi_S) + g_4 \xi_0 \xi^2 + g_5 \xi_0 \xi \tilde{\xi} + g_6 \xi_0 \tilde{\xi}^2
\end{aligned}
$$

and the potential $V = \sum_i \left| \frac{\partial w}{\partial \phi_i} \right|^2 + m_i^2 |\phi_i|^2 + \ldots$

Alignment

One more singlet is needed for vacuum alignment

The superpotential (at leading order):

This term defines $\tilde{\xi}$ (ξ twiddle)

$$
\begin{aligned}
w_d &= M(\varphi_0^T \varphi_T) + g(\varphi_0^T \varphi_T \varphi_T) \\
&+ g_1(\varphi_0^S \varphi_S \varphi_S) + g_2 \tilde{\xi}(\varphi_0^S \varphi_S) + g_3 \xi_0 (\varphi_S \varphi_S) + g_4 \xi_0 \xi^2 + g_5 \xi_0 \xi \tilde{\xi} + g_6 \xi_0 \tilde{\xi}^2
\end{aligned}
$$

and the potential $V = \sum_i \left| \frac{\partial w}{\partial \phi_i} \right|^2 + m_i^2 |\phi_i|^2 + \ldots$

The θ vev arises from minimizing the D-term

$$
V_D = \frac{1}{2}(M_{FI}{}^2 - g_{FN} |\theta|^2 + \ldots..)^2
$$

The driving field have zero VEV. So the minimization is:

$$\frac{\partial w}{\partial \varphi^T_{01}} = M\varphi_{T\,1} + \frac{2g}{3}(\varphi_{T\,1}^2 - \varphi_{T\,2}\varphi_{T\,3}) = 0 \qquad \frac{\partial w}{\partial \varphi^S_{01}} = g_2\tilde{\xi}\varphi_{S\,1} + \frac{2g_1}{3}(\varphi_{S\,1}^2 - \varphi_{S\,2}\varphi_{S\,3}) = 0$$

$$\frac{\partial w}{\partial \varphi^T_{02}} = M\varphi_{T\,3} + \frac{2g}{3}(\varphi_{T\,2}^2 - \varphi_{T\,1}\varphi_{T\,3}) = 0 \qquad \frac{\partial w}{\partial \varphi^S_{02}} = g_2\tilde{\xi}\varphi_{S\,3} + \frac{2g_1}{3}(\varphi_{S\,2}^2 - \varphi_{S\,1}\varphi_{S\,3}) = 0$$

$$\frac{\partial w}{\partial \varphi^T_{03}} = M\varphi_{T\,2} + \frac{2g}{3}(\varphi_{T\,3}^2 - \varphi_{T\,1}\varphi_{T\,2}) = 0 \qquad \frac{\partial w}{\partial \varphi^S_{03}} = g_2\tilde{\xi}\varphi_{S\,2} + \frac{2g_1}{3}(\varphi_{S\,3}^2 - \varphi_{S\,1}\varphi_{S\,2}) = 0$$

$$\frac{\partial w}{\partial \xi_0} = g_4\xi^2 + g_5\xi\tilde{\xi} + g_6\tilde{\xi}^2 + g_3(\varphi_{S\,1}^2 + 2\varphi_{S\,2}\varphi_{S\,3}) = 0$$

Solution:

$$\varphi_T = (v_T, 0, 0) \quad , \qquad v_T = -\frac{3M}{2g}$$

$$\tilde{\xi} = 0$$

$$\xi = u$$

$$\varphi_S = (v_S, v_S, v_S) \quad , \qquad v_S^2 = -\frac{g_4}{3g_3}u^2$$

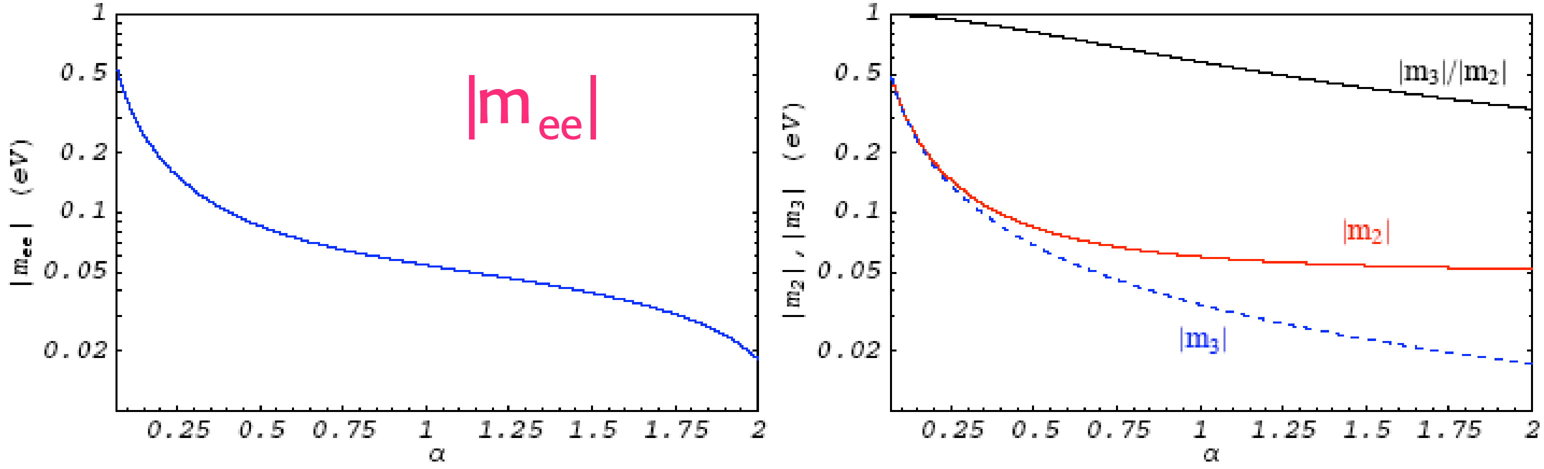
Light neutrino eigenvalues (complex masses)

$$m_1 = \frac{y^2}{A+B}\frac{v_u^2}{u} \quad , \qquad m_2 = \frac{y^2}{A}\frac{v_u^2}{u} \quad , \qquad m_3 = \frac{y^2}{-A+B}\frac{v_u^2}{u}$$

A sum rule typical of A4

$$\frac{1}{m_3} = \frac{1}{m_1} - \frac{2}{m_2}$$

Both normal and inverse hierarchy possible



Here is the inverse hierarchy case

Figure 1: *Behaviour of neutrino masses in the inverted hierarchy case (at fixed Δm^2_{atm} and r) as a function of α in the range between 0.07 and 2 (the lower bound on α corresponds to an upper bound on $|m_i|$). Left panel: $|m_{ee}|$. Right panel: $|m_2|$, $|m_3|$ and the ratio $|m_3|/|m_2|$.*

So, at LO TB mixing is exact

$r \sim \Delta m^2_{sol}/\Delta m^2_{atm}$

The only fine-tuning needed is to account for $r \sim 1/30$
[In most A4 models $r \sim 1$ would be expected as l, $\nu^c \sim 3$]

When NLO corrections are included from operators of higher dimension in the superpotential each mixing angle receives generically corrections of the same order $\delta\theta_{ij} \sim o(VEV/\Lambda)$

As the maximum allowed corrections to θ_{12} (and also to θ_{23}) are $o(\lambda_C^2)$, we need $VEV/\Lambda \sim o(\lambda_C^2)$ and we expect:

$\theta_{13} \sim o(\lambda_C^2)$ measurable in next run of exp's

(T2K starts at the end of '09)

This generic prediction can be altered in ad hoc versions
e.g. Lin '09 has a model where $\theta_{13} \sim o(\lambda_C)$

Why A4 works?

TB mixing corresponds to m
in the basis where
charged leptons are diagonal

$$m = \begin{pmatrix} x & y & y \\ y & x+v & y-v \\ y & y-v & x+v \end{pmatrix}$$

m is the most general matrix invariant under
SmS = m and $A_{23}mA_{23}$= m with:

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$$

$$A_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

2-3
symmetry

Invariance under S can be made automatic in A4 while
invariance under A_{23} happens if 1′ and 1″ flavons are absent.

Charged lepton masses are a generic diagonal matrix, invariant under T (or ηT with η a phase):

$$m_l = v_T \frac{v_d}{\Lambda} \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}$$

$$m_l^+ m_l = T^+ m_l^+ m_l T$$

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}$$

$$\langle \varphi_T \rangle = (v_T, 0, 0)$$
$$\langle \varphi_S \rangle = (v_S, v_S, v_S)$$
$$\langle \xi \rangle = u \quad , \quad \langle \tilde{\xi} \rangle = 0$$

The aligment occurs because is based on A4 group theory:

ϕ_T breaks A4 down to G_T
ϕ_S breaks A4 down to G_S
(G_T, G_S: subgroups generated by T, S)

Note that for TB mixing in A4 it is important that no flavons transforming as 1′ and 1″ exist

Recently Lam claimed that for "a natural" TB model the smallest group is S4 (instead A4 is a subgroup of S4)

This is because he calls "natural" a model only if all possible flavons are introduced

We do not accept this criterium:

In physics we call natural a model if the lagrangian is the most general given the symmetry and the representations of the fields
(for example the SM is natural even if only Higgs doublets are present)

Many versions of A4 models exist by now

- with dim-5 effective operators or with see-saw
- with SUSY or without SUSY
- in 4 dimensions or in extra dimensions

  e.g G.A., Feruglio'05; G.A., Feruglio, Lin '06;
  Csaki et al '08.....

- with different solutions to the alignment problem

  e.g Hirsch, Morisi, Valle '08

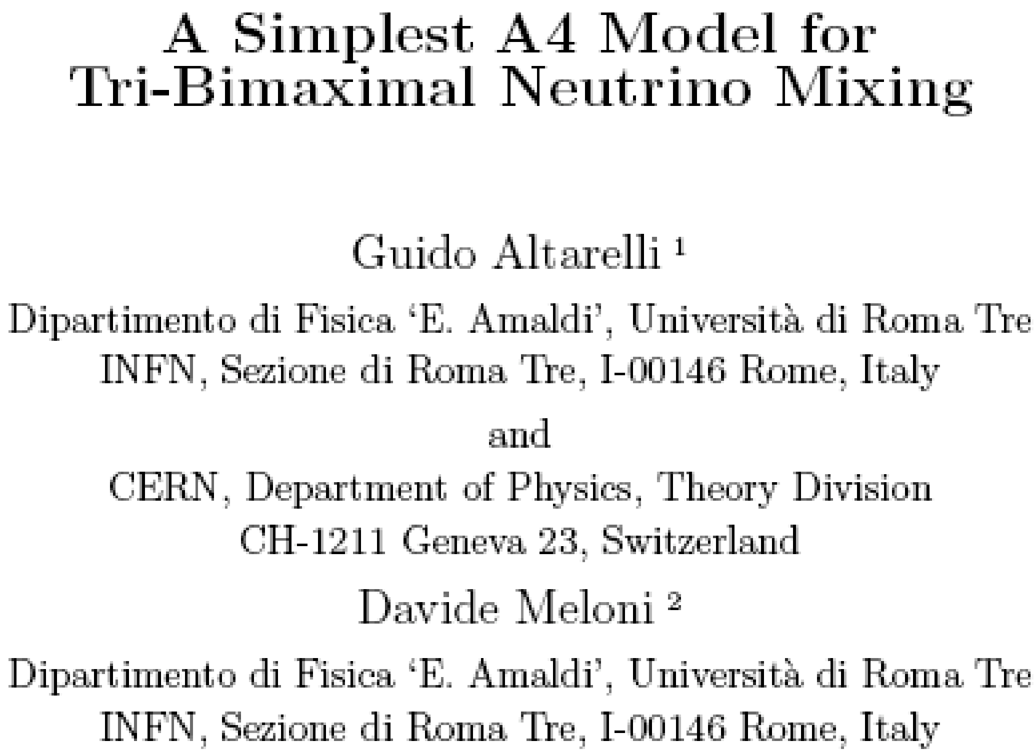
- with sequential (or form) dominance

  e.g King'07 ; Chen, King '09

- with charged lepton hierarchy also following from a special alignment (no $U(1)_{FN}$) Lin'08; GA, Meloni'09
- extension to quarks, possibly in a GUT context

An economic version: ch. lepton hierarchy with no need of $U(1)_{FN}$

Y. Lin '08, '09



A Simplest A4 Model for
Tri-Bimaximal Neutrino Mixing

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arXiv:0905.0620

The idea is to take a different alignment

$$\langle \varphi_T \rangle = (v_T, 0, 0) \longrightarrow \langle \varphi_T \rangle = (0, v_T, 0)$$

$(1,0,0)^n \sim (1,0,0)$

$(0,1,0)^2 \sim (0,0,1)$

$(0,1,0)^3 \sim (1,0,0)$

$$\begin{pmatrix} 2\psi_1\varphi_1 - \psi_2\varphi_3 - \psi_3\varphi_2 \\ 2\psi_3\varphi_3 - \psi_1\varphi_2 - \psi_2\varphi_1 \\ 2\psi_2\varphi_2 - \psi_1\varphi_3 - \psi_3\varphi_1 \end{pmatrix} \sim 3_S$$

Profit of this fact to arrange that τ, μ and e take mass at $o(\phi_T)$, $o(\phi_T^2)$ and $o(\phi_T^3)$ respectively

The model:

Field	ν^c	ℓ	e^c	μ^c	τ^c	h_d	h_u	φ_T	ξ'	φ_S	ξ	φ_0^T	φ_0^S	ξ_0
A_4	3	3	1	1	1	1	1	3	1′	3	1	3	3	1
Z_4	-1	i	1	i	-1	1	i	i	i	1	1	-1	1	1
$U(1)_R$	1	1	1	1	1	0	0	0	0	0	0	2	2	2

$$
\begin{aligned}
w_l \ = \ & \frac{y_\tau}{\Lambda}\tau^c(\ell\varphi_T)\,h_d + \\
& \frac{y_\mu}{\Lambda^2}\mu^c(\ell\varphi_T\varphi_T)\,h_d + \frac{y'_\mu}{\Lambda^2}\mu^c(\ell\varphi_T)''\,\xi'\,h_d + \\
& \frac{y_e}{\Lambda^3}e^c(\ell\varphi_T\varphi_T)''\,\xi'\,h_d + \frac{y'_e}{\Lambda^3}e^c(\ell\varphi_T)'\xi'^2\,h_d + \frac{y''_e}{\Lambda^3}e^c(\ell\varphi_T)'(\varphi_T\varphi_T)''\,h_d + \\
& \frac{y'''_e}{\Lambda^3}e^c(\ell\varphi_T)''(\varphi_T\varphi_T)'\,h_d + \frac{y^{\mathrm{iv}}_e}{\Lambda^3}e^c(\ell\varphi_T)_1(\varphi_T\varphi_T)_1\,h_d + ...
\end{aligned}
$$

At LO ch. leptons are diagonal and hierarchical

$$
m_\ell = \begin{pmatrix} \frac{v_T v_d}{\Lambda^3}\left(2y_e v_T u' + y'_e u'^2 + y''_e v_T^2\right) & 0 & 0 \\ 0 & \frac{v_T v_d}{\Lambda^2}\left(2y_\mu v_T + y'_\mu u'\right) & 0 \\ 0 & 0 & \frac{y_\tau v_d v_T}{\Lambda} \end{pmatrix}
$$

The ν sector is as usual

Alignment

$$
\begin{aligned}
w_d &= M(\varphi_0^S \varphi_S) + g_1(\varphi_0^S \varphi_S \varphi_S) + g_2 \xi(\varphi_0^S \varphi_S) + g_3 \xi_0 (\varphi_S \varphi_S) + g_4 \xi_0 \xi^2 + M_\xi \xi_0 \xi \\
&+ M_0^2 \, \xi_0 + h_1 \xi' (\varphi_0^T \varphi_T)'' + h_2 (\varphi_0^T \varphi_T \varphi_T) \; .
\end{aligned}
$$

This ξ' term is crucial

φ_T	ξ'	φ_S	ξ
3	1'	3	1
i	i	1	1
0	0	0	0

The minimum conditions in the ϕ_T sector are:

$$
\begin{aligned}
\frac{\partial w}{\partial \varphi_{01}^T} &= 2h_2({\varphi_T}_1^2 - \varphi_{T2}\varphi_{T3}) + h_1 \, \xi' \, \varphi_{T3} = 0 \\
\frac{\partial w}{\partial \varphi_{02}^T} &= 2h_2({\varphi_T}_2^2 - \varphi_{T1}\varphi_{T3}) + h_1 \, \xi' \, \varphi_{T2} = 0 \\
\frac{\partial w}{\partial \varphi_{01}^T} &= 2h_2({\varphi_T}_3^2 - \varphi_{T1}\varphi_{T2}) + h_1 \, \xi' \, \varphi_{T1} = 0
\end{aligned}
$$

with solution:

$$
\langle \xi' \rangle = u' \neq 0 \; , \qquad \langle \varphi_T \rangle = (0, v_T, 0) \; , \qquad v_T = -\frac{h_1 u'}{2h_2} \; .
$$