Neutrinos, the Standard Model, and the LHC

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Standard Model

	SU(3)	SU(2)	$U(1)_Y$
$Q_L^i = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix}$	3	2	1/6
$u_R^i = u_R, c_R, t_R$	3	1	2/3
$d_R^i = d_R, s_R, b_R$	3	1	-1/3
$L_L^i = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}, \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}, \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$	1	2	-1/2
$e_R^i = e_R, \mu_R, \tau_R$	1	1	-1
$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$	1	2	1/2

Standard Model

$$L = -\tfrac{1}{4} F^{\mu\nu} F_{\mu\nu} - \tfrac{1}{4} G^{A\mu\nu} G^{A}_{\mu\nu} - \tfrac{1}{4} W^{A\mu\nu} W^{A}_{\mu\nu}$$ Gauge

Standard Model

$$L = -\tfrac{1}{4} F^{\mu\nu} F_{\mu\nu} - \tfrac{1}{4} G^{A\mu\nu} G^{A}_{\mu\nu} - \tfrac{1}{4} W^{A\mu\nu} W^{A}_{\mu\nu} \qquad \text{Gauge}$$

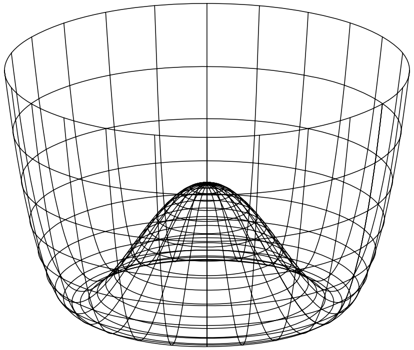
$$+ i\overline{Q}_L^i \not{D} Q_L^i + i\overline{u}_R^i \not{D} u_R^i + i\overline{d}_R^i \not{D} d_R^i + i\overline{L}_L^i \not{D} L_L^i + i\overline{e}_R^i \not{D} e_R^i \qquad \text{Matter}$$

Standard Model

$$L = -\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} - \tfrac{1}{4}G^{A\mu\nu}G^{A}_{\mu\nu} - \tfrac{1}{4}W^{A\mu\nu}W^{A}_{\mu\nu}$$ Gauge

$$+ i\overline{Q}^{i}_{L}\not{D}Q^{i}_{L} + i\overline{u}^{i}_{R}\not{D}u^{i}_{R} + i\overline{d}^{i}_{R}\not{D}d^{i}_{R} + i\overline{L}^{i}_{L}\not{D}L^{i}_{L} + i\overline{e}^{i}_{R}\not{D}e^{i}_{R}$$ Matter

$$+\left(D^{\mu}\phi\right)^{\dagger}D_{\mu}\phi + \mu^{2}\phi^{\dagger}\phi - \lambda\left(\phi^{\dagger}\phi\right)^{2}$$ Higgs



$$\langle\phi^{0}\rangle = \frac{v}{\sqrt{2}}$$

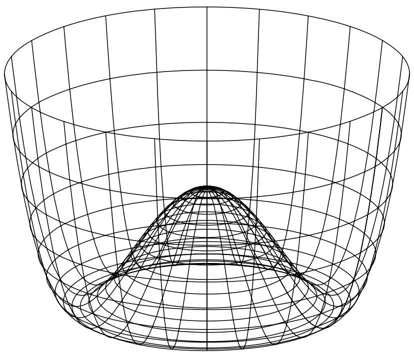
Standard Model

$$L = -\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} - \tfrac{1}{4}G^{A\mu\nu}G^{A}_{\mu\nu} - \tfrac{1}{4}W^{A\mu\nu}W^{A}_{\mu\nu}$$ Gauge

$$+ i\overline{Q}^{i}_{L}\not{D}Q^{i}_{L} + i\overline{u}^{i}_{R}\not{D}u^{i}_{R} + i\overline{d}^{i}_{R}\not{D}d^{i}_{R} + i\overline{L}^{i}_{L}\not{D}L^{i}_{L} + i\overline{e}^{i}_{R}\not{D}e^{i}_{R}$$ Matter

$$-\Gamma^{ij}_{u}\overline{Q}^{i}_{L}\varepsilon\phi^{*}u^{j}_{R} - \Gamma^{ij}_{d}\overline{Q}^{i}_{L}\phi d^{j}_{R} - \Gamma^{ij}_{e}\overline{L}^{i}_{L}\phi e^{j}_{R} + h.c.$$ Yukawa

$$+\left(D^{\mu}\phi\right)^{\dagger}D_{\mu}\phi + \mu^{2}\phi^{\dagger}\phi - \lambda\left(\phi^{\dagger}\phi\right)^{2}$$ Higgs



$$\langle\phi^{0}\rangle = \frac{v}{\sqrt{2}}$$

Standard Model

- Quantum Field Theory
  - Most general Lagrangian consistent with gauge symmetry
  - Chiral gauge theory

$$\psi_{L,R} = \frac{1 \mp \gamma_5}{2} \psi$$

  - No $\nu_R^i = \nu_{eR}, \nu_{\mu R}, \nu_{\tau R}$ (Why?)

- Accidental global symmetries
  - Baryon number B
  - Lepton number L

Standard Model

	SU(3)	SU(2)	$U(1)_Y$	$U(1)_B$	$U(1)_L$
Q_L^i	3	2	1/6	1/3	0
u_R^i	3	1	2/3	1/3	0
d_R^i	3	1	-1/3	1/3	0
L_L^i	1	2	-1/2	0	1
e_R^i	1	1	-1	0	1
ϕ	1	2	1/2	0	0

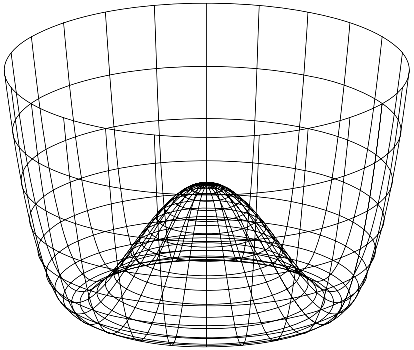
Standard Model

$$L = -\tfrac{1}{4} F^{\mu\nu} F_{\mu\nu} - \tfrac{1}{4} G^{A\mu\nu} G^{A}_{\mu\nu} - \tfrac{1}{4} W^{A\mu\nu} W^{A}_{\mu\nu}$$ Gauge

$$+ i\overline{Q}{}^{i}_{L} \not{D} Q^{i}_{L} + i\overline{u}{}^{i}_{R} \not{D} u^{i}_{R} + i\overline{d}{}^{i}_{R} \not{D} d^{i}_{R} + i\overline{L}{}^{i}_{L} \not{D} L^{i}_{L} + i\overline{e}{}^{i}_{R} \not{D} e^{i}_{R}$$ Matter

$$- \Gamma^{ij}_{u} \overline{Q}{}^{i}_{L} \varepsilon \phi^{*} u^{j}_{R} - \Gamma^{ij}_{d} \overline{Q}{}^{i}_{L} \phi d^{j}_{R} - \Gamma^{ij}_{e} \overline{L}{}^{i}_{L} \phi e^{j}_{R} + h.c.$$ Yukawa

$$+ \left(D^{\mu}\phi\right)^{\dagger} D_{\mu}\phi + \mu^{2}\phi^{\dagger}\phi - \lambda\left(\phi^{\dagger}\phi\right)^{2}$$ Higgs



$$\langle \phi^{0} \rangle = \frac{v}{\sqrt{2}}$$

Standard Model

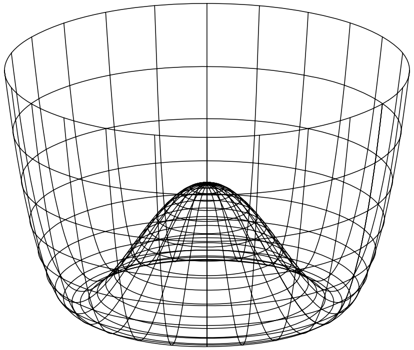
$$L = -\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} - \tfrac{1}{4}G^{A\mu\nu}G^{A}_{\mu\nu} - \tfrac{1}{4}W^{A\mu\nu}W^{A}_{\mu\nu}$$ Gauge

$$+ i\overline{Q}^{i}_{L}\not{D}Q^{i}_{L} + i\overline{u}^{i}_{R}\not{D}u^{i}_{R} + i\overline{d}^{i}_{R}\not{D}d^{i}_{R} + i\overline{L}^{i}_{L}\not{D}L^{i}_{L} + i\overline{e}^{i}_{R}\not{D}e^{i}_{R}$$ Matter

$$-M^{ij}_{u}\overline{u}^{i}_{L}u^{j}_{R} - M^{ij}_{d}\overline{d}^{i}_{L}d^{j}_{R} - M^{ij}_{e}\overline{e}^{i}_{L}e^{j}_{R} + h.c.$$ Fermion masses

$$+M_{W}{}^{2}W_{\mu}{}^{+}W^{\mu-} + \frac{1}{2}M_{Z}{}^{2}Z_{\mu}Z^{\mu} - \frac{1}{2}m_{h}{}^{2}h^{2} + \ldots$$ Boson masses

$$M^{ij} = \frac{v}{\sqrt{2}}\Gamma^{ij}$$



$$\langle\phi^{0}\rangle = \frac{v}{\sqrt{2}}$$

$$M_{W} = \frac{1}{2}gv$$

Standard Model

- All masses proportional to v
  - Explains why masses << M_{Planck}, M_{GUT}
- Fermion masses
  - Dirac mass $\qquad m_D \bar{\psi}_L \psi_R + h.c.$
- Massless neutrinos

SM: Quark mixing

$$L = -\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} - \tfrac{1}{4}G^{A\mu\nu}G^{A}_{\mu\nu} - \tfrac{1}{4}W^{A\mu\nu}W^{A}_{\mu\nu}$$ Gauge

$$+i\overline{Q}_L^i \not{D} Q_L^i + i\overline{u}_R^i \not{D} u_R^i + i\overline{d}_R^i \not{D} d_R^i + i\overline{L}_L^i \not{D} L_L^i + i\overline{e}_R^i \not{D} e_R^i$$ Matter

$$-M_u^{ij}\overline{u}_L^i u_R^j - M_d^{ij}\overline{d}_L^i d_R^j - M_e^{ij}\overline{e}_L^i e_R^j + h.c.$$ Fermion masses

$$+M_W{}^2 W_\mu{}^+ W^{\mu-} + \frac{1}{2}M_Z{}^2 Z_\mu Z^\mu - \frac{1}{2}m_h{}^2 h^2 + \ldots$$ Boson masses

$$u_L^i \to A_{u_L}^{ij} u_L^j \qquad d_L^i \to A_{d_L}^{ij} d_L^j$$

$$u_R^i \to A_{u_R}^{ij} u_R^j \qquad d_R^i \to A_{d_R}^{ij} d_R^j$$

Field redefinitions

SM: Quark mixing

$$L = -\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} - \tfrac{1}{4}G^{A\mu\nu}G^{A}_{\mu\nu} - \tfrac{1}{4}W^{A\mu\nu}W^{A}_{\mu\nu}$$ Gauge

$$+i\overline{Q}^{i}_{L}\not{D}Q^{i}_{L} + i\overline{u}^{i}_{R}\not{D}u^{i}_{R} + i\overline{d}^{i}_{R}\not{D}d^{i}_{R} + i\overline{L}^{i}_{L}\not{D}L^{i}_{L} + i\overline{e}^{i}_{R}\not{D}e^{i}_{R}$$ Matter

$$-M^{ij}_{Du}\overline{u}^{i}_{L}u^{j}_{R} - M^{ij}_{Dd}\overline{d}^{i}_{L}d^{j}_{R} - M^{ij}_{e}\overline{e}^{i}_{L}e^{j}_{R} + h.c.$$ Fermion masses

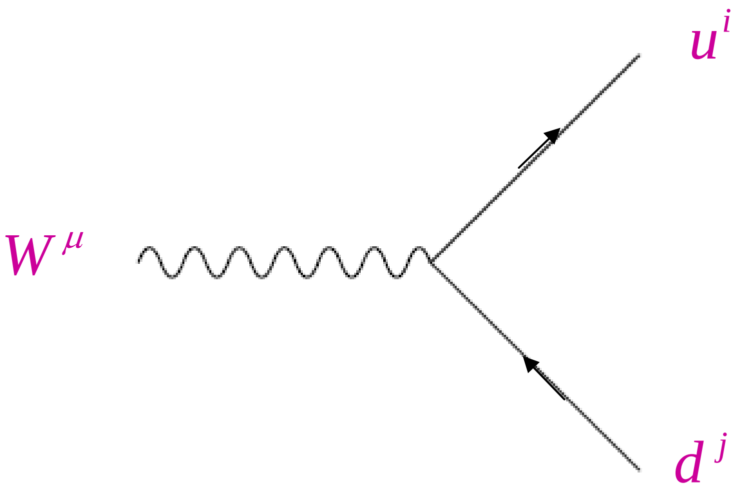
$$+M_W^{\;2}W_{\mu}^{\;+}W^{\mu-} + \frac{1}{2}M_Z^{\;2}Z_{\mu}Z^{\mu} - \frac{1}{2}m_h^{\;2}h^2 + \ldots$$ Boson masses

$$u^{i}_{L} \to A^{ij}_{u_L}u^{j}_{L} \qquad d^{i}_{L} \to A^{ij}_{d_L}d^{j}_{L}$$

$$u^{i}_{R} \to A^{ij}_{u_R}u^{j}_{R} \qquad d^{i}_{R} \to A^{ij}_{d_R}d^{j}_{R}$$

$$\to \quad M_{Du} = A^{\dagger}_{u_L}M_{u}A_{u_R}, \qquad M_{Dd} = A^{\dagger}_{d_L}M_{d}A_{d_R}$$

CKM matrix



$$= -i\frac{g}{2\sqrt{2}} V_{ij}\gamma^\mu (1-\gamma_5)$$

$$V \equiv A^\dagger_{u_L} A_{d_L}$$

CKM matrix

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

where $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$

Four parameters: $\theta_{12}, \theta_{23}, \theta_{13}, \delta$

Neutrino masses

- Dirac masses
  - Add $\nu_R^i = \nu_{eR}, \nu_{\mu R}, \nu_{\tau R}$ to the SM

Standard Model + ν_R^i

	SU(3)	SU(2)	$U(1)_Y$	$U(1)_B$	$U(1)_L$
Q_L^i	3	2	1/6	1/3	0
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L_L^i	1	2	-1/2	0	1
e_R^i	1	1	-1	0	1
ν_R^i	1	1	0	0	1

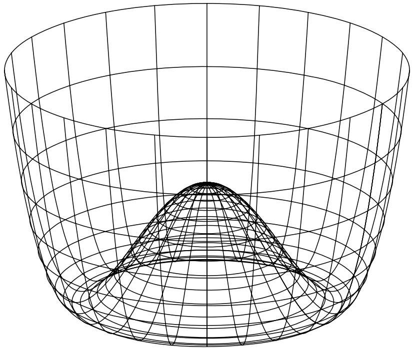
Standard Model + ν_R^i

$$L = -\tfrac{1}{4} F^{\mu\nu} F_{\mu\nu} - \tfrac{1}{4} G^{A\mu\nu} G^A_{\mu\nu} - \tfrac{1}{4} W^{A\mu\nu} W^A_{\mu\nu}$$ Gauge

$$+ i\overline{Q}_L^i \not{D} Q_L^i + i\overline{u}_R^i \not{D} u_R^i + i\overline{d}_R^i \not{D} d_R^i + i\overline{L}_L^i \not{D} L_L^i + i\overline{e}_R^i \not{D} e_R^i$$ Matter

$$-\Gamma_u^{ij} \overline{Q}_L^i \varepsilon \phi^* u_R^j - \Gamma_d^{ij} \overline{Q}_L^i \phi d_R^j - \Gamma_e^{ij} \overline{L}_L^i \phi e_R^j - \Gamma_\nu^{ij} \overline{L}_L^i \varepsilon \phi^* \nu_R^j + h.c.$$ Yukawa

$$+ \left(D^\mu \phi\right)^\dagger D_\mu \phi + \mu^2 \phi^\dagger \phi - \lambda \left(\phi^\dagger \phi\right)^2$$ Higgs



$$\langle \phi^0 \rangle = \frac{v}{\sqrt{2}}$$

Standard Model + ν_R^i

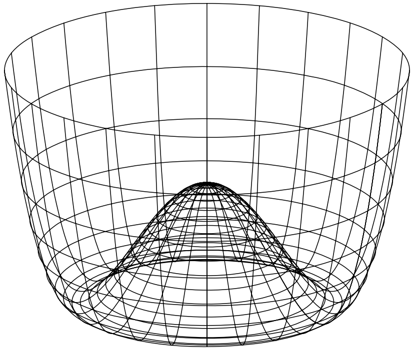
$$L = -\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} - \tfrac{1}{4}G^{A\mu\nu}G^{A}_{\mu\nu} - \tfrac{1}{4}W^{A\mu\nu}W^{A}_{\mu\nu}$$ Gauge

$$+ i\overline{Q}_L^i \not{D} Q_L^i + i\overline{u}_R^i \not{D} u_R^i + i\overline{d}_R^i \not{D} d_R^i + i\overline{L}_L^i \not{D} L_L^i + i\overline{e}_R^i \not{D} e_R^i$$ Matter

$$- M_u^{ij}\overline{u}_L^i u_R^j - M_d^{ij}\overline{d}_L^i d_R^j - M_e^{ij}\overline{e}_L^i e_R^j - M_\nu^{ij}\overline{\nu}_L^i \nu_R^j + h.c.$$ Fermion masses

$$+ M_W{}^2 W_\mu^+ W^{\mu-} + \frac{1}{2}M_Z{}^2 Z_\mu Z^\mu - \frac{1}{2}m_h{}^2 h^2 + \ldots$$ Boson masses

$$M^{ij} = \frac{v}{\sqrt{2}}\Gamma^{ij}$$



$$\langle \phi^0 \rangle = \frac{v}{\sqrt{2}}$$

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Quark and lepton mixing

$$L = -\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} - \tfrac{1}{4}G^{A\mu\nu}G^{A}_{\mu\nu} - \tfrac{1}{4}W^{A\mu\nu}W^{A}_{\mu\nu}$$ Gauge

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$$-M_{u}^{ij}\overline{u}^{i}_{L}u^{j}_{R} - M_{d}^{ij}\overline{d}^{i}_{L}d^{j}_{R} - M_{e}^{ij}\overline{e}^{i}_{L}e^{j}_{R} - M_{\nu}^{ij}\overline{\nu}^{i}_{L}\nu^{j}_{R} + h.c.$$ Fermion masses

$$+M_{W}^{2}W_{\mu}^{+}W^{\mu-}\frac{1}{2}M_{Z}^{2}Z_{\mu}Z^{\mu} - \frac{1}{2}m_{h}^{2}h^{2} + \ldots$$ Boson masses

$$u^{i}_{L} \to A^{ij}_{u_L}u^{j}_{L} \qquad d^{i}_{L} \to A^{ij}_{d_L}d^{j}_{L} \qquad e^{i}_{L} \to A^{ij}_{e_L}e^{j}_{L} \qquad \nu^{i}_{L} \to A^{ij}_{\nu_L}\nu^{j}_{L}$$

$$u^{i}_{R} \to A^{ij}_{u_R}u^{j}_{R} \qquad d^{i}_{R} \to A^{ij}_{d_R}d^{j}_{R} \qquad e^{i}_{R} \to A^{ij}_{e_R}e^{j}_{R} \qquad \nu^{i}_{R} \to A^{ij}_{\nu_R}\nu^{j}_{R}$$

Quark and lepton mixing

$$L = -\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} - \tfrac{1}{4}G^{A\mu\nu}G^{A}_{\mu\nu} - \tfrac{1}{4}W^{A\mu\nu}W^{A}_{\mu\nu}$$ Gauge

$$+ i\overline{Q}^{i}_{L}\not{D}Q^{i}_{L} + i\overline{u}^{i}_{R}\not{D}u^{i}_{R} + i\overline{d}^{i}_{R}\not{D}d^{i}_{R} + i\overline{L}^{i}_{L}\not{D}L^{i}_{L} + i\overline{e}^{i}_{R}\not{D}e^{i}_{R}$$ Matter

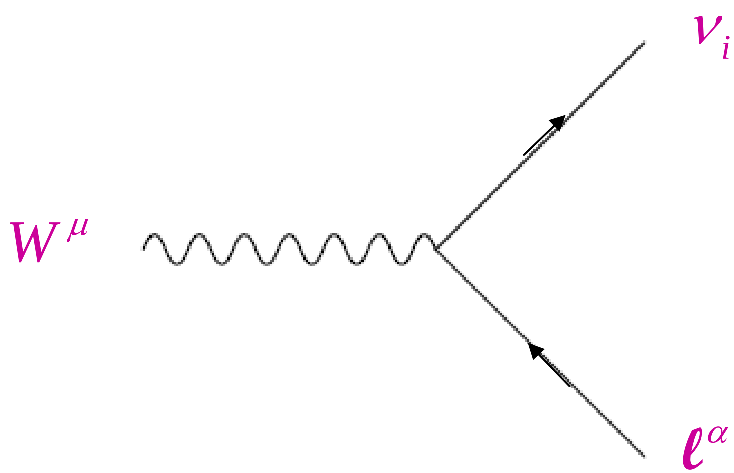
$$-M^{ij}_{Du}\overline{u}^{i}_{L}u^{j}_{R} - M^{ij}_{Dd}\overline{d}^{i}_{L}d^{j}_{R} - M^{ij}_{De}\overline{e}^{i}_{L}e^{j}_{R} - M^{ij}_{D\nu}\overline{\nu}^{i}_{L}\nu^{j}_{R} + h.c.$$ Fermion masses

$$+M_W^{\,2}W_{\mu}^{+}W^{\mu-}\frac{1}{2}M_Z^{\,2}Z_{\mu}Z^{\mu} - \frac{1}{2}m_h^{\,2}h^2 + \ldots$$ Boson masses

$$M_{Du} = A^{\dagger}_{u_L}M_u A_{u_R} \qquad M_{De} = A^{\dagger}_{e_L}M_e A_{e_R}$$

$$M_{Du} = A^{\dagger}_{u_L}M_u A_{u_R} \qquad M_{D\nu} = A^{\dagger}_{\nu_L}M_\nu A_{\nu_R}$$

MNS matrix



$$= -i\frac{g}{2\sqrt{2}}U^*_{\alpha i}\gamma^\mu\left(1-\gamma_5\right)$$

$$U^\dagger \equiv A^\dagger_{\nu_L} A_{e_L}$$

MNS matrix

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

where $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$

Four parameters: $\theta_{12}, \theta_{23}, \theta_{13}, \delta$

Neutrino masses

- Dirac masses
  - Add $\nu_R^i = \nu_{eR}, \nu_{\mu R}, \nu_{\tau R}$ to the SM
    - Mass eigenstates ν_i with masses m_i
    - L conserved
    - No explanation of why $m_i \ll m_e^i, m_d^i, m_u^i$

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- Dirac masses
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    - No explanation of why $m_i \ll m_e^i, m_d^i, m_u^i$

$\sim m_i / E$

Neutrino masses

- Dirac masses
  - Add $\nu_R^i = \nu_{eR}, \nu_{\mu R}, \nu_{\tau R}$ to the SM
    - Mass eigenstates ν_i with masses m_i
    - L conserved
    - No explanation of why $m_i \ll m_e^i, m_d^i, m_u^i$
- Majorana masses
  - Add dim 5 operator to the SM

SM: Dimensional Analysis

$$L = -\tfrac{1}{4} F^{\mu\nu} F_{\mu\nu} - \tfrac{1}{4} G^{A\mu\nu} G^{A}_{\mu\nu} - \tfrac{1}{4} W^{A\mu\nu} W^{A}_{\mu\nu}$$ Gauge

$$+ i\overline{Q}^i_L \not{D} Q^i_L + i\overline{u}^i_R \not{D} u^i_R + i\overline{d}^i_R \not{D} d^i_R + i\overline{L}^i_L \not{D} L^i_L + i\overline{e}^i_R \not{D} e^i_R$$ Matter

$$-\Gamma^{ij}_u \overline{Q}^i_L \varepsilon \phi^* u^j_R - \Gamma^{ij}_d \overline{Q}^i_L \phi d^j_R - \Gamma^{ij}_e \overline{L}^i_L \phi e^j_R + h.c.$$ Yukawa

$$+\left(D^\mu \phi\right)^\dagger D_\mu \phi + \mu^2 \phi^\dagger \phi - \lambda \left(\phi^\dagger \phi\right)^2$$ Higgs

$$\dim A^\mu = 1$$

$$\dim \psi = \frac{3}{2}$$

$$\dim \phi = 1$$

All terms dim 4 or less

Standard Model + dim 5

$$L = -\tfrac{1}{4} F^{\mu\nu} F_{\mu\nu} - \tfrac{1}{4} G^{A\mu\nu} G^{A}_{\mu\nu} - \tfrac{1}{4} W^{A\mu\nu} W^{A}_{\mu\nu}$$ Gauge

$$+ i\overline{Q}_L^i \not{D} Q_L^i + i\overline{u}_R^i \not{D} u_R^i + i\overline{d}_R^i \not{D} d_R^i + i\overline{L}_L^i \not{D} L_L^i + i\overline{e}_R^i \not{D} e_R^i$$ Matter

$$-\Gamma_u^{ij} \overline{Q}_L^i \varepsilon \phi^* u_R^j - \Gamma_d^{ij} \overline{Q}_L^i \phi d_R^j - \Gamma_e^{ij} \overline{L}_L^i \phi e_R^j + h.c.$$ Yukawa

$$+ \left(D^\mu \phi\right)^\dagger D_\mu \phi + \mu^2 \phi^\dagger \phi - \lambda \left(\phi^\dagger \phi\right)^2$$ Higgs

$$+ \frac{c^{ij}}{M} (L_L^{iT} \varepsilon \phi) C (\phi^T \varepsilon L_L^j) + h.c. + \frac{1}{M^2} \dim 6 + \ldots$$ Dim 5

$$\dim A^\mu = 1$$

$$\dim \psi = \frac{3}{2}$$

$$\dim \phi = 1$$

Violates L

Standard Model + dim 5

$$L = -\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} - \tfrac{1}{4}G^{A\mu\nu}G^{A}_{\mu\nu} - \tfrac{1}{4}W^{A\mu\nu}W^{A}_{\mu\nu}$$ Gauge

$$+ i\overline{Q}^{i}_{L}\not{D}Q^{i}_{L} + i\overline{u}^{i}_{R}\not{D}u^{i}_{R} + i\overline{d}^{i}_{R}\not{D}d^{i}_{R} + i\overline{L}^{i}_{L}\not{D}L^{i}_{L} + i\overline{e}^{i}_{R}\not{D}e^{i}_{R}$$ Matter

$$-M^{ij}_{u}\overline{u}^{i}_{L}u^{j}_{R} - M^{ij}_{d}\overline{d}^{i}_{L}d^{j}_{R} - M^{ij}_{e}\overline{e}^{i}_{L}e^{j}_{R} + h.c.$$ Fermion masses

$$+M_W^{\,2}W_\mu^{\,+}W^{\mu-} \frac{1}{2}M_Z^{\,2}Z_\mu Z^\mu - \frac{1}{2}m_h^{\,2}h^2 + \ldots$$ Boson masses

$$+\frac{1}{2}M^{ij}_{\nu}\nu^{iT}_{L}C\nu^{j}_{L} + h.c.$$ Neutrino masses

$$M^{ij} = \frac{v}{\sqrt{2}}\Gamma^{ij} \qquad M^{ij}_{\nu} = \frac{v^2}{M}c^{ij} \qquad \langle\phi^0\rangle = \frac{v}{\sqrt{2}}$$

$$M_W = \frac{1}{2}gv$$

Standard Model + dim 5

- All Dirac masses proportional to v
  - Explains why Dirac masses << M_{Planck}, M_{GUT}
- Neutrino Majorana masses proportional to v^2/M
  - Tiny neutrino masses if $v \ll M$
    - This is the most general statement of the Seesaw Mechanism

Dirac and Majorana masses

- Fermion masses
  - Dirac mass $m_D \bar{\psi}_L \psi_R + h.c.$
  - Majorana mass $\frac{1}{2} m_M \psi_L^T C \psi_L + h.c.$

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  - Dirac mass $m_D \bar{\psi}_L \psi_R + h.c.$
  - Majorana mass $\frac{1}{2} m_M \psi_L^T C \psi_L + h.c.$
- Neutrino masses
  - Dirac mass $m_D \bar{\nu}_L \nu_R + h.c.$ Conserves L
  - Majorana mass $\frac{1}{2} m_M \nu_L^T C \nu_L + h.c.$ Violates L

SM + dim 5: Lepton mixing

$$L = -\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} - \tfrac{1}{4}G^{A\mu\nu}G^{A}_{\mu\nu} - \tfrac{1}{4}W^{A\mu\nu}W^{A}_{\mu\nu}$$ Gauge

$$+i\overline{Q}{}_L^i \not{D} Q_L^i + i\overline{u}{}_R^i \not{D} u_R^i + i\overline{d}{}_R^i \not{D} d_R^i + i\overline{L}{}_L^i \not{D} L_L^i + i\overline{e}{}_R^i \not{D} e_R^i$$ Matter

$$-M_{Du}^{ij}\overline{u}{}_L^i u_R^j - M_{Dd}^{ij}\overline{d}{}_L^i d_R^j - M_e^{ij}\overline{e}{}_L^i e_R^j + h.c.$$ Fermion masses

$$+M_W{}^2 W_\mu{}^+ W^{\mu-} \frac{1}{2} M_Z{}^2 Z_\mu Z^\mu - \frac{1}{2} m_h{}^2 h^2 + \ldots$$ Boson masses

$$+\frac{1}{2} M_\nu^{ij} \nu_L^{iT} C \nu_L^j + h.c.$$ Neutrino masses

$$e_L^i \to A_{e_L}^{ij} e_L^j \qquad \nu_L^i \to A_{\nu_L}^{ij} \nu_L^j$$

$$e_R^i \to A_{e_R}^{ij} e_R^j$$

Field redefinitions

SM + dim 5: Lepton mixing

$$L = -\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} - \tfrac{1}{4}G^{A\mu\nu}G^{A}_{\mu\nu} - \tfrac{1}{4}W^{A\mu\nu}W^{A}_{\mu\nu}$$ Gauge

$$+i\overline{Q}^{i}_{L}\not{D}Q^{i}_{L} + i\overline{u}^{i}_{R}\not{D}u^{i}_{R} + i\overline{d}^{i}_{R}\not{D}d^{i}_{R} + i\overline{L}^{i}_{L}\not{D}L^{i}_{L} + i\overline{e}^{i}_{R}\not{D}e^{i}_{R}$$ Matter

$$-M^{ij}_{Du}\overline{u}^{i}_{L}u^{j}_{R} - M^{ij}_{Dd}\overline{d}^{i}_{L}d^{j}_{R} - M^{ij}_{De}\overline{e}^{i}_{L}e^{j}_{R} + h.c.$$ Fermion masses

$$+M_W{}^2 W_\mu{}^+ W^{\mu-} \frac{1}{2}M_Z{}^2 Z_\mu Z^\mu - \frac{1}{2}m_h{}^2 h^2 + \ldots$$ Boson masses

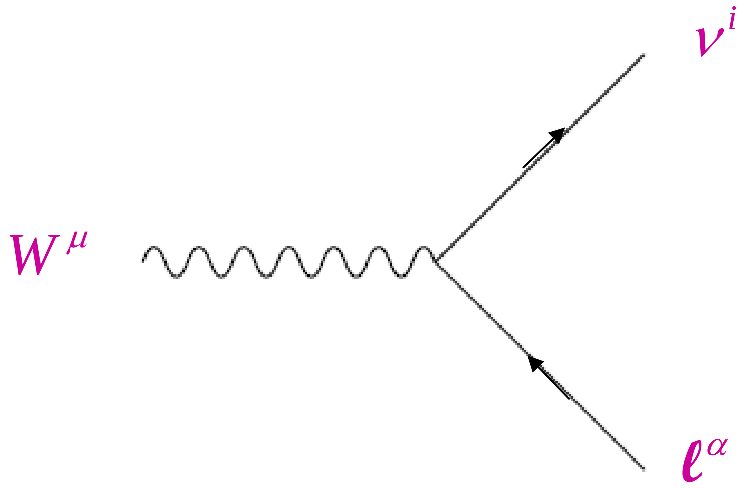
$$+\frac{1}{2}M^{ij}_{D\nu}\nu^{iT}_{L}C\nu^{j}_{L} + h.c.$$ Neutrino masses

$$e^{i}_{L} \to A^{ij}_{e_L} e^{j}_{L} \qquad \nu^{i}_{L} \to A^{ij}_{\nu_L}\nu^{j}_{L}$$

$$e^{i}_{R} \to A^{ij}_{e_R} e^{j}_{R}$$

$$\to \quad M_{De} = A^{\dagger}_{e_L} M_e A_{e_R}, \qquad M_{D\nu} = A^{T}_{\nu_L} M_\nu A_{\nu_L}$$

MNS matrix



$$= -i\frac{g}{2\sqrt{2}}U^*_{\alpha i}\gamma^\mu\left(1-\gamma_5\right)$$

$$U^\dagger \equiv A^\dagger_{\nu_L} A_{e_L}$$

MNS matrix

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$$\times\ \mathrm{diag}(e^{i\alpha_1/2},\ e^{i\,\alpha_2/2},\ 1)$$

where $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$

Six parameters: $\theta_{12}, \theta_{23}, \theta_{13}, \delta, \alpha_1, \alpha_2$

Neutrino masses

- Dirac masses
  - Add $\nu_R^i = \nu_{eR}, \nu_{\mu R}, \nu_{\tau R}$ to the SM
    - Mass eigenstates ν_i with masses m_i
    - L conserved
    - No explanation of why $m_i \ll m_e^i, m_d^i, m_u^i$
- Majorana masses
  - Add dim 5 operator to the SM
    - L violated
    - Neutrino = Antineutrino
    - Explains why $m_i \ll m_e^i, m_d^i, m_u^i$

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Neutrino masses

- Dirac masses
  - Add $\nu_R^i = \nu_{eR}, \nu_{\mu R}, \nu_{\tau R}$ to the SM: $\Gamma_\nu^{ij} \overline{L}_L^i \varepsilon \phi^* \nu_R^j$
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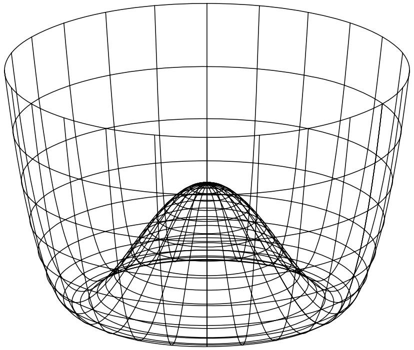
Standard Model + ν_R^i

$$L = -\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} - \tfrac{1}{4}G^{A\mu\nu}G^{A}_{\mu\nu} - \tfrac{1}{4}W^{A\mu\nu}W^{A}_{\mu\nu}$$ Gauge

$$+ i\overline{Q}_L^i \not{D} Q_L^i + i\overline{u}_R^i \not{D} u_R^i + i\overline{d}_R^i \not{D} d_R^i + i\overline{L}_L^i \not{D} L_L^i + i\overline{e}_R^i \not{D} e_R^i$$ Matter

$$-\Gamma_u^{ij}\overline{Q}_L^i \varepsilon\phi^* u_R^j - \Gamma_d^{ij}\overline{Q}_L^i \phi d_R^j - \Gamma_e^{ij}\overline{L}_L^i \phi e_R^j - \Gamma_\nu^{ij}\overline{L}_L^i \varepsilon\phi^* \nu_R^j + h.c.$$ Yukawa

$$+\left(D^\mu\phi\right)^\dagger D_\mu\phi + \mu^2\phi^\dagger\phi - \lambda\left(\phi^\dagger\phi\right)^2$$ Higgs



$$\langle\phi^0\rangle = \frac{v}{\sqrt{2}}$$

Standard Model + ν_R^i

$$L = -\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} - \tfrac{1}{4}G^{A\mu\nu}G^{A}_{\mu\nu} - \tfrac{1}{4}W^{A\mu\nu}W^{A}_{\mu\nu}$$ Gauge

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$$-\Gamma_u^{ij}\overline{Q}_L^i \varepsilon\phi^* u_R^j - \Gamma_d^{ij}\overline{Q}_L^i \phi d_R^j - \Gamma_e^{ij}\overline{L}_L^i \phi e_R^j - \Gamma_\nu^{ij}\overline{L}_L^i \varepsilon\phi^* \nu_R^j + h.c.$$ Yukawa

$$-\frac{1}{2}M_R^{ij}\nu_R^{iT} C \nu_R^j + h.c.$$ Majorana mass

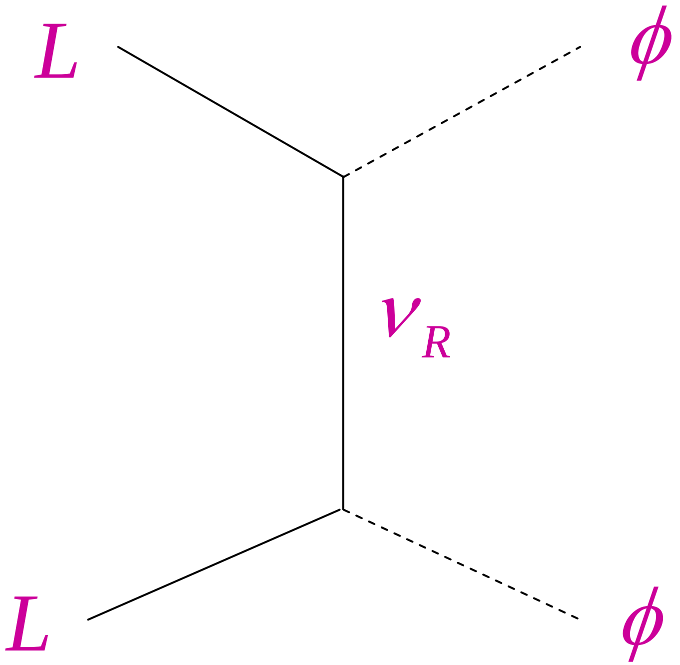
$$+\left(D^\mu\phi\right)^\dagger D_\mu\phi + \mu^2\phi^\dagger\phi - \lambda\left(\phi^\dagger\phi\right)^2$$ Higgs

Majorana mass for ν_R^i

Neutrino masses

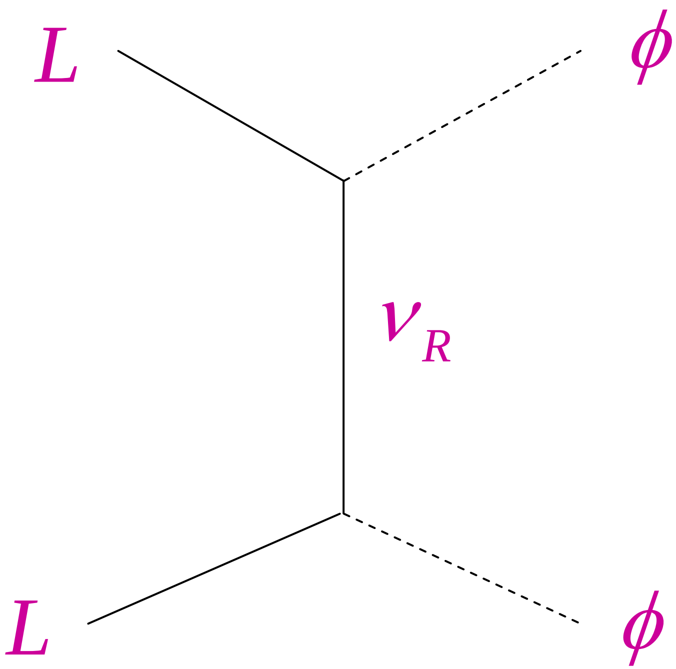
- Majorana mass for ν_R^i
  - Violates L
  - M_R^{ij} is unrelated to ν
    - vanishes if L is imposed $\rightarrow$ Dirac neutrinos
    - might be as large as the GUT scale

Neutrino masses

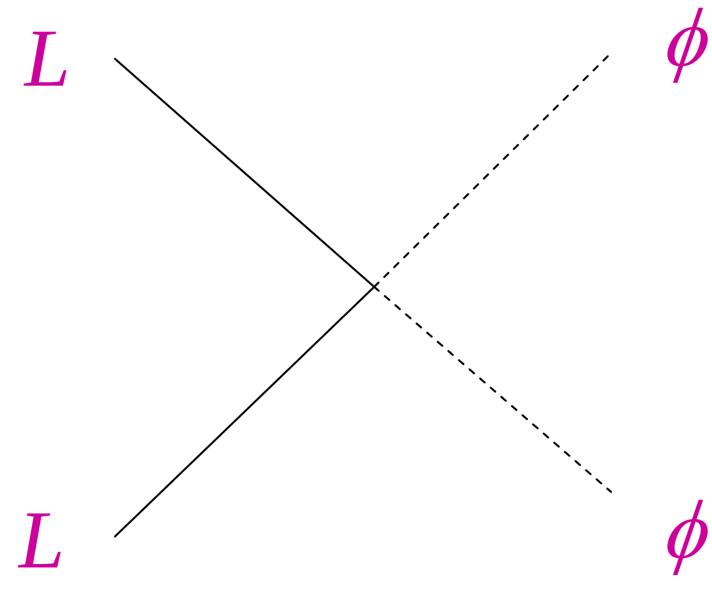


$$\Gamma_{\nu}^{ij}\overline{L}_L^i\varepsilon\phi^*\nu_R^j$$

Neutrino masses



$$\xrightarrow[M_R \gg v]{}$$



$$\Gamma_\nu^{ij} \overline{L}_L^i \varepsilon \phi^* \nu_R^j$$

$$\frac{c^{ij}}{M} (L_L^{iT} \varepsilon \phi) C (\phi^T \varepsilon L_L^j)$$

where $\dfrac{c}{M} \sim \dfrac{\Gamma^2}{M_R}$

Type I Seesaw

Majorana ———————— M_R

Dirac ———————— v

Majorana ———————— $\frac{v^2}{M_R}$

$M_R = 0$

ν_R

$M_R \gg v$

$\nu_R = N_R$

Type I Seesaw

Maj. ———— M_R

Dirac ———— v Maj. ————

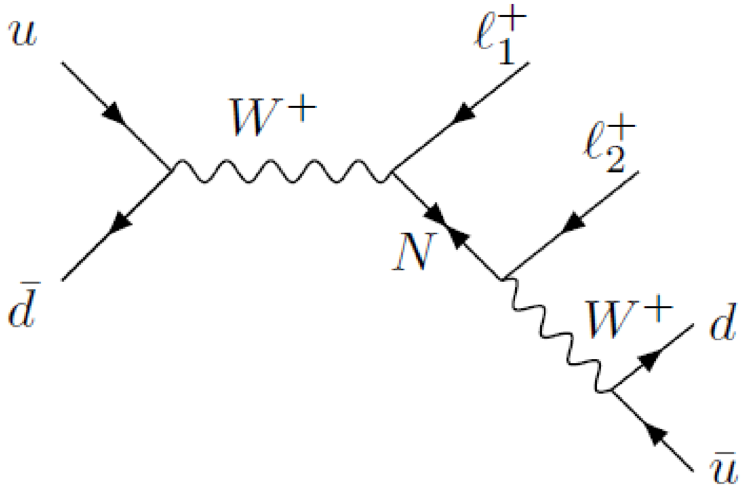
Maj. ————

Maj. ———— $\frac{v^2}{M_R}$

$M_R = 0$ $M_R \sim v$ $M_R \gg v$

ν_R $\nu_R = \nu_s$ $\nu_R = N_R$

Neutrinos and the LHC



$N \approx N_R$

Atre, Han, Pascoli, Zhang 09

Neutrinos and the LHC

- Dirac masses
  - Add $\nu_R^i = \nu_{eR}, \nu_{\mu R}, \nu_{\tau R}$ to the SM: $\Gamma_\nu^{ij} \overline{L}_L^i \varepsilon \phi^* \nu_R^j$
    - Mass eigenstates ν_i with masses m_i
    - L conserved
    - No explanation of why $m_i \ll m_e^i, m_d^i, m_u^i$
- Majorana masses
  - Add dim 5 operator to the SM: $\frac{c^{ij}}{M}(L_L^{iT} \varepsilon \phi) C (\phi^T \varepsilon L_L^j)$
    - L violated
    - Neutrino = Antineutrino
    - Explains why $m_i \ll m_e^i, m_d^i, m_u^i$

Questions

- In the SM, lepton number conservation
  - is accidental
  - is imposed
  - follows from baryon number conservation

Questions

- The mass of the muon neutrino is
  - Zero
  - Nonzero
  - Undefined
  - Oscillatory

Questions

- The MNS matrix relates
  - Charged lepton mass eigenstates to neutrino flavor eigenstates
  - Charged lepton mass eigenstates to neutrino mass eigenstates
  - Neutrino flavor eigenstates to neutrino mass eigenstates

Questions

- Assume that neutrinos are Majorana. Antineutrino beams produced at Fermilab
  - don’t exist
  - contain mostly positive helicity neutrinos
  - have positive chirality

Questions

- There are no dim 5 operators in the SM
  - by definition
  - because they are nonrenormalizable
  - because they yield divergences in loop diagrams

Questions

- The cross on the internal fermion line represents
  - a Majorana mass term
  - a L violating interaction
  - a helicity flip operator