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Models of Neutrino Masses and Mixings

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Why A4 works?

TB mixing corresponds to m in the basis where charged leptons are diagonal

$$m = \begin{pmatrix} x & y & y \\ y & x+v & y-v \\ y & y-v & x+v \end{pmatrix}$$

m is the most general matrix invariant under SmS = m and $A_{23} m A_{23} = m$ with:

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} \qquad A_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

2-3 symmetry

Invariance under S can be made automatic in A4 while invariance under A_{23} happens if 1′ and 1″ flavons are absent.

Three singlet inequivalent represent'ns:

$$\omega = \exp i\frac{2\pi}{3} = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$$
$$\omega^3 = 1$$
$$1 + \omega + \omega^2 = 0$$
$$\omega^2 = \omega^*$$

Recall:
$S^2 = T^3 = (ST)^3 = 1$

$$\begin{cases} 1: \; S=1,\, T=1 \\ 1': \; S=1,\, T=\omega \\ 1'': \; S=1,\, T=\omega^2 \end{cases}$$

The only irreducible 3-dim represent'n is obtained by:

$$S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \qquad T = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

(S-diag basis)

An equivalent form:

$$VV^\dagger = V^\dagger V = 1$$

$$S' = \frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix} = VSV^\dagger \qquad T' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{bmatrix} = VTV^\dagger \qquad V = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{bmatrix}$$

(T-diag basis)

Cabibbo '78

Charged lepton masses are a generic diagonal matrix, invariant under T (or ηT with η a phase):

$$m_l = v_T \frac{v_d}{\Lambda} \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}$$

$$m_l^+ m_l = T^+ m_l^+ m_l T$$

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}$$

$$\langle \varphi_T \rangle = (v_T, 0, 0)$$
$$\langle \varphi_S \rangle = (v_S, v_S, v_S)$$
$$\langle \xi \rangle = u \quad , \quad \langle \tilde{\xi} \rangle = 0$$

The aligment occurs because is based on A4 group theory:

ϕ_T breaks A4 down to G_T
ϕ_S breaks A4 down to G_S
(G_T, G_S: subgroups generated by T, S)

Note that for TB mixing in A4 it is important that no flavons transforming as 1′ and 1″ exist

Recently Lam claimed that for "a natural" TB model the smallest group is S4 (instead A4 is a subgroup of S4)

This is because he calls "natural" a model only if all possible flavons are introduced

We do not accept this criterium:

In physics we call natural a model if the lagrangian is the most general given the symmetry and the representations of the fields
(for example the SM is natural even if only Higgs doublets are present)

A baseline A4 model (a 4-dim SUSY version with see-saw)

$$w_l = y_e e^c(\varphi_T l) + y_\mu \mu^c(\varphi_T l)' + y_\tau \tau^c(\varphi_T l)'' + y(\nu^c l) +$$ ch. leptons

$$+(x_A \xi + \tilde{x}_A \tilde{\xi})(\nu^c \nu^c) + x_B(\varphi_S \nu^c \nu^c) + h.c. + ...$$ neutrinos

shorthand: Higg, U(1) flavon θ, and cut-off scale Λ omitted, e.g.:

$$y_e e^c(\varphi_T l) \sim y_e e^c(\varphi_T l) h_d \theta^4/\Lambda^5$$

Fields and their transformation properties

	l	e^c	μ^c	τ^c	ν^c	$h_{u,d}$	θ	φ_T	φ_S	ξ	φ_0^T	φ_0^S	ξ_0
A_4	3	1	1"	1'	3	1	1	3	3	1	3	3	1
Z_3	ω	ω^2	ω^2	ω^2	ω^2	1	1	1	ω^2	ω^2	1	ω^2	ω^2
$U(1)_{FN}$	0	4	2	0	0	0	-1	0	0	0	0	0	0
$U(1)_R$	1	1	1	1	1	0	0	0	0	0	2	2	2

In the T-diagonal basis we have:

$$VV^\dagger = V^\dagger V = 1$$

$$S' = \frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix} = VSV^\dagger \qquad T' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{bmatrix} = VTV^\dagger \qquad V = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{bmatrix}$$

Cabibbo '78

For $3_1=(a_1,a_2,a_3)$, $3_2=(b_1,b_2,b_3)$ we have in $3_1 \times 3_2$:

$$1 = a_1b_1 + a_2b_3 + a_3b_2$$

$$1' = a_3b_3 + a_1b_2 + a_2b_1$$

$$1'' = a_2b_2 + a_1b_3 + a_3b_3$$

We will see that in this basis the charged leptons are diagonal

$$3_{symm} \sim \frac{1}{3}(2a_1b_1 - a_2b_3 - a_3b_2, 2a_3b_3 - a_1b_2 - a_2b_1, 2a_2b_2 - a_1b_3 - a_3b_1)$$

$$3_{antisymm} \sim \frac{1}{2}(a_2b_3 - a_3b_2, a_1b_2 - a_2b_1, a_1b_3 - a_3b_1)$$

Many versions of A4 models exist by now

- with dim-5 effective operators or with see-saw
- with SUSY or without SUSY
- in 4 dimensions or in extra dimensions

  e.g G.A., Feruglio'05; G.A., Feruglio, Lin '06; Csaki et al '08.....

- with different solutions to the alignment problem

  e.g Hirsch, Morisi, Valle '08

- with sequential (or form) dominance

  e.g King'07 ; Chen, King '09

- with charged lepton hierarchy also following from a special alignment (no $U(1)_{FN}$) Lin'08; GA, Meloni'09
- extension to quarks, possibly in a GUT context

Extension to quarks

If we take all fermion doublets as 3 and all singlets as 1, 1′, 1″ (as for leptons): $Q_i \sim 3$, $u^c, d^c \sim 1$, $c^c, s^c \sim 1'$, $t^c, b^c \sim 1''$

Then u and d quark mass matrices, like for charged leptons, are BOTH diagonal in the T-diagonal basis

As a result V_{CKM} is unity: $V_{CKM} = U_u^+ U_d \sim 1$

So, in first approx. (broken by loops and higher dim operators), ν mixings are TB and quark mixings ~identity

Corrections are too small to reproduce quark mixings e.g. λ_C (for leptons, corrections cannot exceed $o(\lambda_C^2)$). But even those are essentially the same for u and d quarks)

A4 is simple and economic for leptons

Aranda, Carone, Lebed
Carr, Frampton
Feruglio et al
Chen, Mahanthappa

One would like to extend the model to quarks

Also one would like a GUT model with all fermion masses and mixings reproduced, which includes TB mixing for ν's from A4

The assignments $Q_i \sim 3$, $u^c, d^c \sim 1$, $c^c, s^c \sim 1'$, $t^c, b^c \sim 1''$ are not compatible with A4 commuting with SU(5).

For A4 to commute with SU(5) one needs

If $l \sim 3$ then all $F_i \sim 5_i^* \sim 3$, so that $d^c_i \sim 3$
if $e^c, \mu^c, \tau^c \sim 1, 1'', 1'$ then all $T_i \sim 10_i \sim 1, 1'', 1'$

Widespread feeling that A4 cannot be unified in a satisfactory way.
We have produced a counterexample

Here is our A4 GUT model (0802.0090[hep-ph])

A SUSY SU(5) Grand Unified Model of Tri-Bimaximal Mixing from A_4

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Abstract

We discuss a grand unified model based on SUSY SU(5) in extra dimensions and on the flavour group $A_4 \times U(1)$ which, besides reproducing tri-bimaximal mixing for neutrinos with the accuracy required by the data, also leads to a natural description of the observed pattern of quark masses and mixings.

SUSY-SU(5) GUT with A4

Key ingredients:

- SUSY

  In general SUSY is crucial for hierarchy, coupling unification and p decay
  Specifically it makes simpler to implement the required alignment

- GUT's in 5 dimensions

  In general GUT's in ED are most natural and effective
  Here also contribute to produce fermion hierarchies

- Extended flavour symmetry: $A4 \times U(1) \times Z_3 \times U(1)_R$

  $U(1)_R$ is a standard ingredient of SUSY GUT's in ED

  Hall-Nomura'01

GUT's in extra dimensions

- Minimal SUSY-SU(5), -SO(10) models are in trouble
- More realistic models are possible but they tend to be baroque (e.g. large Higgs representations)

Recently a new idea has been developed and looks promising: unification in extra dimensions

Kawamura
GA, Feruglio
Hall, Nomura;
Hebecker, March-Russell;
Hall, March-Russell, Okui, Smith
Asaka, Buchmuller, Covi
....

Factorised metric

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + h_{ij}(y) dy^i dy^j$$

The compactification radius $R \sim 1/M_{GUT}$ (not so large!)

Virtues:

- No baroque large Higgs representations
- SUSY and SU(5) breaking by orbifolding
- Doublet-triplet splitting problem solved
- New handles for p decay, flavour hierarchies

SUSY-SU(5) GUT with A4

Key ingredients:

- GUT's in 5 dimensions
- Extended flavour symmetry: $A4 \times U(1) \times Z_3 \times U(1)_R$

Froggatt-Nielsen

Reduces to R-parity when SUSY is broken at m_{soft}

Keeps ϕ_S and ϕ_T separate

Field	N	F	T_1	T_2	T_3	H_5	$H_{\bar{5}}$	φ_T	φ_S	$\xi,\ \tilde{\xi}$	θ	θ''	φ_0^T	φ_0^S	ξ_0
SU(5)	1	$\bar{5}$	10	10	10	5	$\bar{5}$	1	1	1	1	1	1	1	1
A_4	3	3	$1''$	$1'$	1	1	$1'$	3	3	1	1	$1''$	3	3	1
U(1)	0	0	3	1	0	0	0	0	0	0	-1	-1	0	0	0
Z_3	ω	ω	ω	ω	ω	ω	ω	1	ω	ω	1	1	1	ω	ω
$U(1)_R$	1	1	1	1	1	0	0	0	0	0	0	0	2	2	2

U(1) breaking flavons

driving fields for alignment

: in bulk

ED effects contribute to the fermion mass hierarchies

A bulk field is related to its zero mode by: $B = \frac{1}{\sqrt{\pi R}} B^0 + \ldots$

This produces a suppression parameter for couplings with bulk fields $s \equiv \frac{1}{\sqrt{\pi R \Lambda}} < 1$

Λ: UV cutoff

- In bulk: N=2 SUSY Yang-Mills fields + H_5, H_5^{bar} + T_1, T_2, T_1', T_2'

(doubling of bulk fermions to obtain chiral massless states at y=0)

also crucial to avoid too strict mass relations for 1,2 families:

(b-τ unification only for 3rd family)

- All other fields on brane at y=0 (in particular N, F, T_3)

Superpotential terms on the brane
($T_{1,2}$ represent either $T_{1,2}$ or $T'_{1,2}$)

Up masses

$$w_{up} = \frac{1}{\Lambda^{1/2}} H_5 T_3 T_3 + \frac{\theta''}{\Lambda^2} H_5 T_2 T_3 + \frac{\theta''^2}{\Lambda^{7/2}} H_5 T_2 T_2 + \frac{\theta\theta''^2}{\Lambda^4} H_5 T_1 T_3$$
$$+ \frac{\theta^4}{\Lambda^{11/2}} H_5 T_1 T_2 + \frac{\theta\theta''^3}{\Lambda^{11/2}} H_5 T_1 T_2 + \frac{\theta^5\theta''}{\Lambda^{15/2}} H_5 T_1 T_1 + \frac{\theta^2\theta''^4}{\Lambda^{15/2}} H_5 T_1 T_1$$

Down and charged lepton masses

$$w_{down} = \frac{1}{\Lambda^{3/2}} H_{\bar{5}} (F\varphi_T)'' T_3 + \frac{\theta}{\Lambda^3} H_{\bar{5}} (F\varphi_T)' T_2 + \frac{\theta^3}{\Lambda^5} H_{\bar{5}} (F\varphi_T) T_1 + \frac{\theta''^3}{\Lambda^5} H_{\bar{5}} (F\varphi_T) T_1$$
$$+ \frac{\theta''}{\Lambda^3} H_{\bar{5}} (F\varphi_T)'' T_2 + \frac{\theta^2\theta''}{\Lambda^5} H_{\bar{5}} (F\varphi_T)' T_1 + \frac{\theta\theta''^2}{\Lambda^5} H_{\bar{5}} (F\varphi_T)'' T_1 + \ldots \quad ,$$

Neutrino masses from see-saw
(correct relation bewteen m_ν and M_{GUT})

$$w_\nu = \frac{y^D}{\Lambda^{1/2}} H_5 (NF) + (x_a \xi + \tilde{x}_a \tilde{\xi})(NN) + x_b(\varphi_S NN)$$

$$m_u = \begin{pmatrix} s^2t^5t'' + s^2t^2t''^4 & s^2t^4 + s^2tt''^3 & stt''^2 \\ s^2t^4 + s^2tt''^3 & s^2t''^2 & st'' \\ stt''^2 & st'' & 1 \end{pmatrix} sv_u^0 \sim \begin{pmatrix} \lambda^8 & \lambda^6 & \lambda^4 \\ \lambda^6 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} \lambda v_u^0$$

dots=0 in 1st approx
fixed by higher dim operators & corrections to alignment (see later)

$$m_d = \begin{pmatrix} st^3 + st''^3 & \dots & \dots \\ st^2t'' & st & \dots \\ stt''^2 & st'' & 1 \end{pmatrix} v_T sv_d^0 \sim \begin{pmatrix} \lambda^4 & \dots & \dots \\ \lambda^4 & \lambda^2 & \dots \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} v_T \lambda v_d^0$$

$$m_e = \begin{pmatrix} st^3 + st''^3 & st^2t'' & stt''^2 \\ \dots & st & st'' \\ \dots & \dots & 1 \end{pmatrix} v_T sv_d^0 \sim \begin{pmatrix} \lambda^4 & \lambda^4 & \lambda^4 \\ \dots & \lambda^2 & \lambda^2 \\ \dots & \dots & 1 \end{pmatrix} v_T \lambda v_d^0$$

with

$$\frac{\langle\varphi_T\rangle}{\Lambda} = (v_T, 0, 0) \quad , \quad \frac{\langle\varphi_S\rangle}{\Lambda} = (v_S, v_S, v_S) \quad , \quad \frac{\langle\xi\rangle}{\Lambda} = u \qquad \frac{\langle\theta\rangle}{\Lambda} = t \quad , \qquad \frac{\langle\theta''\rangle}{\Lambda} = t''$$

$s \sim t \sim t'' \sim \lambda \sim 0.22$ $\qquad$ $v_T \sim \lambda^2 \sim m_b/m_t$ $\qquad$ v_S, $u \sim \lambda^2$

For ν's after see-saw

$$m_\nu = \frac{1}{3a(a+b)} \begin{pmatrix} 3a+b & b & b \\ b & \frac{2ab+b^2}{b-a} & \frac{b^2-ab-3a^2}{b-a} \\ b & \frac{b^2-ab-3a^2}{b-a} & \frac{2ab+b^2}{b-a} \end{pmatrix} \frac{s^2(v_u^0)^2}{\Lambda}$$

with

$$a \equiv \frac{2x_a u}{(y^D)^2} \quad , \qquad b \equiv \frac{2x_b v_S}{(y^D)^2}$$

m_ν is of the form

$$m = \begin{pmatrix} x & y & y \\ y & x+v & y-v \\ y & y-v & x+v \end{pmatrix} \longrightarrow U = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & +1/\sqrt{2} \end{pmatrix}$$

charged lepton diagonalization for dots=0 contributes λ^4, λ^8, λ^4 terms to 12, 13, 23

with

$$m_1 = \frac{1}{(a+b)} \quad , \qquad m_2 = \frac{1}{a} \quad , \qquad m_3 = \frac{1}{(b-a)}$$

or

$$\frac{2}{m_2} = \frac{1}{m_1} - \frac{1}{m_3}$$

Finally:

By taking $s \sim t \sim t'' \sim \lambda \sim 0.22$ $v_T \sim \lambda^2 \sim m_b/m_t$ $v_S, u \sim \lambda^2$

a good description of all quark and lepton masses is obtained.
As for all U(1) models only $o(\lambda^p)$ predictions can be given (modulo o(1) coeff.s)

TB mixing for neutrinos is reproduced in first approximation

Quark hierarchies force corrections to TB mixing to be $o(\lambda^2)$
(in particular we predict $\theta_{13} \sim o(\lambda^2)$, accessible at T2K).

A moderate fine tuning is needed to fix λ_C and r (nominally of $o(\lambda^2)$ and 1 respectively)

Normal or inverse hierarchy are possible, degenerate ν's are excluded

Thus:

The A4 approach to TB neutrino mixing is shown to be compatible with quark masses and mixings in a GUT model

The unification with quarks fixes the size of the expected deviations from TB mixing: all mixing angles should deviate by o(λ^2) from the TB values

A normal or inverse hierarchy spectrum is indicated with

$$\frac{2}{m_2} = \frac{1}{m_1} - \frac{1}{m_3}$$

But agreement with TB mixing could be accidental

If θ_{13} is found near its present bound this would hint that TB is accidental and bimaximal mixing (BM) could be a better first approximation

There is an intriguing empirical relation:

$$\theta_{12} + \theta_C = (47.0 \pm 1.7)^o \sim \pi/4$$

Raidal'04

Suggests bimaximal mixing in 1st approximation, corrected by charged lepton diagonalization.

Recall that

$$\lambda_C \approx 0.22 \text{ or } \sqrt{\frac{m_\mu}{m_\tau}} \approx 0.24$$

$\lambda_C = \sin\theta_C$

While $\theta_{12} + o(\theta_C) \sim \pi/4$ is easy to realize, exactly $\theta_{12} + \theta_C \sim \pi/4$ is more difficult: no compelling model

Minakata, Smirnov'04

Taking the "complementarity" relation seriously:

$\theta_{12} + \theta_C = (47.0 \pm 1.7)^o \sim \pi/4$ Raidal'04

leads to consider models that give $\theta_{12} = \pi/4$ but for corrections from the diag'tion of charged leptons

$$U_{PMNS} = U_\ell^\dagger U_\nu$$

Recall:

$\lambda_C \approx 0.22$ or $\sqrt{\frac{m_\mu}{m_\tau}} \approx 0.24$

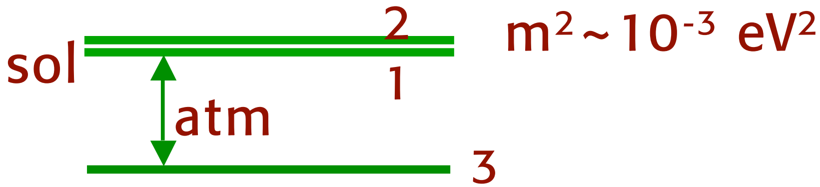
Examples:

- L_e-L_μ-L_τ symmetry
- Bimaximal mixing (BM)

Normally one obtains $\theta_{12} + o(\theta_C) \sim \pi/4$ "weak compl." rather than $\theta_{12} + \theta_C \sim \pi/4$

L_e-L_μ-L_τ symm. & inverted Hierarchy

Zee, Joshipura et al;
Mohapatra et al; Jarlskog et al;
Frampton,Glashow; Barbieri et al
Xing; Giunti, Tanimoto.......



An interesting model:

An exact U(1) L_e-L_μ-L_τ symmetry for m_ν predicts:

(a good 1st approximation)

$$m_\nu = U m_{\nu diag} U^T = m \begin{bmatrix} 0 & 1 & x \\ 1 & 0 & 0 \\ x & 0 & 0 \end{bmatrix} \quad \text{with} \quad m_{\nu diag} = \begin{bmatrix} m' & 0 & 0 \\ 0 & -m' & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

- $\theta_{13} = 0$
- $\theta_{12} = \pi/4$ → θ_{sun} maximal!
- $\tan^2\theta_{23} = x^2$ → θ_{atm} generic

Bimixing would also need μ–τ symm. in m_ν

Can arise from see-saw or dim-5 $L^T H H^T L$

- 1-2 degeneracy stable under rad. corr.'s

1st approximation

$$m_{\nu diag} = \begin{bmatrix} m' & 0 & 0 \\ 0 & -m' & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad m_\nu = U m_{\nu diag} U^T = m \begin{bmatrix} 0 & 1 & x \\ 1 & 0 & 0 \\ x & 0 & 0 \end{bmatrix}$$

- Data? This texture prefers θ_{sol} closer to maximal than θ_{atm}

In fact: 12-> $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \longrightarrow$ Pseudodirac θ_{12} maximal 23-> $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \longrightarrow \theta_{23} \sim o(1)$

With HO corrections: $\begin{bmatrix} 0 & 1 & x \\ 1 & 0 & 0 \\ x & 0 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} \delta & 1 & 1 \\ 1 & \eta & \eta \\ 1 & \eta & \eta \end{bmatrix}$ (modulo o(1) coeff.s)

one gets $1 - tg^2\,\theta_{12} \sim o(\delta+\eta) \sim (\Delta m^2_{sol}/\Delta m^2_{atm})$

Exp. (3σ): 0.46-0.70 0.025-0.039

- In principle one can use the charged lepton mixing to go away from θ_{12} maximal.
  In practice constraints from θ_{13} small ($\delta\theta_{12} \sim \theta_{13}$)
  Frampton et al; GA, Feruglio, Masina '04

Suggests that deviations from BiMaximal mixing arise from charged lepton diagonalisation (BM: $\theta_{12}= \theta_{23} =\pi/4 \ \ \theta_{13}=0$)

$$\tilde{U} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$U = U_e^\dagger U_\nu = \underbrace{\tilde{U}_e^\dagger \mathrm{diag}(-e^{-i(\alpha_1+\alpha_2)}, -e^{-i\alpha_2}, 1)\tilde{U}_\nu}_{=\bar{U}}$$

GA, Feruglio, Masina
Frampton et al
Petcov et al
King
Antusch et al........

$$\bar{U}_{12} = -\frac{e^{-i(\alpha_1+\alpha_2)}}{\sqrt{2}} + \frac{s_{12}^e e^{-i\alpha_2} + s_{13}^e e^{i\delta_e}}{2}$$

$$\bar{U}_{13} = \frac{s_{12}^e e^{-i\alpha_2} - s_{13}^e e^{i\delta_e}}{\sqrt{2}}$$

$$\bar{U}_{23} = -e^{-i\alpha_2}\frac{1 + s_{23}^e e^{i\alpha_2}}{\sqrt{2}}$$

Corr.'s from s^e_{12}, s^e_{13} to U_{12} and U_{13} are of first order (2nd order to U_{23})

For the corrections from the charged lepton sector, typically $|\sin\theta_{13}| \sim (1- \tan^2\theta_{12})/4\cos\delta \sim 0.15$

Needs θ_{13} near its upper bound

Here we construct a model where BM mixing holds in 1st approximation and is then corrected by terms $o(\lambda_C)$ from diagonalisation of charged leptons

Revisiting Bimaximal Neutrino Mixing in a Model with S_4 Discrete Symmetry

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appeared on the web arXiv: 0903.1940

BM mixing

$$\theta_{12} = \theta_{23} = \pi/4, \ \theta_{13} = 0$$

$$U_{BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

By adding $\sin^2\theta_{12} \sim 1/2$ to $\theta_{13} \sim 0$, $\theta_{23} \sim \pi/4$:

Bimaximal Mixing

$$m_\nu = \begin{bmatrix} x & y & y \\ y & z & w \\ y & w & z \end{bmatrix} \rightarrow m_{\nu BM} = \begin{pmatrix} x & y & y \\ y & z & x-z \\ y & x-z & z \end{pmatrix}$$

$$\sin^2 2\theta_{12} = \frac{8y^2}{(x-w-z)^2 + 8y^2}$$

$$m_1 = x + \sqrt{2}y$$

$$m_2 = x - \sqrt{2}y$$

$$m_3 = 2z - x$$

BM corresponds to $\tan^2\theta_{12}=1$
while exp.: $\tan^2\theta_{12} = 0.45 \pm 0.04$
so a large correction is needed

The 3 remaining parameters are the mass eigenvalues

Bimaximal Mixing

$$U_{BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

In the basis of diagonal ch. leptons:

$m_\nu = U \, \mathrm{diag}(m_1, m_2, m_3) \, U^T$

$$m_{\nu BM} = \left[\frac{m_3}{2} M_3 + \frac{m_2}{4} M_2 + \frac{m_1}{4} M_1\right]$$

$$M_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}, \; M_2 = \begin{pmatrix} 2 & -\sqrt{2} & -\sqrt{2} \\ -\sqrt{2} & 1 & 1 \\ -\sqrt{2} & 1 & 1 \end{pmatrix}, \; M_1 = \begin{pmatrix} 2 & \sqrt{2} & \sqrt{2} \\ \sqrt{2} & 1 & 1 \\ \sqrt{2} & 1 & 1 \end{pmatrix}$$

Eigenvectors: $(\sqrt{2}, 1, 1)/2, \; (-\sqrt{2}, 1, 1)/2 \;, \; (0, 1, -1)/\sqrt{2}$

BM mixing corresponds to m
in the basis where
charged leptons are diagonal

$$m_{\nu BM} = \begin{pmatrix} x & y & y \\ y & z & x-z \\ y & x-z & z \end{pmatrix}$$

m is the most general matrix invariant under
$SmS = m$ and $A_{23} m A_{23} = m$ with:

$$S = \begin{pmatrix} 0 & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{2} & \frac{1}{2} \end{pmatrix} \qquad A_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

2-3 symmetry

Invariance under S can be made automatic in S4 while
invariance under A_{23} happens if the flavon content is suitable

S4: Group of permutations of 4 objects (24 transformations)

Irreducible representations: 1, 1', 2, 3, 3'

$$S^2 = T^4 = (ST)^3 = (TS)^3 = 1$$

1 $\quad T = 1 \qquad S = 1$

2 $\quad T = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad S = \frac{1}{2}\begin{pmatrix} -1 & \sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$

3 $\quad T = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -i & 0 \\ 0 & 0 & i \end{pmatrix} \qquad S = \begin{pmatrix} 0 & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$

1 <-> 1' and 3<-> 3' by changing S, T <-> -S, -T

Symmetry: S4xZ4xU(1)$_{FN}$xU(1)$_R$

	l	e^c	μ^c	τ^c	ν^c	$h_{u,d}$	θ	φ_l	χ_l	ψ_l^0	χ_l^0	ξ_ν	φ_ν	ξ_ν^0	φ_ν^0
S_4	3	1	1′	1	3	1	1	3	3′	2	3′	1	3	1	3
Z_4	1	-1	-i	-i	1	1	1	i	i	-1	-1	1	1	1	1
$U(1)_{FN}$	0	2	1	0	0	0	-1	0	0	0	0	0	0	0	0
$U(1)_R$	1	1	1	1	1	1	0	0	0	2	2	0	0	2	2

$$w_l = \frac{y_e^{(1)}}{\Lambda^2}\frac{\theta^2}{\Lambda^2}e^c(l\varphi_l\varphi_l) + \frac{y_e^{(2)}}{\Lambda^2}\frac{\theta^2}{\Lambda^2}e^c(l\chi_l\chi_l) + \frac{y_e^{(3)}}{\Lambda^2}\frac{\theta^2}{\Lambda^2}e^c(l\varphi_l\chi_l) + + \frac{y_\mu}{\Lambda}\frac{\theta}{\Lambda}\mu^c(l\chi_l)' + \frac{y_\tau}{\Lambda}\tau^c(l\varphi_l) + \dots$$

$$w_\nu = y(\nu^c l) + M\Lambda(\nu^c\nu^c) + a(\nu^c\nu^c\xi_\nu) + b(\nu^c\nu^c\varphi_\nu) + \dots$$ ← see-saw

$$\frac{\langle\varphi_\nu\rangle}{\Lambda} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} C \qquad \frac{\langle\xi_\nu\rangle}{\Lambda} = D \qquad \frac{\langle\varphi_l\rangle}{\Lambda} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} A \qquad \frac{\langle\chi_l\rangle}{\Lambda} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} B$$

⊕ Alignment along minimum of most general potential in LO

In leading order charged leptons are diagonal

$$m_l = \begin{pmatrix} (y_e^{(1)} B^2 - y_e^{(2)} A^2 + y_e^{(3)} AB)t^2 & 0 & 0 \\ 0 & y_\mu B t & 0 \\ 0 & 0 & y_\tau A \end{pmatrix} v_d$$

$$\frac{\langle \theta \rangle}{\Lambda} = t$$

$U(1)_{FN}$ flavon VEV

and neutrinos show BM mixing

$$m_\nu^D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} y v_u$$

Dirac

$$M_N = \begin{pmatrix} 2M + 2aD & -2bC & -2bC \\ -2bC & 0 & 2M + 2aD \\ -2bC & 2M + 2aD & 0 \end{pmatrix} \Lambda$$

Majorana

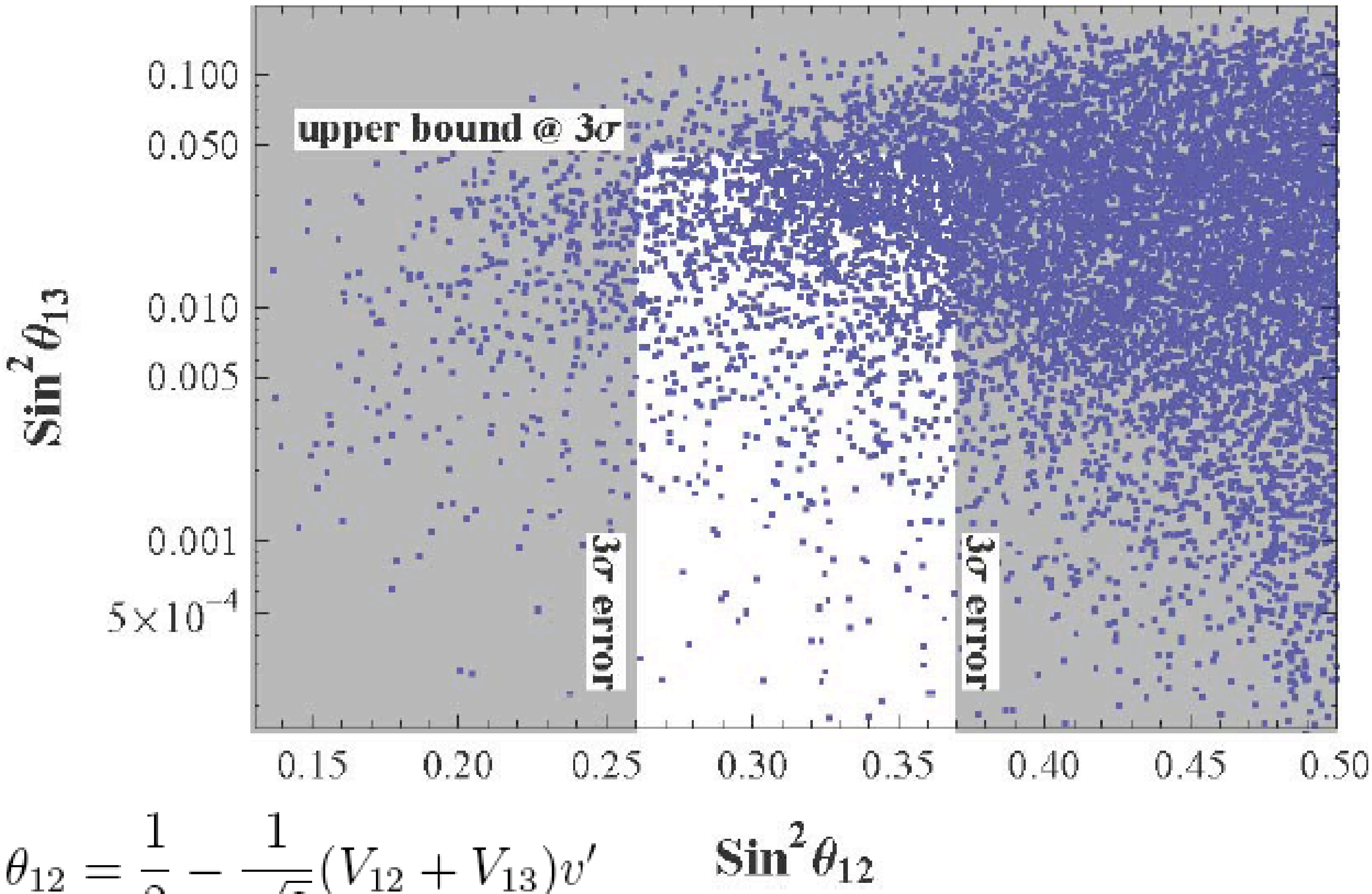
$$|m_1| = \frac{|y^2| v_u^2}{2|M + aD - \sqrt{2} bC|} \frac{1}{\Lambda} \qquad |m_2| = \frac{|y^2| v_u^2}{2|M + aD + \sqrt{2} bC|} \frac{1}{\Lambda} \qquad |m_3| = \frac{|y^2| v_u^2}{2|M + aD|} \frac{1}{\Lambda}$$

$$A \sim B \sim v \, , \qquad C \sim D \sim v'$$

In this model BM mixing is exact at LO

For the special flavon content chosen, at NLO θ_{12} and θ_{13} are corrected only from the charged lepton sector by terms of $o(\lambda_C)$ (large correction!) while θ_{23} gets smaller corrections at NNLO(great!)
[for a generic flavon content also $\delta\theta_{23} \sim o(\lambda_C)$]

An experimental indication for this model would be that θ_{13} is found near its present bound at T2K, CHOOZ2......



$$\sin^2\theta_{12} = \frac{1}{2} - \frac{1}{\sqrt{2}}(V_{12}+V_{13})v'$$

$$\sin\theta_{13} = \frac{1}{\sqrt{2}}(V_{12}-V_{13})v'$$

$|V_{ij}|$ =0-2, v'=0.15

Conclusion

Model building covers a wide spectrum.
Extremes:

No order -> Anarchy

No symmetry, no dynamics assumed, only chance

Maximum order -> Tri-bimaximal mixing

$$U=\begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Specific flavour symmetry: e.g. A4

Indeed the observed pattern of neutrino masses can be accommodated in different models

For example, TB mixing from A4 with small corrections or BM with large corrections from charged lepton diag.

Quark and lepton mixings can be described together and GUT schemes are also possible

But, with many different alternatives that may work, no compelling illumination about the dynamics of flavour has emerged so far.