

# Neutrino Interaction Modeling Survey

May 4, 2020

## 1 Which neutrino interaction processes do you model?

Authors	Processes
Saori Pastore et al.	nu-nucleus qe using quantum monte carlo methods to solve the many-body nuclear problem supported by the short-time-approximation (sta) which allows to extract info on the two-nucleon dynamics involved in the scattering process QE and MEC
Gil Paz et al.	QE
Artur Ankowski et al.	QE
Alessandro Lovato et al.	Elastic scattering, low energy transition, QE
Luis Alvarez et al.	QE, (coherent) pion, eta production and photon emission
Noemi Rocco et al.	QE, MEC, 1 and 2 pion production
Raul jimenez et al.	QE
Minoo Kabirnezhad. et al.	Single pion production
Natalie Jachowicz et al.	Coherent low-energy above nucleon emission threshold (CRPA), quasi-elastic (CRPA), SRC, MEC single pion production
Toru Sato et al.:	Meson(pion,kaon,eta,2pi) production for nucleon in nucleon resonance region
Huma Haider et al.	Deep Inelastic Scattering
Juan Nieves et al.	QE+SpectralFunctions+RPA+2p2h+ pion production (Delta, chiral background, some other N*)
Maria Barbaro et al. (SuSA)	Quasi-elastic scattering, two-nucleon emission (2p2h), pion production, higher resonances, deep inelastic scattering, both CC and NC processes.

## 2 What are the theoretical inputs to these models? (For example Form Factors, PDFs...)

1. Saori Pastore et al.: Nucleonic form factors, many-nucleon interactions (AV18 two-body force + IL7 three-body force), two-body currents are constructed from the given nucleon-nucleon interaction
2. Gil Paz et al.: Form factors and parameters of the Correlated Fermi Gas model from arXiv:1408.0772
3. Artur Ankowski et al.: Form factors, spectral function, optical potential, folding function
4. Alessandro Lovato et al.: Nuclear potentials and consistent electroweak currents; nucleon form factors.

5. Luis Alvarez et al.: Nucleon and nucleon-to-resonance vector and axial form factors, baryon resonance properties: masses, branching ratios to pions, photons, nuclear-density distributions, nuclear form factors
6. Noemi Rocco et al.: Form Factors, Nucleon-Nucleon interactions
7. Raul Jimenez et al.: The model describes the nucleons as independent particles under the influence of relativistic mean field potentials.

Standard phenomenological optical potentials for the p-A system are included, similar to the ones employed to tune the cascade models. Also, phenomenological mean field potentials to describe the initial nucleus are needed. These are available for a large range of nuclei and are reasonably accurate for  $A \geq 4$ .

8. Minoo Kabirnezhad et al.: It has Form Factor, but they are not theoretical input. The new version of MK model has new form factors extracted from electron and pion scattering data.
9. Natalie Jachowicz et al.: Coherent, Mean-field wave functions (coherent, LE, QE, and even for SRC and MEC) i.e. binding and momentum distributions, Pauli blocking are naturally included ; long-range interaction (CRPA) i.e. collective excitations can be described ; correlation parameters (SRC,MEC). The influence of SRC and MEC on 1p1h are also studied.

Natalie Jachowicz et al.: On the single-nucleon level we use the tree-level background diagrams derived from a Chiral Lagrangian that describes the interaction of pion with the nucleon and external gauge bosons (W, Z and photon), the inputs are then the pion-nucleon coupling constant and the vector and axial nucleon form-factors. The vector form factors are well constrained by electron scattering data. For the axial form factor we use a dipole, and the pseudoscalar form factor is then given PCAC and pion-pole dominance. In addition to the background several resonances are included in the model [P33(1232) (Delta), D13(1520), S11(1535) and P11(1440) (Roper)]. Inputs for the resonances are the mass, decay width, and vector and axial form factors. The masses and decay widths are well known, vector form factors are constrained by electron scattering data, we use the parametrizations from different analyses for the vector and axial form factors of the resonances, an overview can be found in [PRD 95, 113007 (2017)]. For higher invariant mass ( $W > 1.6 GeV$ ) a Regge approach is used to model single pion production. The Regge-model requires input of the Regge trajectories of the exchanged Reggeons (Pion-like and rho-like) which are the standard linear trajectories that are extracted from Chew-Frautschi plots. The residual t-dependence of the Regge model is taken from the equivalent exchanges in the background diagrams. The high energy model thus uses the same coupling and form factors as the low energy background. In addition to this an 'off-shell' form factor for the nucleon exchange is included to extend the model to higher values of -t and  $Q^2$ , which requires a single energy cutoff-scale which was fitted to the high-energy neutrino SPP data of [Nucl.Phys. B264 (1986)] as detailed in [PRD 95, 113007 (2017)].

We implement this model in the nucleus by describing the nucleons as independent particles under the influence of a relativistic mean-field potential. For the initial state mean field models are available for a large range of nuclei and are reasonably accurate for  $A \geq 4$ . For the final-state nucleons we can use the same potential as the initial state, and standard phenomenological potentials extracted from p-A scattering.

10. Toru Sato et al.: resonance mass, coupling constants, form factors which describes resonant and non-resonant interactions.
11. Huma Haider et al.: Nuclear Structure Functions

12. Juan Nieves et al.: Free space inputs and well established many body techniques, which are used to (microscopically) calculate nuclear response functions within a systematic scheme.
13. Maria Barbaro et al. (SuSA): Electromagnetic form factors for the 1p1h model: Gari-Krumpelmann extended parametrization (VMD model) is the commonly used but we also have implemented in our codes the usual Galster parametrization and other 7 additional parametrizations (see <https://hdl.handle.net/11441/74826> for details). Axial form factors for the 1p1h model: We employ the common dipole axial form factor. We have recently implemented other 2 parametrizations in our codes based on the two-component model (see Phys. Rev. C 101, 025501 (2020) for details)

PDFs and inelastic nucleon form factors in the SuSAv2-inelastic model: We employ the Bosted-Christy parametrization for the inelastic single nucleon form factors as they provide a very good agreement with  $(e, e')$  data in the full inelastic regime. Other options such as the Bodek-Ritchie parametrization of PDFs (GRV98) are also implemented but they provide a poor agreement with  $(e, e')$  data (see <https://hdl.handle.net/11441/74826> for details)

### 3 What free parameters are there in the model? Are the parameters constrained by any data set, and if so, which data set?

1. Saori Pastore et al.: none (besides the given inputs - see previous question)
2. Gil Paz et al.: no free parameters
3. Artur Ankowski et al.: no free parameters
4. Alessandro Lovato et al.: The nuclear parameters are taken from arXiv:1408.0772. The form factors inputs can be taken from any source
5. Luis Alvarez et al.: electron- and neutrino-nucleon (QE and pion production) , pion-nucleon and electron-nucleus scattering data directly or indirectly help fixing several quantities in the model. Nucleon to resonance axial transition form factors at finite  $Q^2$  remain largely unknown.
6. Noemi Rocco et al.: The nuclear potentials used in the calculations are either the semi-phenomenological AV18-IL7 or determined from chiral effective field theory. The parameters in both cases are determined by NN scattering data and properties of light nuclei.
7. Raul Jimenez et al.: Nucleon form factors: EM ones constrained by elastic electron nucleon scattering, and the axial one... less constrained but as any other model, taken from universal fits (pion decay) or from similar weak interaction experiments (beta decay, miniboone,...)
8. Mino Kabirnezhad et al.: In the original published MK model there are two free parameters for axial form factor but the new version has many parameters for vector form factor, using electron data, and axial form factor, fitted to the pion scattering data.
9. Natalie Jachowicz et al.: nucleon form factors, nuclear interaction parameters (constrained by nuclear radii, electron-scattering data )

Natalie Jachowicz et al.: Vector form factors of the nucleon and its resonances are constrained by electron scattering data. For the vector form factors of the nucleon we use the standard Galster dipole form. The vector form factors of the resonances are taken from the fit in Ref. [Phys.Rev. D74 (2006)]. As well as the axial form factors except for the Delta contribution. For the axial form factors of the spin 3/2 resonances using the Rarita-Schwinger formalism in

general 4 form factors are needed. For the D13 the two suppressed form factors are set to zero and PCAC is used to constrain the C5 coupling at  $Q^2 = 0$  by relating it to the decay rate of the resonance, the C6 pseudoscalar coupling is related to C5 by pion-pole dominance. For the spin  $\hat{A}_{\frac{1}{2}}$  resonances only two axial couplings are needed, these are determined analogously as in the spin 3/2 case. For all the axial form factors (except the delta) the  $Q^2$  dependence is given by a faster than dipole form and the PS coupling is determined by pion-pole dominance. For the delta we include the Watson phases to partially restore unitarity in the isospin 3/2 channel by matching the pi-nucleon scattering phase shifts for the dominant partial waves. The phases and form factors obtained by a fit to ANL-BNL data are given in [Phys.Rev. D93 (2016)], here the  $Q^2$  dependence is a dipole. In addition to this, multi-dipole gaussian form-factors can be included to regularize the unphysical behavior of the tree-level resonances far away from their peak these are given by the prescription of [Phys.Rev. C84 (2011) 045201], the only input to these form-factors is the resonance decay width. Clearly the axial couplings and especially their  $Q^2$  dependence are the largest uncertainties and least well constrained quantities, one could therefore treat them as 'free parameters within certain limits. This uncertainty can only be resolved in any foreseeable way by detailed neutrino induced SPP data which is currently unavailable.

For the high energy Regge model the main free parameter is the cut-off in the axial off-shell form factors. It was fitted to the data of [Nucl.Phys. B264 (1986)]. In addition to this, the  $t$ -dependence of the vector current could be taken from electron scattering data and varied within its error margin. In the current approach the  $t$ -dependence of the Regge exchanges is modelled by the background diagrams, yielding a satisfactory description of electron scattering data. The potentials used to describe the nucleus contain parameters that are fitted so that the resulting ground state reproduces well measured nuclear properties of different nuclei, such as: charge mean radius, binding energies, and nuclear neutron radius. The optical potentials for the final-state nucleon are built to describe proton-nucleus elastic observables, plus neutron-nucleus cross-sections plus proton-nucleus reaction cross-sections. The success of this approach in reproducing both inclusive as exclusive electron scattering data over the past couple of decades lends significant confidence to this model.

10. Toru Sato et al.: electromagnetic(form factors) and strong interaction parameters(resonance mass,coupling constants) are constrained from data of pi-N, gamma-N, gamma\*-N reactions(Jlab, Mainz,MIT..).
11. Huma Haider et al.: None
12. Juan Nieves et al.: No free parameters when producing neutrino cross sections. All parameters were fitted in the 80's and 90's to pion-, photo-, electro-nuclear inclusive and semiexclusive reactions and others processes like nuclear inclusive muon capture.
13. Maria Barbaro et al. (SuSA): In the SuSAv2 models two parameters are present in the quasi-elastic region, associated to the transition from RMF to RPWIA. Two more parameters of the same kind are associated to the inelastic region. These parameters are fitted to electron scattering data. Two more parameters are the Fermi momentum and the energy shift, fitted to the position and width of the quasi-elastic peak for electron scattering and also based on the analysis of RMF results for electrons. The 2p2h calculation has no free parameters.

## 4 Do you have plans to estimate uncertainties for the model? How will the uncertainties be derived?

1. Saori Pastore et al.: yes, two kinds of uncertainties 1) model dependence (use different inputs and study the sensitivity of the results w.r.t. input's variation) 2) use of chiral effective field theory interactions and currents to estimate the theoretical error due to the truncation in the chiral expansion
2. Gil Paz et al.: There are errors bars in arXiv:1408.0772 for the nuclear parameters. These can be combined with form factors uncertainties
3. Artur Ankowski et al.: The uncertainty of double differential cross sections is estimated to be 5% in the paper that describes the approach.
4. Alessandro Lovato et al.: There are errors bars in arXiv:1408.0772 for the nuclear parameters. These can be combined with form factors uncertainties
5. Luis Alvarez et al.: There is a relatively well defined error budget for the relevant parameters. This however does not validate approximations in the model
6. Noemi Rocco et al.: Yes. Use different chiral interactions to obtain the nuclear Spectral Function. Assess uncertainty on the nuclear interactions and use different interactions
7. Raul Jimenez et al.: If the model gets implemented in a MC, then it would be possible to assign weights to the parameters as well as using different potentials. The parameters of the model are generally fit to reproduce known and measured properties of nucleus, such as binding energies, separation energies, nuclear sizes, momentum distributions, nucleon-nucleus elastic observables. There are accepted ranges of variations for these parameters which fit the data equally well. The variation/uncertainty in the neutrino observables cross-sections induced by this parameters can be easily estimated.
8. Mino Kabirnezhad et al.: In the new version of the MK model, the systematic error band is constructed by finding the range in each bin containing 68% of the throws, excluding the same fraction of throws above and below.
9. Natalie Jachowicz et al.: We have used different form factors (dipole, BBBA), mean-field wave functions (Hartree-Fock vs Woods-Saxon, RMF), nuclear interactions (Skyrme vs Landau Migdal); we have tested various relativistic corrections for the CRPA calculations-uncertainties depend on the kind of information you are looking at (e.g. total or differential cross sections), but could be estimated to be around 5 – 10% for normal kinematic conditions

Natalie Jachowicz et al., As discussed in the previous section the largest uncertainties are due to the resonance couplings and their  $Q^2$  dependence. These can be varied within natural boundaries to estimate their effect on neutrino observables. However since there is no clear benchmark for the axial couplings except for bubble chamber data which is not very detailed and has already been used to constrain the delta contributions, there is no real target with respect to which uncertainties can be estimated. For the vector current much more data is available, and thereby uncertainties could be derived by confronting the results with these datasets.

For the nuclear modelling, the parameters of the model are generally fit to reproduce known and measured properties of nucleus, such as binding energies, separation energies, nuclear sizes, momentum distributions, nucleon-nucleus elastic observables. There are accepted ranges of variations for these parameters which fit the available data equally well. The variation/uncertainty in the neutrino cross-sections induced by these parameters can be easily estimated.

10. Toru Sato et al.: large uncertainty in axial form factors of nucleon resonances which is difficult to estimate uncertainties
11. Huma Haider et al.: No
12. Juan Nieves et al.: We have always made efforts in that direction, and in most of our works we provide theoretical uncertainties affecting our results.
13. Maria Barbaro et al. (SuSA): This could be done for the SuSAv2 model making a complete  $\chi^2$  analysis within the determination of the optimum Fermi momentum value, the energy-shift and the transition parameter from RMF to RPWIA.

**5 What nuclei are you modeling? Please include any plans for adding other nuclei, especially argon. For nuclei not explicitly treated in your calculations, is there a way that we can extrapolate?**

1. Saori Pastore et al.: at the moment 4He and we are working to have the c12, the plan is to export the short-time-approximation to 40Ar. scaling can be used to extrapolate to larger nuclear systems.
2. Gil Paz et al.: arXiv:1408.0772 considered symmetric nuclear matter. It's not clear how it applies to argon. We'll have to contact the authors of arXiv:1408.0772
3. Artur Ankowski et al.: Carbon, can be extended to argon in the future.
4. Alessandro Lovato et al.: We can exactly model nuclei up to Carbon-12. In the near future, we plan to reach Oxygen-16. We are also developing more approximate approaches to tackle Argon.
5. Luis Alvarez et al.: With the Local Fermi gas one can make predictions for any medium-heavy nucleus (from Carbon12 on) for whom empirical density distributions are available or can be constrained by interpolation.
6. Noemi Rocco et al.: 4He, 12C, 16O, 40Ar. There is no accurate way to extrapolate
7. Raul Jimenez et al.: Using mean field theory one can easily model any nucleus larger than let say helium 4. Even-even target nucleus would be more accurately described, and generally the predictions will be better for spherical, closed-shell, stable nuclei. For argon 40, the predictions will still be OK: for QE scattering, with energy transfer typically larger than 100 MeV, a very detailed description of shell structure is not important, apart from counting the right number of protons and neutrons, argon is very close to calcium 40. We have published predictions for QE scattering on 12C, 16O, 40Ca, 40Ar, 46Ti, 208Pb, 56Fe, ...
8. Mino Kabirnezhad et al.: Free nucleon
9. Natalie Jachowicz et al.: 12C, 16O, 40Ar, 40Ca, 56Fe, 208Pb  
 Natalie Jachowicz, Using mean field theory one can easily model any nucleus larger than let say helium 4. Even-even target nuclei would be more accurately described, and generally the predictions will be better for spherical, closed-shell, stable nuclei. For argon 40, the predictions will still be reliable. For SPP calculations and QE scattering, where the outgoing and/or intermediary nucleon in general gets an energy of more than 100 MeV and the momentum transfer is

larger than around 200 MeV, a very detailed description of shell structure is not important, apart from counting the right number of protons and neutrons, argon is very close to calcium 40. Pion production calculations with and without nucleon FSI have been published for  $^{12}\text{C}$ ,  $^{16}\text{O}$ ,  $^{46}\text{Ti}$  and  $^{40}\text{Ar}$ . We have published predictions and comparisons for QE scattering in the same nuclear model, for  $^{12}\text{C}$ ,  $^{16}\text{O}$ ,  $^{40}\text{Ca}$ ,  $^{40}\text{Ar}$ ,  $^{46}\text{Ti}$ ,  $^{208}\text{Pb}$ ,  $^{56}\text{Fe}$ , which describe data in a satisfactory way.

10. Toru Sato et al.: deuteron, no progress for complex nuclei
11. Huma Haider et al.: We have obtained the nuclear structure functions for several Nuclei from lighter nuclei to heavier nuclei including Argon
12. Juan Nieves et al.: We can make predictions for any spherical nucleus
13. Maria Barbaro et al. (SuSA): For the QE region we can model any nucleus by fitting the above parameters to electron scattering data. The  $2p2h$  response can be evaluated for any nucleus.

## 6 What is the range of neutrino energies for your model?

1. Saori Pastore et al.: below 1 GeV.
2. Gil Paz et al.: CCQE
3. Artur Ankowski et al.: Above 0.1 GeV, no upper limit.
4. Alessandro Lovato et al.: Roughly up to 1.5 GeV, but we need to carefully gauge relativistic effects.
5. Luis Alvarez et al.: Neutrino energy is not a good variable to quantify the limitation of models. Generically one can argue that the validity is from 100 MeV to a few GeV.
6. Noemi Rocco et al.: The region of validity of the Spectral Function method (based on a Factorization scheme) is better defined in terms of the 3-momentum transfer:  $q$ . The spatial resolution of the probe (i.e. the  $W$  boson in CC processes) goes as  $1/|q|$  it has to be  $\ll d$ , where  $d$  is the average nucleon nucleon separation distance. Hence, the Spectral Function+Factorization scheme is applicable in the intermediate-high momentum transfer region.
7. Raul Jimenez et al.: We prefer to talk about the energy transfer\* ( $\omega$ ,  $w$ ) instead of neutrino energy. Our model works for QE scattering from  $w$  around the one nucleon knockout threshold to no limitation in principle: if the energy transfer is very large the QE contribution will be small, and the model does that. For  $w$  around threshold (10-40 MeV) the model may be less accurate in the fine details. In general, in the neutrino energy range where the QE process is significant the model is accurate.
8. Minoo Kabirnezhad et al.: Single pion production has a large contribution around 1-3 GeV.
9. Natalie Jachowicz et al.: Depends on energy/momentum transfer rather than incoming neutrino energy ; so roughly 10 MeV (nucleon knockout threshold) to 1.5-2 GeV

Natalie Jachowicz , On the single nucleon level it is more natural to think about the hadronic invariant mass  $W$  instead of incoming energy. For the low-energy side ( $W < 1.5$ ) the model performs well, given its resonance content and the uncertainties on the axial couplings, the  $Q^2$  behavior is dominated by the Delta and compares well with the ANL-BNL data. In the high-energy region ( $W > 2\text{GeV}$ ) the model also compares well to detailed electron scattering data and in the  $Q^2 = 0$  limit gives a magnitude comparable to pion-nucleon scattering data

as dictated by PCAC. The high-energy model in general performs better as  $W$  becomes larger. It is most applicable to small  $-t$  values (forward pions in the CMS), which is where the cross section peaks. The SPP cross section shows an exponential fall-off with increasing  $-t$  in this kinematic region which is well reproduced in the model, which means that the integrated cross section should only slightly underpredict the total reaction strength as the backward angle cross section that is underpredicted is in general orders of magnitude smaller than the forward one. In the intermediate  $W$ -region, a smooth interpolation between the high and low-energy models is performed. When it comes to the nuclear model it is best understood in terms of the energy transfer  $w'$  and momentum transferred  $q'$  to the single-nucleon state. i.e.  $w' = w - E_{pi}$  and  $q' = q - p_{pi}$ . The nuclear modelling then is applicable from around the single nucleon knock-out threshold up to 'arbitrarily' large values. For large values the cross section will naturally go to zero as is expected. Note that this is true for the primed variables, this does not mean that the cross section disappears for large energy and momentum transfers  $w$  and  $q$ , as then mostly higher energy pions will be produced at higher invariant mass  $W$ .

10. Toru Sato et al.: Above 0.1 GeV, no upper limit.
11. Huma Haider et al.: Our model works for wide range of energies
12. Juan Nieves et al.: What really matters is not the neutrino energy, but instead the energy and momentum transferred to the nucleus. The approach performs best for scales below 500-600 MeV
13. Maria Barbaro et al. (SuSA): In principle any range, but we do not consider our model reliable for energy transfers  $\omega$  smaller than about 40-50 MeV.

## 7 What are the other limits of validity in which your model applies? For example, $\omega > 30 MeV$ or $q_3^2 > (100 MeV/c)^2$ etc.

1. Saori Pastore et al.:  $q_3 < 700 MeV$ ,  $\omega < 1 GeV$
2. Gil Paz et al.: no clear yet
3. Artur Ankowski et al.: Excitation energy above 26 MeV.
4. Alessandro Lovato et al.: With relativistic corrections included, the region of applicability is  $0 MeV < \omega < 1.5(q_3^2)/(2m_n)$  where  $m$  is the mass of the nucleon  $m_n$  1 GeV and  $0 < q < 900 MeV$
5. Luis Alvarez et al.: At the nucleon level, high invariant masses of the produced meson/photon-nucleon pair are challenging so  $W < 1.6 GeV$ . At the nuclear level, low energy transfers  $q_0 < 50 MeV$  cannot be described within the local Fermi gas picture.
6. Noemi Rocco et al.:  $q > 500 MeV$
7. Raul Jimenez et al.:  $W < 2 GeV$ ,  $Q^2 < 2 \cdot 3 (GeV/c)^2$
8. Mino Kabirnezhad et al.: Most important limitation would be  $\omega < 30 MeV$  (around), and on the high energy region the main limitation is that QE contribution gets negligible.

The model is built to describe QE scattering, meaning that there is one nucleon knocked out in the primary vertex. Meson exchange currents and correlations in the initial state may produce events with more than one-nucleon knocked-out, this can be estimated from the model itself, meaning that the ratio of events with additional nucleons due to these effects compared to events with only nucleon knocked out after the interaction can be estimated quite accurately.



---

There may be further nucleons emitted, than the one knocked-out by the elementary vertex interaction. That is, multi-nucleon emission due to FSI.

Our model can adequately describe the following channels:

1. The channel where due to FSI no further nucleons are emitted, that is, essentially the knocked-out nucleon is the one and only one that exits the nucleus.
2. Similar as the one before, but the contribution of p-n and n-p exchanges due to FSI, as these can also be easily computed within the model.

These would be labeled 'elastic FSI and charge-exchange FSI' of the model, and would be similar to cascade elastic and charge-exchange channels, only that the calculation of this model does not include any classical approximations as in the cascade case,

These two cases can be labeled as 'only one nucleon in the final state' predictions and would be very accurate.

3. The aggregation of channels where, besides the knocked out nucleon, due to FSI, other nucleons may be further knocked out from the nucleus. These may be thought as the 'at least one nucleon in the final state estimations', and again, these would be rather accurate in the energy domain where the QE process dominate.
4. The 'inclusive' calculation, where only the final lepton is detected, is also well described by the model, where no pions are produced in the final state.
5. The main limitation of the model is that, for final states with 'at least one nucleon', no further break down of the final state into subgroups is possible, apart from the 'just one nucleon and nothing else' subchannel.

Notice that the model predicts accurately the relevant (e,e') data and (e,e'p) available data.

---

Important, as said in the first paragraph, a version of this model in which we focus on pion production, but with a similar nuclear part description is also available and being described elsewhere. These alternate model would have a similar content, just incorporating the contribution where one pion is produced at the elementary vertex, besides one nucleon.

9. Natalie Jachowicz et al.:  $\Omega=0$  -around 1200 MeV
10. Toru Sato et al.:  $W < 2GeV, Q^2 < 2 \cdot 3(GeV/c)^2$
11. Huma Haider et al.: We have limits on  $Q^2 > 1$
12. Juan Nieves et al.: Please read my answer above
13. Maria Barbaro et al. (SuSA): See previous answer.

## 8 How much of the phase space is covered by the model for final state lepton? Is there something missing? What is the best way (consistency) to cover any remaining missing phase space?

1. Saori Pastore et al.: pion-production is missing in the model, knockout processes the short-time-approximation has been developed to allow for a natural incorporation of two-body dynamics including meson-production and two-nucleon knockout. the plan is to account for this kind of phenomena.

2. Gil Paz et al.: no clear yet
3. Artur Ankowski et al.: Taking 100% uncertainty for excitation energies below 26 MeV, all phase space for quasielastic scattering is covered.
4. Alessandro Lovato et al.: The model can be complemented by approaches based on the factorization of the final state, provided that they retain a realistic description of the initial target state of the nucleus.
5. Luis Alvarez et al.: The fraction of the phase space covered depends on the neutrino energy. I might be failing to understand the question...
6. Noemi Rocco et al.: The low momentum transfer region,  $q < 500$  MeV can not be accurately described within the Spectral Function+Factorization Scheme as the interactions between the struck particle and the spectator system are not negligible. Quantum Monte Carlo calculations based on the same nuclear dynamical input entering in the spectral function can be used to cover the remaining missing phase space in a consistent fashion.
7. Raul Jimenez et al.: The model covers essentially all the phase space corresponding to quasielastic scattering: one nucleon knocked out in the primary vertex. As said before, a complementary calculation exists where the contribution of the processes where one pion is produced in the elementary vertex is computed.
8. Mino Kabirnezhad et al.: It covers full phase space.
9. Natalie Jachowicz et al.: full phase space is included  
Natalie Jachowicz, The lepton-phase space is fully included for the SPP interaction on a bound nucleon as described in the previous sections.
10. Toru Sato et al.: as far as energy and momentum transfer to nucleon is within the range described above, there is no limitation for lepton side.
11. Huma Haider et al.: We use spectral hole function for our DIS calculations which means we are inside the Fermi Sea.
12. Juan Nieves et al.: Please read my answers above
13. Maria Barbaro et al. (SuSA): This is related to the previous answer. The final lepton phase space corresponding to small energy transfers should not be taken too seriously.

## **9 Is the model optimized for some observables? Which ones? In which regions has the model has been tested?**

1. Saori Pastore et al.: the sta has been tested on electromagnetic observables in the qe region, and compared with available electromagnetic experimental data
2. Gil Paz et al.: It hasn't been tested yet.
3. Artur Ankowski et al.: Only lepton kinematics is considered, and nucleon final states are integrated out. Tasted against electron scattering data for energies from 160 MeV to 3.6 GeV.
4. Alessandro Lovato et al.: The model has been tested against: 1) electron scattering data off Helium-4 and Carbon-12; 2) Inclusive muon-capture rates of Helium-4 and Helium-3; 3) Inclusive neutrino scattering rates induced by charged-current transitions on Carbon-12

5. Luis Alvarez et al.: This kind of models are applicable for observables that are inclusive in the final state of the residual nucleus or when it is left in its ground state (coherent reactions).
6. Noemi Rocco et al.:The model has been widely validated in the electron sector. Accurately reproduces cross sections in the quasielastic region
7. Raul Jimenez et al.: There are two situations where the model is easier to compare to former electron-nucleus scattering (both (e,e') and (e,e'p) cases and thus its validity has been tested more in depth:
  - 1) the inclusive cross section (only the lepton is detected), and
  - 2) the fully exclusive cross section, where the experiment is conducted in a way that it is known that there is only one nucleon in the final state.
8. Minoo Kabirnezhad et al.: It has been tested in the full phase space.
9. Natalie Jachowicz et al.:CRPA, SRC, MEC : no optimization, tested against broad set of electron scattering data
 

Natalie Jachowicz, The model is implemented in a fully exclusive manner, and can thereby provide cross sections for the lepton, outgoing nucleon, and pion simultaneously. However, FSI for the outgoing pion is not yet included within our framework. Still, in general, the more 'integrated' the cross section is, the more reliable the prediction will be. The exclusive nature of the calculation means that all relevant kinematic cuts on the kinematic variables can be made ( e.g pion energy above 200 MeV and lepton scattering angles between 0 and 20 degrees etc..). The pion production model without FSI for the outgoing nucleon has been compared to neutrino data from T2K, Minerva and MiniBooNE. The SPP model including nucleon FSI has been confronted with inclusive electron scattering data for several nuclei ( $^{12}\text{C}$ ,  $^{40}\text{Ca}$ ,  $^{46}\text{Ti}$ , and  $^{40}\text{Ar}$ ).

This nuclear model has been successfully applied to describe electron scattering data over the past few decades.

There are two situations where it is easier to compare to former electron-nucleus scattering (both (e,e') and (e,e'p) cases and thus its validity has been tested more in depth:

  - 1) the inclusive cross section (only the lepton is detected), and
  - 2) the fully exclusive cross section, where the experiment is conducted in a way that it is known that there is only one nucleon in the final state.
10. Toru Sato et al.: GA of N to Delta transition tested against neutrino data on total ( $\nu p \rightarrow \mu p + n$ ) cross section.
11. Huma Haider et al.: No
12. Juan Nieves et al.: Inclusive observables. We cannot address reactions where the final nucleus is left in some particular excited state. Our scheme works much better when the sum over different final nuclear states is required, including configurations with the nucleus are broken and there are (ejected) nucleons in the continuum.
13. Maria Barbaro et al. (SuSA): The best description is given in the quasielastic peak, where the more accurate validation versus electron scattering data has been performed.

**10 Does your model include corrections for final state interactions (FSI)? If so, how are these included? Please include information about these 3 categories: (1) Leptonic Coulomb interaction, (2) Soft hadron FSI (corrections at the vertex), (3) Hard hadron FSI (rescattering in the nuclear medium), (4) De-excitations of the residual nucleus**

1. Saori Pastore et al.: hard fsi
2. Gil Paz et al.: The model includes and is motivated by short-range correlations between neutron-proton pairs. This can probably be considered as FSI.
3. Artur Ankowski et al.: It includes (1) Coulomb effects, (2) soft FSI, and (3) hard FSI. Deexcitations are not relevant for the lepton kinematics.
4. Alessandro Lovato et al.: 1) No; 2) Yes, 3) Yes, 4) Yes
5. Luis Alvarez et al.: (1) never included, but could be implemented; (2) I guess in-medium  $\Delta(1232)$  modifications can be included in this category; (3) not directly although some of the developments have been implemented in cascade and transport (GiBUU) models; (4) no.
6. Noemi Rocco et al.: Yes, FSI in the quasielastic region can be included accounting for points (2) and (3): the energy spectrum of the outgoing particle is modified including the real part on an optical potential and the effect of rescattering processes is parametrized in terms of a folding function.
7. Raul Jimenez et al.: 1) Our calculation may include Leptonic coulomb interaction exactly by solving Dirac equation in presence of the Coulomb potential for the lepton (this is usually called Coulomb distortion). It was done in the past by our group for electron scattering, see e.g. Phys. Rev. C48, 2731 (1993). In most of the energy range where the quasielastic process dominates, and for nuclei as Argon-40 or lighter, this is a relatively small effect and it would be wise not to consider it in a exact calculation as it increase the computational time a lot. The full, exact, calculation of the lepton FSI however can be performed for selected benchmark calculations to verify the approximate, standard approaches to lepton FSI such as the effective momentum approach which are included in the model with no computational cost, and which should be more than enough in more current scenarios for neutrino experiments  
2) The model solves the relativistic wave equation in presence of mean field potentials we include all sort of this soft FSI, and in a proper quantum mechanical way.  
3) All kind of re-scattering in the nuclear medium are included in the previous step (correction at vertex). What our model does not give is the multiplicity of hadrons in the final state. Work is in progress to facilitate a smooth integration of our model with cascade approaches to the FSI for the hadron in a coherent way.  
4) We do not consider de-excitation of the residual nucleus, that is, the fact that the residual nucleus may appear in an excited states after the primary interaction is of course considered. De-excitation of these excited nucleus by gamma emission is very simple to incorporate, though. More complex deexcitations mechanism are not contemplated.
8. Mino Kabirnezhad et al.: No

9. Natalie Jachowicz et al.:(1): yes (Fermi function for very low energies (correction of the wave function with a factor that provides the ratio between the plane wave and the s-wave solution of the Coulomb potential, MEMA at higher energies, which is implemented very easily as a shift in momentum transfer and a corresponding modification of the lepton prefactors with an overall phase-space factor) (2) yes the vertex calculation includes the full nuclear model, with distorted outgoing wave function and (3) this means that elastic FSI are automatically included (but e.g. no absorption) (4) no

Natalie Jachowicz 1) Leptonic coulomb interaction can be treated within the Modified Effective Momentum approximation, which is implemented very easily as a shift in momentum transfer and a corresponding modification of the lepton prefactors with an overall phase-space factor. For light nuclei ( $Z$  smaller than around 40 or 50) the effect is only appreciable for low energy leptons, and rather small. The full, exact, calculation of the lepton FSI by solving the Dirac equation in a coulomb potential however can be performed for selected benchmark calculations to verify this approximate, standard approach to lepton FSI. This has been done multiple times for electron scattering calculations and neutrino interactions yielding mostly satisfactory results.

2) The model solves the relativistic wave equation in presence of mean field potentials for the nucleon, thereby we include all sort of this soft FSI, in a proper quantum mechanical way. For the outgoing pion this has not yet been done.

3) Using distorted outgoing nucleon wave functions, the cross section is corrected for elastic FSI in the calculation of the vertex. By using a complex potential in the final state all strength lost to inelastic processes, i.e. Hard FSI, is removed from the cross section, and only the genuine elastic FSI is retained. It is exactly this strength that in Monte-Carlo event generators that use cascade programs is distributed over the specific inelastic channels that are open, thereby giving the hadron multiplicity in the final state. Our model does not provide the multiplicity of hadrons in the final state. Again, for the outgoing pion this has not been done.

4) We do not consider de-excitation of the residual nucleus. The fact that the residual nucleus may appear in an excited state thereby carrying some energy and momentum after the primary interaction is of course considered.

10. Toru Sato et al.: 1) N (planning to examine for low energy nuclear reaction, for deuteron reaction FSI are included.
11. Huma Haider et al.: No
12. Juan Nieves et al.: (1) yes for the QE (2) I do not quite understand the question. Nevertheless, we have a complete model for production at the first step (weak vertex) (3) Yes, we use MonteCarlo (4) No
13. Maria Barbaro et al. (SuSA):Leptonic Coulomb corrections are included in the Effective Momentum Approximation scheme. Hadronic FSI are included in the Relativistic Mean Field Model, where they are originated by the same potential acting on the initial state. In our calculation FSI can be switched off, yielding the RPWIA result.

## 11 What element of the calculation is reusable - for example, if one calculates the effects from soft hadron FSI, or de-excitations of residual nuclei, could that part of the calculation be factored out so that it can be used as part of other calculations?

1. Saori Pastore et al.: Na
2. Gil Paz et al.: Not clear yet.
3. Artur Ankowski et al.: cross section, structure function, amplitudes of neutrino-nucleon reaction as a input for neutrino-nuclear reactions.
4. Alessandro Lovato et al.: Unfortunately not. But we can provide quantities such as single- and two-nucleon momentum and density distributions that are relevant for neutrino event generators and for approaches based on the factorization of the nuclear final state.
5. Luis Alvarez et al.: Unfortunately not. But we can provide quantities such as single- and two-nucleon momentum and density distributions that are relevant for neutrino event generators and for approaches based on the factorization of the nuclear final state.
6. Noemi Rocco et al.: The phenomenological description of FSI discussed in the previous point can be factored out and used in other calculations
7. Raul Jimenez et al.: What element of the calculation is reusable - for example, if one calculates the effects from soft hadron FSI, or de-excitations of residual nuclei, could that part of the calculation be factored out so that it can be used as part of other calculations?

The FSI part of the model can be factored-out to some extent and put into play to compare with cascade and other approaches, but it is more worthy when coupled with the elementary vertex, as our model is essentially non-factorized.

A factorized version of the model can be extracted, which can be coupled into cascade of other models for FSI.

8. Mino Kabirnezhad et al.: Na
9. Natalie Jachowicz et al.: To a certain extend yes, but switching off the distortion in the final state will violate the consistency of the calculation

Natalie Jachowicz For the single nucleon knockout mechanism the FSI part of the model can be factored-out to some extent and put into play to compare with cascade and other approaches, but it is more worthy when coupled with the elementary vertex, as our model is essentially non-factorized.

For SPP further complications arise as the single-particle operator is more complicated, especially when pion FSI would be considered (which currently it is not). In principle the procedure for extracting a spectral function from the model should be similar as in the QE case, however as the factorization of the SPP model has not yet been studied within our framework, it is difficult to make any direct assertions.

10. Toru Sato et al.: cross section, structure function, amplitudes of neutrino-nucleon reaction as a input for neutrino-nuclear reactions.
11. Huma Haider et al.: May be don't know

12. Juan Nieves et al.: The use of the local density approximation provides a probabilistic profile with information on the hadron configuration at the primary step (I guess what you call soft hadron FSI), which includes the hadrons produced with kinematics, and the position ( $r$ ) of the nucleus where this first interaction has taken place. This profile can be easily used in any Monte Carlo to account for the rescattering in the nuclear medium of the produced hadrons in the first step (absorption of the weak probe, W or Z)
13. Maria Barbaro et al. (SuSA):The model is generally consistent, so it is difficult to extract one piece of the calculation and implement it in a different model. Only the 2p2h response is calculated in a Relativistic Fermi Gas basis and could be used in other models based on the RFG.

## 12 What output information is available? (Differential cross sections, leptonic and hadronic tensor, hard events a la LHC)

1. Saori Pastore et al.: we provide nuclear response functions obtained from the hadronic tensor. in addition we provide nuclear response densities. these are functions of the relative and C.M. energies of the two-nucleon pair struck by the probe, they include information on the electroweak vertex. upon integrating the response densities once recover the response function entering the differential cross-sections.
2. Gil Paz et al.: Differential cross sections
3. Artur Ankowski et al.: Differential cross sections
4. Alessandro Lovato et al.: Response functions, differential cross sections, flux-folded cross sections;
5. Luis Alvarez et al.: Differential cross sections.
6. Noemi Rocco et al.:All of the above
7. Raul Jimenez et al.: Differential cross sections with a lepton and one nucleon in the final state. Since the model is for QE scattering it can give this 6 differential cross section, or of course any other integrated cross section derived from it. Accelerated versions of the codes for the fully inclusive cases (only the lepton or only the nucleon is detected) are available, and these can also be easily given as tables.
8. Minoo Kabirnezhad et al.: full kinematic (5d) differential cross section.
9. Natalie Jachowicz et al.:Total cross sections, differential cross sections in lepton kinematics for CRPA, lepton and hadron kinematics for 2p2h, hadron and lepton tensor  
 Natalie Jachowicz The differential cross section in terms of the independent kinematic variables (e.g. Lepton energy and solid angle, pion energy and solid angle, nucleon energy and solid angle or any other combination that can be reconstructed thereof in the nucleus case) can be computed. Again we stress that pion FSI is not yet included, although results in which the pion is described by a plane wave are available. Different versions of the codes exist that are more efficient for producing the inclusive response functions, or the RPWIA cross section neglecting nucleon FSI a.o.
10. Toru Sato et al.: structure functions( $W_i$ ) and simple code to calculate cross sections
11. Huma Haider et al.: Nuclear Structure functions and Double Differential Scattering cross section.

12. Juan Nieves et al.: This is a microscopic approach, so I imagine that with close collaboration any output information can be made available. For instance since 2012 or 2013, We have made public QE+RPA and 2p2h hadron tensors. This year, we have recalculated (confirming the bulk or our 2012 predictions) the 2p2h cross sections for neutrinos (2002.08302), and we are currently working for antineutrinos.
13. Maria Barbaro et al. (SuSA): Differential cross sections with respect to the leptonic variables, leptonic and hadronic tensors. Work is in progress to get differential cross sections with respect to the ejected hadron(s) variables. A general study on semi-inclusive neutrino scattering processes with several models is under way.

### **13 Is it possible to produce the leptonic tensor $L_{\mu\nu}$ and/or hadronic tensor $W_{\mu\nu}$ ?**

1. Saori Pastore et al.: Yes
2. Gil Paz et al.: Yes
3. Artur Ankowski et al.: The prescription is formulated at the level of the double differential cross section.
4. Alessandro Lovato et al.: Yes
5. Luis Alvarez et al.: Yes
6. Noemi Rocco et al.,The prescription is formulated at the level of the double differential cross section.
7. Raul Jimenez et al.: Yes
8. Mino Kabirnezhad et al.: Yes
9. Natalie Jachowicz et al.:Yes  
Natalie Jachowicz
10. Toru Sato et al.: Yes
11. Huma Haider et al.: Yes
12. Juan Nieves et al.: By construction, we obtain in this way the cross sections.
13. Maria Barbaro et al. (SuSA): Yes

### **14 In which language is the model written?**

1. Saori Pastore et al.: Fortran
2. Gil Paz et al.: There are a number of codes build upon essentially the same model for the quasielastic processes. The most up to date is in C++ but it uses some important routines or functions written in FORTRAN. A Fortran-only version is also available.
3. Artur Ankowski et al.: C++
4. Alessandro Lovato et al.: Fortran 2003



5. Luis Alvarez et al.: Different developments have been performed with different collaborators and students, so it varies: fortran77, 90, C++
6. Noemi Rocco et al.:Fortran 90
7. Raul Jimenez et al.: There are a number of codes build upon essentially the same model for the quasielastic processes. The most up to date is in C++ but it uses some important routines or functions written in FORTRAN. A Fortran-only version is also available.
8. Mino Kabirnezhad et al.: C
9. Natalie Jachowicz et al.:Fortran  
Natalie Jachowicz Except for one FORTRAN routine that computes the outgoing nucleon wave-functions which is linked in, the code is completely written in C++. An alternative routine for the wave functions in C++ exists, but is less efficient.
10. Toru Sato et al.: Fortran
11. Huma Haider et al.: c++
12. Juan Nieves et al.: fortran, and the new calculation of 2002.08302 y guess that is written in C
13. Maria Barbaro et al. (SuSA): Fortran. Python in the case of semi-inclusive processes.

## 15 Please provide the most relevant reference paper(s) or preprint(s)

1. Saori Pastore et al.: <https://inspirehep.net/literature/1754168>
2. Gil Paz et al.: arXiv:1408.0772
3. Artur Ankowski et al.: PHYSICAL REVIEW D 91, 033005 (2015)
4. Alessandro Lovato et al.: arXiv:2003.07710, Phys.Rev. C100 (2019) no.3, 035502, Phys.Rev. C97 (2018) no.2, 022502, Phys.Rev.Lett. 117 (2016) no.8, 082501
5. Luis Alvarez et al.: <https://doi.org/10.1103/PhysRevC.99.025204>;  
<https://doi.org/10.1103/PhysRevD.93.014016>;  
<https://doi.org/10.1103/PhysRevC.89.015503>;  
<https://doi.org/10.1103/PhysRevC.79.034601>;  
<https://doi.org/10.1103/PhysRevC.75.055501>;  
<https://doi.org/10.1103/PhysRevC.87.015503>
6. Noemi Rocco et al.:<https://doi.org/10.1103/PhysRevC.99.025204>;  
<https://doi.org/10.1103/PhysRevD.93.014016>;  
<https://doi.org/10.1103/PhysRevC.89.015503>;  
<https://doi.org/10.1103/PhysRevC.79.034601>;  
<https://doi.org/10.1103/PhysRevC.75.055501>;  
<https://doi.org/10.1103/PhysRevC.87.015503>

7. Raul Jimenez et al.: PHYSICAL REVIEW C, VOLUME 64, 024614 (2001). PHYSICAL REVIEW C 100, 045501 (2019). PHYSICAL REVIEW LETTERS 123, 052501 (2019). And references there in.
8. Mino Kabirnezhad et al.: [1] M. Kabirnezhad. Improvement of Single Pion Production for T2K experiment simulation tools. PhD thesis, NCBJ, Warsaw, 2017. [2] M. Kabirnezhad. Single pion production in neutrino-nucleon Interactions. Phys. Rev., D97(1):013002, 2018.
9. Natalie Jachowicz et al.: Phys. Rev. C 94, 054609 (2016), Phys. Rev. C 94, 024611 (2016) , arXiv:1702.06402, Phys. Rev. C 97, 044616 (2018), Phys. Rev. Lett. 123, 052501 (2019), arXiv:1912.10714  
Natalie Jachowicz Phys. Rev. D 95, 113007 (2017), Phys. Rev. D 97, 013004 (2018) , Phys. Rev. D 97, 093008 (2018)
10. Toru Sato et al.: PRD99 031301(R) 2019, S. X. Nakamura, H. Kamano, T. Sato
11. Huma Haider et al.: F. Zaidi, H. Haider, M. Sajjad Athar, S. K. Singh and I. Ruiz Simo, “Weak structure functions in  $\nu_l - N$  and  $\nu_l - A$  scattering with nonperturbative and higher order perturbative QCD effects, Phys. Rev. D **101**, no. 3, 033001 (2020).
12. Juan Nieves et al.: Neutrinos: 1102.2777 [hep-ph], nucl-th/0408005, 1106.5374 [hep-ph], hep-ph/0701149, 1307.8105 [hep-ph], 1204.5404 [hep-ph], hep-ph/0511204 [hep-ph], 1001.4416 [hep-ph], 1302.0703 [hep-ph], 0811.1421 [hep-ph], 1304.1320 [hep-ph], 0903.5285 [hep-ph], hep-ph/0604042, 1007.3685 [hep-ph], 1510.06266 [hep-ph], 1701.03628 [nucl-th], 1311.2151 [nucl-th], 1407.6060 [hep-ph], 1612.02343 [hep-ph], 1807.11281 [hep-ph], 1507.02446 [hep-ph], nucl-th/0606042 [nucl-th], hep-ph/0608119 [hep-ph], 1701.05151 [nucl-th], 1906.05656 [nucl-th], 1901.10192 [nucl-th], 2002.08302 [nucl-th]. PhD Thesis of M. Valverde (2007), E. Wang (2014) and Joanna.E.Sobczyk (2019)  
and many works in the 90’s and 80’s (including the PhD Thesis of A. Gil (1996) on electronuclear reactions and my own PhD thesis (1992), supervised by E. Oset, on pion-nucleus interactions.
13. Maria Barbaro et al. (SuSA): J.E. Amaro et al., ”Electron- versus neutrino-nucleus scattering”, arXiv:1912.10612 , to appear on the JPG focus issue ”Neutrino-nucleus scattering” (review article, under revision). See references therein.

## 16 What could you provide to facilitate integration in existing Monte Carlo?

1. Saori Pastore et al.: Do you prefer using a file interface, using some standardized interface? Yes
2. Gil Paz et al.: We are pretty flexible. All the formulas will be available analytically and publicly. We are willing to contribute towards implementations within generators.
3. Artur Ankowski et al.: Would you be willing to contribute towards producing full, native implementations within generators? Yes
4. Alessandro Lovato et al.: Would you be willing to contribute towards producing full, native implementations within generators? Yes
5. Luis Alvarez et al.: Would you be willing to contribute towards producing full, native implementations within generators? Yes

6. Noemi Rocco et al.: Do you prefer to maintain packages for your calculation that event generator interface to? Yes
7. Raul Jimenez et al.: We have no preferences. We are open to discuss and try to figure out what would be the best way to do it.
8. Mino Kabirnezhad et al.: The MK-model is implemented in NEUT and it is being implemented in GENIE.
9. Natalie Jachowicz et al.: Do you prefer to maintain packages for your calculation that event generator interface to? Yes  
Natalie Jachowicz Do you prefer to maintain packages for your calculation that event generator interface to? Yes
10. Toru Sato et al.: we can provide data files of structure functions
11. Huma Haider et al.: Do you prefer to maintain packages for your calculation that event generator interface to? Yes
12. Juan Nieves et al.: Would you be willing to contribute towards producing full, native implementations within generators? Yes
13. Maria Barbaro et al. (SuSA): As a first step, it would be very easy to implement pre-computed tables of the leptonic and hadronic tensor in generators but in the long term the best option would be to implement the full code. This could be done just including the code in generators with an interface to it.