Stochastic ODEs for Spin-Orbit Dynamics of Polarized e+ e- Beams [1] [2]

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Outline

- Topic: Spin-polarization in electron storage rings like FCC-ee, CEPC, EIC
- Single-particle effects to be taken into account:
  1. Thomas-BMT precession
  2. Spin diffusion (radiative depolarization)
  3. Sokolov-Ternov effect (radiative polarization) + Baier-Katkov correction
  4. Kinetic polarization
- Many-particle effects to be taken into account:
  1. Beam-beam effects
  2. Intra-beam scattering
  3. Collective beam effects
- Issue 1: Computation of spin-orbit motion and maximization of polarization due to single-particle effects
- Issue 2: Computation of spin-orbit motion and maximization of polarization due many-particle effects

Conventional Approach 1 of computation

- Derbenev-Kondratenko formulas for equilibrium polarization and depolarization time (1973-present) [3]
    - Addresses all single-particle effects: Good!
    - Projects polarization density onto invariant spin field ($=n$-axis): Sufficient?

[3] K. Heinemann et al. White paper of invited talk, Hong Kong, January 17, 2019. To be published in IJMPA. See also: `www.math.unm.edu/~ellison`.

Conventional Approach 2 of computation

- Monte Carlo spin-orbit tracking + Baier-Katkov-Strakhovenko equation (1982-present) [4]
    - Addresses all single-particle effects: Good!
    - The Baier-Katkov-Strakhovenko part requires projection of polarization density onto closed-orbit invariant spin field ($=n_0$-axis): Sufficient?
    - Monte Carlo part does not require projection of polarization density: Good!
    - Monte Carlo part is effectively based on an incomplete system of stochastic ODEs (spin dynamics only includes Thomas-BMT precession): Not so good!
    - Conventional Approach 2 is most popular one. Related software:
      SITROS by J. Kewisch (BNL)
      SLICKTRACK by D.P. Barber (DESY)
      Zgoubi by F. Meot (BNL)
      Bmad: PTC/FPP code by E. Forest (KEK)

[4] Z. Duan, M. Bai, D.P. Barber, Q. Qin, *A Monte-Carlo simulation of the equilibrium beam polarization in ultra-high energy electron (positron) storage rings*, Nucl. Instr. Meth. A793 (2015), pp.81-91. On archive at arXiv:1505.02392 [physics.acc-ph].

Broad Research Problem

- Complete system of spin-orbit stochastic ODEs (2019) containing all single-particle effects [5]

$$\begin{pmatrix} Y \\ \vec{S} \end{pmatrix}' = H(t, Y, \vec{S}) + N(t, Y)\xi(t)$$

- Remark: $\xi(t)$=White noise $\Longrightarrow$ Markov process
- Remark: Stochastic ODEs in Ito sense (Ito Calculus)
- Stochastic ODEs $\Longrightarrow$ spin-orbit Fokker-Planck equation for spin-orbit joint probability density $\mathcal{P}(t, y, \vec{s})$
- Polarization vector $\vec{P}$ related to spin-orbit probability density

$$\vec{P}(t) =< \vec{S}(t) >= \int \vec{s}\, \mathcal{P}(t, z, \vec{s}) dz d\vec{s}$$

[5] K. Heinemann, D. Appelö, D. P. Barber, O. Beznosov, J. A. Ellison, *The Bloch equation for spin dynamics in electron storage rings: Computational and theoretical aspects*, International Journal of Modern Physics A 34, 1942032 (2019).

New Stochastic approach to compute polarization vector based on stochastic ODEs

- Development of ultrafast Monte-Carlo spin-orbit tracking code
  - Needed for large storage rings
  - Based on E. Forest's stochastic one turn map (part of Beznosov's PhD thesis)
  - To be implemented into Bmad
- Development of a multi-level stochastic ODE solver
- Merits:
  - Addresses all single-particle effects: Good!
  - Requires no projection of polarization density: Good!
  - Gives important insights into both conventional approaches: Good!

New PDE approach to compute polarization vector

- Spin-orbit joint probability density $\Longrightarrow$ polarization density $\vec{\eta}$:

$$\vec{\eta}(t, y) = \int \vec{s}\, \mathcal{P}(t, y, \vec{s}) d\vec{s}$$

- Stochastic ODEs $\Longrightarrow$ spin-orbit Fokker-Planck equation $\Longrightarrow$ PDE for polarization density $\vec{\eta}$
- Remark: PDE for polarization density introduced in 1975 (thesis of A.M. Kondratenko)
- Remark: We call PDE "Bloch equation"
- Remark: Physics of new PDE approach equivalent to new stochastic approach $\Longrightarrow$ same merits as new stochastic approach

- Bloch equation numerically complex $\Longrightarrow$ need effective Bloch equation for analysis and numerics
- Effective Bloch equation
    - Method of Averaging (much progress)
    - Multiple-Scales Method (future work)
- State-of-the-art numerics to solve effective Bloch equation:
    - Algorithm discretizes orbital phase space by pseudospectral method $\Longrightarrow$ memory efficient
    - Algorithm discretizes time by implicit-explicit time stepping $\Longrightarrow$ large time steps
    - Parallelized code (MPI Fortran)
    - Algorithm and code part of Beznosov's PhD thesis

Spin matching (maximizing polarization)

- Standard approach [6]: Minimizing spin perturbations in linear approximation leading to so-called spin transparency
- Future Approach:
  - Based on new computation methods (e.g., ultrafast tracking)
  - Using quasi-Newton methods, e.g., LBFG-S algorithm
  - Remark: To be used in a second step, i.e., on top of standard approach

[6] D.P. Barber, G. Ripken, *Handbook of Accelerator Physics and Engineering*. Eds. A.W. Chao and M. Tigner, 1st edition, 3rd printing, World Scientific, 2006. See also arXiv:physics/9907034v2.

Strategic Road Map for spin-polarization-1 in electron storage rings

- Continue work on new stochastic and PDE approaches
- Apply to EIC, FCC-ee, CEPC
- Putting codes into Bmad (toolkit of Cornell): Need to combine computation and optimization tools under same rubric
- Mysterious correction terms to Derbenev-Kondratenko formulas:
  Are they of Markovian nature?
  Are they contained in stochastic and PDEs approach?
  Or are they additional single-particle effect?
  Are they related to Froissart-Stora effect?
  Remark: Due to synchrotron radiation, electron can cross spin-orbit resonances
  $\Longrightarrow$ Froissart-Stora spin flips possible?

Strategic Road Map for spin-polarization-2 in electron storage rings

- Investigate Markov approximation of single-particle orbital motion due to synchrotron radiation
  - Remark: This is an orbital dynamics issue and may impact other work of Accelerator and Beam Physics (ABP) at GARD
- Address many-particle effects
- One aspect of collective beam effects:
  - Replace orbital single-particle approach by using Vlasov-Fokker-Planck equation

    See, e.g., work of R. Warnock (SLAC) and G.Bassi (BNL)

    An issue is the validity of the Vlasov approximation [7]

[7] J.A. Ellison, G. Bassi, K. Heinemann, *Random N-Particle Klimontovich-Maxwell System: Probabilistic Analysis, Fluctuations from Mean and Ecker Hierarchy*, IPAM Beam Dynamics Workshop, UCLA, January 25, 2017

Relation to DOE missions

- Connection to Office of Science:
  - Nuclear Physics - NP
  - High Energy Physics - HEP
  - Advanced Scientific Computing Research - ASCR
- Grand challenges - GC
  - Because of connection to ASCR there is an obvious relation to GC4
  - Because spin polarized beams can be used for beam energy monitoring there is also a relation to GC3
  - There is also a weak relation to GC1,GC2 since there is a distant connection to spin polarized beam studies in laser accelerators as conducted at University of Michigan
- Connection to workshop groups
  - GARD 1: Workshop group 1
  - GARD 1: Workshop group 3

Early history

(1) Discovery of radiative polarization by Sokolov and Ternov in 1964 (Sokolov-Ternov effect).

(2) Recognition of radiative depolarization by Baier and Orlov in 1965.

(3) Discovery of the Baier-Katkov correction to the Sokolov-Ternov effect by Baier and Katkov in 1967. This was observed while rederiving the Sokolov-Ternov effect by using a technique of Schwinger from the 1950s.

(4) Discovery of the Baier-Katkov-Strakhovenko equation by Baier, Katkov and Strakhovenko in 1969. This merges the Thomas-BMT equation, the Sokolov-Ternov effect and the Baier-Katkov correction into an ODE for the polarization vector of the bunch but it does not include depolarization from (2).

(5) Discovery of the kinetic polarization effect by Derbenev and Kondratenko around 1972. Discovery of the Derbenev-Kondratenko formulas (DKFs) by Derbenev and Kondratenko around 1972 which merges the Thomas-BMT equation, the Sokolov-Ternov effect, the Baier-Katkov correction, the spin diffusion and the kinetic polarization effect.

(6) Discovery of the Bloch equation for the polarization density by Derbenev and Kondratenko in 1975.

Reviews

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