

# Alternative subtraction scheme using Nagy-Soper dipoles

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## Aim of the talk

- will not talk for hours on importance of NLO at LHC (you know this anyway); instead:
- give a schematic overview on sources of divergencies and treatment for NLO calculations
- explain idea and setup of subtraction scheme(s)
- show this explicitly for single  $W$  at LHC
- explain why we develop a new scheme
- (only very few words about parton showers)

- 1 NLO calculations in the LHC era
- 2 NLO calculations and subtraction schemes
  - Structure of NLO calculations
  - Subtraction Schemes
- 3 Nagy Soper subtraction scheme
  - Applications
- 4 Summary and Outlook

# NLO @ LHC: importance and pitfalls

- era of LHC (approaching):  
    "real" data-taking (hopefully) some time this year
- LHC: hadron collider, processes governed by QCD:  
    large NLO corrections (up to 100%), also need parton showers
- if we want to do everything correctly: match showers and  $N^m L^n O$  contributions (at the moment,  $m = n = 1$ )
- for this:  
    need "smart" treatments of behavior in singular regions
- here:  
    talk about treatment of **NLO corrections on parton level**
- few words about **parton showers**

## Mini presummary (for experts and others)

### Main motivation for new scheme

- different matching between  $m$  and  $m + 1$  phase spaces
- ⇒ leads to a much smaller number of transformations
- especially important for large number of external particles
- dipoles: derived from splitting functions of "Parton shower with quantum interference" (Nagy, Soper, 2007)
- ⇒ once shower and scheme are implemented: facilitates matching with NLO calculations
- here: concentrate on subtraction scheme

# Structure of NLO calculations

# NLO corrections: general structure

## Masterformula

for  $m$  particles in the final state

$$\sigma_{\text{NLO,tot}} = \sigma_{\text{LO}} + \sigma_{\text{NLO}},$$

$$\sigma_{\text{LO}} = \int d\Gamma_m |\mathcal{M}_{\text{Born}}^{(m)}|^2(s) \quad \text{leading order contribution}$$

$$\sigma_{\text{NLO}} = \sigma_{\text{real}} + \sigma_{\text{virt}},$$

$$\sigma_{\text{real}} = \int d\Gamma_{m+1} |\mathcal{M}^{(m+1)}|^2 \quad \text{real emission}$$

$$\sigma_{\text{virt}} = \int d\Gamma_m 2 \text{Re}(\mathcal{M}_{\text{Born}}^{(m)} (\mathcal{M}_{\text{virt}}^{(m)})^*) \quad \text{virtual contribution}$$

with  $d\Gamma$ : phase space integral,  $\mathcal{M}$  matrix elements  
(here: flux factors etc implicit)

# Infrared divergencies in NLO corrections

- source of infrared divergence: integration over phase space of emitted massless particles in real and virtual contribution (poles cancel in  $\sigma_{\text{real}} + \sigma_{\text{virt}}$ )
- appear in matrix elements as terms

$$\frac{1}{p_i p_j} = \frac{1}{E_i E_j (1 - \cos \theta_{ij})}$$

$E_j \rightarrow 0$ : soft divergence,  $\cos \theta_{ij} \rightarrow 1$ : collinear divergence

- poles arise from **integration** of phase space of  $p_j$
- cancellations:

$\Rightarrow$  **need to have a good (analytical) parametrization of the singularity structure**



# Infrared divergencies: Treatments

- one option: go from  $D = 4$  to  $D = 4 - 2\varepsilon$  dimensions
- poles then appear as

$$\sigma_{\text{real}} = \frac{A}{\varepsilon^2} + \frac{B}{\varepsilon} + \dots$$

- $A, B$  depend on the splitting process
- eg in QCD  $\tilde{p}_i \rightarrow p_i + p_j$  (omitted color factors etc)

$$q \rightarrow qg : \propto \frac{1}{\varepsilon^2} + \frac{3}{2\varepsilon}$$

$$g \rightarrow q\bar{q} : \propto -\frac{1}{3\varepsilon}$$

- important: **this behaviour is the same for all processes**

# Subtraction Schemes

# Dipole subtraction: general idea

- know that pole structure always the same
- matrix element level: in the singular limits,

$$|\mathcal{M}^{(m+1)}|^2 \longrightarrow D_{ij}(p_i, p_j) |\mathcal{M}^{(m)}|^2, \quad D_{ij} \sim \frac{1}{p_i p_j} \quad (1)$$

- $D_{ij}$ : **dipoles**, contain complete singularity structure
- also means that

$$\int d\Gamma_{m+1} \left( |\mathcal{M}^{(m+1)}|^2 - \sum_{ij} D_{ij} |\mathcal{M}^{(m)}|^2 \right) = \text{finite}$$

- **general idea of dipole subtraction:** make use of (1), shift singular parts from  $m + 1$  to  $m$  particle phase space

# Dipole subtraction for total cross sections

## Master formula

$$\begin{aligned} \sigma &= \sigma^{LO} + \sigma^{NLO} \\ \sigma^{NLO} &= \int_{m+1} d\sigma^R + \int_m d\sigma^V + \int d\sigma^C \\ &= \int_{m+1} (d\sigma^R - d\sigma^A) + \int_m (d\sigma^{\tilde{A}} + d\sigma^V + d\sigma^C), \end{aligned}$$

⇒ effectively added "0"; both integrals finite

$$\begin{aligned} \sigma_m^{NLO}(s) &= \int_m \left\{ |\tilde{\mathcal{M}}_{\text{virt}}(s; \varepsilon)|^2 + \mathbf{I}(\varepsilon) |\mathcal{M}_{\text{Born}}(s)|^2 \right. \\ &\quad \left. + \int_0^1 dx (\mathbf{K}(x) + \mathbf{P}(x; \mu_F)) |\mathcal{M}_{\text{Born}}(x, s)|^2 \right\} \end{aligned}$$

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# Ingredients for subtraction schemes: momentum matching

- previous slide: add and subtract "0" in terms of

$$\int d\Gamma_m \tilde{F}_{\text{sing}} |\mathcal{M}_{\text{Born}}^{(m)}|^2 - \int d\Gamma_{m+1} F_{\text{sing}} |\mathcal{M}_{\text{Born}}^{(m)}|^2$$

- addition and subtraction takes place in different phase spaces

$$m \longleftrightarrow m + 1$$

- want all external particles to be onshell:  
need to define a matching  $(m + 1) \Rightarrow (m)$

$$p_{\tilde{a}}^{(m)} = F \left( p_a^{(m+1)}, p_b^{(m+1)}, \dots \right)$$

- also need to keep total energy/ momentum conserved:

$$\sum_m p_{\tilde{a}} \stackrel{!}{=} \sum_{m+1} p_a$$

(sum over outgoing particles only)



## Second ingredient: Parametrization of integration variables

- again: remember you have

$$F_{\text{sing}} \propto D_{ij}, \quad \tilde{F}_{\text{sing}} = \int d\Gamma_1 D_{ij}, \quad d\Gamma_1 \propto d^4 p_j \delta(p_j^2)$$

$$\implies \tilde{F}_{\text{sing}} \propto \int d^4 p_j \delta(p_j^2) D_{ij}$$

- 3 free variables (in  $D$  dimensions:  $D - 1$ )  
!! need to be written in terms of  $m$  particle variables !!
- now all ingredients:  
**total energy momentum conservation, onshellness of external particles, need for integration variables**

## So far:

- tool to handle cancellation of infrared divergencies:  
**dipole subtraction schemes**
- infinities are "shifted" around in  $m$ ,  $m+1$  contributions of  $\sigma_{\text{tot}}$
- complications: energy momentum conservation, onshellness or external particles, parametrization of integration variables

### master formula

$$\sigma_{NLO} = \int d\Gamma_m \left( |\mathcal{M}|_{\text{Born}}^2 + 2 \text{Re}(\mathcal{M}_{\text{Born}} \mathcal{M}_{\text{virt}}^*) + \tilde{F}_{\text{sing}} |\mathcal{M}|_{\text{Born}}^2 \right) J^{(m)} + \int d\Gamma_{m+1} \left( |\mathcal{M}|_{\text{real}}^2 - F_{\text{sing}} |\mathcal{M}|_{\text{Born}}^2 \right) J^{(m+1)}$$

## In the following:

- talk about Catani Seymour dipole scheme
- talk about (alternative) Nagy Soper dipole scheme
- show some results and comparisons for the latter
- will NOT talk about parton showers (work done by collaborators)

...work in progress...

# Catani Seymour and Nagy Soper subtraction schemes

# Subtraction schemes

- many different subtraction schemes are around
- best known: Catani, Seymour, 1996

## important message:

poles have to be the same; finite parts can differ

⇒ **behaviour in the singular regions is unique** ⇐

# Nagy Soper subtraction scheme

## Main motivation for new scheme

- basic idea: can use the splitting functions in the parton shower as dipole subtraction terms
  - ⇒ have same behaviour in singular limits
- "turn around" of idea suggested by Nagy, Soper (hep-ph/0503053): use Catani Seymour Dipoles for shower algorithm
- introduce new matching between  $m$  and  $m + 1$  phase spaces
  - ⇒ leads to a much smaller number of subtraction terms
    - especially important for large number of external particles
  - ⇒ same dipoles in shower and subtraction scheme: facilitates matching with NLO calculations

# Difference 1: Shifting momenta

- matching between  $m$  and  $m + 1$  particle spaces requires reshuffling of momenta

- for

$$p_{\text{mother}}^{(m)} = p_{\text{daughter}, 1}^{(m+1)} + p_{\text{daughter}, 2}^{(m+1)}$$

not all particles can be onshell simultaneously

⇒ need additional spectators to take over additional momenta

- Catani Seymour: define emitter-spectator pair, momentum goes to 1 additional particle only

⇒ quite easy integrations; however, for increasing number of particles, huge number of transformations necessary

- Nagy Soper:

shift momenta to **all** non-emitting external particles

- number of transformations = number of emitters

- leads to more complicated integrals during framework setup

- in general: # of transformations: CS  $\sim N_{\text{jets}}^3/2$ , NS  $\sim N_{\text{jets}}^2/2$

# Final state $g \rightarrow q \bar{q}$ : Catani Seymour vs Nagy Soper (1)

- $g(\tilde{p}_i) \rightarrow q(p_i) + \bar{q}(p_j)$ ,  
spectator: any other final state parton,  $p_k$
- Dipole (in terms of integration variables):

$$D_{\text{NS, CS}}^{ij,k} \propto \underbrace{\frac{1}{y}}_{\text{sing}} \left[ 1 - \frac{z(1-z)}{1-\epsilon} \right]$$

- NS definitions

$$y_{\text{NS}} = \frac{p_i p_j}{(p_i + p_j)Q - p_i p_j}, \quad z_{\text{NS}} = \frac{p_j \tilde{n}}{p_i \tilde{n} + p_j \tilde{n}}$$

$$\tilde{n} = \frac{1+y+\lambda}{2\lambda} Q - \frac{a}{\lambda} (p_i + p_j), \quad \lambda = \sqrt{(1+y)^2 - 4ay}, \quad a = \frac{Q^2}{(p_i + p_j)Q - p_i p_j}$$

- CS definitions:

$$y_{\text{CS}} = \frac{p_i p_j}{p_i p_j + p_i p_k + p_j p_k}, \quad z_{\text{CS}} = \frac{p_i p_k}{p_i p_k + p_j p_k}$$



# Final state $g \rightarrow q \bar{q}$ : Catani Seymour vs Nagy Soper (2)

- CS matching (all other final state particles untouched)

$$\tilde{p}_i = p_i + p_j - \frac{y}{1-y} p_k, \quad \tilde{p}_k^\mu = \frac{1}{1-y} p_k^\mu$$

- NS matching

$$\tilde{p}_i = \frac{1}{\lambda} (p_i + p_j) - \frac{1-\lambda+y}{2\lambda a} Q, \quad \tilde{p}_k^\mu = \Lambda^\mu{}_\nu p_k^\nu \quad \text{all fs particles}$$

$$\Lambda^{\mu\nu} = g^{\mu\nu} - \frac{2(K+\tilde{K})^\mu(K+\tilde{K})^\nu}{(K+\tilde{K})^2} + \frac{2K^\mu\tilde{K}^\nu}{K^2}, \quad K=Q-p_i-p_j, \quad \tilde{K}=Q-\tilde{p}_i$$

- integration measure (identical, same pole structure)

$$[dp_j]_{\text{CS}} = \frac{(2\tilde{p}_i\tilde{p}_k)^{1-\epsilon}}{16\pi^2} \frac{d\Omega_{d-3}}{(2\pi)^{1-\epsilon}} dz dy (1-y)^{1-2\epsilon} y^{-\epsilon} [z(1-z)]^{-\epsilon},$$

$$[dp_j]_{\text{NS}} = \frac{(2\tilde{p}_i Q)^{1-\epsilon}}{16\pi^2} \frac{d\Omega_{d-3}}{(2\pi)^{1-\epsilon}} dz dy \lambda^{1-2\epsilon} y^{-\epsilon} [z(1-z)]^{-\epsilon}$$

# Final state $g \rightarrow q \bar{q}$ : Catani Seymour vs Nagy Soper (3)

- result CS

$$\mu^{2\epsilon} \int [dp_j] D^{ij,k} = \frac{\alpha_s}{2\pi} \frac{1}{\Gamma(1-\epsilon)} T_R \left( \frac{2\mu^2\pi}{\tilde{p}_i \tilde{p}_k} \right)^\epsilon \left[ -\frac{2}{3\epsilon} - \frac{16}{9} \right]$$

- result NS

$$\mu^{2\epsilon} \int [dp_j] D^{ij} = T_R \frac{\alpha_s}{2\pi} \frac{\alpha_s}{\Gamma(1-\epsilon)} \left( \frac{2\pi\mu^2}{p_i Q} \right)^\epsilon$$

$$\times \left[ -\frac{2}{3\epsilon} - \frac{16}{9} + \frac{2}{3} [(a-1) \ln(a-1) - a \ln a] \right],$$

- for  $a = 1$ , reduces completely to Catani Seymour result
- (reason:  $a = 1$  implies only 2 particles in the final state,  $\tilde{n} \rightarrow p_k$ ,  $\Rightarrow$  complete equivalence)
- tradeoff: all final state particles get additional momenta: integral more complicated, but fewer transformations necessary

## Difference 2: Matching with parton showers

- double counting: hard real emissions are described in both shower and "real emission" matrix element
- avoid double counting

$$- \int_{m+1} d\sigma^{\text{PS}}|_{m+1} + \int_{m+1} d\sigma^{\text{PS}}|_m$$

details eg in hep-ph/0204244: "Matching NLO QCD computations and parton shower simulations" (Frixione, Webber), MC@NLO

- important: have new terms in  $m + 1$  phase space

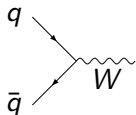
$$\int_{m+1} \left( d\sigma^R - \underbrace{d\sigma^A + d\sigma^{\text{PS}}|_m}_{=0} - d\sigma^{\text{PS}}|_{m+1} \right)$$

- same splitting functions: second and third term cancel analytically !!

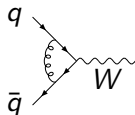
⇒ improves numerical efficiency

# Applications

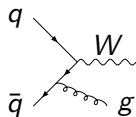
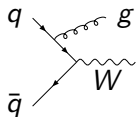
# Single W production (slide by C. Chung)



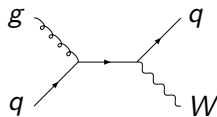
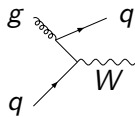
Tree level:  $q\bar{q} \rightarrow W$



Virtual corrections:  $q\bar{q} \rightarrow W$



Real corrections:  $q\bar{q} \rightarrow Wg$



$gq \rightarrow Wq$  (+ 2 more diagrams)

$$\frac{1}{4} \frac{1}{9} |\mathcal{M}_B|^2 = \frac{g^2}{12} |V_{qq'}|^2 M_W^2, \quad \frac{1}{4} \frac{1}{9} \sum |\mathcal{M}_R|^2 = \frac{8g^2\pi\alpha_s}{9} |V_{qq'}|^2 \frac{\hat{t}^2 + \hat{u}^2 + 2M_W^2 \hat{s}}{\hat{t}\hat{u}}$$

$$|\mathcal{M}_V|^2 = |\mathcal{M}_B|^2 \frac{\alpha_s}{2\pi} C_F \frac{1}{\Gamma(1-\epsilon)} \left( \frac{4\pi\mu^2}{Q^2} \right)^\epsilon \left\{ -\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \pi^2 + \mathcal{O}(\epsilon) \right\}$$

## Subtraction terms à la Nagy Soper

- 2 particle phase space (real emission)

$$\mathcal{D}^{14,2} + \mathcal{D}^{24,1} = \frac{8}{9} \pi \alpha_s g^2 \left( \frac{t^2 + u^2 + 2s p_3^2}{t u} \right) = \underbrace{\frac{1}{4} \frac{1}{9} \sum |\mathcal{M}_{\text{real}}|^2}_{\text{singular}}$$

- 1 particle phase space (virtual contribution)

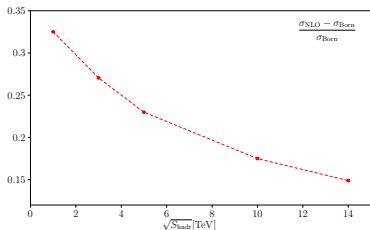
$$\mathbf{I}(\epsilon) |\mathcal{M}_b|^2 = \underbrace{\frac{2\alpha_s}{3\pi} \frac{1}{\Gamma(1-\epsilon)} (-8 + \frac{2}{3}\pi^2) |\mathcal{M}_b|^2}_{\text{finite}} - \underbrace{|\widetilde{\mathcal{M}}_v|^2}_{\text{singular (+finite)}}$$

$$\mathbf{K}^a(x p_a) = \frac{\alpha_s}{2\pi} C_F \frac{1}{\Gamma(1-\epsilon)} \left[ -(1-x) \ln x + 2(1-x) \ln(1-x) + 4x \left( \frac{\ln 1-x}{1-x} \right)_+ - \frac{2x \ln x}{(1-x)_+} - \left( \frac{1+x^2}{1-x} \right)_+ \ln \left( \frac{4\pi\mu^2}{2x p_a \cdot p_b} \right) \right]$$

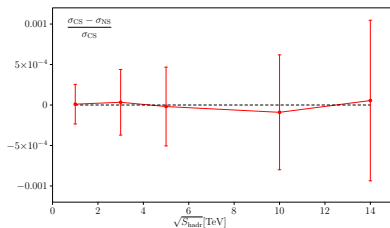
$$\mathbf{P}(x, \mu_F^2) = \frac{\alpha_s}{2\pi} C_F \frac{1}{\Gamma(1-\epsilon)} \left( \frac{1+x^2}{1-x} \right)_+ \ln \left( \frac{4\pi\mu^2}{\mu_F^2} \right)$$

# Numerical results for single W ( slide by C. Chung)

input:  $M_W = 80.35$  GeV, PDF  $\Rightarrow$  cteq6m,  $\alpha_s(M_W) = 0.120299$



$\frac{\sigma_{NLO} - \sigma_{LO}}{\sigma_{LO}}$  as a function of  $\sqrt{S_{hadr}}$   
 corrections up to 30%



relative difference between CS and NS:  $\frac{\sigma_{CS} - \sigma_{NS}}{\sigma_{CS}}$   
 agree on the sub-permill level ✓

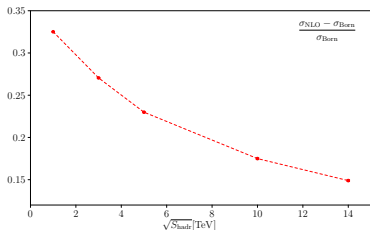
difference between schemes:

subtraction in  $m$  particle phase space,  $K(x)$  terms

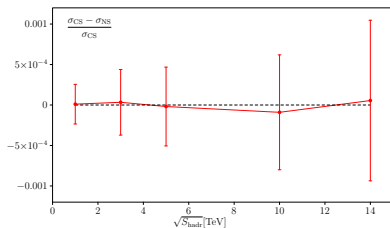
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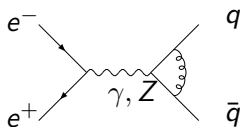
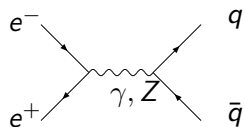
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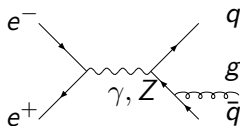
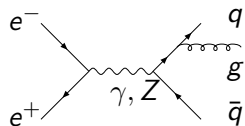
**pole structure the same, finite terms shifted around ✓**



Applications:  $e^+e^- \rightarrow 2 \text{ jets (1)}$  (slide by C.Chung)

Tree level diagram:  
 $e^+e^- \rightarrow q(p_1) + \bar{q}(p_2)$

Virtual corrections:  
 $e^+e^- \rightarrow q(p_1) + \bar{q}(p_2)$



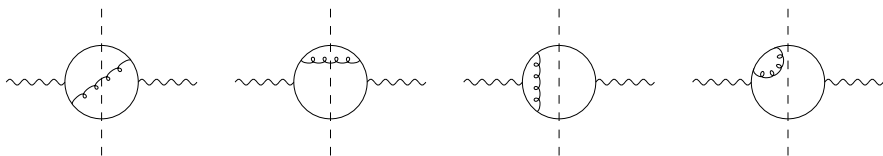
Real corrections:  
 $e^+e^- \rightarrow$   
 $q(p_1) + \bar{q}(p_2) + g(p_3)$

The matrix element for NLO real emission (three particle ps):

$$|\mathcal{M}_3(p_1, p_2, p_3)|^2 = C_F \frac{8\pi\alpha_s}{Q^2} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)} |\mathcal{M}_2|^2, \quad x_i = \frac{2p_i \cdot Q}{Q^2}$$

( $\mathcal{M}_2$ ,  $\mathcal{M}_3$  averaged over angles)

soft/ collinear singularities from  $x_i \rightarrow 1$

Applications:  $e^+e^- \rightarrow 2 \text{ jets}$  (2) (slide by C. Chung)

2 dipole contributions  $\mathcal{D}_1$  and  $\mathcal{D}_2$  (in 3 particle ps):

$$\begin{aligned} \mathcal{D}_1 &= v_{qgq}^2 - v_{\text{soft}}^2 = (v_{qgq}^2 - v_{\text{eik}}^2) + (v_{\text{eik}}^2 - v_{\text{soft}}^2) \\ &= \frac{4}{Q^2} \left\{ \left( \frac{1}{x_2} \right) \left[ 2 \left( \frac{x_1}{2-x_1-x_2} - \frac{1-x_2}{(2-x_1-x_2)^2} \right) + \frac{1-x_1}{1-x_2} \right] \right. \\ &\quad \left. + 2 \left( \frac{x_1+x_2-1}{1-x_2} \right) \frac{x_1}{(1-x_1)x_1+(1-x_2)x_2} \right\} \end{aligned}$$

Integration over dipole

$$2 \left( \frac{4\pi\alpha_s}{2} \right) \mu^{2\epsilon} C_F \int d\zeta_p \mathcal{D}_1 = \frac{\alpha_s}{2\pi} C_F \frac{1}{\Gamma(1-\epsilon)} \left( \frac{4\pi\mu^2}{Q^2} \right)^\epsilon \left( \frac{2}{\epsilon^2} + \frac{3}{\epsilon} - 2 + \frac{\pi^2}{3} \right)$$

$$\sigma^{NLO} = \sigma^{NLO\{2\}} + \sigma^{NLO\{3\}} = \frac{3}{4} \frac{\alpha_s}{\pi} C_F \sigma^{LO} \quad (\checkmark)$$

# Status Quo and Outlook

# Status quo (instead of Summary)

- goal: establish NS dipole formalism
- all integrals are done ✓
- need to countercheck a) singularities, b) finite terms
- a) almost completely done  
(missing: processes w more than 2 partons in the final state)
- b) counterchecked for all processes with initial state partons only as well as  $q \rightarrow qg$  in final state, rest needs checks

## Checked processes

- single W at hadron colliders:  
complete equivalence, agreement with MCFM
- Dijet production at lepton colliders: complete equivalence  
(analytic)
- deep inelastic scattering and  $gg \rightarrow H$ :  
singularity cancellation for virtual parts checked, rest underway

# Outlook

## Outlook

- continue checks by application to simple processes for unchecked splitting functions ( $g \rightarrow gg$ ,  $m > 2$  in final state)
- implement on matrix element level
- match with parton shower (Z. Nagy; underway)
- apply in (new) higher order calculations
- .... (more to come)

! Thanks for listening !

# Appendix

# Processes at hadron colliders: general

- hadron colliders (as Tevatron, LHC) collide **hadrons**
- QCD: talks about **partons**
- transition: parton distribution functions (PDFs)  $f_l(x, \mu_F)$ ;  
 $l = q, \bar{q}, g$  flavour,  $x$  momentum fraction, ( $\mu_F$  factorization scale)

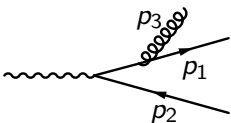
## masterformula

$$\sigma_{\text{hadr}}(p \bar{p} \rightarrow X) = \sum_{l_1, l_2} \int dx_1 \int dx_2 f_{l_1}(x_1) f_{l_2}(x_2) \sigma_{\text{part}}(x_1, x_2; l_1 l_2 \rightarrow X)$$

- **perturbative**, **nonperturbative** part

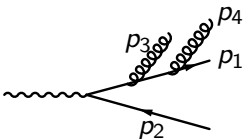
# Shifting momenta: Example (1)

$$\gamma^* \longrightarrow q(p_1)\bar{q}(p_2)g(p_3) \text{ (@ NLO)}$$



part of Born contribution

**real gluon emissions for this diagramm:**



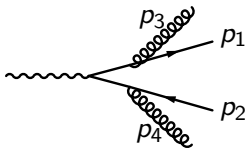
CS: 1 momentum shift/ spectator

$p_2, p_3$ : 2 transformations

NS: 1 total transformation



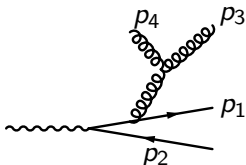
## Shifting momenta: Example (2)



CS: 1 momentum shift/ spectator

$p_1, p_3$ : 2 transformations

NS: 1 total transformation



CS: 1 momentum shift/ spectator

$p_1, p_2$ : 2 transformations

NS: 1 total transformation

$\Rightarrow$  from simple counting:

**12 transformations using CS vs 6 using NS dipoles !!**

of course many more contributions (eg  $g \rightarrow q \bar{q}$ , other Born terms, ... )

# Dipole subtraction: Real master formula

## Real Masterformula ( $s = (p_a + p_b)^2$ )

$$\begin{aligned}
 \sigma(s) = & \int_m d\Phi^{(m)}(s) \frac{1}{n_c(a)n_c(b)} |\mathcal{M}^{(m)}|^2(s) F_J^{(m)} \\
 & + \int d\Phi^{(m+1)}(s) \left\{ \frac{1}{n_c(a)n_c(b)} |\mathcal{M}^{(m+1)}|^2(s) F_J^{(m+1)} - \sum_{\text{dipoles}} (\mathcal{D} \cdot F_J^{(m)}) \right\} \\
 & + \int d\Phi^{(m)}(s) \left\{ \frac{1}{n_c(a)n_c(b)} |\mathcal{M}^{(m)}|^2_{\text{loop}}(p_a, p_b) + \mathbf{I}(\varepsilon) |\mathcal{M}^{(m)}|^2(s) \right\}_{\varepsilon=0} F_J^{(m)} \\
 & + \left\{ \int dx_a dx_b \delta(x - x_a) \delta(x_b - 1) \int d\Phi^{(m)}(x_a p_a, x_b p_b) |\mathcal{M}^{(m)}|^2(x_a p_a, x_b p_b) \right. \\
 & \left. \times \left( \mathbf{K}^{a,a'}(x) + \mathbf{P}^{a,a'}(x_a p_a, x_b p_b, x; \mu_F^2) \right) \right\} + (a \leftrightarrow b)
 \end{aligned}$$

where all colour/ phase space factors have been accounted for

# Integrated Dipoles in more details: $I, K, P$ (1)

$m + 1$  phase space: in principle easy

$$\int d\Gamma_{m+1} \left( |\mathcal{M}_{\text{real}}|^2 - \sum D \right), \text{ finite}$$

$m$  particle phase space: more complicated

need integration variables (emission from  $p_1$ ):

$$x = 1 - \frac{p_4(p_1 + p_2)}{p_1 p_2} \text{ softness, } \tilde{\nu} = \frac{p_1 p_4}{p_1 p_2} \text{ collinearity}$$

## Integrated Dipoles in more details: $I, K, P$ (2)

- in principle, obtain  $\int d\Gamma_1 D = \int_0^1 dx \left( \mathbf{I}(\varepsilon) + \tilde{\mathbf{K}}(x, \varepsilon) \right)$
- $\mathbf{I}(\varepsilon) \propto \delta(1-x)$ : corresponds to loop part
- $\tilde{\mathbf{K}}(x, \varepsilon)$  contains finite parts as well as **collinear singularities**
- latter need to be cancelled by adding **collinear counterterm**

$$\frac{1}{\epsilon} \left( \frac{4\pi\mu^2}{\mu_F^2} \right)^\epsilon P^{qq}(x)$$

depends on factorization scale  $\mu_F$  ( $P^{qq}(x)$  splitting function)

- PDFs come in again: term already accounted for by folding w PDF, needs to be subtracted
- for  $qg \rightarrow Wq$  like processes, only singularity which appears

# $q \rightarrow qg$ for initial state quarks: Catani Seymour (1)

- $q(\tilde{p}_1) \rightarrow q(p_1) + g(p_4)$ ,  $q$  enters hard interaction
- Dipole:

$$D^{14,2} = -\frac{8\pi\mu^2\alpha_s C_F}{s+t+u} \left( \frac{2s(s+t+u)}{t(t+u)} + (1-\varepsilon)\frac{t+u}{t} \right)$$

- matching ( $\tilde{p}_2 = p_2$ )

$$\tilde{p}_1 = x p_1, \quad x = 1 - \frac{p_4(p_1 + p_2)}{(p_1 p_2)}$$

$$\tilde{p}_k^\mu = \Lambda^\mu{}_\nu p_k^\nu, \quad (k: \text{final state particles})$$

$$\Lambda^{\mu\nu} = -g^{\mu\nu} - \frac{2(K + \tilde{K})^\mu(K + \tilde{K})^\nu}{(K + \tilde{K})^2} + \frac{2K^\mu \tilde{K}^\nu}{K^2}$$

$$K = p_1 + p_2 - p_4, \quad \tilde{K} = \tilde{p}_1 + p_2$$

## $q \rightarrow qg$ for initial state quarks: Catani Seymour (2)

- integration variables:

$$v = \frac{p_1 p_4}{p_1 p_2}, \quad x = 1 - \frac{p_4 (p_1 + p_2)}{(p_1 p_2)}$$

- in  $p_1, p_2$  cm system:  $E_4 \rightarrow 0 \Rightarrow x \rightarrow 1$  (softness)  
 $\cos \theta_{14} \rightarrow 1 \Rightarrow v \rightarrow 0$  (collinearity)
- Dipole in terms of integration variables

$$D^{14,2} = -\frac{8\pi\alpha_s C_F}{v x s} \left( \frac{1+x^2}{1-x} - \varepsilon(1-x) \right)$$

- integration measure

$$[dp_j] = \frac{(2p_1 p_2)^{1-\varepsilon}}{16\pi^2} \frac{d\Omega_{d-3}}{(2\pi)^{1-\varepsilon}} dv dx (1-x)^{-2\varepsilon} \left[ \frac{v}{1-x} \left( 1 - \frac{v}{1-x} \right) \right]^{-\varepsilon}$$

where  $v \leq 1 - x$  and all integrals between 0 and 1

# $q \rightarrow qg$ for initial state quarks: Catani Seymour (3)

- result

$$\mu^{2\epsilon} \int [dp_j] D^{14,2} = \frac{\alpha_s}{2\pi} \frac{1}{\Gamma(1-\epsilon)} C_F \left( \frac{2\mu^2\pi}{p_1 p_2} \right)^\epsilon$$

$$\times \int_0^1 dx \left( \mathbf{I}(\epsilon)\delta(1-x) + \tilde{\mathbf{K}}(x, \epsilon) \underbrace{- \frac{1}{\epsilon} P^{qq}(x)}_{\text{killed by coll CT}} \right)$$

with

$$\mathbf{I}(\epsilon) = \frac{1}{\epsilon^2} + \frac{3}{2\epsilon} - \frac{\pi^2}{6}$$

$$\mathbf{K}(x) = (1-x) - 2(1+x)\ln(1-x) + \left( \frac{4}{1-x} \ln(1-x) \right)_+$$

$$P^{qq}(x) = \left( \frac{1+x^2}{1-x} \right)_+ \quad \text{regularized splitting function}$$

$q \rightarrow qg$  for initial state quarks: Nagy Soper (1)

- $q(\tilde{p}_1) \rightarrow q(p_1) + g(p_4)$ ,  $q$  enters hard interaction
- Dipole:

$$D^{14,2} = -\frac{8\pi\mu^2\alpha_s C_F}{s+t+u} \left( \frac{2su(s+t+u)}{t(t^2+u^2)} + (1-\varepsilon)\frac{u}{t} \right)$$

as CS, same pole structure as CS

- matching, integration variables, integration measure: as Catani Seymour ( $v \leftrightarrow y$ )
- Dipole in terms of integration variables

$$D^{14,2} = -\frac{8\pi\alpha_s C_F}{xs} \times \left( \frac{1-x-y}{y}(1-\varepsilon) + \frac{2x}{y(1-x)} - \frac{2x[2y-(1-x)]}{(1-x)[y^2+(1-x-y)^2]} \right)$$



# $q \rightarrow qg$ for initial state quarks: Nagy Soper (2)

- result

$$\mu^{2\varepsilon} \int [dp_j] D^{14,2} = \frac{\alpha_s}{2\pi} \frac{1}{\Gamma(1-\varepsilon)} C_F \left( \frac{2\mu^2\pi}{p_1 p_2} \right)^\varepsilon$$

$$\times \int_0^1 dx \left( \mathbf{I}(\varepsilon) \delta(1-x) + \tilde{\mathbf{K}}(x, \varepsilon) \underbrace{-\frac{1}{\varepsilon} P^{qq}(x)}_{\text{killed by coll CT}} \right)$$

with

$\mathbf{K}(x) =$

$$(1-x) - 2(1+x) \ln(1-x) + \left( \frac{4}{1-x} \ln(1-x) \right)_+ - (1-x)$$

- equivalence of dipoles schemes checked analytically

# Catani Seymour subtraction terms for single W (1)

- 2 particle phase space (real emission)

$$\begin{aligned}
 \mathcal{D}^{14,2} + \mathcal{D}^{24,1} &= \frac{8}{9} \pi \alpha_s g^2 \left( \frac{s^2 + (s+t+u)^2}{tu} \right) \\
 &= \underbrace{\frac{1}{4} \frac{1}{9} \sum |\mathcal{M}_{\text{real}}|^2}_{\text{singular}} + \underbrace{\frac{16}{9} g^2 \alpha_s \pi}_{\text{finite}}
 \end{aligned}$$

- 1 particle phase space (virtual contribution)

$$\mathbf{I}(\epsilon) |\mathcal{M}_b|^2 = \underbrace{\frac{2\alpha_s}{3\pi} \frac{1}{\Gamma(1-\epsilon)} (-8 + \frac{2}{3}\pi^2)}_{\text{finite}} |\mathcal{M}_b|^2 - \underbrace{|\widetilde{\mathcal{M}}_v|^2}_{\text{singular (+finite)}}$$

**all singular terms will disappear in subtraction**

# Catani Seymour subtraction terms for single W (2)

$$\begin{aligned}
 \mathbf{K}^a(xp_a) &= \frac{\alpha_s}{2\pi} C_F \frac{1}{\Gamma(1-\epsilon)} \left[ \left( \frac{4}{1-x} \ln(1-x) \right)_+ + (1-x) \right. \\
 &\quad \left. - 2(1+x) \ln(1-x) - \left( \frac{1+x^2}{1-x} \right)_+ \ln \left( \frac{4\pi\mu^2}{2xp_a \cdot p_b} \right) \right] \\
 \mathbf{P}(x, \mu_F^2) &= \frac{\alpha_s}{2\pi} C_F \frac{1}{\Gamma(1-\epsilon)} \left( \frac{1+x^2}{1-x} \right)_+ \ln \left( \frac{4\pi\mu^2}{\mu_F^2} \right)
 \end{aligned}$$