The Planck Scale from Top Condensation

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The hierarchy problem

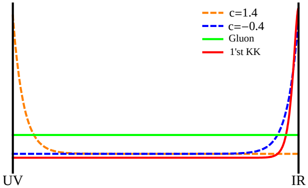
- In the standard model, the weak scale is not stable against radiative corrections:

$$\frac{v_{\rm EW}}{M_{\rm Planck}} \approx 10^{-16} \quad ???$$

- Proposed possible solutions:
  - Technicolor model (or its Ads/CFT version: Higgsless model)
  - Supersymmetry
  - Warped Extra Dimension

Randall-Sundrum model

- The 5-dimensional geometry is: $ds^2 = e^{-2ky}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$



- Need a Goldberger-Wise field ϕ propagating in the bulk to stabilize the distance between the IR and UV branes

- The large hierarchy is given:

$$\frac{\sim \mathrm{TeV}}{M_{\mathrm{Planck}}} = e^{-\frac{4k^2}{m^2}}\,\frac{v_{\mathrm{IR}}}{v_{UV}}$$

Randall-Sundrum model

A list of unsatisfied points in the RS model:

- It does not address how the electroweak symmetry is broken
- There is no real dynamical linkage between the M_{Planck} and $\langle H \rangle = v_{EW} = 175$ GeV
- There exists a little hierarchy between the KK-scale M_{KK} and v_{EW}. $M_{KK} > 2$ TeV [Carena, Pontón, Santiago, Wagner] :

Little hierarchy problem

How the electroweak symmetry is broken?

In the standard model

- a Higgs doublet with a potential

$$V(H) = m^2 H H^\dagger + \frac{\lambda}{2}(H H^\dagger)^2 \qquad m^2 < 0$$

$$v_{EW} = \sqrt{\frac{-m^2}{\lambda}} \approx 174\ \mathrm{GeV} \qquad H = (0, v_{EW} + \frac{h}{\sqrt{2}})^T$$

- the Higgs boson $\leftrightarrow$ an elementary scalar
- Questions:

$$m^2 < 0\,? \qquad v_{EW} \ll M_{\mathrm{Planck}}\,?$$

Strong Dynamical Symmetry Breaking

Technicolor Model [Weinberg and Susskind]

- $\langle \bar{Q}_L Q_R \rangle \approx \Lambda_{TC}^3$ breaks the chiral symmetry $\Lambda_{TC} = O(100)\,\mathrm{GeV}$
- Q_L weak doublet and Q_R singlet $v_{EW} \sim \Lambda_{TC}$
- $v_{EW}/M_{\mathrm{Planck}}$ is due to “dimensional transmutation”
- No fundamental scalar

Difficulties (for QCD-like dynamics)

- satisfying electroweak precision observables like the S parameter
- generate the top quark mass large enough

Possible Solution:

- Walking Technicolor (not QCD-like dynamics) [Appelquist, Shrock, Sannio $\cdots$]

Top Condensation [Nambu; Miransky, Tanabashi and Yamawaki]

Top quark is peculiar:

- the heaviest quark in the SM
- the top quark mass $m_t = 172.4$ GeV is not far from $v_{EW} \approx 174$ GeV
- the RG running equation of the top Yukawa coupling shows an infrared fixed point for the top quark mass [Pendleton and Ross; Hill]

Therefore

- the electroweak breaking may be from $\langle \bar{t}_L \, t_R \rangle$

$$\Longrightarrow \quad \text{Top Condensation}$$

Nambu Jona Lasinio (NJL) model

An effective lagrangian with 4-fermion interactions

$$\mathcal{L} = i\overline{\psi}_L \not{D}\psi_L + i\overline{\psi}_R \not{D}\psi_R + \frac{g^2}{M^2}(\overline{\psi}_L \psi_R)(\overline{\psi}_R \psi_L)$$

Using an auxiliary "Higgs" field, H, rewrite

$$\mathcal{L}(M) = i\overline{\psi}_L \not{D}\psi_L + i\overline{\psi}_R \not{D}\psi_R - M^2 H^\dagger H + (g\, H \overline{\psi}_L \psi_R + \mathrm{h.c.})$$

Renormalization group running to a lower scale

$$\begin{aligned}\mathcal{L}(\mu) \quad &= \quad \mathcal{Z}_{\mathrm{L}}\, i\overline{\psi}_L \not{D}\psi_L + \mathcal{Z}_{\mathrm{R}}\, i\overline{\psi}_R \not{D}\psi_R + (\mathcal{Z}_g\, g\, H \overline{\psi}_L \psi_R + \mathrm{h.c.}) \\ &\quad + \mathcal{Z}_H\, \partial_\mu H^\dagger \partial^\mu H - m_H^2 H^\dagger H - \frac{\lambda}{2}(H^\dagger H)^2 \\ \text{with} \quad &\quad \mathcal{Z}_H = \frac{N_c\, g^2}{16\,\pi^2} \ln\left(\frac{M^2}{\mu^2}\right)\end{aligned}$$

H is a composite particle

NJL model (cont.)

$$m_H^2 \approx M^2 \left[1 - \frac{g^2 N_c}{8\pi^2}\left(1 - \frac{\mu^2}{M^2}\right)\right] \qquad \lambda \approx \frac{g^4 N_c}{8\pi^2} \ln\left(\frac{M^2}{\mu^2}\right)$$

H acquires a non-vanishing VEV if $g^2 > G_c^2 \equiv \frac{8\pi^2}{N_c}$ (valid at large N_c limit)

The Higgs potential is

$$V(H) \quad = \quad \overline{m}_H^2 H^\dagger H + \frac{\bar{\lambda}}{2}(H^\dagger H)^2 \quad \text{with} \quad \overline{m}_H^2 = \frac{m_H^2}{\mathcal{Z}_H} \quad \bar{\lambda} = \frac{\lambda}{\mathcal{Z}_H^2}$$

The electroweak scale $v_{EW} = \sqrt{-\overline{m}_H^2/\bar{\lambda}}$

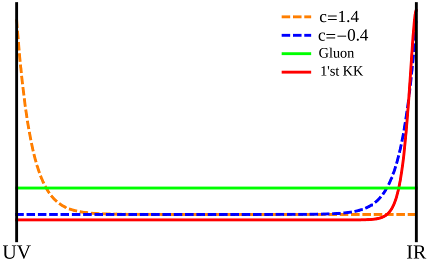
- Origin of the strong coupling ($g^2 \gg 1$) or the UV theory? Topcolor[Hill]
- Fine-tuning problem? For $v_{EW} \ll M$, need to adjust g^2 very close to the critical value G_c^2. So far, no solution

Wave-function

Consider an $SU(N_c)$ gauge theory in the AdS$_5$ background ($0 \leq y \leq L$)

$$ds^2 = e^{-2ky}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$$

$$f_c(y) = \sqrt{\rho_c}\; e^{(\frac{1}{2}-c)ky} \qquad \text{UV(IR) localized for} \quad c > \frac{1}{2}\ (c < \frac{1}{2})$$



other fermions

top quark

Four-fermion Operator

The 4-d $SU(N_c)$ gauge coupling is $4\,\pi\,\alpha_s = \frac{g_5^2\,k}{k\,L}$

Integrating the 1'st KK mode leads to 4-fermion operators of the form

$$-\frac{g_{c_1} g_{c_2}}{M_{\rm KK}^2}(\overline{\psi}_{1L}\,T^A\gamma^\mu\,\psi_{1L})(\overline{\psi}_{2R}\,T^A\gamma^\mu\,\psi_{2R}) = \frac{g_{c_1} g_{c_2}}{M_{\rm KK}^2}(\overline{\psi}_{1L}\,\psi_{2R})(\overline{\psi}_{2R}\,\psi_{1L}) + \mathcal{O}(1/N_c)$$

The 1'st KK gluon mass is $M_{\rm KK} = x_1\,k\,e^{-kL}$, where $x_1 \approx 2.45$.

$$g_{c_1} g_{c_2} \approx g_5^2 k\, x_1^2\left[f_1(c_1, c_2) - \frac{f_2(c_1, c_2)}{kL}\right] \qquad \frac{g_{c_1} g_{c_2}}{4\,\pi\,\alpha_s} \approx 30 \quad (\text{for } c_1 = c_2 = -0.5)$$

Randall-Sundrum Model $\rightsquigarrow$ Top Condensation:

- The coupling of the KK gluon to fermions can be much larger than the zero mode.
- The 4-fermion operator generated by integrating out the first KK gluon can be over the critical coupling in the NJL model and induce top condensation, if the top quark is localized very close to the IR brane.

Radion and Higgs Potential

Choosing the interval L between UV and IR branes as a free parameter

Referring back to the NJL analysis, we have a radion and Higgs coupled potential

$$
\begin{aligned}
V(H,L) &= \overline{m}_H^2(L)\, H^\dagger H + \frac{\bar{\lambda}(L)}{2}\, (H^\dagger H)^2 \\
&= \frac{\bar{\lambda}(L)}{2}\left[H^\dagger H + \frac{\overline{m}_H^2(L)}{\bar{\lambda}(L)}\right]^2 - \frac{\overline{m}_H^4(L)}{2\bar{\lambda}(L)}
\end{aligned}
$$

$$
m_H^2 \approx M_{\mathrm{KK}}^2\left[1 - \frac{g_\psi^2 N_c}{8\pi^2}\left(1 - \frac{\mu^2}{M_{\mathrm{KK}}^2}\right)\right] \qquad \lambda \approx \frac{g_\psi^4 N_c}{8\pi^2} \ln\left(\frac{M_{\mathrm{KK}}^2}{\mu^2}\right)
$$

$$
\overline{m}_H^2 = \frac{m_H^2}{\mathcal{Z}_H} \qquad \bar{\lambda} = \frac{\lambda}{\mathcal{Z}_H^2} \qquad \mathcal{Z}_H = \frac{N_c\, g_\psi^2}{16\,\pi^2} \ln\left(\frac{M_{\mathrm{KK}}^2}{\mu^2}\right) \qquad M_{\mathrm{KK}} \equiv x_1\, k\, e^{-k\,L}
$$

Radion and Higgs Potential

$$g_\psi^2 \equiv g_{c_1} g_{c_2} \approx g_5^2 k\, x_1^2 \left[f_1(c_1, c_2) - \frac{f_2(c_1, c_2)}{kL} \right]$$

Choosing c_1 and c_2 to require $g_5^2 k\, x_1^2 f_1(c_1, c_2) > G_c^2$

There is a critical value, L_c, defined by

$$g_5^2 k\, x_1^2 \left[f_1(c_1, c_2) - \frac{f_2(c_1, c_2)}{kL_c} \right] = G_c^2$$

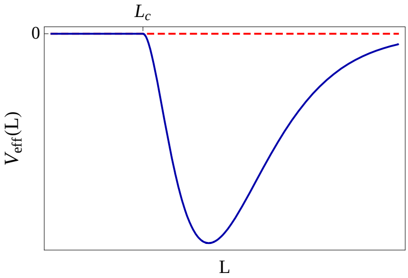
such that when $L > L_c$, $\langle H \rangle \neq 0$ and $L < L_c$, $\langle H \rangle = 0$.

$$V_{\rm eff}(L) \quad = \quad -\frac{\overline{m}_H^4(L)}{2\bar{\lambda}(L)}\, \theta(L - L_c) \approx -\frac{M_{\rm KK}^4 \left(\frac{1}{G_c^2} - \frac{1}{g_\psi^2} \right)^2}{\frac{N_c}{4\pi^2} \log \frac{M_{\rm KK}^2}{\mu^2}}\, \theta(L - L_c)$$

$L < L_c$, $V_{\rm eff} = 0$; $L \to \infty$, $M_{\rm KK} \equiv x_1\, k\, e^{-k\,L}$ goes to zero and $V_{\rm eff} \to 0$

There is a minimum for $V_{\rm eff}(L)$

Radion Potential



The fermion condensation can stabilize
the relative distance of two branes

A novel way to realize the Goldberger-Wise mechanism to stabilize the radion

Top Condensation $\rightsquigarrow$ Randall-Sundrum Model

Scales: Leading Order Analysis

The minimum of the radion potential is

$$kL_{\rm min} \approx \frac{\bar f_2}{\bar f_1 - G_{\rm c}^2} + \frac{1}{2} \qquad \bar f_i \equiv g_5^2 k\, x_1^2 f_i(c_1, c_2)$$

The 4-fermion coupling $\quad g_\psi^2 \approx G_{\rm c}^2 + \frac{(\bar f_1 - G_{\rm c}^2)^2}{2\bar f_2}$

The electoweak scale is

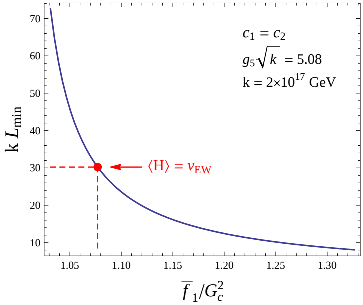
$$v_{\rm EW} \approx M_{\rm KK} \left(\frac{\bar f_1 - G_{\rm c}^2}{G_{\rm c}^2}\right) \sqrt{\frac{1}{4\,\bar f_2}} = x_1\, k\, e^{-k\,L_{\rm min}} \left(\frac{\bar f_1 - G_{\rm c}^2}{G_{\rm c}^2}\right) \sqrt{\frac{1}{4\,\bar f_2}}$$

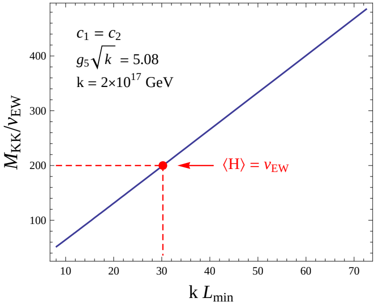
If $\bar f_1$ is 10% close to $G_{\rm c}^2$, we have $\frac{g_\psi^2}{G_{\rm c}^2} - 1 \approx 10^{-3}$ and $v_{EW} = O(\frac{M_{\rm KK}}{100})$

v_{EW} is two orders of magnitude below $M_{\rm KK}$ (Little Hierarchy)

A consequence of the radion stabilization using the strong dynamics

Scales: Leading Order Analysis





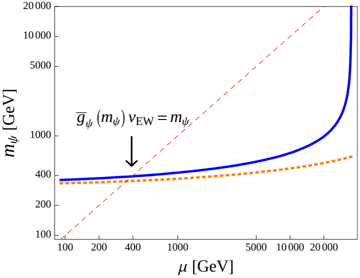
M_{KK} is predicted to be around 35 TeV

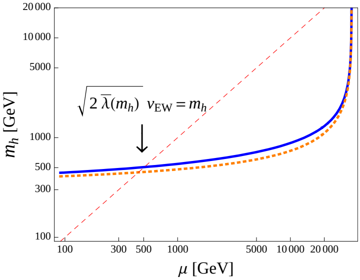
Scales: Improved Analysis

$$16\pi^2 \frac{d\,\bar{g}_\psi}{d\,\ln\mu} = \bar{g}_\psi \left[\frac{9}{2}\,\bar{g}_\psi^2 - 8\,\bar{g}_3^2 - \frac{9}{4}\,\bar{g}_2^2 - \frac{17}{12}\,\bar{g}_Y^2\right]$$

$$16\pi^2 \frac{d\,\bar{\lambda}}{d\,\ln\mu} = 12\left[\bar{\lambda}^2 + (\bar{g}_\psi^2 - \frac{1}{4}\,\bar{g}_Y^2 - \frac{3}{4}\,\bar{g}_2^2)\,\bar{\lambda} + \frac{1}{16}\,\bar{g}_Y^4 + \frac{1}{8}\,\bar{g}_Y^2\,\bar{g}_2^2 + \frac{3}{16}\bar{g}_2^4 - \bar{g}_\psi^4\right]$$

Using the "compositeness conditions" $\bar{g}_\psi = \infty$ and $\bar{\lambda} = \infty$ at the M_{KK} scale [Bardeen, Hill, Lindner]





$m_\psi = 375 \pm 25$ GeV and $m_h = 475 \pm 25$ GeV

Caveats

- The NJL analysis is only trustable in the large N_c approximation.
- The Casimir energy generated by gauge boson loops can dominate the potential from top condensation and shift the VEV of radion. This can be suppressed by introducing “inert" fermions to match the d.o.f of bosons.
- The non-calculable contributions to the radion potential through the IR tension, which are sensitive to the UV completion of the 5D theory, may be linked to the cosmological constant problem and suppressed by unknown 5D UV physics.
- How to understand the interplay of Higgs and radion (dilaton) fields in terms of a 4d effective field theory is unknown now.

The realistic model

In the "top sector", Ψ_{Q_L} charged as $(3,2)_{1/3}$, Ψ_1 and Ψ_2 as $(3,1)_{4/3}$

- choosing mixed boundary conditions for Ψ_1 and Ψ_2
- one linear combination $\cos\theta\,\Psi_{1R} + \sin\theta\,\Psi_{2R}$ has right-handed zero mode t_R
- another linear combination has ultralight Dirac KK mode: χ_L and χ_R, with a mass $m_d(c_1, c_2, \theta)$
- the low-energy fermions are Q_L, t_R, χ_L and χ_R

In a new basis: $t'_R \equiv \cos\alpha\, t_R + \sin\alpha\, \chi_R$ and $\chi'_R \equiv -\sin\alpha\, t_R + \cos\alpha\, \chi_R$

$$(\bar{t}'_R \quad \bar{\chi}'_R) \begin{pmatrix} g_{c_1} & 0 \\ 0 & g_{c_2} \end{pmatrix} G^A_\mu\, T^A\, \gamma^\mu \begin{pmatrix} t'_R \\ \chi'_R \end{pmatrix}$$

$$\alpha = -\tan^{-1}\left(\sqrt{\rho_{c_1}/\rho_{c_2}}\,\tan\theta\right)$$

Top quark mass and EWSB

The top sector mass matrix is (“Top Seesaw” [Dobrescu and Hill])

$$\begin{pmatrix} \bar{t}_L & \overline{\chi}_L \end{pmatrix} \begin{pmatrix} -\sin\alpha\, \bar{g}_{Q\chi_R} \langle H \rangle & \cos\alpha\, \bar{g}_{Q\chi_R} \langle H \rangle \\ 0 & m_d \end{pmatrix} \begin{pmatrix} t_R \\ \chi_R \end{pmatrix}$$

In the limit that the Dirac mass is large, $m_d \gg \bar{g}_{Q\chi_R} \langle H \rangle$, the physical top quark mass is

$$\begin{aligned} m_t^2 &\approx \sin^2\alpha\, (\bar{g}_{Q\chi_R} \langle H \rangle)^2 \\ m_\chi^2 &\approx m_d^2 + \cos^2\alpha\, (\bar{g}_{Q\chi_R} \langle H \rangle)^2 \end{aligned}$$

Using the RG improved value of $\bar{g}_{Q\chi_R} \langle H \rangle = 375$ GeV, we need $\sin\alpha \approx 0.46$ to fit the top quark mass.

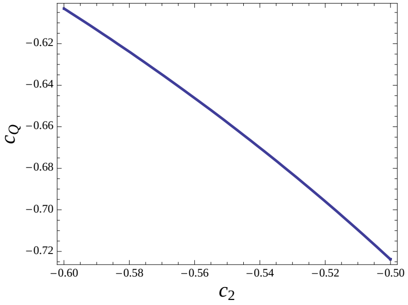
We have 5 model parameters: g_5, c_Q, c_1, c_2 and $\theta(\alpha)$ to fit three observed quantities: α_s, v_{EW} and m_t. We are left with two free parameters in the model: c_1 and c_2.

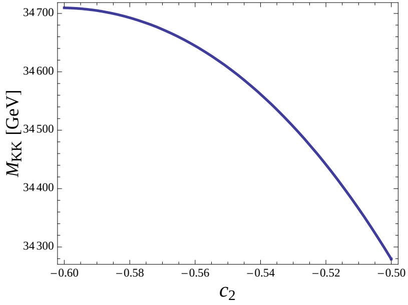
Top quark mass and EWSB

$$\frac{1}{M_{\rm KK}^2}\left[g^2_{\chi_L\chi_R}(\overline{\chi}_L\chi'_R)(\overline{\chi'}_R\chi_L)+g^2_{\chi_L t}(\overline{\chi}_L t'_R)(\overline{t'}_R\chi_L)+g^2_{Q\chi_R}(\overline{Q}_L\chi'_R)(\overline{\chi'}_R Q_L)+g^2_{Qt}(\overline{Q}_L t'_R)(\overline{t'}_R Q_L)\right]$$

Choosing $c_Q \le c_2 < c_1 < -1/2, \quad g^2_{\chi_L t}, g^2_{\chi_L\chi_R} < 0 < g^2_{Qt} < G_c^2 < g^2_{Q\chi_R}$ [χ_L is UV localized]

- the condensation and radion stabilization happen in the channel $\langle \bar{Q}_L\, \chi'_R\rangle$





Radion

The radion-dependent terms arise from the 5D Einstein-Hilbert action

$$S = \frac{M_5^3}{k} \int d^4x \sqrt{-g} \left(1 - \frac{\phi^2}{F^2}\right) \mathcal{R}_4 + \frac{1}{2} \int d^4x \sqrt{-g} \left\{ \partial_\mu \phi \partial^\mu \phi - V\left(H, -k^{-1} \ln \phi / F\right) \right\}$$

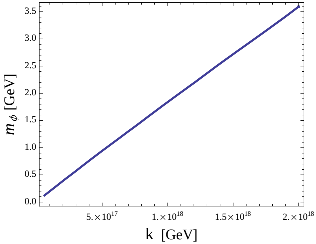
where $\phi(x) \equiv Fe^{-k\,T(x)}$ with $\langle T \rangle = L$ and $F = \sqrt{12 M_5^3 / k}$.

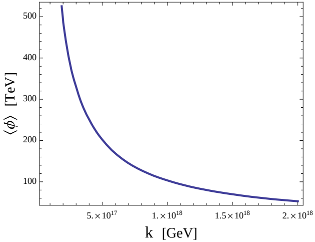
$$\langle \phi \rangle = \tilde{F} = Fe^{-k\,L_{\text{min}}}$$

4D Planck mass is $M_P^2 \approx M_5^3/k \sim (2 \times 10^{18}\ \text{GeV})^2$, we have $F \approx 2\sqrt{3} M_P \approx 6.9 \times 10^{18}\ \text{GeV}$
The mass of the radion takes the approximate form

$$m_\varphi \approx \frac{3\, x_1\, \bar{f}_2\, k\, M_{\text{KK}}}{64\, \pi^3 \log^2\left(\frac{x_1\, k}{M_{\text{KK}}}\right) \log^{\frac{1}{2}}\left(\frac{M_{\text{KK}}}{\mu}\right) M_P} \approx \frac{k}{M_P} (4\ \text{GeV})$$

Radion (cont.)





LEP imposes bounds on the couplings of a light scalar to Z gauge bosons: $v_{\rm EW}/\langle\phi\rangle < 10^{-1}$

$$\mathcal{L}_{int} = \frac{\phi}{\langle\phi\rangle}\left[\sum_\psi F_{c_\psi} m_\psi \bar{\psi}\psi + M_Z^2 Z^\mu Z_\mu + 2M_W^2 W^{+\mu}W_{-\mu} + \frac{\beta(g_s)}{2\,g_s}G^{a\mu\nu}G_{a\mu\nu} + \frac{\beta(e)}{2\,e}F^{\mu\nu}F_{\mu\nu}\right]$$

In our case: $v_{\rm EW}/\langle\phi\rangle = \text{few} \times 10^{-4}$

[arXiv:0903.3410: Davoudiasl, Pontón]

Electroweak Precision Constraints

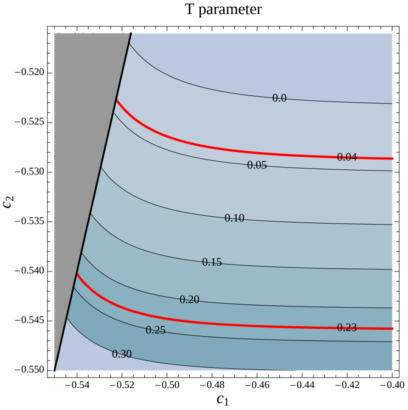
[He, Hill and Tait]

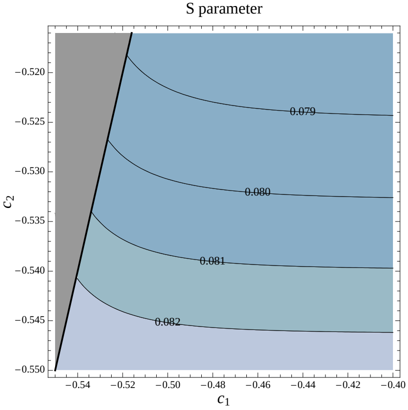
- the T parameter
  - the 475 GeV Higgs boson contributes ~ -0.2 to ΔT
  - the χ fermion loop provides a positive and non-negligible contribution
  - Therefore, there is an upper bound on m_χ
- the S parameter
  - the 475 GeV Higgs boson contributes ~ 0.07 to ΔS
  - while χ contributes ~ 0.01
  - $\Delta S = 0.08$ for a wide range of mode parameter space
- the $Z\, \bar{b}_L\, b_L$ coupling

$$\delta g_{b_L}^{loop} \approx \frac{e^2}{64\pi^2 s_W^2 M_W^2} \frac{(\bar{g}_{Q\chi_R} \cos\alpha \langle H \rangle)^4}{m_\chi^2} \left[1 + 2 \frac{m_t^2}{(\bar{g}_{Q\chi_R} \cos\alpha \langle H \rangle)^2} \left(\log \frac{m_\chi^2}{m_t^2} - 1 \right) \right]$$

$\delta g_{b_L}^{loop} / g_{b_L} \approx -2.4 \times 10^{-3}$ for $m_\chi = 2$ TeV, $\bar{g}_{Q\chi_R} \langle H \rangle = 375$ GeV and $\alpha = 0.48$

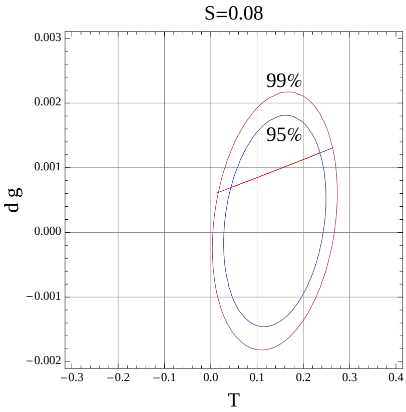
T and S parameters

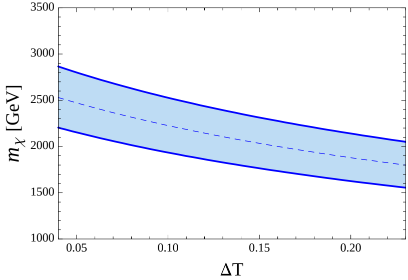




Fermilab

Constrains on the fermion χ mass





$1.6\ \mathrm{TeV} < m_\chi < 2.9\ \mathrm{TeV}$

Fermilab

LHC Collider Phenomenology

- The heavy Higgs boson with a mass around 475 GeV and SM-like
  - a total width of $\Gamma_h^{total} \approx 56.6$ GeV;
    $Br(h \to \bar{t}t) \approx 19.5\%$, $Br(h \to W^+W^-) \approx 54.5\%$ and $Br(h \to ZZ) \approx 26.0\%$,
  - the decay $h \to ZZ \to 4\,\ell$ is the "gold-plated" mode for discovery at the LHC

- The vector-like fermion χ with a mass around 2 TeV;
  - The mixing angle of the χ_L and t_L is $\beta_L \approx 0.16$; its total width is around 140 GeV

$$\Gamma(\chi \to t\,h) = \Gamma(\chi \to t\,Z) = \frac{1}{2}\,\Gamma(\chi \to b\,W) = \frac{\cot^2\alpha\, m_t^2}{64\,\pi\, v^2}\, m_\chi$$

  - can be discovered up to $m_\chi = 2.5$ TeV with 300 fb^{-1} at $5\,\sigma$ in the decay chain $\chi \to b\,W \to \ell\,\nu\,b$ [G. Azuelos et al.]
  - $\chi \to h\,t \to Z\,Z\,W\,b$, $3\,W\,b$ and $3\,W\,3\,b$ would provide interesting signal topologies for the presence of the heavy quark and Higgs boson

Conclusions

Randall-Sundrum Model $\rightleftharpoons$ Top Condensation

- From left to right : the flavor structure of the RS model naturally provides a 4-fermion operator with a large coefficient, which is necessary for top quarks to condense and to break the electroweak symmetry dynamically.
- From right to left : the top condensation can stabilize the radion (the distance between the UV and IR branes). Therefore directly links the Planck and the eletroweak scale or the top condensation scale.

All together

- A complete story from the Planck scale to the weak scale is known.
- The Kaluza-Klein scale ($\sim$ 35 TeV) is predicted to be about two orders of magnitude above the electroweak scale.
- In a realistic model constructed, three new particles below 10 TeV: heavy Composite Higgs ($\sim$ 500 GeV), heavy top quark ($\sim$ 2 TeV) and light radion ($\sim$ 1 GeV).