Efficient Circuit Decompositions Using Intermediate Qudits

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Outline

1. Basics of Multivalued Quantum Computing
2. Generalized Toffoli
3. Arithmetics Part 1
4. Qubit-Qudit Compression
5. Arithmetics Part 2

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1. **Basics of Multivalued Quantum Computing**
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From Qubits to Qu*dits*

Classically:

Bits - $b_2 \in \{0, 1\}$

Trits - $b_3 \in \{0, 1, 2\}$

Dits - $b_d \in \{0, 1, 2, ..., d-1\}$

From Qubits to Qu*dits*

Classically:

Bits - $b_2 \in \{0, 1\}$

Trits - $b_3 \in \{0, 1, 2\}$

Dits - $b_d \in \{0, 1, 2, ..., d-1\}$

Quantumly:

Qubits - $|\psi\rangle_2 = \alpha|0\rangle + \beta|1\rangle$

Qu*trits* - $|\psi\rangle_3 = \alpha|0\rangle + \beta|1\rangle + \gamma|2\rangle$

Qu*dits* - $|\psi\rangle_d = \alpha_0|0\rangle + \alpha_1|1\rangle + ... + \alpha_{d-1}|d-1\rangle$

Logical Manipulation of Qudit States

Classically Reversible Operations - Single Qudit Operators

$$X\left|\psi_2\right\rangle = X(\alpha\left|0\right\rangle + \beta\left|1\right\rangle) = \beta\left|0\right\rangle + \alpha\left|1\right\rangle$$

Logical Manipulation of Qudit States

Classically Reversible Operations - Single Qudit Operators

$$X |\psi_2\rangle = X(\alpha |0\rangle + \beta |1\rangle) = \beta |0\rangle + \alpha |1\rangle$$

$$X_{ij} |\psi_d\rangle = X_{ij}(\alpha_0 |0\rangle + \ldots + \alpha_i |i\rangle + \ldots + \alpha_j |j\rangle + \ldots + \alpha_{d-1} |d-1\rangle)$$
$$= \alpha_0 |0\rangle + \ldots + \alpha_j |i\rangle + \ldots + \alpha_i |j\rangle + \ldots + \alpha_{d-1} |d-1\rangle$$

Logical Manipulation of Qudit States

Classically Reversible Operations - Single Qudit Operators

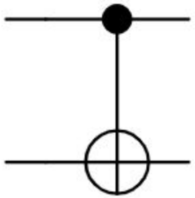
$$X\left|\psi_2\right\rangle = X(\alpha\left|0\right\rangle + \beta\left|1\right\rangle) = \beta\left|0\right\rangle + \alpha\left|1\right\rangle$$

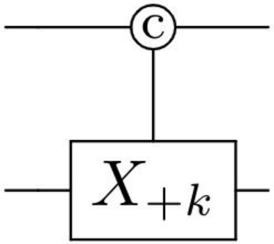
$$\begin{aligned} X_{ij}\left|\psi_d\right\rangle &= X_{ij}(\alpha_0\left|0\right\rangle + ... + \alpha_i\left|i\right\rangle + ... + \alpha_j\left|j\right\rangle + ... + \alpha_{d-1}\left|d-1\right\rangle) \\ &= \alpha_0\left|0\right\rangle + ... + \alpha_j\left|i\right\rangle + ... + \alpha_i\left|j\right\rangle + ... + \alpha_{d-1}\left|d-1\right\rangle \end{aligned}$$

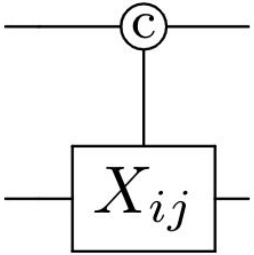
$$\begin{aligned} X_{+k}\left|\psi_d\right\rangle &= X_{+k}(\alpha_0\left|0\right\rangle + ... + \alpha_{d-1}\left|d-1\right\rangle) \\ &= \alpha_0\left|k\right\rangle + \alpha_1\left|k+1\right\rangle ... + \alpha_{d-1}\left|k-1\right\rangle \end{aligned}$$

Logical Manipulation of Qudit States

Classically Reversible Operations - Controlled Qudit Operators

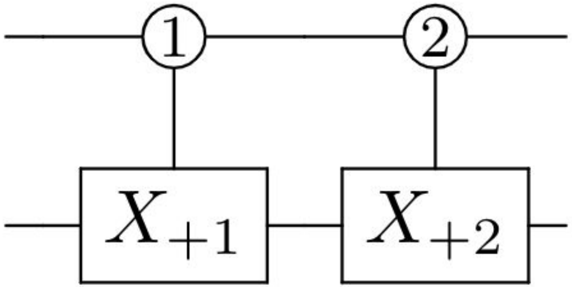






Logical Manipulation of Qudit States

Qutrit CSUM can be written using our set of gates



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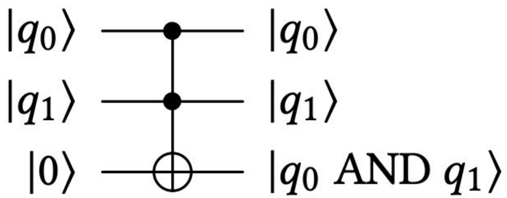
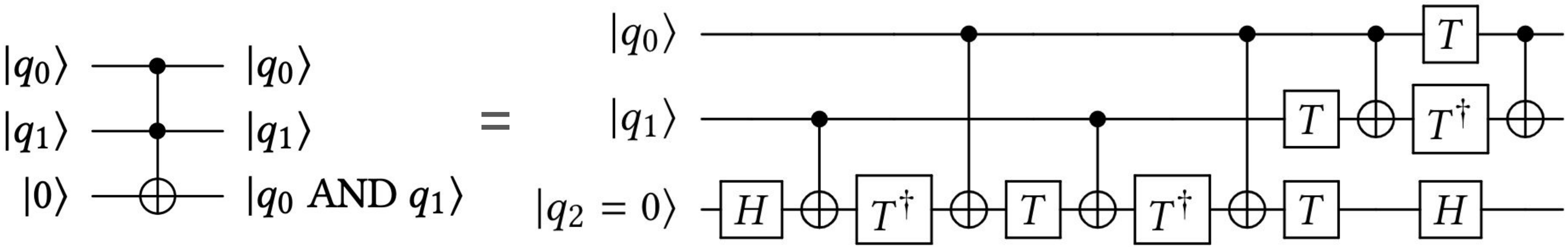
2-Control Toffoli



Nielsen and Chuang

Gokhale et al. (https://arxiv.org/abs/1905.10481)

Generalized Toffoli



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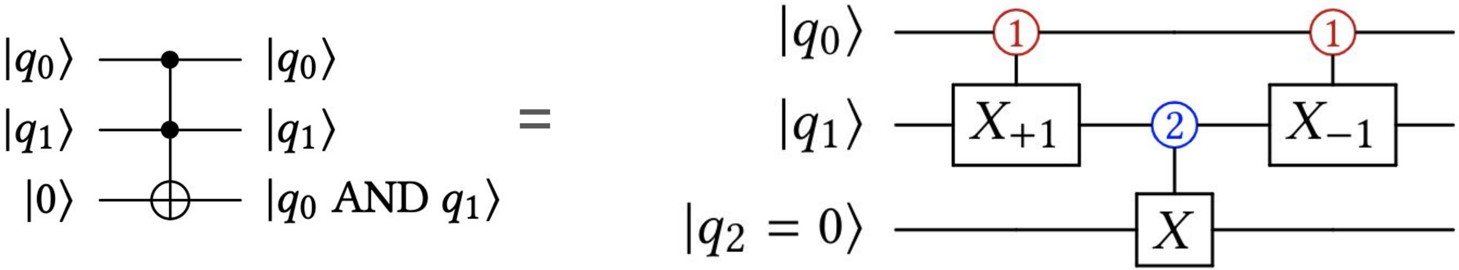
Generalized Toffoli



Nielsen and Chuang

Gokhale et al. (https://arxiv.org/abs/1905.10481)

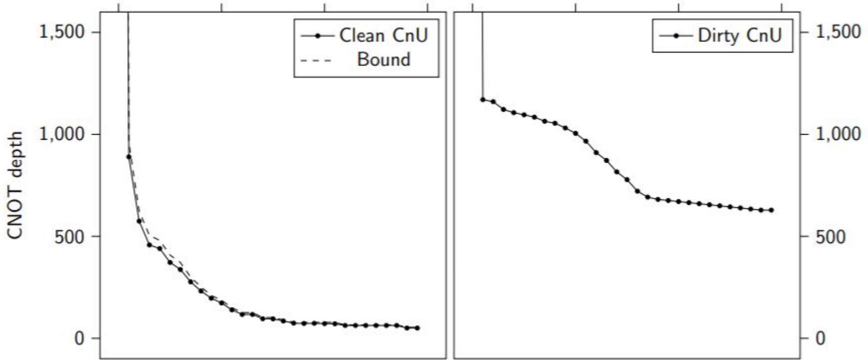
Generalized Toffoli

	This Work	Gidney [28]	He [29]	Barenco [30]	Wang [25]	Lanyon [31], Ralph [32]
Depth	$\log N$	N	$\log N$	N^2	N	N
Ancilla	0	0	N	0	0	0
Control Type	Qutrits	Qubits	Qubits	Qubits	Qutrits	Qubits
Target Type	Qubit	Qubit	Qubit	Qubit	Qubit	$d = N$-level qudit
Constants	Small	Large	Small	Small	Small	Small

Intermediate Qutrits

Generalized Toffoli

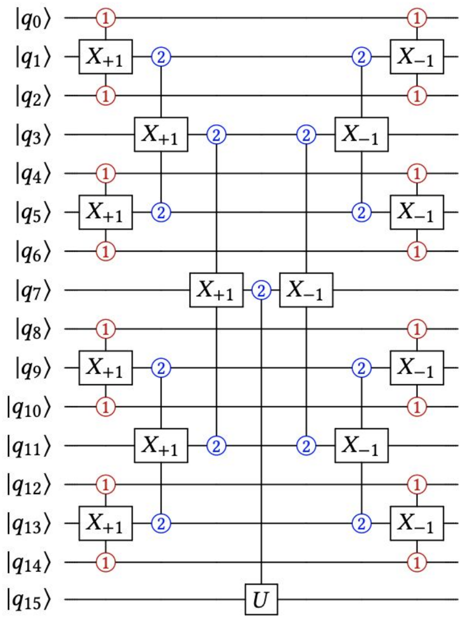
Bridging the gap:



Number of Ancilla (30 Controls)

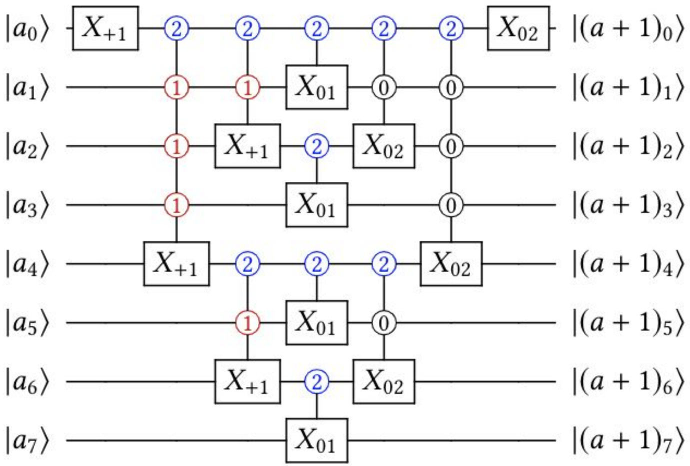
https://arxiv.org/abs/1904.01671

Generalized Toffoli



Gokhale et al. (https://arxiv.org/abs/1905.10481)

Generalized Toffoli Use:



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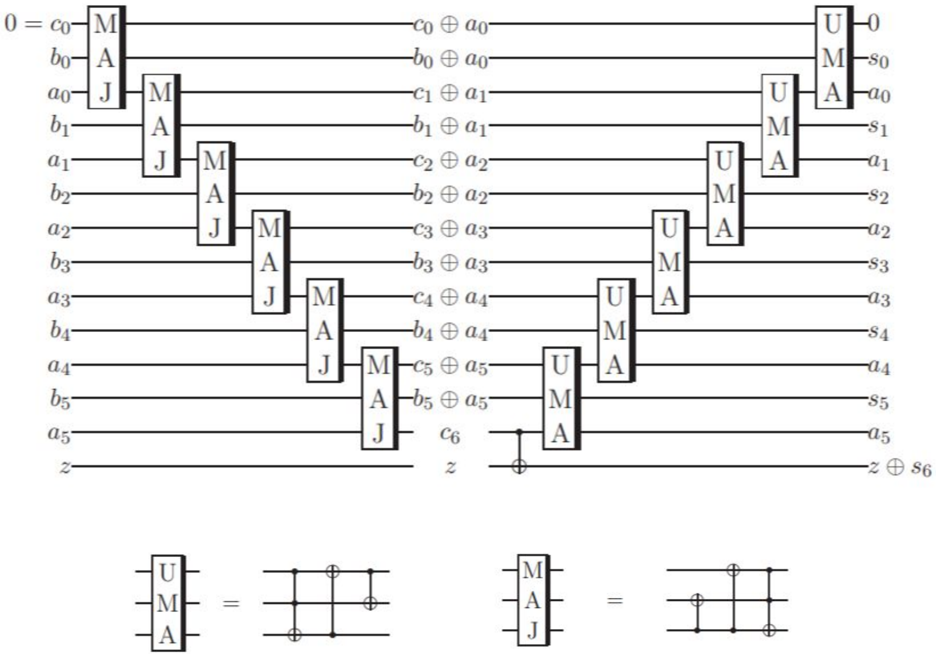
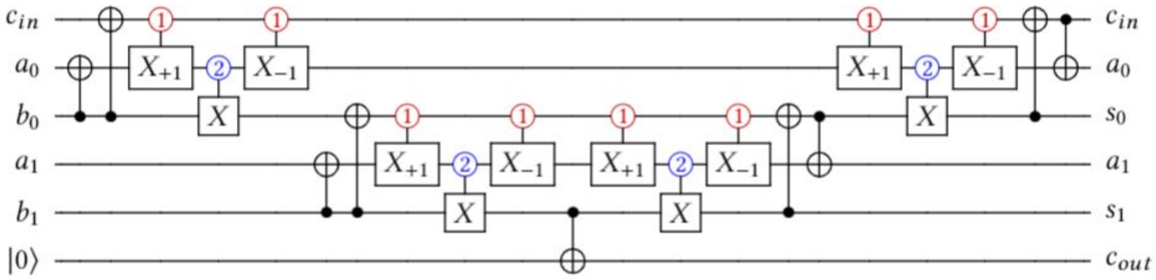
Ripple Carry Adder

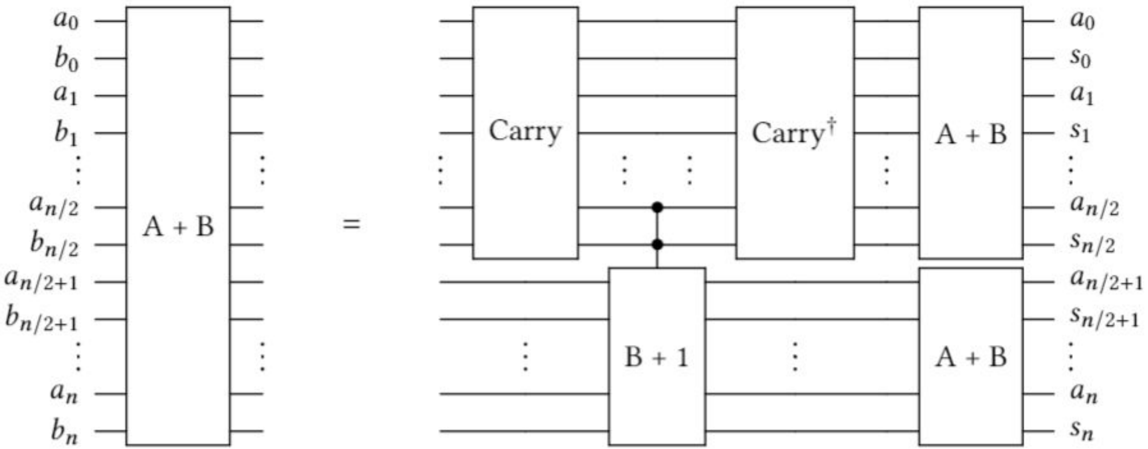


https://arxiv.org/abs/quant-ph/0410184

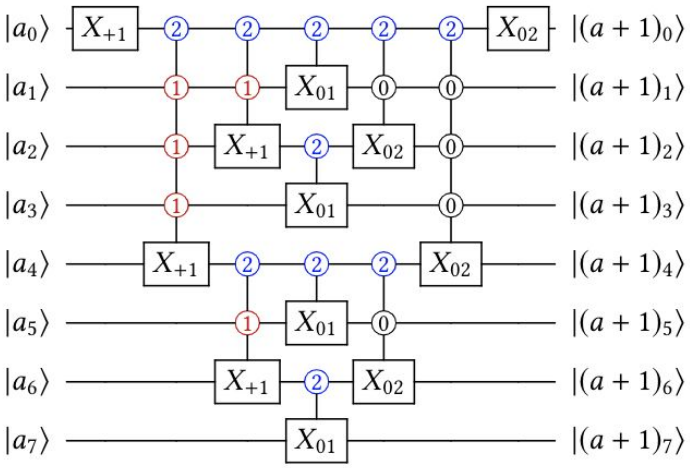
Ripple Carry Adder - w/ Qutrits



Adder with Temp. Qutrits



Adder with Temp. Qutrits

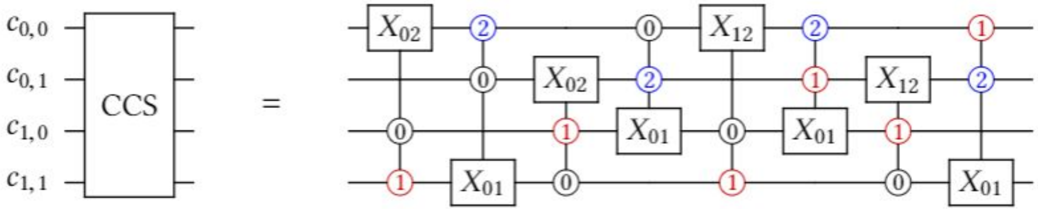


Problem: With only two *bits*, irreversible

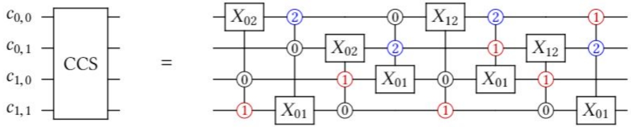
A	B	Carry Status
0	0	k
0	1	p
1	0	p
1	1	g

C_i	C_j	C_{out}
k	k	k
k	p	k
k	g	g
p	k	k
p	p	p
p	g	g
g	k	k
g	p	g
g	g	g

Fix: Use Temp. Qutrits



Fix: Use Temp. Qutrits

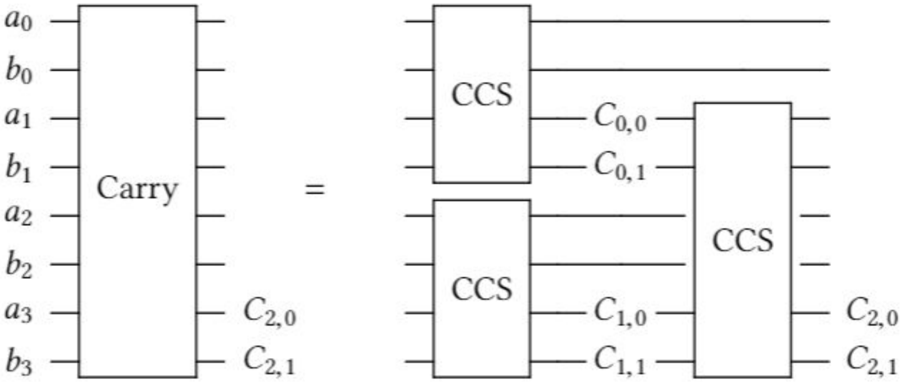


A	B	Carry Status
0	0	k
0	1	p
1	0	p
1	1	g

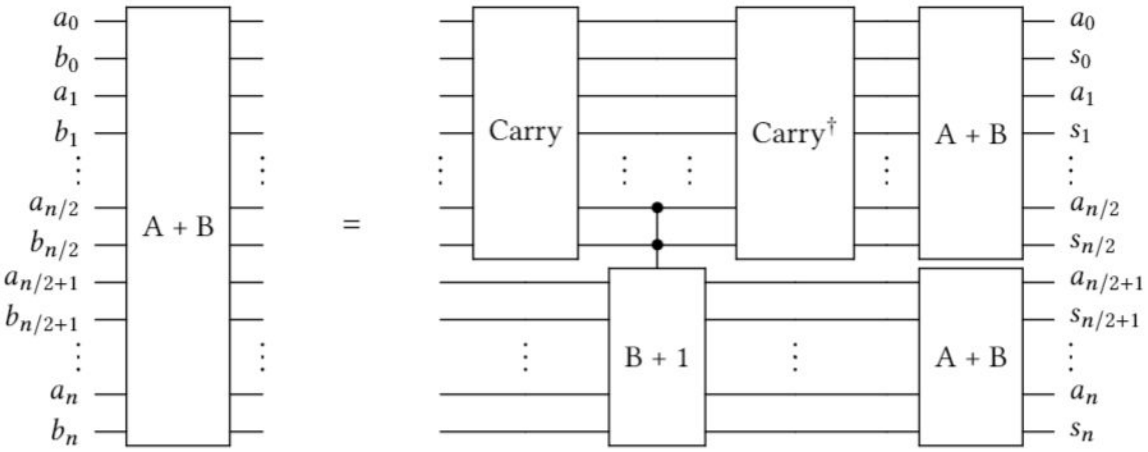
C_i	C_j	C_{out}
k	k	k
k	p	k
k	g	g
p	k	k
p	p	p
p	g	g
g	k	k
g	p	g
g	g	g

$c_{0,0}$	$c_{0,1}$	$c_{1,0}$	$c_{1,1}$	$c'_{0,0}$	$c'_{0,1}$	$c'_{1,0}$	$c'_{1,1}$
0	0	0	0	(0	0)	0	0
0	0	0	1	(2	0)	0	0
0	0	1	0	(0	1)	0	0
0	0	1	1	(0	0)	1	1
0	1	0	0	(0	2)	0	0
0	1	0	1	(1	1)	0	1
0	1	1	0	(0	2)	1	0
0	1	1	1	(0	1)	1	1
1	0	0	0	(1	0)	0	0
1	0	0	1	(2	0)	0	1
1	0	1	0	(1	1)	1	0
1	0	1	1	(1	0)	1	1
1	1	0	0	(2	1)	0	1
1	1	0	1	(2	2)	1	1
1	1	1	0	(1	1)	1	1
1	1	1	1	(1	2)	1	1

Fix: Use Temp. Qutrits



Adder with Temp. Qutrits



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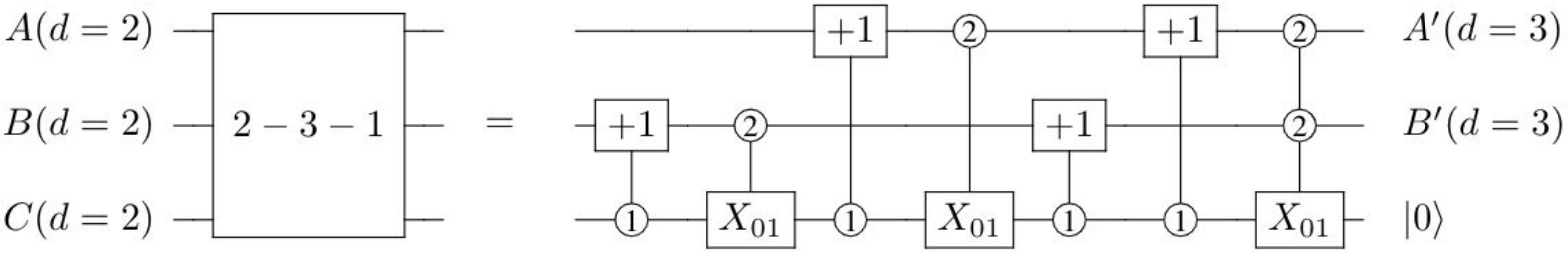
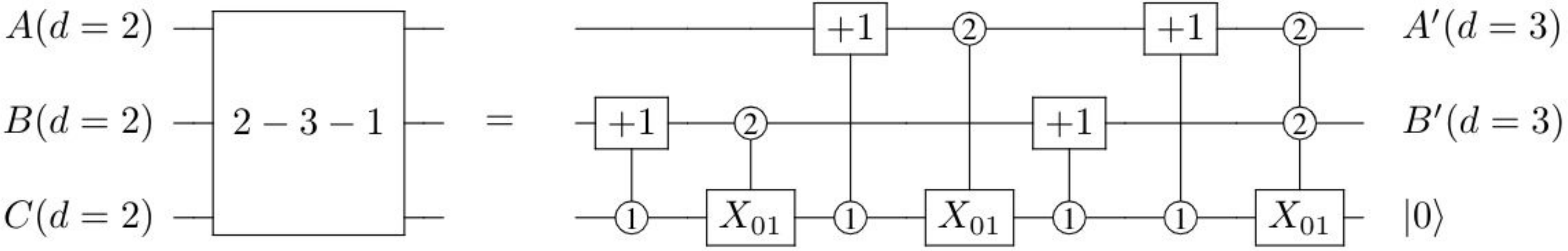
Qubit-Qudit Compression



https://arxiv.org/pdf/2002.10592.pdf

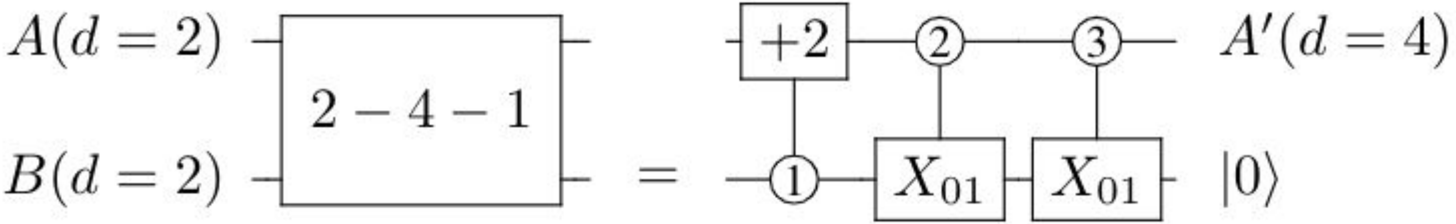
Use Cases: Qubit-Qudit Compression



A	B	C	A'	B'	C'
0	0	0	0	0	0
0	0	1	2	2	0
0	1	0	0	1	0
0	1	1	0	2	0
1	0	0	1	0	0
1	0	1	2	1	0
1	1	0	1	1	0
1	1	1	1	2	0

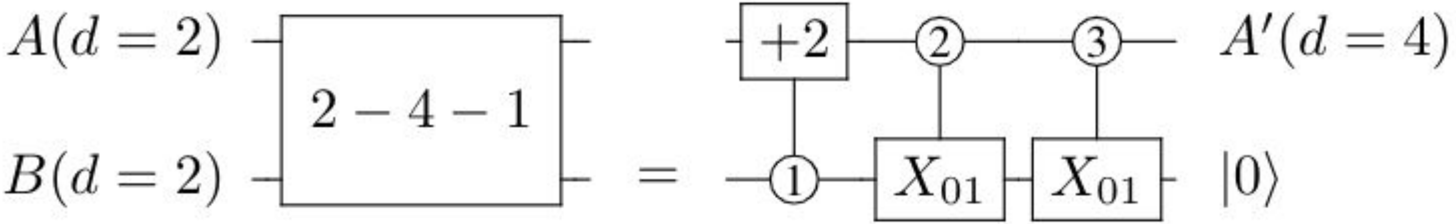
https://arxiv.org/pdf/2002.10592.pdf

Qubit-Qudit Compression



https://arxiv.org/pdf/2002.10592.pdf

Qubit-Qudit Compression



A	B	A'	B'
0	0	0	0
0	1	2	0
1	0	1	0
1	1	3	0

https://arxiv.org/pdf/2002.10592.pdf

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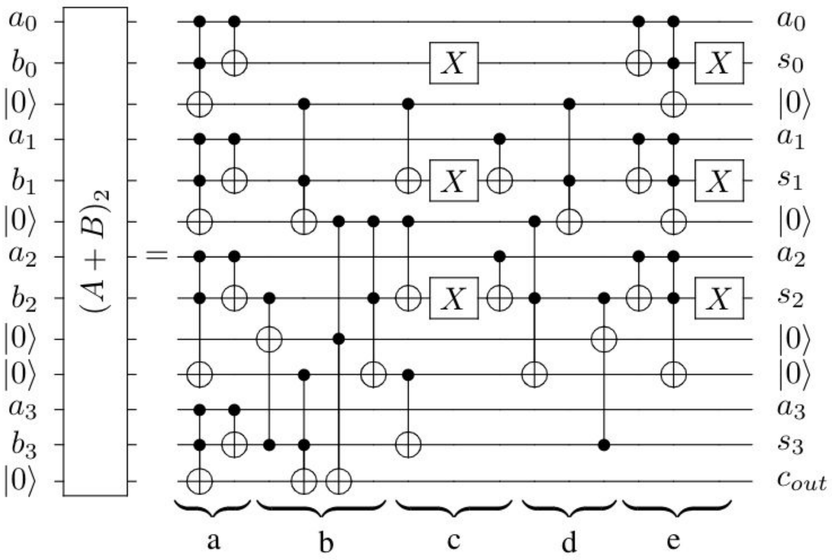
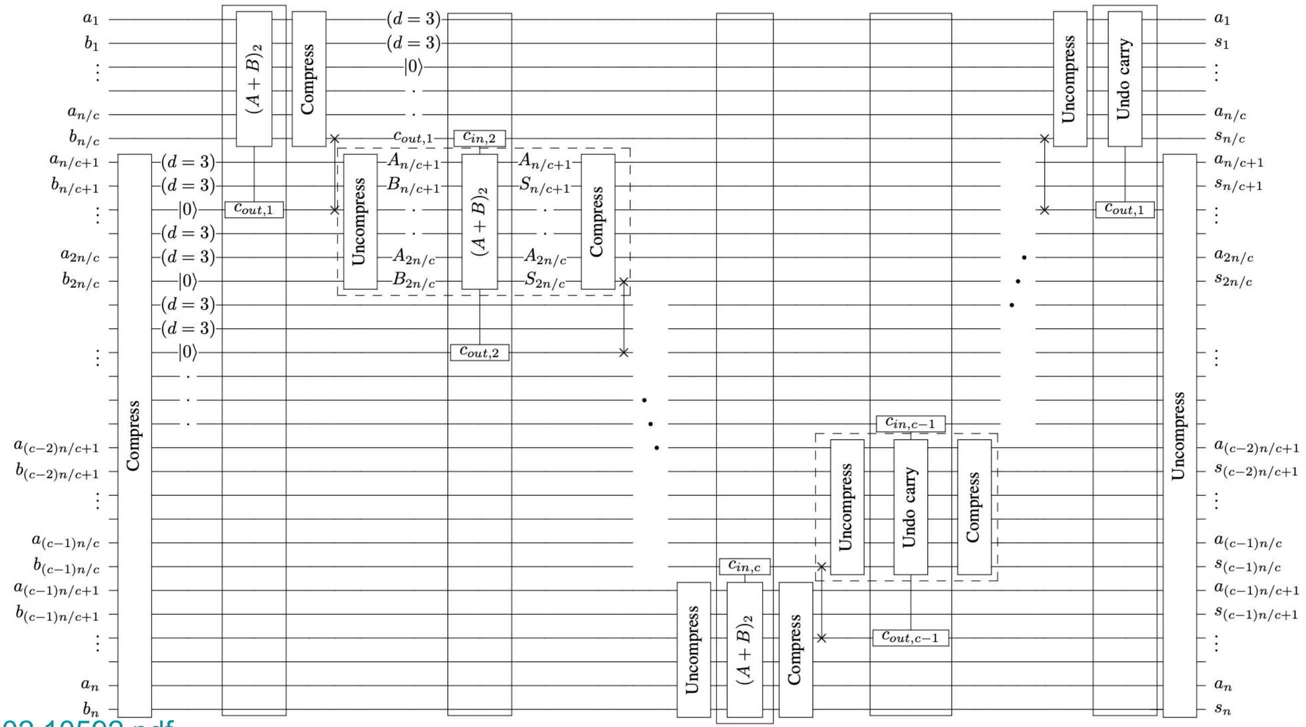
A Log Depth Adder (All Binary w/ Ancilla)



https://arxiv.org/abs/quant-ph/0406142

A Log Depth Adder (Temp. Qudits, No Ancilla)



https://arxiv.org/pdf/2002.10592.pdf

Questions?