PHYSICS AT THE TERA SCALE
Helmholtz Alliance

Review of Parton Showers

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DESY

Many thanks to Dave Soper

ISMD 2013

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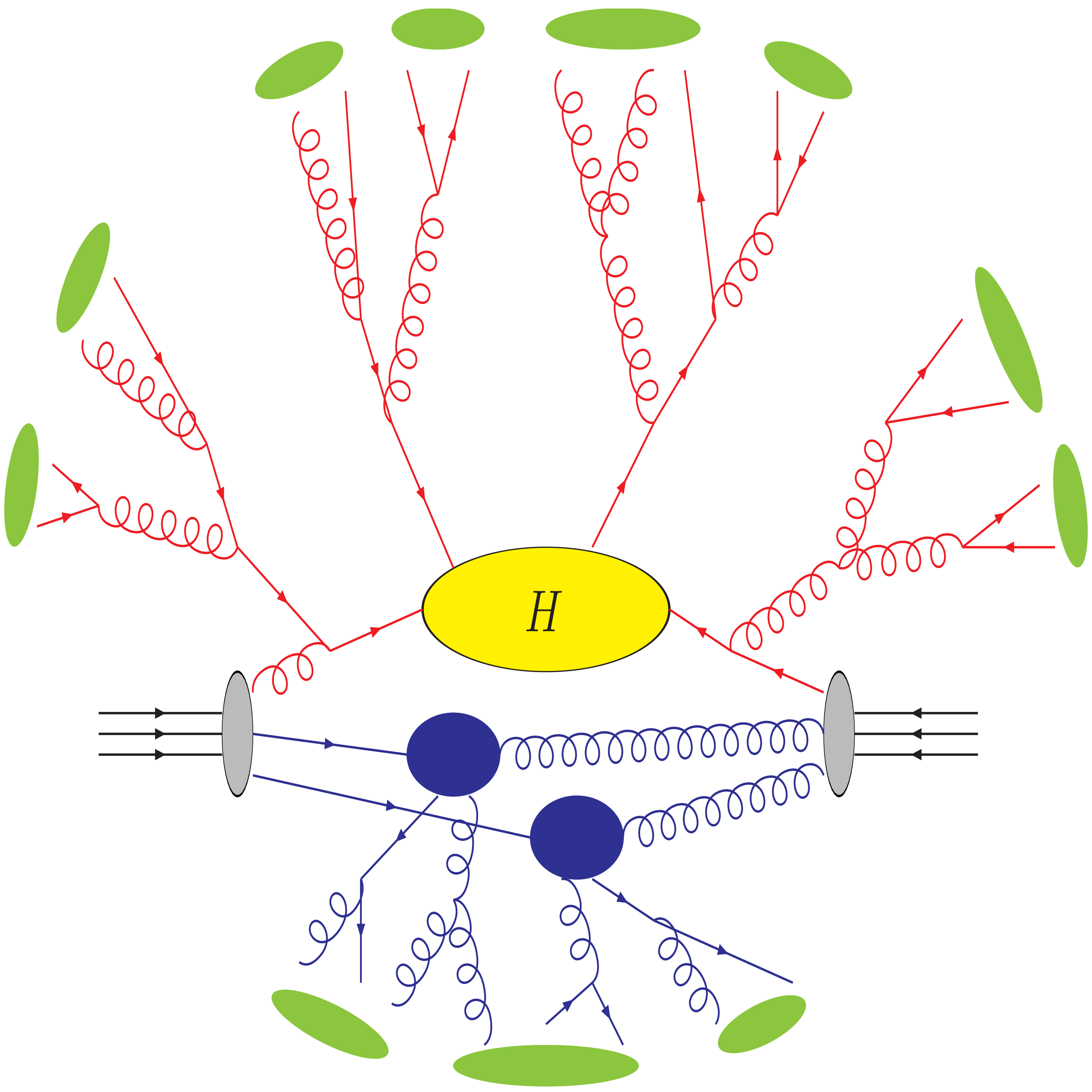
Structure of MC Programs

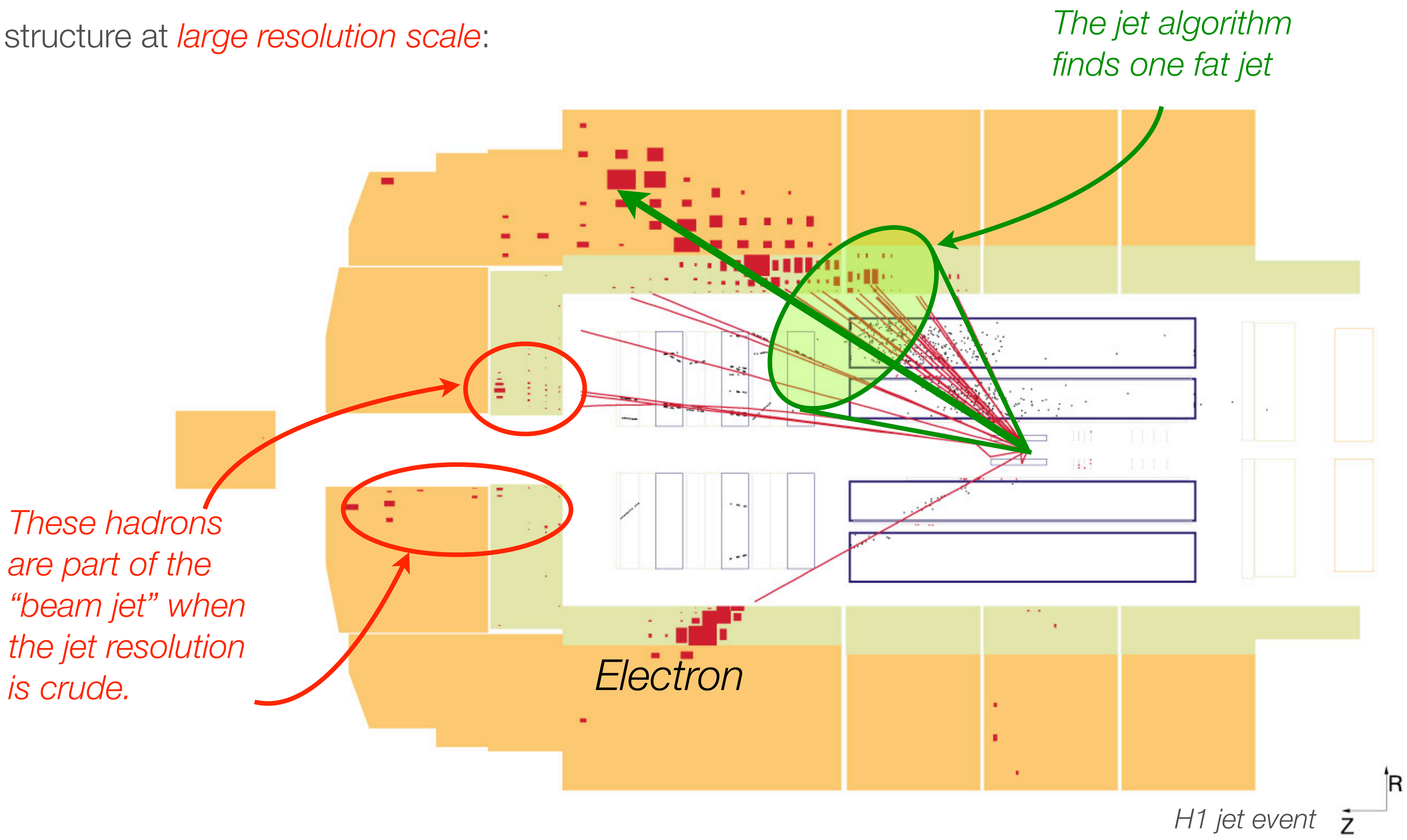
From theory point of view this event looks very complicated



1. Incoming hadron (gray bubbles)
   - Parton distribution function
   - Multi parton distribution functions
2. Hard part of the process (yellow bubble)
   - Matrix element calculation, cross sections at LO, NLO, NNLO level
3. Radiations (red graphs)
   - Parton shower calculation
   - Partonic decay
   - Matching to NLO, NNLO
4. Underlying event (blue graphs)
   - Models based on multiple interaction
   - Diffraction
5. Hardonization (green bubbles)
   - Universal models
   - Hadronic decay
   - ....

Jet event in DIS process

Jet structure at *large resolution scale*:

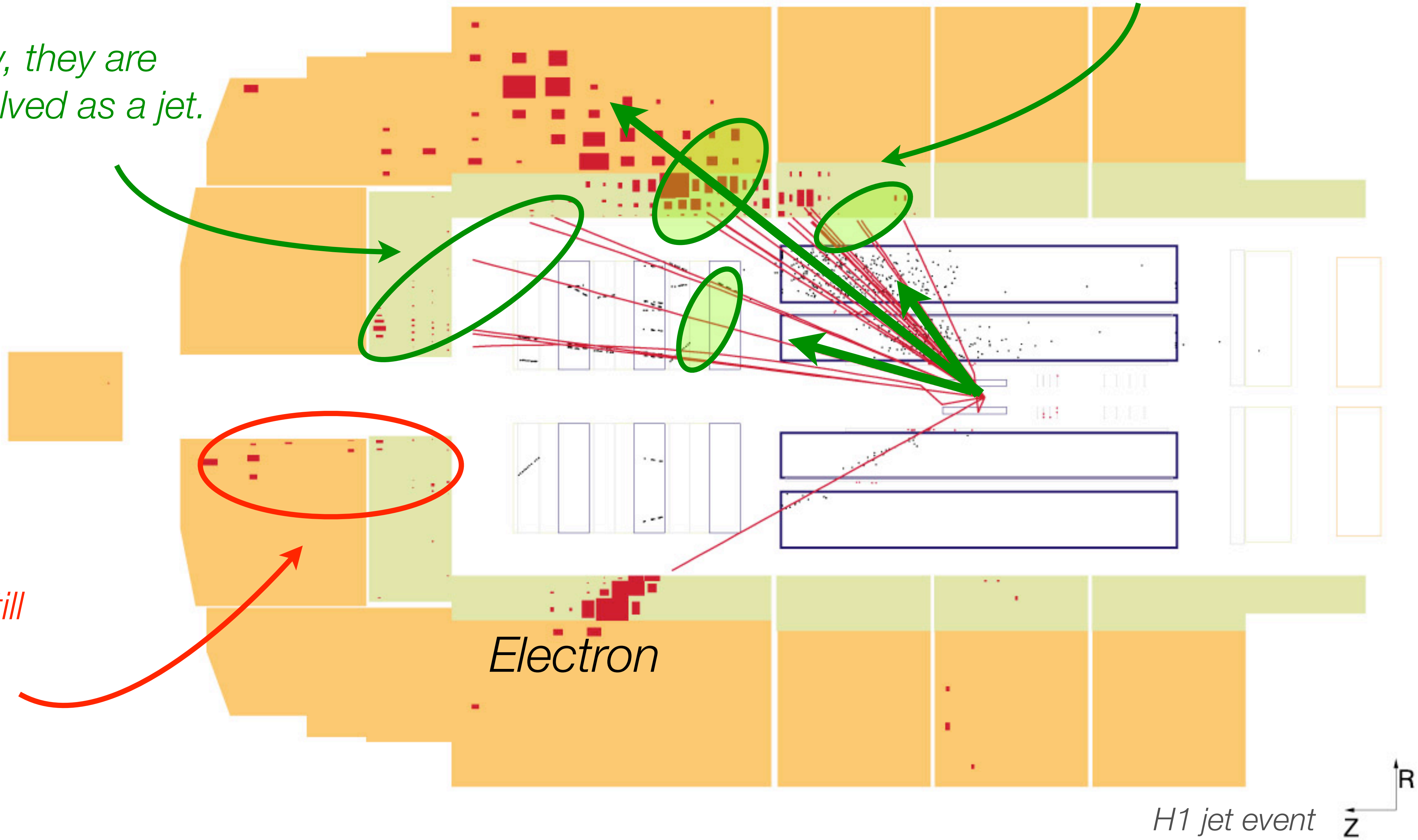


Jet event in DIS process

Jet structure at *small resolution scale*:

The jet algorithm find one fat jet

Now, they are resolved as a jet.

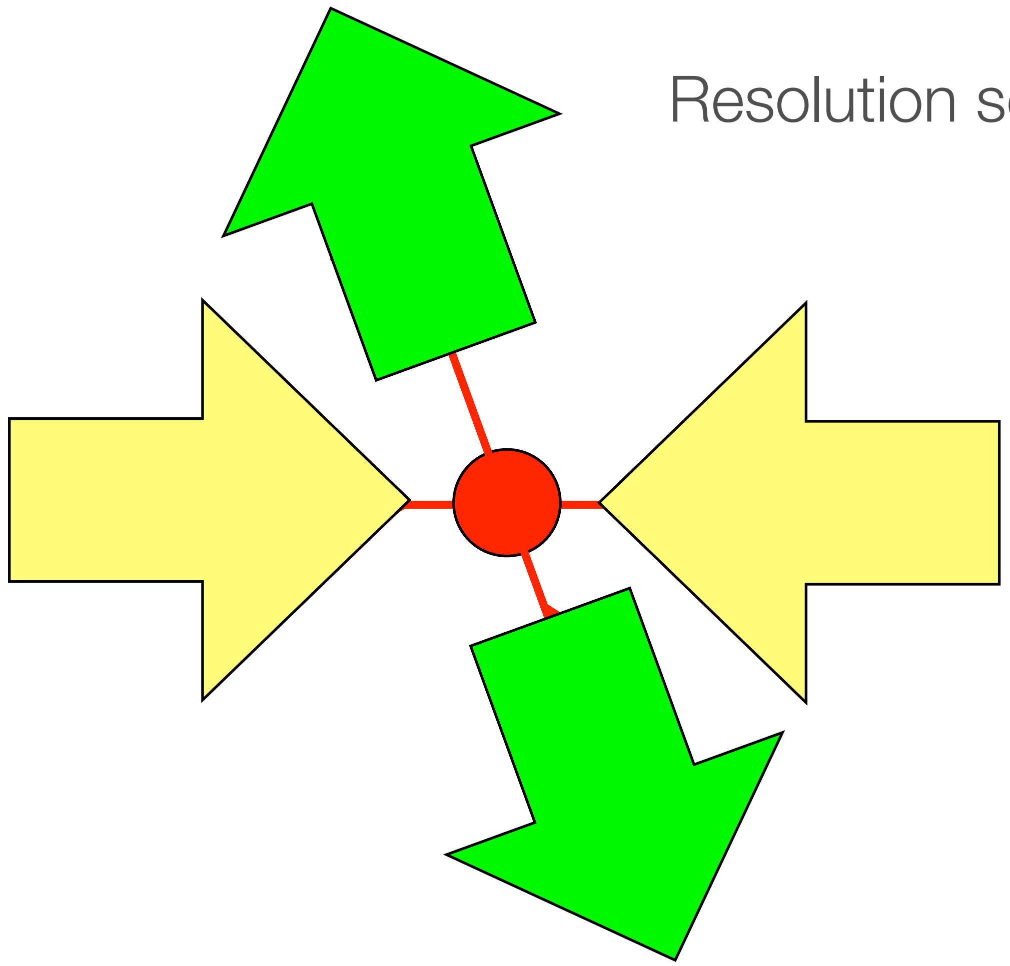


These are still part of the beam jet.

H1 jet event

Hadron-Hadron Collision

In hadron-hadron collision the picture is more complicated.



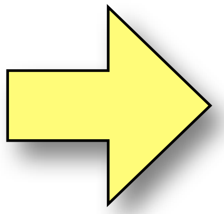
Resolution scale: 400 GeV

Decreasing the resolution scale more and more partons are visible and less absorbed by the incoming hadrons and the final state jets.

Important observation: The total cross section *is independent of* the resolution of the measurement (or detector).

We have to also consider the evolution of the final state jets.

Does perturbative QCD support this nice intuitive picture?



Hadron-Hadron Collision

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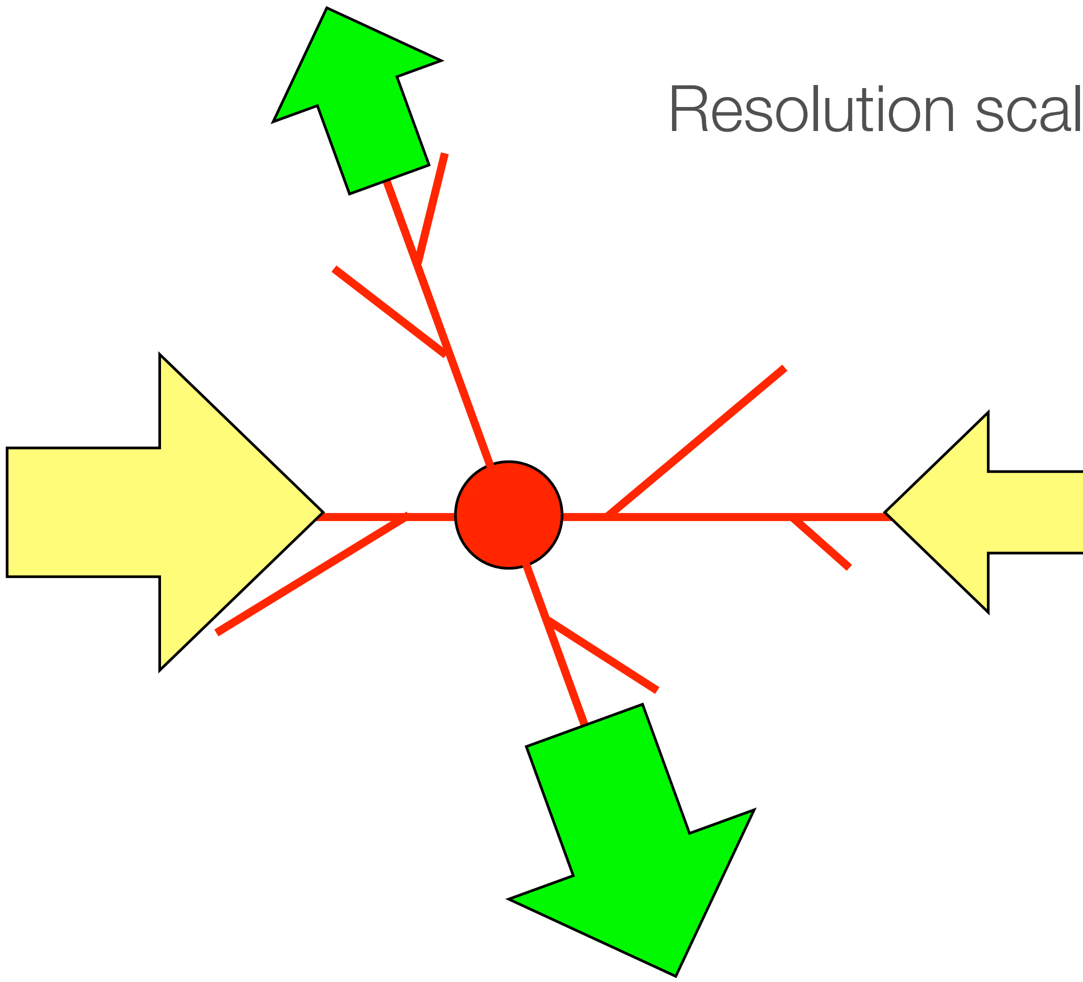

Resolution scale: 100 GeV

Decreasing the resolution scale more and more partons are visible and less absorbed by the incoming hadrons and the final state jets.

Important observation: The total cross section *is independent of* the resolution of the measurement (or detector).

We have to also consider the evolution of the final state jets.

Does perturbative QCD support this nice intuitive picture?

What do we want?

A general purpose parton shower program must generate partonic final states ready for hadronization

- in a *FULLY exclusive way* (momentum, flavor, spin and color are fully resolved)
- as *precisely* as possible (*e.g.*: sums up large logarithms at NLL level).

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$$\sigma[F] = \sum_m \int [d\{p,f\}_m]\, \overbrace{f_{a/A}(\eta_{\rm a},\mu_F^2)\, f_{b/B}(\eta_{\rm b},\mu_F^2)}^{\textit{parton distributions}}\, \frac{1}{2\eta_{\rm a}\eta_{\rm b}\, p_A\!\cdot\! p_B}$$

$$\times \big\langle \mathcal{M}(\{p,f\}_m)\big|\, \underbrace{F(\{p,f\}_m)}_{\textit{observable}}\, \underbrace{\big|\mathcal{M}(\{p,f\}_m)\big\rangle}_{\textit{matrix element}}$$

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$$\sigma[F] = \sum_m \int [d\{p,f\}_m] \, \mathrm{Tr}\{\underbrace{\rho(\{p,f\}_m)}_{\text{density operator in color} \otimes \text{spin space}} F(\{p,f\}_m)\}$$

The fully exclusive final state is described by the QCD density operator, that is the basic object in the Monte Carlos

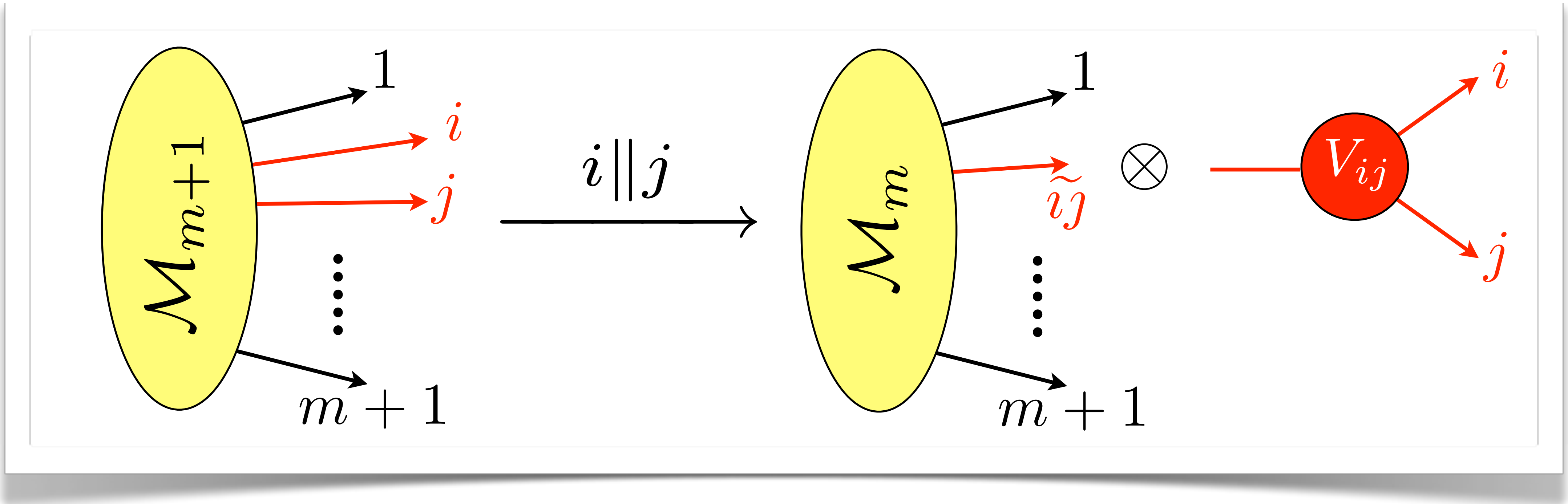
$$\rho = \sum \rho\big(\{p,f\}_m\big) \Leftrightarrow \big|\rho\big) = \sum \big|\rho(\{p,f\}_m)\big)$$

We try to approximate the QCD density operator with the universal factorization properties of the QCD amplitudes.

Factorization: Collinear limit

The QCD matrix elements have universal factorization property when two external partons become collinear

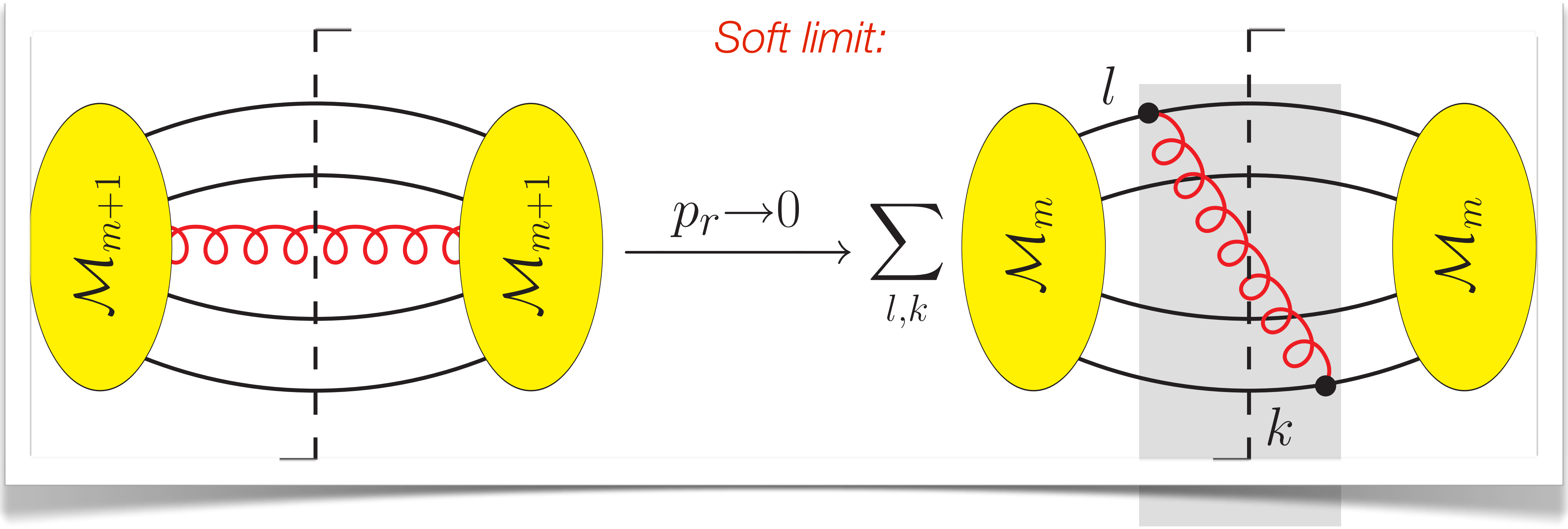
Collinear limit:



- The collinear part of the splitting operator is simple in color.
- It introduces some spin correlation.

Factorization: Soft limit

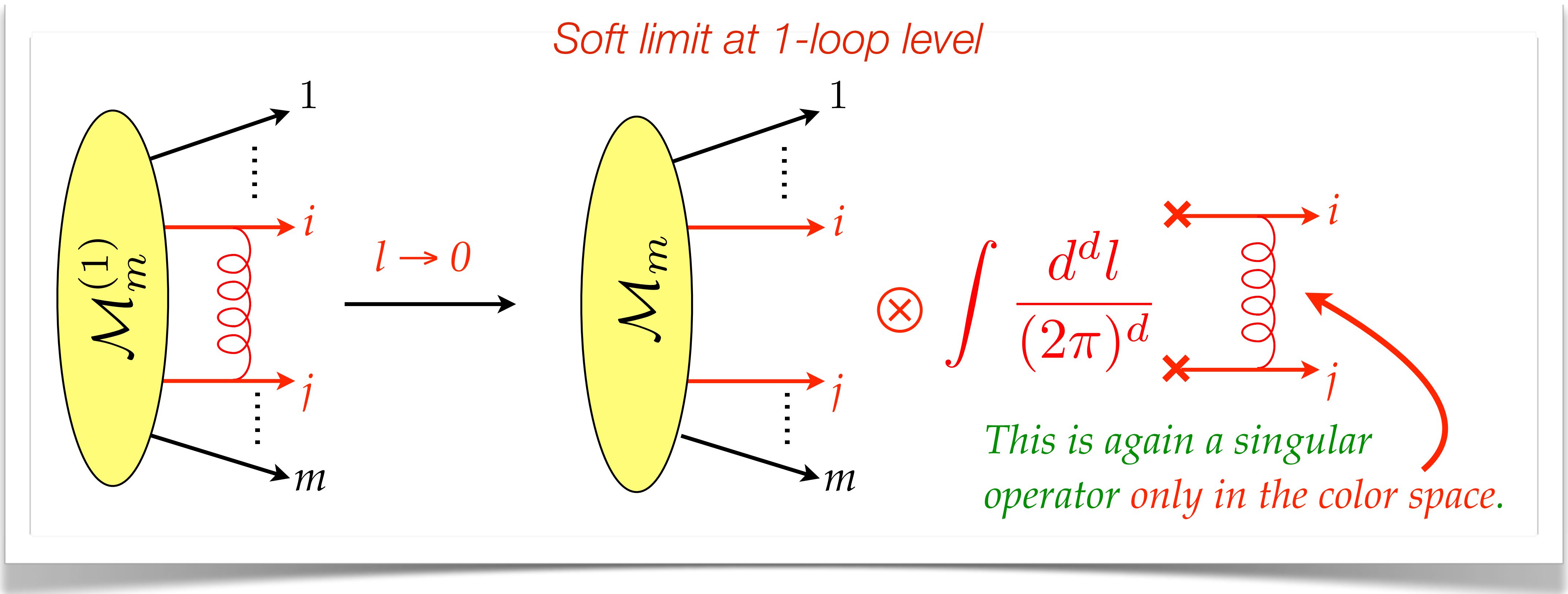
The QCD matrix elements have universal factorization property when an external gluon becomes soft

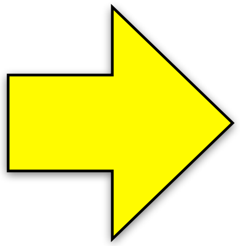


- It is introduces very complicated color interferences.
- In spin space the soft contributions are diagonal and simple.

Factorization: Soft limit (1-loop)

There is another type of the unresolvable radiation, *the virtual (loop graph) contributions.* We have *universal factorization properties* for the loop graphs. E.g.: in the soft limit, when the loop momenta become soft we have





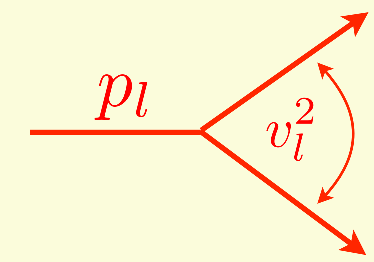
The splitting operators can be obtained from these factorization rules.

Approx. of the Density Operator

Real radiation

$$|\rho^{R}_{\infty}) \approx \int_{t}^{\infty} d\tau\, \mathcal{H}(\tau) |\rho(t))$$

$$\tau = \log \frac{p_l \cdot Q_0}{v_l^2}$$



The shower time can be the virtuality of the splitting divided by the mother parton energy.

Here we impose strong ordering. Only the softer or more collinear radiation are allowed.

Virtual radiation

$$|\rho^{V}_{\infty}) \approx -\int_{t}^{\infty} d\tau \left[\underbrace{\mathcal{V}^{(\epsilon)}(\tau) + \mathrm{i}\pi\widetilde{\mathcal{V}}^{(\epsilon)}(\tau)}_{\text{Singular part}} + \Delta\mathcal{V}(\tau) \right] |\rho(t))$$

Finite contribution

Some of the real emissions are not resolvable. Having a snapshot of the system at shower time t'

$$|\rho^{R}_{\infty}) \approx \underbrace{\int_{t}^{t'} d\tau\, \mathcal{H}(\tau)\, |\rho(t))}_{\text{Resolved emissions}} + \underbrace{\int_{t'}^{\infty} d\tau\, \mathcal{V}^{(\epsilon)}(\tau)\, |\rho(t))}_{\text{Unresolved emissions}}$$

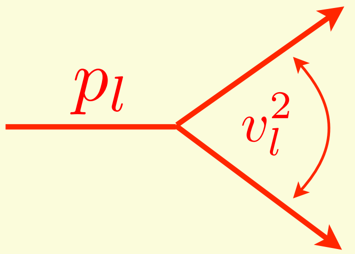
Unresolved emissions
This is a singular contribution

Approx. of the Density Operator

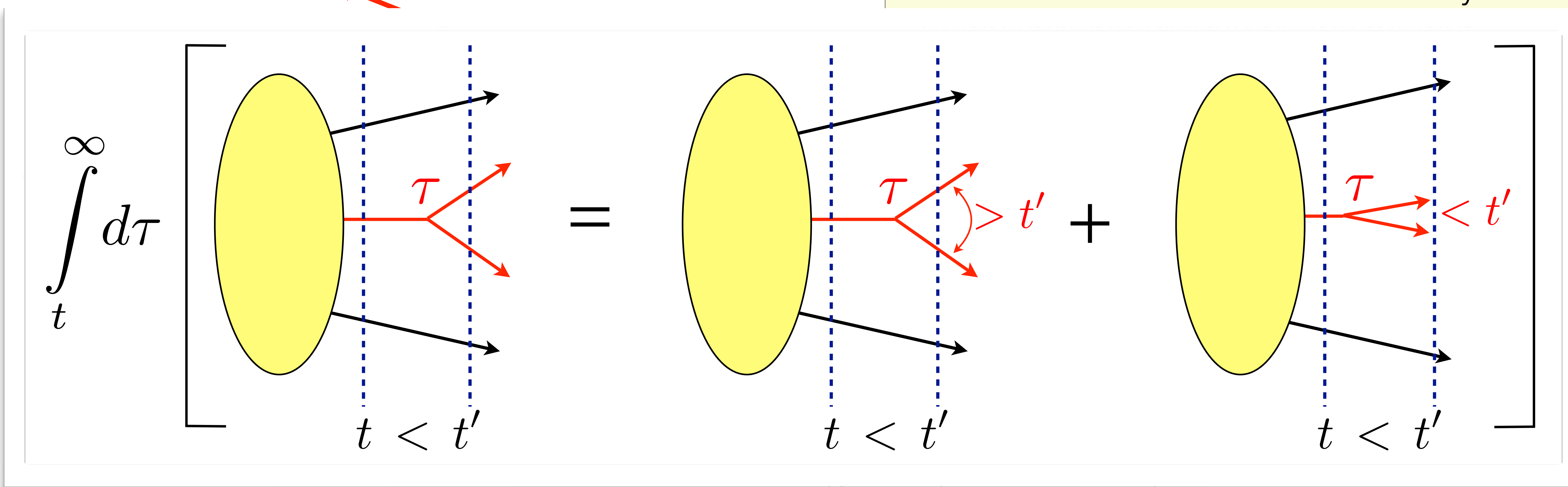
Real radiation

$$|\rho^R_\infty) \approx \int_t^\infty d\tau\, \mathcal{H}(\tau)|\rho(t))$$

$$\tau = \log \frac{p_l \cdot Q_0}{v_l^2}$$



The shower time can be the virtuality



$$|\rho^R_\infty) \approx \underbrace{\int_t^{t'} d\tau\, \mathcal{H}(\tau)\,|\rho(t))}_{\text{Resolved emissions}} + \underbrace{\int_{t'}^\infty d\tau\, \mathcal{V}^{(\epsilon)}(\tau)\,|\rho(t))}_{\text{Unresolved emissions}}$$

Resolved emissions

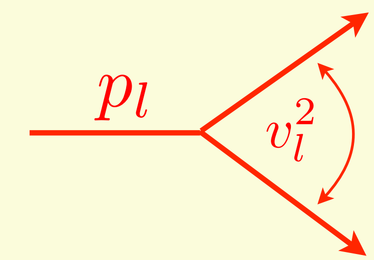
Unresolved emissions
This is a singular contribution

Approx. of the Density Operator

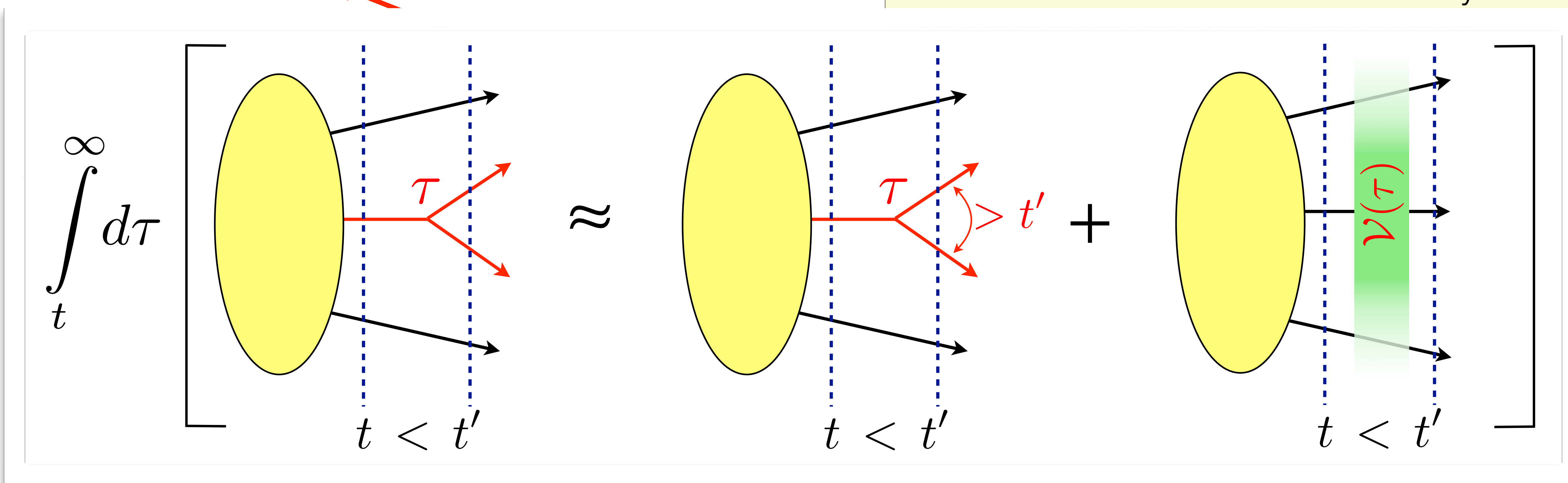
Real radiation

$$|\rho^{R}_{\infty}) \approx \int_{t}^{\infty} d\tau\, \mathcal{H}(\tau) |\rho(t))$$

$$\tau = \log \frac{p_l \cdot Q_0}{v_l^2}$$



The shower time can be the virtuality



$$|\rho^{\mathrm{R}}_{\infty}) \approx \underbrace{\int_{t}^{t'} d\tau\, \mathcal{H}(\tau)\, |\rho(t))}_{\text{Resolved emissions}} + \underbrace{\int_{t'}^{\infty} d\tau\, \mathcal{V}^{(\epsilon)}(\tau)\, |\rho(t))}_{\text{Unresolved emissions}}$$

Resolved emissions

Unresolved emissions
This is a singular contribution

Approx. of the Density Operator

Combining the real and virtual contribution we have got

$$\big|\rho^{\mathrm{R}}_{\infty}\big) + \big|\rho^{\mathrm{V}}_{\infty}\big) = \int_t^{t'} d\tau\, \Big[\mathcal{H}(\tau) - \mathcal{V}(\tau) - \mathrm{i}\pi\widetilde{\mathcal{V}}(\tau) + \Delta\mathcal{V}(\tau)\Big]\, \big|\rho(t)\big)$$

This operator dresses up the physical state with one real and virtual emissions those *are softer or more collinear than the hard state*. Thus the emissions are ordered.

$$\big[\mathcal{H}(\tau) - \mathcal{V}(\tau)\big]$$

Classical shower splitting operator
This part is implemented in every parton shower those are available on maket (HERWIG, PYTHIA, SHERPA)

$$\big(1\big|\,\big[\mathcal{H}(t) - \mathcal{V}(t)\big] = 0$$

Unitary condition

$$-\mathrm{i}\pi\tilde{\mathcal{V}}(\tau)$$

Coulomb gluon contribution
It is not implemented in any publicly available code. It shouldn't be ignored, it can lead to factorization breaking.

$$-\Delta\mathcal{V}(\tau)$$

"Finite" loop contribution
It is alway set to zero in order to maintain unitary. In some cases (*e.g.: new physics searches*) it shouldn't be zero.

Approx. of the Density Operator

Combining the real and virtual contribution we have got

$$|\rho_\infty^{\mathrm{R}}) + |\rho_\infty^{\mathrm{V}}) = \int_t^{t'} d\tau\, \left[\mathcal{H}(\tau) - \mathcal{V}(\tau) - \mathrm{i}\pi\widetilde{\mathcal{V}}(\tau) + \Delta\mathcal{V}(\tau)\right] |\rho(t))$$

This operator dresses up the physical state with one real and virtual emissions those are softer or

$$|\rho(t')) = \underbrace{\mathbb{T}\exp\left\{\int_t^{t'} d\tau\, \left[\mathcal{H}(\tau) - \mathcal{V}(\tau) - \mathrm{i}\pi\widetilde{\mathcal{V}}(\tau) - \Delta\mathcal{V}(\tau)\right]\right\}}_{\mathcal{U}(t',t)\ \textit{shower evolution operator}} |\rho(t))$$

Unitary parton shower is

$$\mathcal{U}_1(t,t') = \mathbb{T}\exp\left\{\int_t^{t'} d\tau\, [\mathcal{H}(\tau) - \mathcal{V}(\tau)]\right\}$$

$$-\Delta\mathcal{V}(\tau)$$

"Finite" loop contribution

It is alway set to zero in order to maintain unitary. In some cases (*e.g.: new physics searches*) it shouldn't be zero.

Evolution Equation

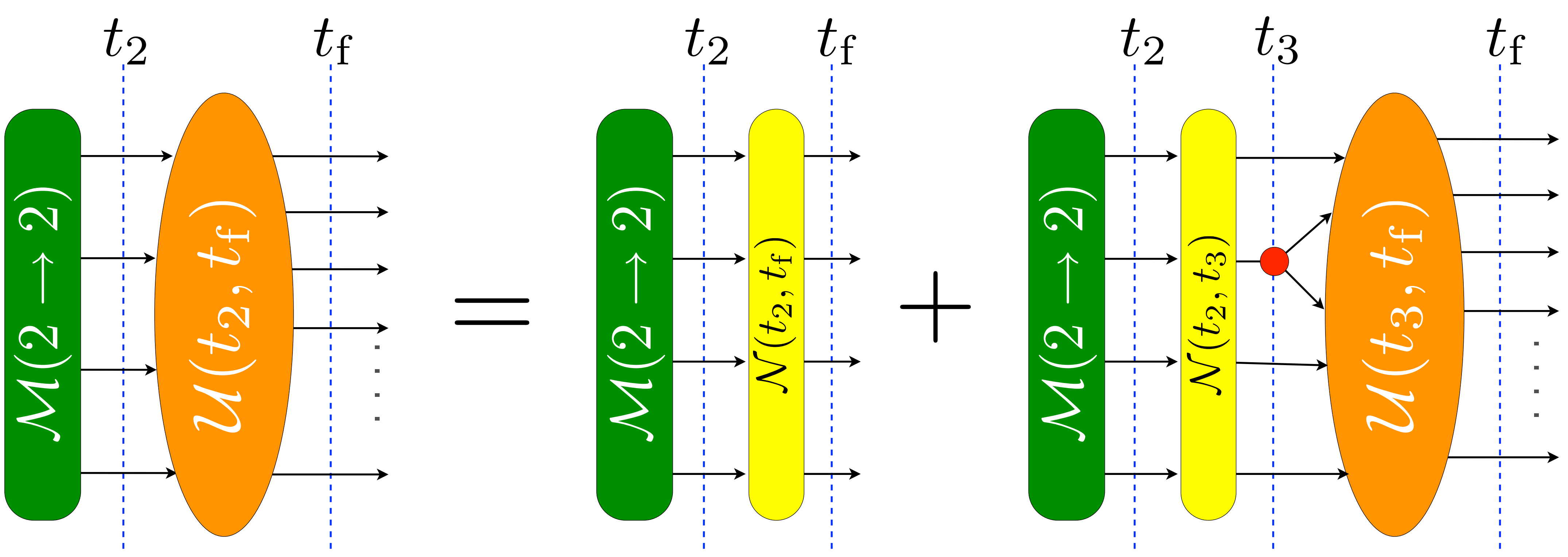
We can write the evolution equation in an integral equation form

$$\mathcal{U}(t_\mathrm{f},t_2) = \underbrace{\mathcal{N}(t_\mathrm{f},t_2)}_{\text{“Nothing happens”}} + \overbrace{\int_{t_2}^{t_\mathrm{f}} dt_3\, \mathcal{U}(t_\mathrm{f},t_3)\mathcal{H}(t_3)\mathcal{N}(t_3,t_2)}^{\text{“Something happens”}}$$

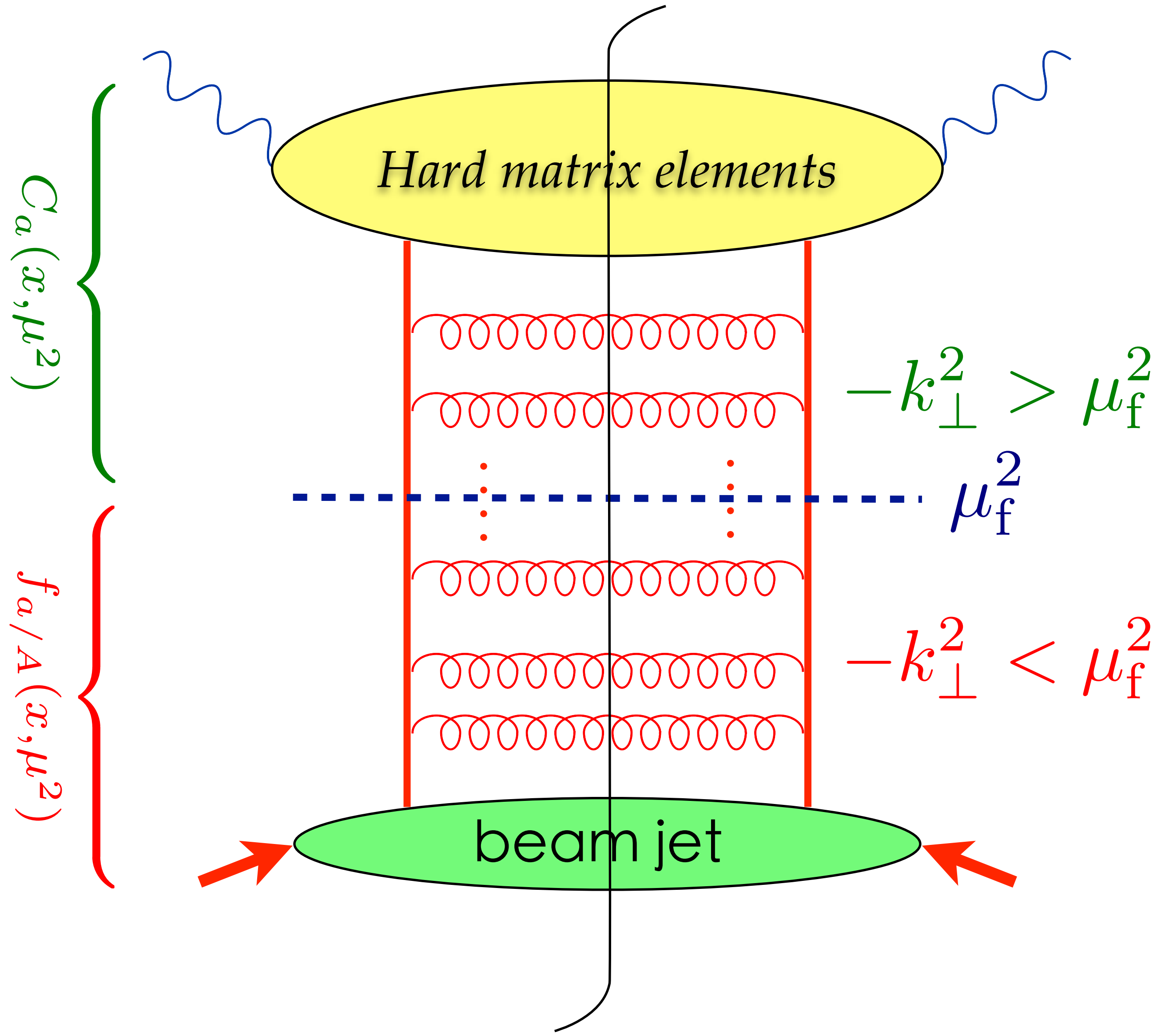
where the non-splitting operator is

$$\mathcal{N}(t',t) = \mathbb{T}\exp\left\{-\int_t^{t'} d\tau \left[\mathcal{V}(\tau) + \mathrm{i}\pi\widetilde{\mathcal{V}}(\tau) + \Delta\mathcal{V}(\tau)\right]\right\}$$

Sudakov operator



DGLAP Evolution of PDFs



Perturbative part (what we calculate)
Completely independent of the PDFs

$$\left|\rho(t_f)\right) = \underbrace{\mathcal{F}(t_f)}_{} \overbrace{\left|\rho_{\rm pert}(t_f)\right)}^{}$$

PDFs: The non-perturbative physics is only here

It MUST BE independent of the PDF, otherwise the perturbative and non-perturbative physics are mixed.

Non-trivial PDF dependence

$$\mathbb{T}\exp\left\{\overbrace{-\int_t^{t'} d\tau\, \mathcal{V}(\tau)}^{}\right\} = \mathcal{N}(t',t) = \mathcal{F}(t')\mathcal{N}_{\rm pert}(t',t)\mathcal{F}^{-1}(t) = \mathcal{F}(t')\,\mathbb{T}\exp\left\{\overbrace{-\int_t^{t'} d\tau\, \mathcal{V}^{\rm pert}(\tau)}^{}\right\}\mathcal{F}^{-1}(t)$$

Leads to the evolution equation of the parton distribution functions.

DGLAP Evolution

In general the incoming parton can be massive, this leads to a slightly modified DGLAP evolution. That is

$$\mu^2 \frac{d}{d\mu^2} f_{a/A}(\eta, \mu^2) = \sum_{\hat{a}} \int_0^1 \frac{dz}{z} \; \frac{\alpha_s(\mu^2)}{2\pi} P_{a\hat{a}}\left(z, z\frac{m^2}{\mu^2}\right) f_{\hat{a}/A}(\eta/z, \mu^2)$$

with the modified evolution kernels:

$$P_{qq}(z, \lambda) = C_F \left[\left(\frac{2}{1-z} - (1+z) - 2\lambda\right) \theta\left(\frac{1}{1-z} > 1 + \lambda\right)\right]_+ \quad ,$$

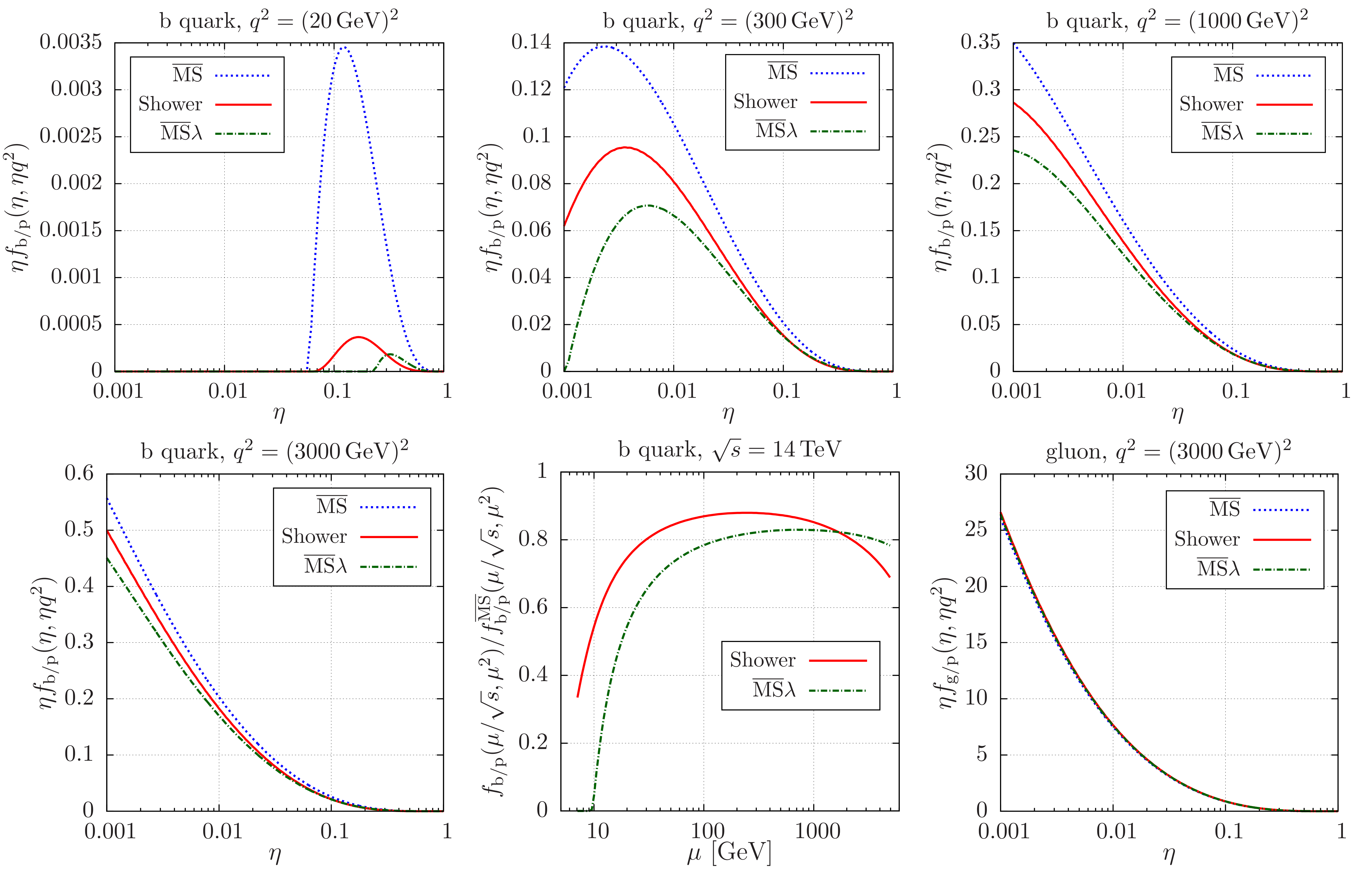
$$P_{gg}(z, \lambda) = 2C_A \left[\frac{1}{(1-z)_+} - 1 + \frac{1-z}{z} + z(1-z)\right] + \gamma_g(\lambda)\, \delta(1-z) \quad ,$$

$$P_{qg}(z, \lambda) = T_R \left[1 - 2\,z\;(1-z) + 2\lambda\right] \theta(z(1-z) > \lambda) \quad ,$$

$$P_{gq}(z, \lambda) = C_F \left[\frac{1+(1-z)^2}{z} - 2\lambda\right] \theta\left(\frac{1}{z} > 1 + \lambda\right) \quad .$$

With different shower time the mass depend parts of the DGLAP kernels are different!

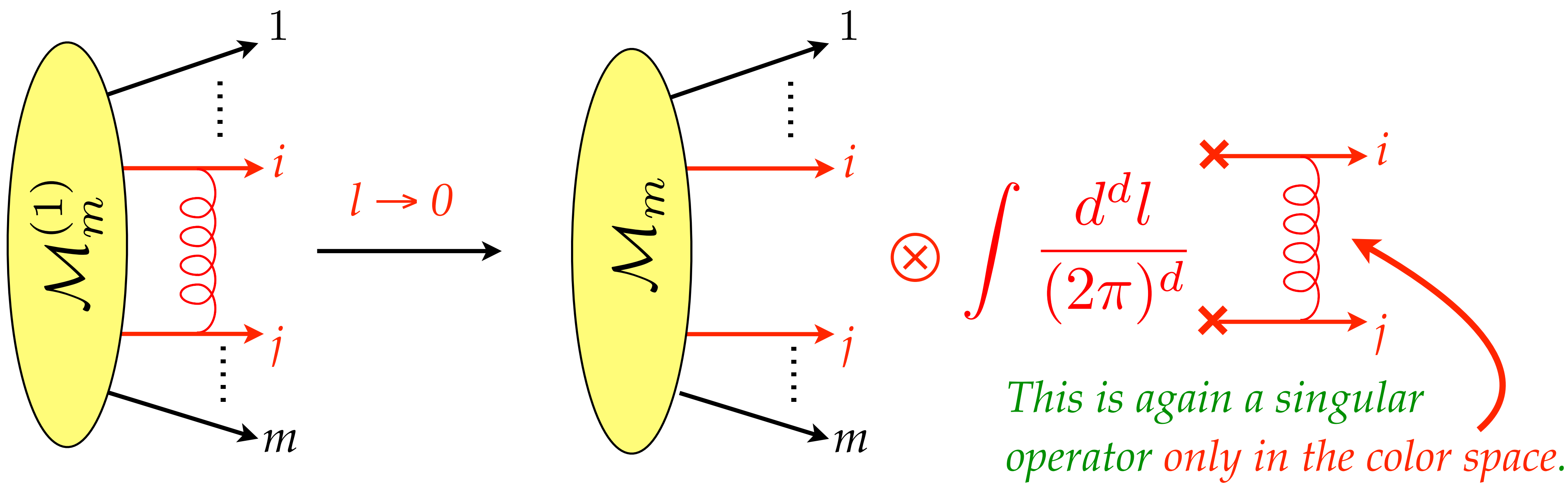
Shower PDFs



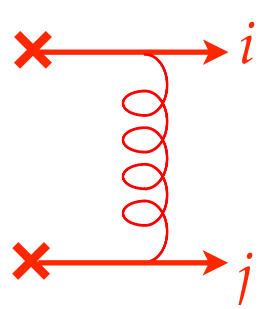
b quark, $q^2 = (20\,\mathrm{GeV})^2$
b quark, $q^2 = (300\,\mathrm{GeV})^2$
b quark, $q^2 = (1000\,\mathrm{GeV})^2$
b quark, $q^2 = (3000\,\mathrm{GeV})^2$
b quark, $\sqrt{s} = 14\,\mathrm{TeV}$
gluon, $q^2 = (3000\,\mathrm{GeV})^2$
$\overline{\mathrm{MS}}$
Shower
$\overline{\mathrm{MS}}\lambda$
$\eta f_{\mathrm{b/p}}(\eta, \eta q^2)$
$\eta f_{\mathrm{g/p}}(\eta, \eta q^2)$
$f_{\mathrm{b/p}}(\mu/\sqrt{s}, \mu^2)\,/\,f^{\overline{\mathrm{MS}}}_{\mathrm{b/p}}(\mu/\sqrt{s}, \mu^2)$
η
μ [GeV]
0.001
0.01
0.1
1
10
100
1000
0
0.0005
0.001
0.0015
0.002
0.0025
0.003
0.0035
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.05
0.15
0.2
0.25
0.3
0.35
0.4
0.5
0.6
0.8
5
15
20
25
30

Soft gluons

The imaginary part of the soft gluon insertion is called Coulomb gluon:



$$\int \frac{d^d l}{(2\pi)^d}$$



$$\propto \mathcal{V}^{(1-loop)}(\tau) + \mathrm{i}\pi\widetilde{\mathcal{V}}(\tau)$$

"Eikonal gluon"

"Coulomb gluon"

What can Coulomb gluon do?

Catani, de Florian, Rodrigo; Forshaw, Kyrieleis, Seymour; Forshaw, Seymour, Siodmok

Well, the simple answer is trouble. In p-p collisions the Coulomb gluon breaks factorization. Without factorization theorem it would be very hard to do any phenomenology at the LHC.

$$\sigma(Q) = \underbrace{f_A(\mu) \otimes f_B(\mu)}_{\text{PDF of the incoming hadrons}} \otimes \overbrace{\hat{\sigma}(Q,\mu)}^{\text{partonic cross section}} + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{Q^2}\right)$$

neglected power suppressed terms

This formulae has been proven only for Drell-Yan total cross section, but one can say it is true for *sufficiently inclusive* observables. At the LHC we are interested in more exclusive measurements than the total cross sections. Every observable has a typical resolution scale Q_0 and this scale dependence doesn't factorize in the usual way.

$$\sigma(Q) = f_A(\mu) \otimes f_B(\mu) \otimes \hat{\sigma}(Q_0, Q; \mu) + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{Q_0^2}\right)$$

If Q_0 is small then it is not a good approximation anymore.

$$\hat{\sigma}(Q_0, Q, \mu) \propto \alpha_s \log \frac{Q^2}{Q_0^2}$$

These large logs has to be summed up!

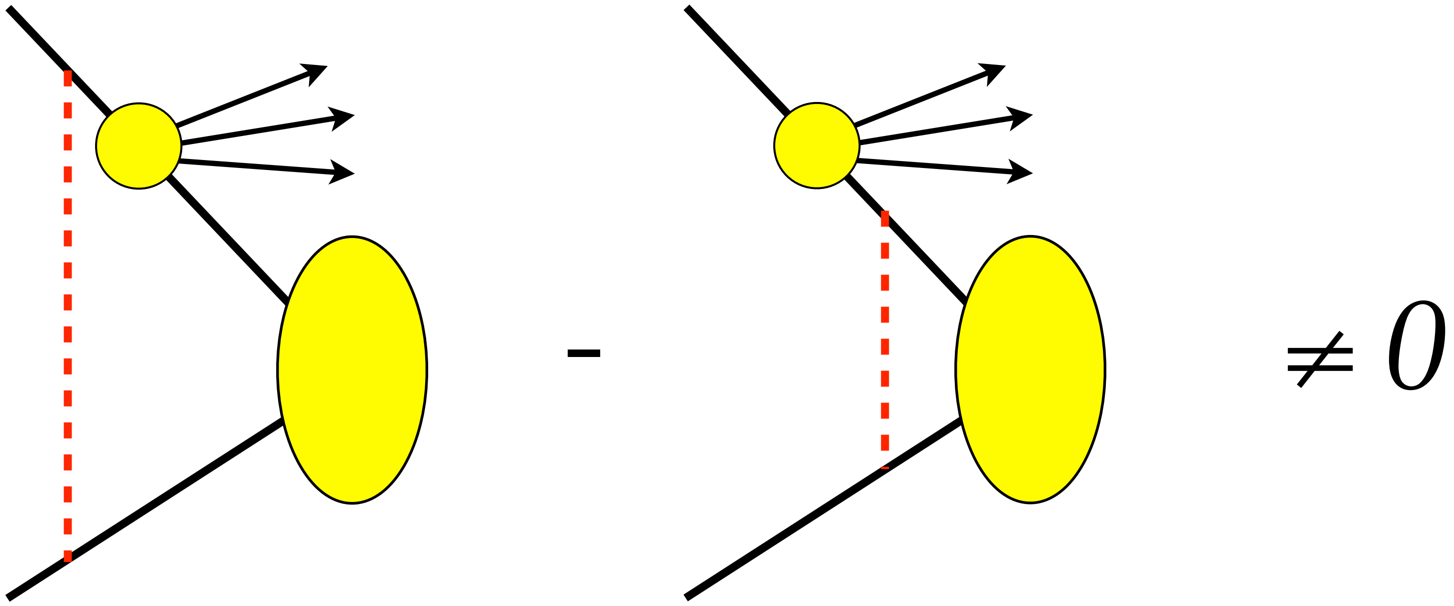
What else it can do?

Just more trouble. Because of Coulomb gluon we cannot do fixed order calculation for processes like:

- $2 \to 2$ at N^4LO level or beyond (inclusive jet or Z+2jet)
- $2 \to 3$ at N^3LO level or beyond (3-jet production)
- $2 \to 4$ at NNLO level or beyond (4-jet production)
- every process with at least one massive parton in the initial state at NNLO level

In these processes the Coulomb gluon leads to un-cancelled soft singularities. (Note, in a general NNLO subtraction scheme this un-cancelled soft singularity should appear explicitly.)

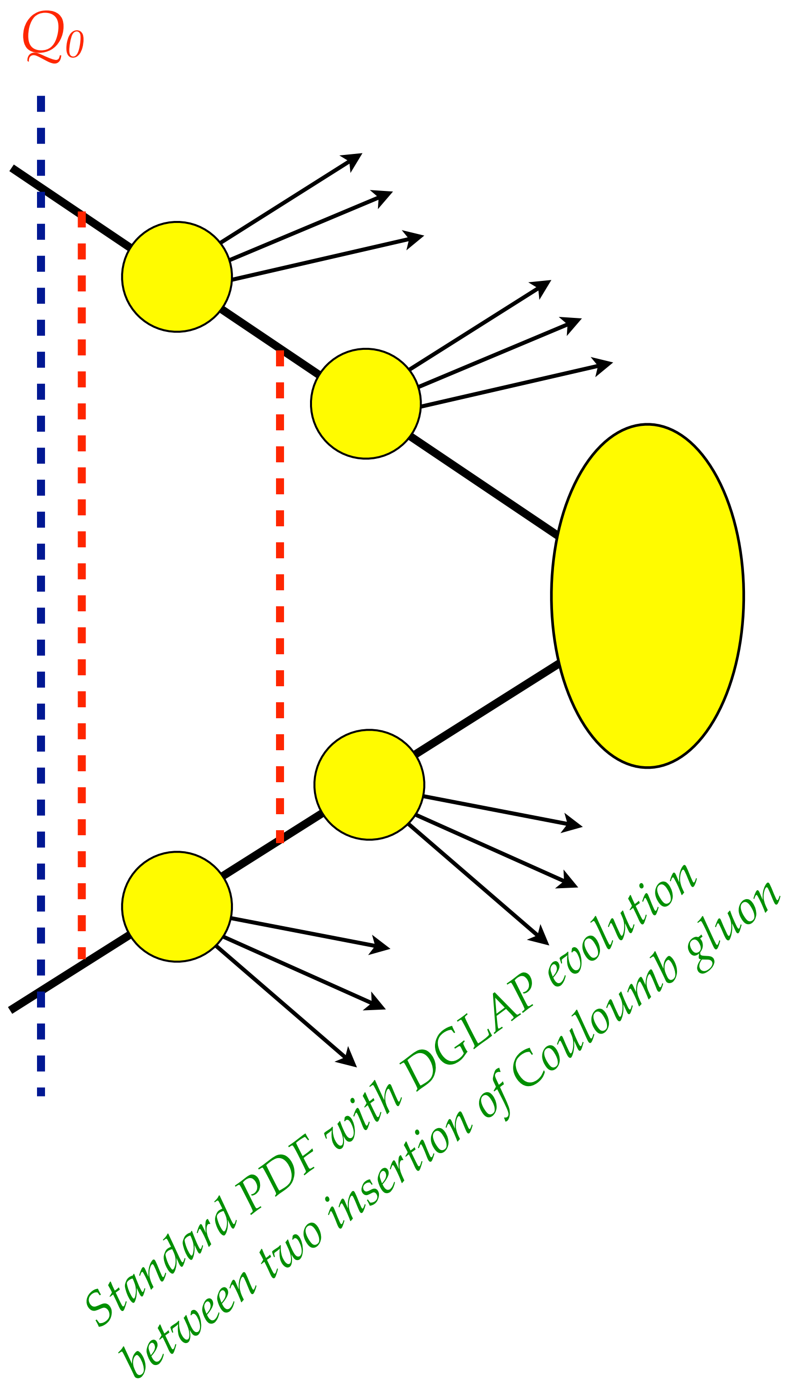
The problem comes from this non commuting insertion of Coulomb gluon:



Can parton shower deal with this problem?

Coulomb gluon in Shower

The origin of Coulomb gluon is pure virtual, thus it doesn't change the total cross section. This let us to define the PDF as before and it still has the standard DGLAP evolution.



Parton shower obeys the following equation:

$$\mathcal{U}(t,t') = \mathcal{U}_1(t,t') + i\pi \int_{t'}^{t} d\tau\, \mathcal{U}(t,t')\widetilde{\mathcal{V}}(\tau)\mathcal{U}_1(t,t')$$

- It is important that Coulomb gluon disappear in leading color approximation.
- Nether angular ordered shower can deal with this problem.
- Color evolution and proper treatment of soft gluon are needed in the parton shower implementation.

Still soft gluon...

$$\mathcal{U}(t,t') = \mathbb{T}\exp\left\{\int_t^{t'} d\tau\left[\mathcal{H}(\tau) - \mathcal{V}(\tau) - \mathrm{i}\pi\widetilde{\mathcal{V}}(\tau) - \Delta\mathcal{V}(\tau)\right]\right\}$$

What is this term?

- It is just another soft gluon contribution, but this part comes from the real eikonal contribution.
- It is labeled as finite contribution but it is supposed to sum up the threshold logarithms.
- Threshold logs appear when some heavy colored objects are produced in the final state and the incoming partons have just enough energy to produce them. In such a scenario the final state particles can radiate only soft gluons and these soft gluons need to be summed up.
- This term fixes the error what have we made by imposing unitarity condition. Unitary condition makes the shower implementation simple but it is not what pQCD tell us to do.
- In processes with massless partons only it is safe to set it to zero.
- The effect of the threshold log are important in new physics searches.
- Implementation of this term is shower leads to weighted shower, but we have already gave up the concept of unweighted shower when we implemented color evolution.

Conclusion

- It is clear that we need more precise tools for the next run of the LHC.
- The large QCD effects are always there.
- In MC we should go from “postdiction” to prediction. This requires more theory development on parton showers and fixed order calculation.
- I think the fixed order (NNLO,...) and parton shower developments shouldn’t happen independently. To learn more about parton shower we need a general NNLO subtraction scheme for fixed order calculations.
- The soft gluon is big obstacle. It is hard to deal with it in analytic resummation as well as in parton shower, but leading order parton shower can provide a nice framework to sum up large logarithms at NLL level for more exclusive and sophisticated observables.
- ... but we are making progress.