The Status of QCD Diffractive Physics in DIS on Protons and Nuclei

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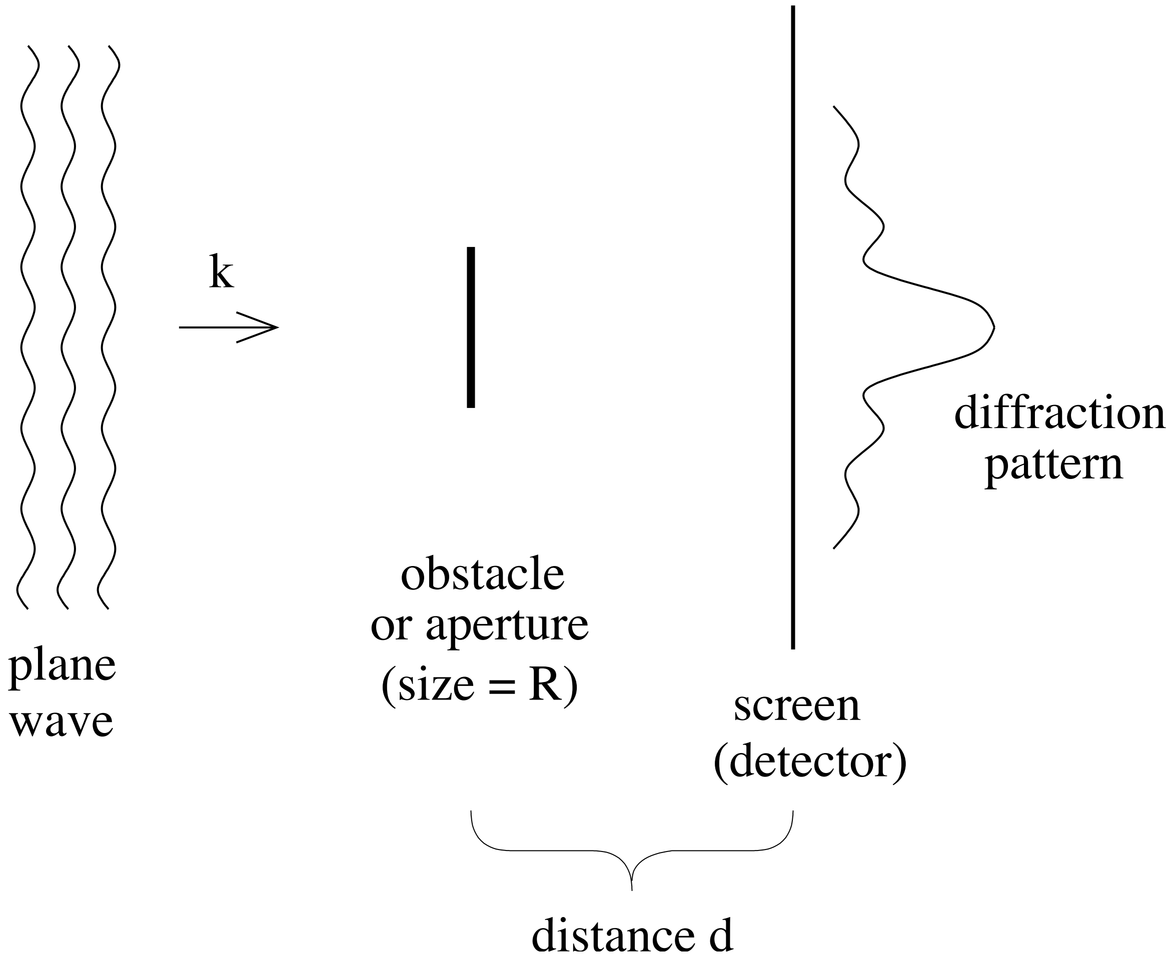
Outline

- Diffraction: general concepts
- Brief review of small-x saturation physics, the concept of a dipole amplitude
- Low-mass diffraction in DIS
  - Elastic scattering: theory and HERA & EIC phenomenology
  - Exclusive vector meson production at EIC
- High-mass diffraction in DIS
  - Theory for high-mass diffraction: nonlinear evolution equation + rc corrections.

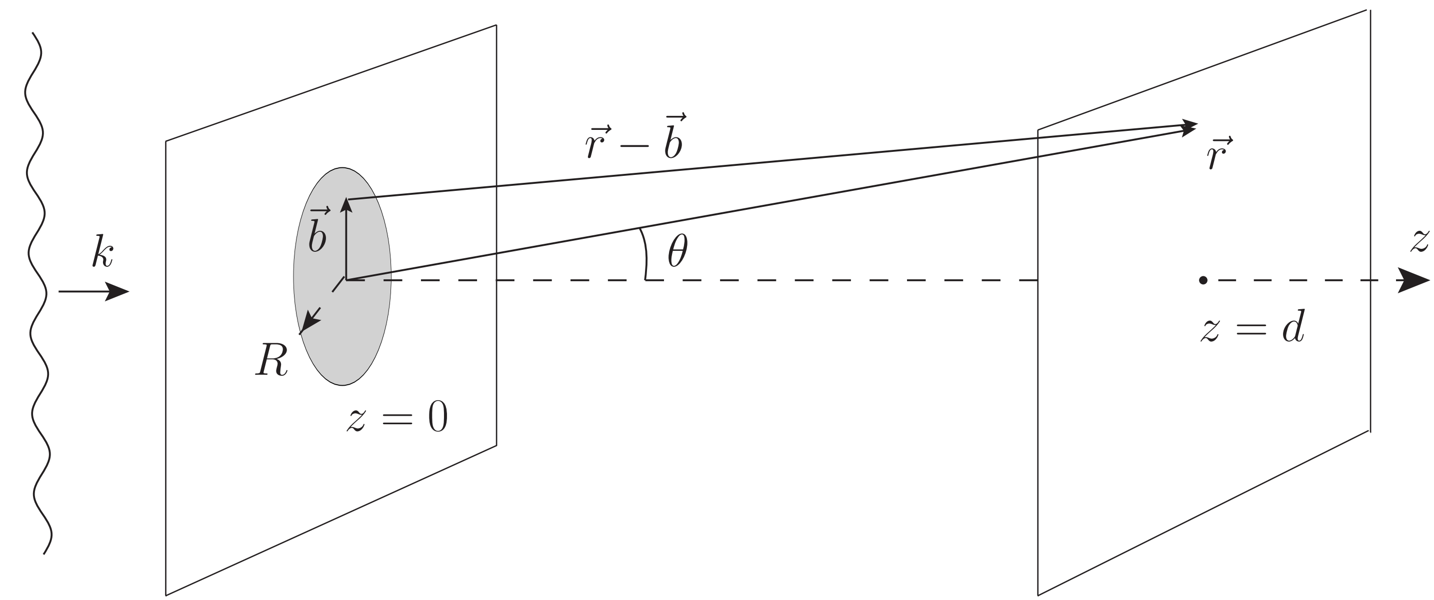
Diffraction: general concepts

Diffraction in optics



Diffraction pattern contains information about the size R of the obstacle and about the optical “blackness” of the obstacle.

Diffraction in optics and QCD



- In optics, diffraction pattern is studied as a function of the angle θ.
- In high energy scattering the diffractive cross sections are plotted as a function of the Mandelstam variable $t = k \sin \theta$.

Diffraction in QCD

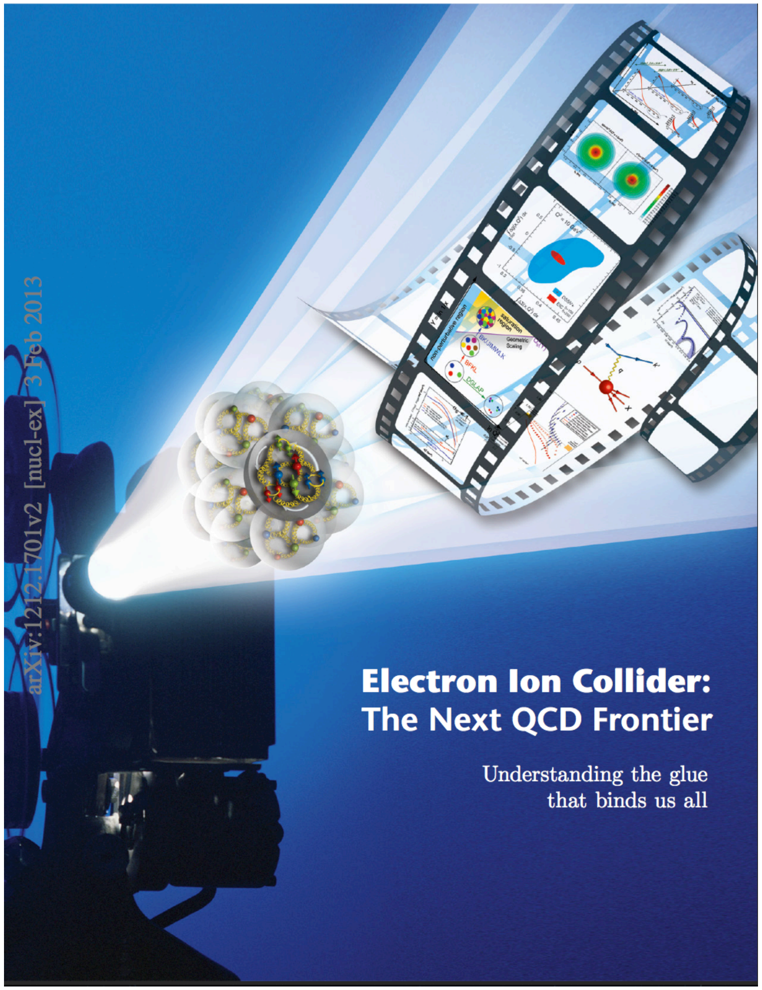
- The goal of my talk is to argue that by studying diffraction in DIS on protons and nuclei we can learn a lot about
  - The effective size of the target = range of interaction
  - The properties of strong interactions: if the target is a 'black disk', interactions are really strong (probably parton saturation), if not it could be a 'gray disk' or even a weakly-interacting 'white disk'.

Diffraction in QCD

- Diffraction DIS data has until recently been collected at HERA.

- There is a proposal in the US for the Electron-Ion Collider (EIC), which would do DIS on nuclei and protons, which would also do diffraction measurements.

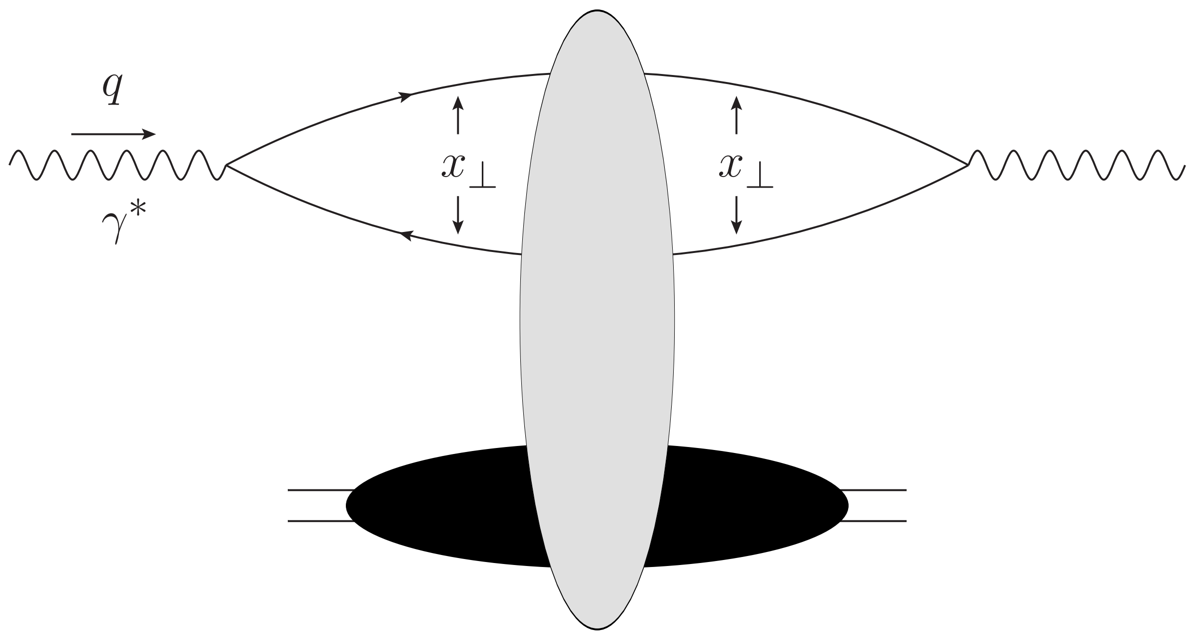
- Thanks to T. Ullrich for collaborating on the chapter of EIC WP and to V. Guzey, M. Lamont, C. Marquet, and T. Toll for making some of the plots I will show today



Dipole approach to DIS

Dipole picture of DIS

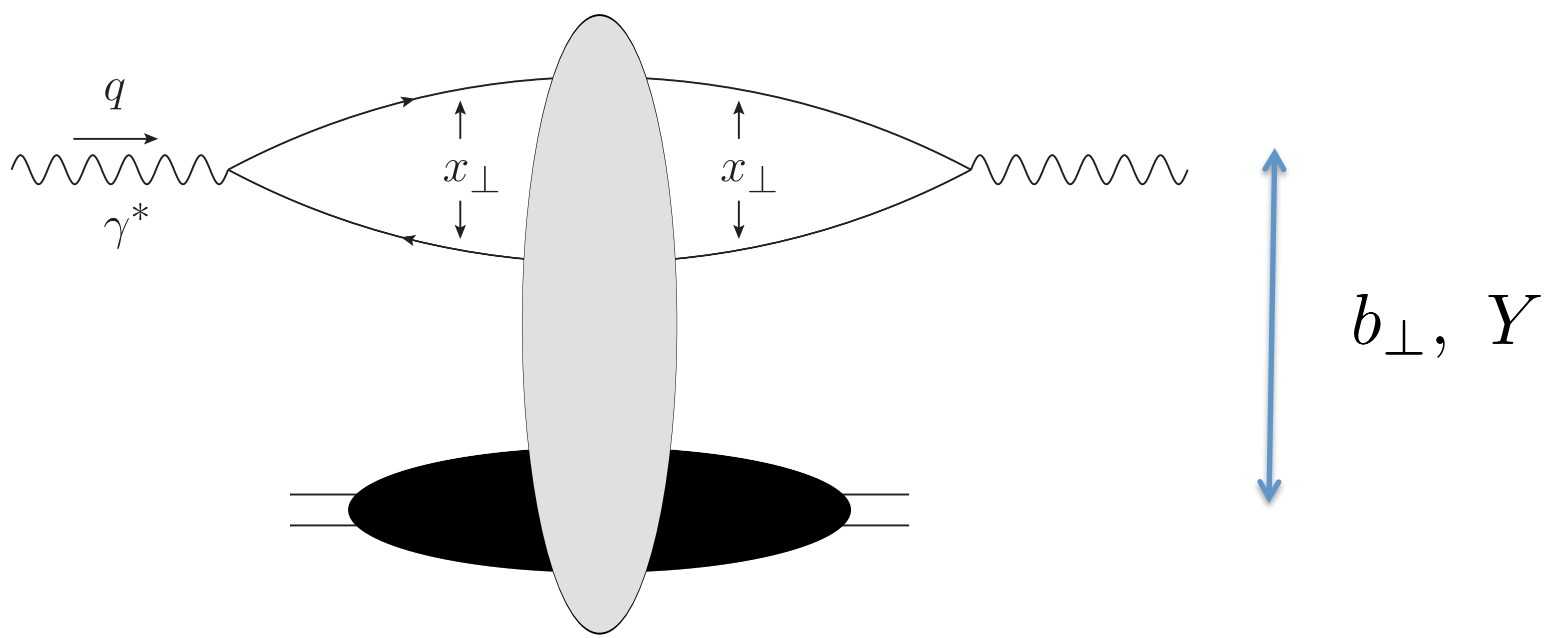
- In the dipole picture of DIS the virtual photon splits into a quark-antiquark pair, which then interacts with the target.
- The total DIS cross section and structure functions are calculated via:



Dipole Amplitude

- The total DIS cross section is expressed in terms of the quark dipole amplitude N:

$$\sigma_{tot}^{\gamma^* A} = \int \frac{d^2 x_\perp}{2\,\pi}\, d^2 b_\perp \int_0^1 \frac{dz}{z\,(1-z)}\ |\Psi^{\gamma^* \to q\bar{q}}(\vec{x}_\perp, z)|^2\ N(\vec{x}_\perp, \vec{b}_\perp, Y)$$



Dipole Amplitude

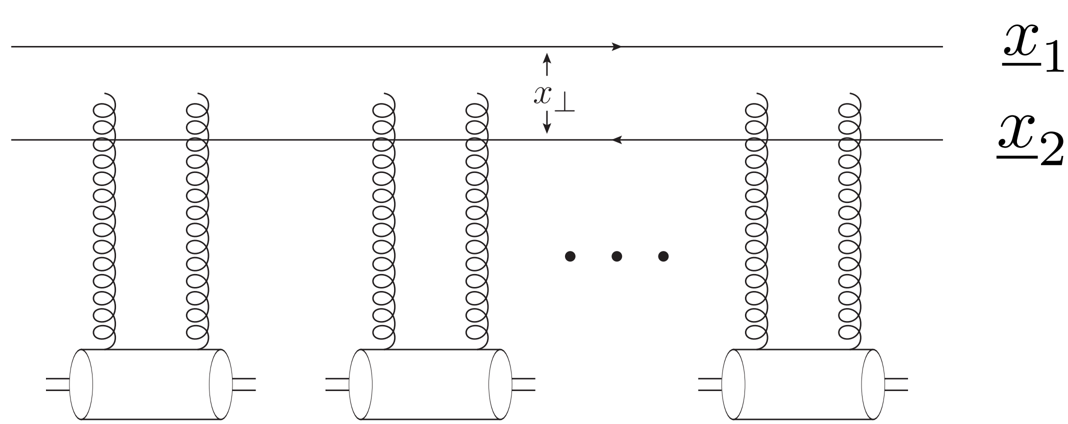
- The quark dipole amplitude is defined by

$$N(\underline{x}_1, \underline{x}_2) = 1 - \frac{1}{N_c} \left\langle \mathrm{tr} \left[V(\underline{x}_1)\, V^\dagger(\underline{x}_2) \right] \right\rangle$$

- Here we use the Wilson lines along the light-cone direction

$$V(\underline{x}) = \mathrm{P} \exp \left[i\, g \int\limits_{-\infty}^{\infty} dx^+ \, A^-(x^+, x^- = 0, \underline{x}) \right]$$

- In the classical Glauber-Mueller/McLerran-Venugopalan approach the dipole amplitude resums multiple rescatterings:

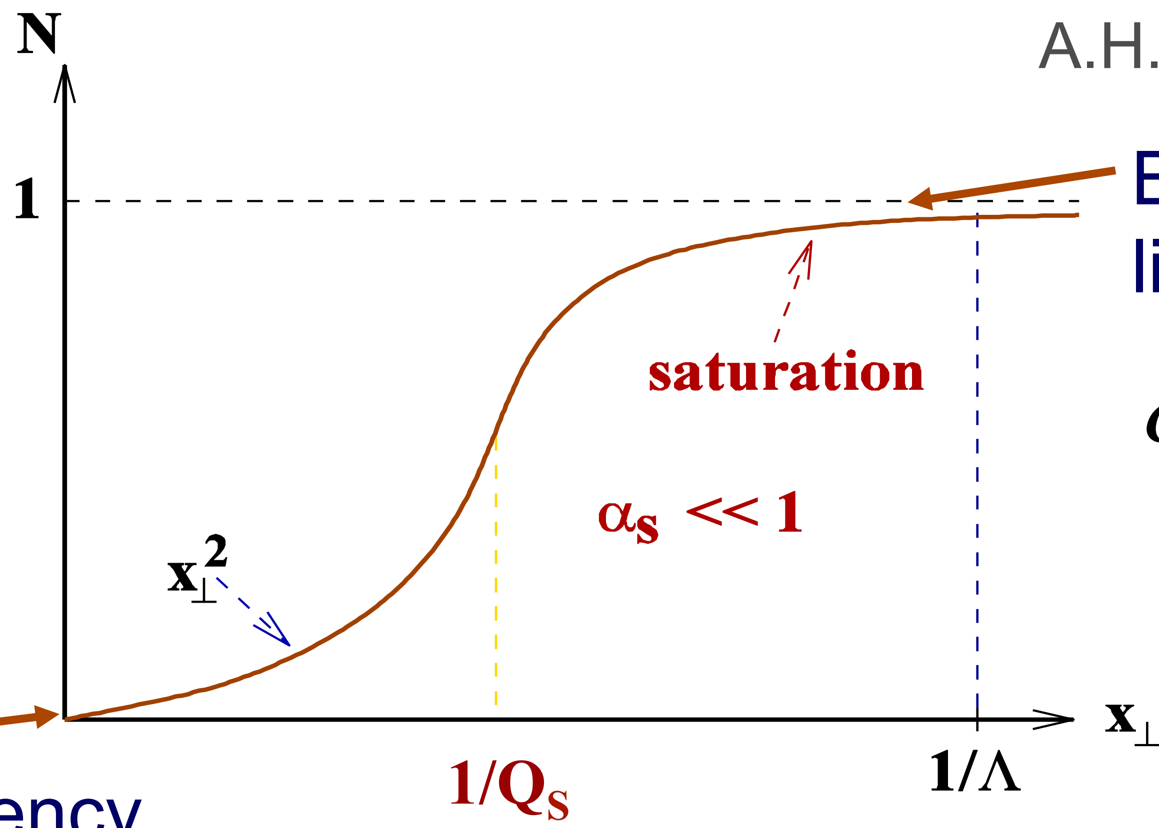


DIS in the Classical Approximation

The dipole-nucleus amplitude in the classical approximation is

$$N(x_\perp, Y) = 1 - \exp\left[-\frac{x_\perp^2\, Q_s^2}{4}\, \ln\frac{1}{x_\perp\, \Lambda}\right]$$

A.H. Mueller, '90



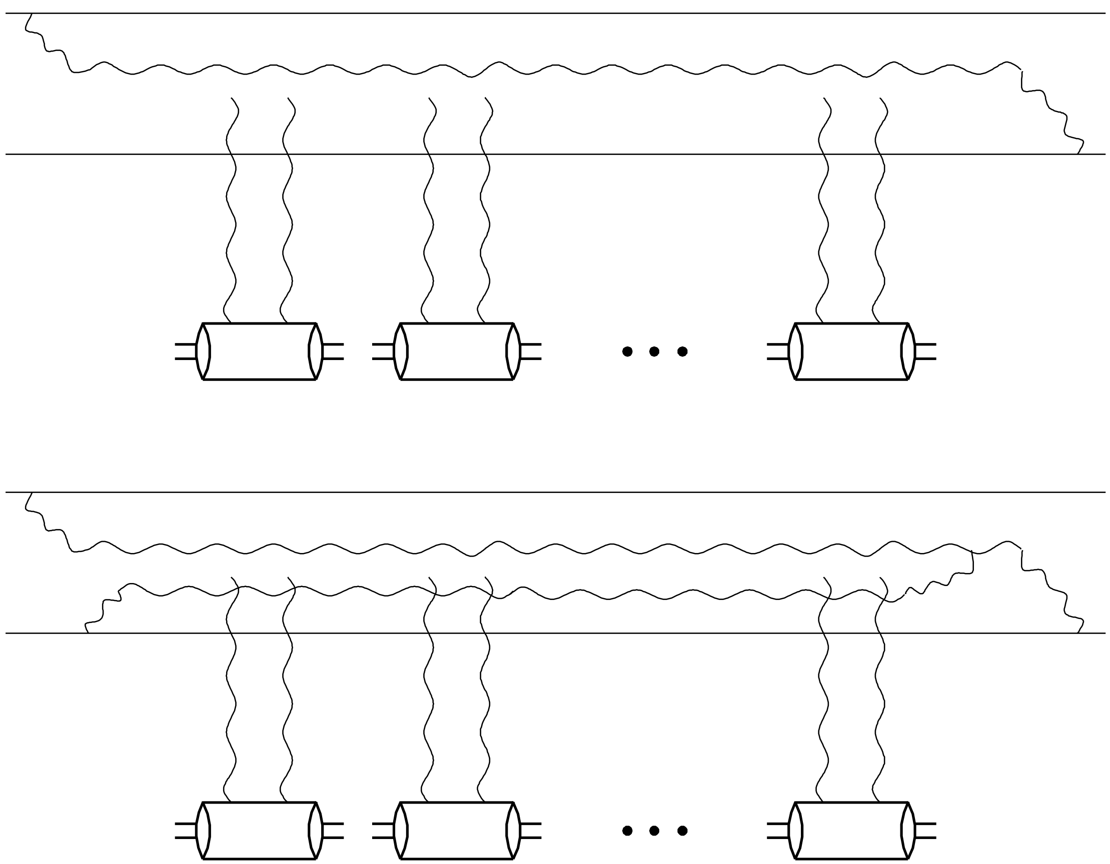
Black disk limit,

$$\sigma_{tot} < 2\pi R^2$$

Color transparency

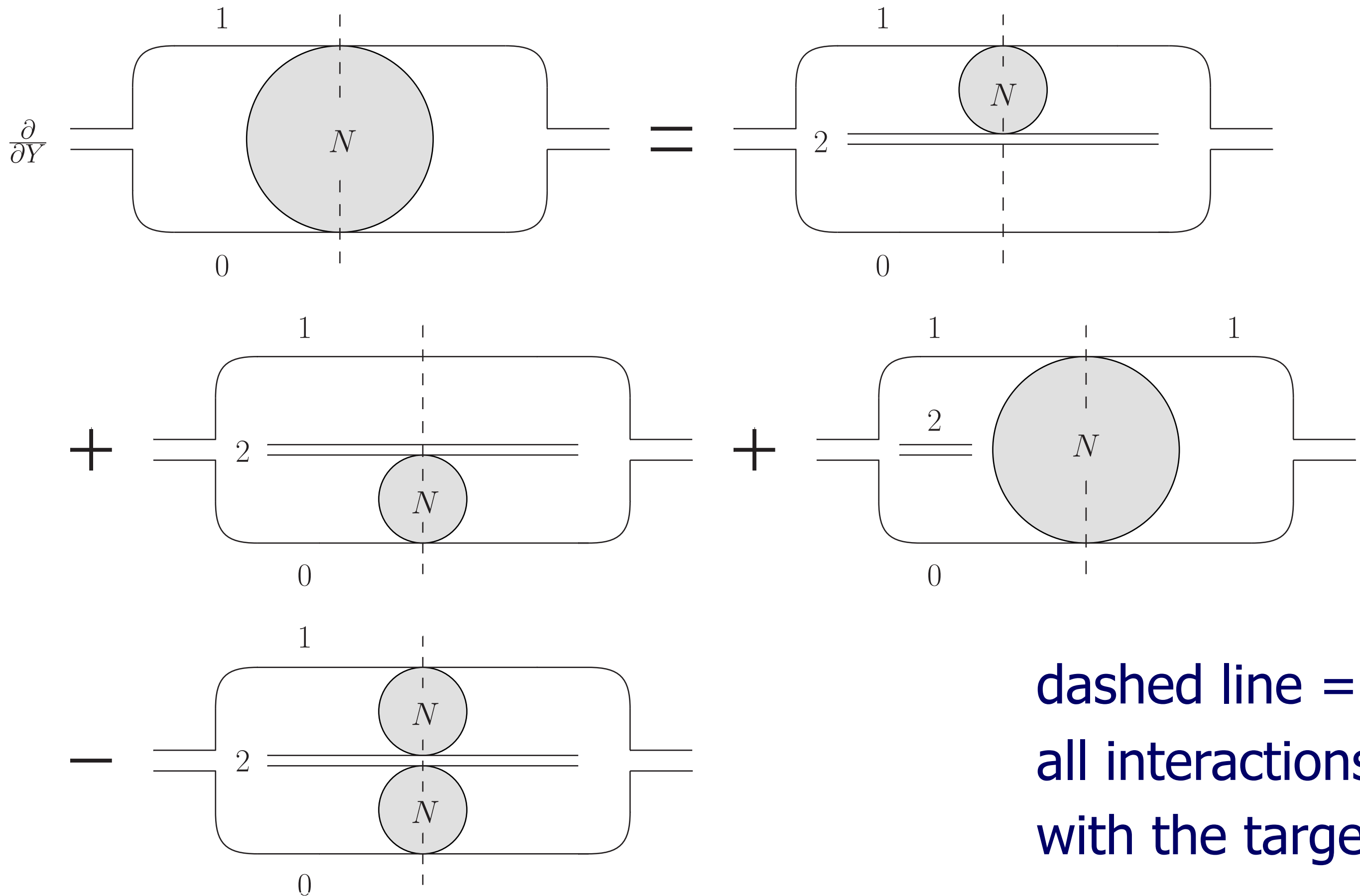
Dipole Amplitude

- The energy dependence comes in through nonlinear small-x BK/JIMWLK evolution, which comes in through the long-lived s-channel gluon corrections:



$$\alpha_s \, \ln \frac{1}{x} \sim \alpha_s \, Y \sim 1$$

Nonlinear evolution at large N_c

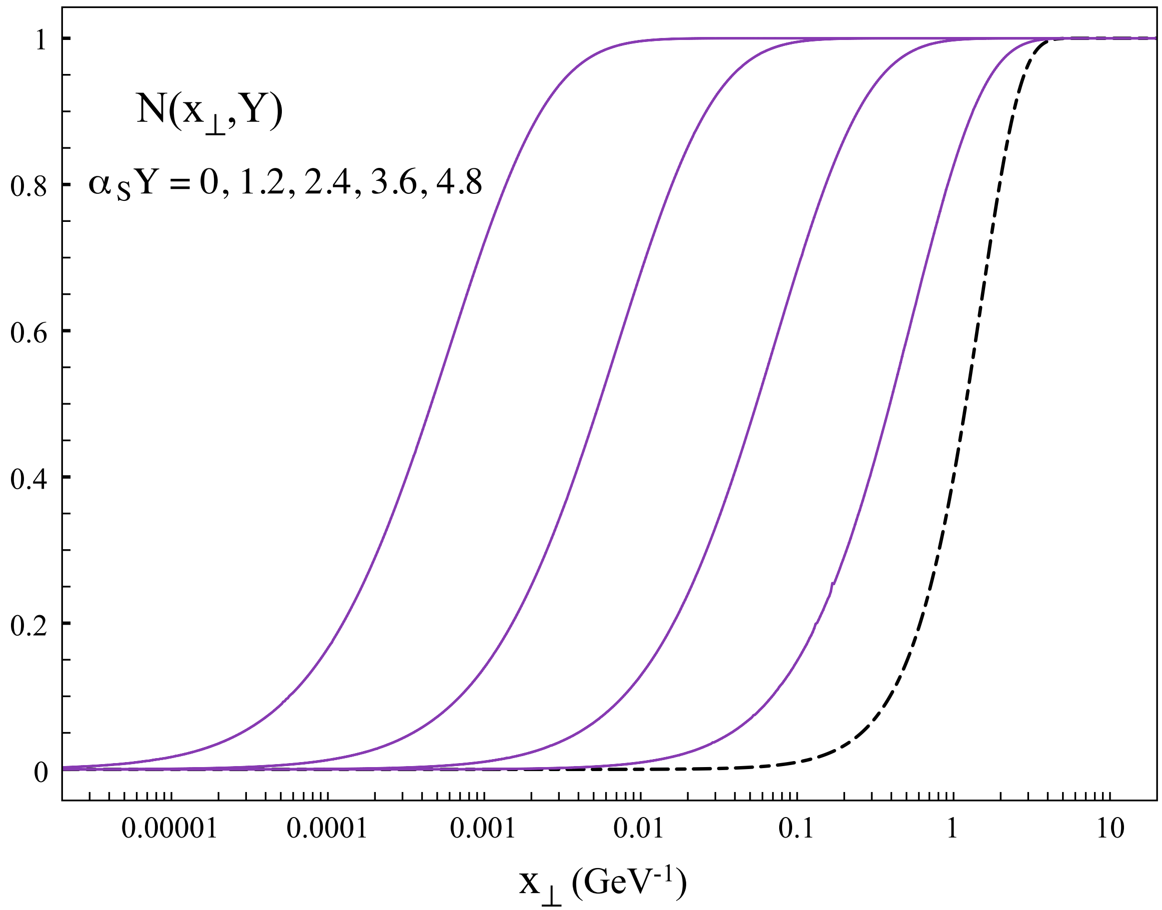


dashed line =
all interactions
with the target

$$\partial_Y N_{\mathbf{x}_0,\mathbf{x}_1}(Y) = \frac{\alpha_s N_c}{2\pi^2} \int d^2x_2 \, \frac{x_{01}^2}{x_{02}^2 \, x_{21}^2} \, [N_{\mathbf{x}_0,\mathbf{x}_2}(Y) + N_{\mathbf{x}_2,\mathbf{x}_1}(Y) - N_{\mathbf{x}_0,\mathbf{x}_1}(Y) - N_{\mathbf{x}_0,\mathbf{x}_2}(Y) \, N_{\mathbf{x}_2,\mathbf{x}_1}(Y)]$$

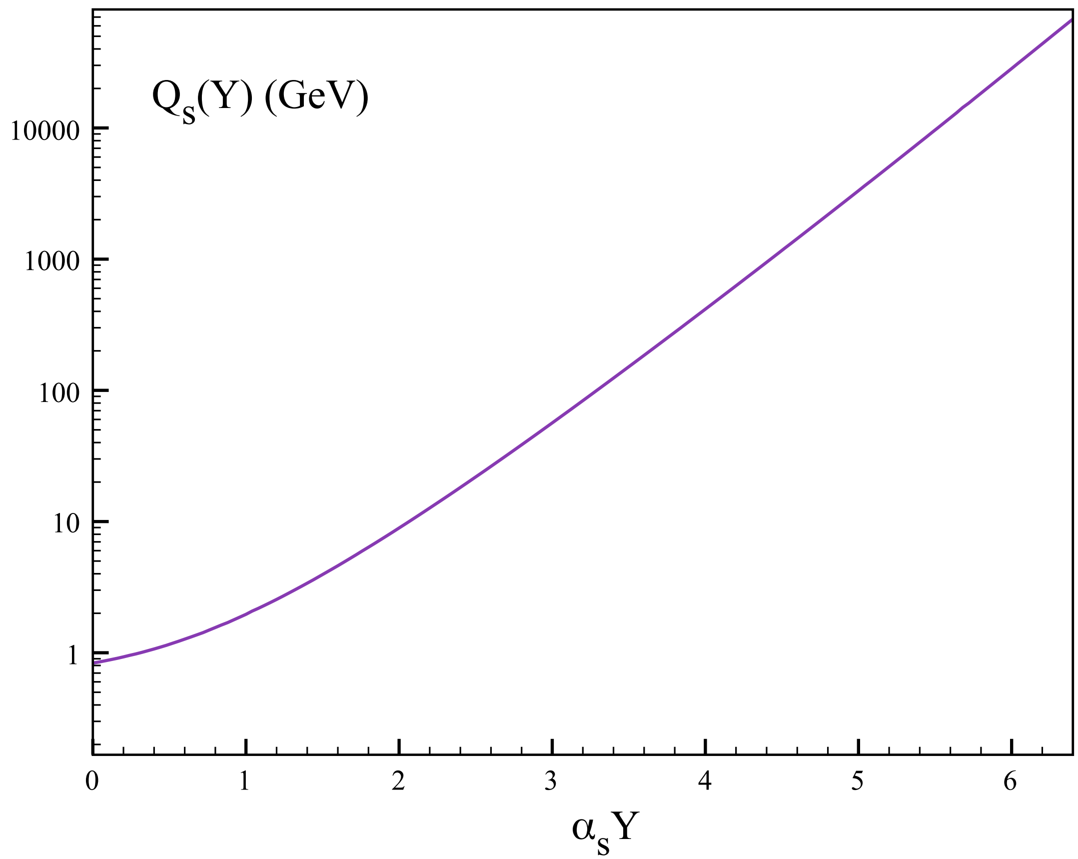
Balitsky '96, Yu.K. '99

Solution of BK equation



numerical solution by J. Albacete

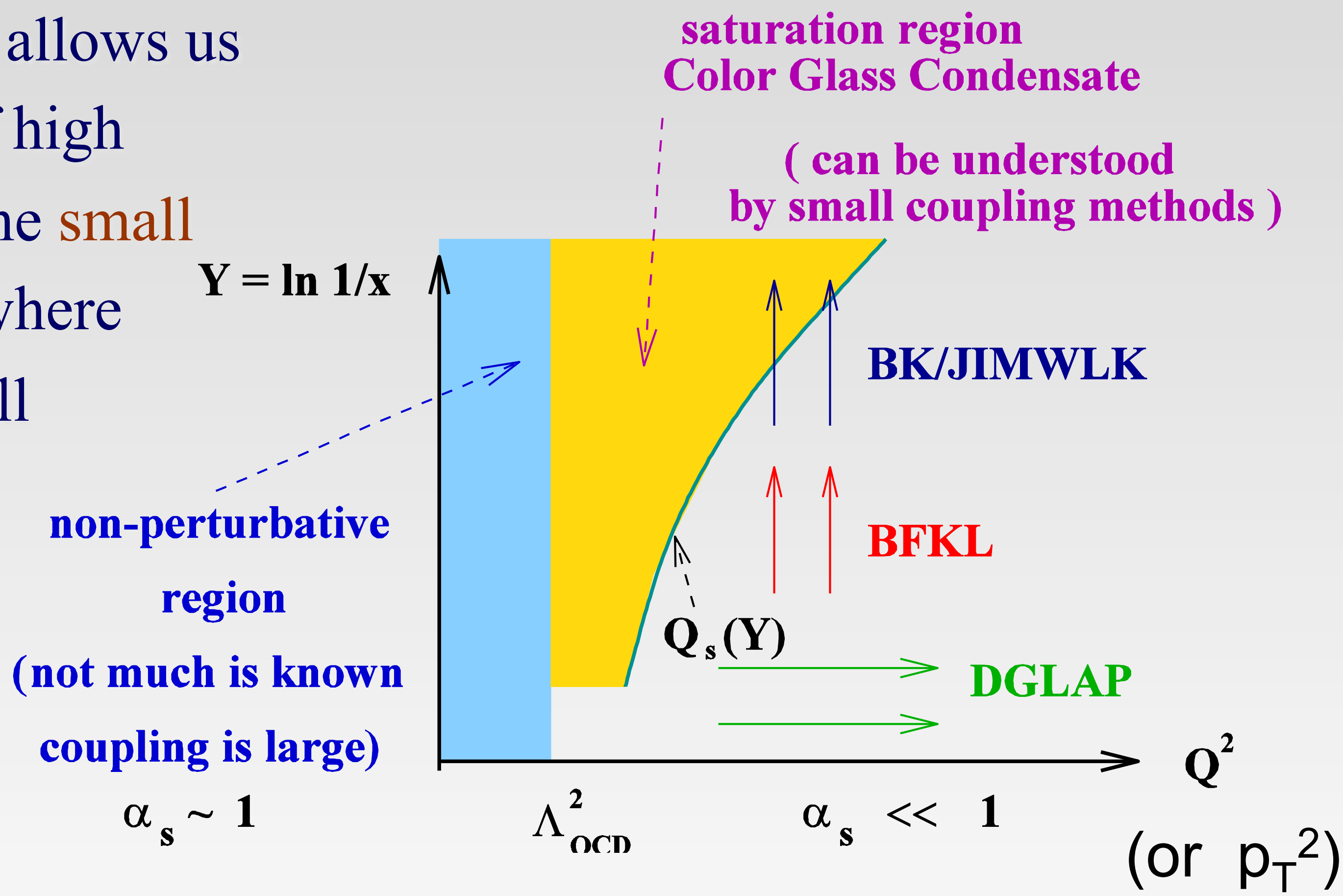
Saturation scale



numerical solution by J. Albacete

Map of High Energy QCD

Saturation physics allows us to study regions of high parton density in the small coupling regime, where calculations are still under control!

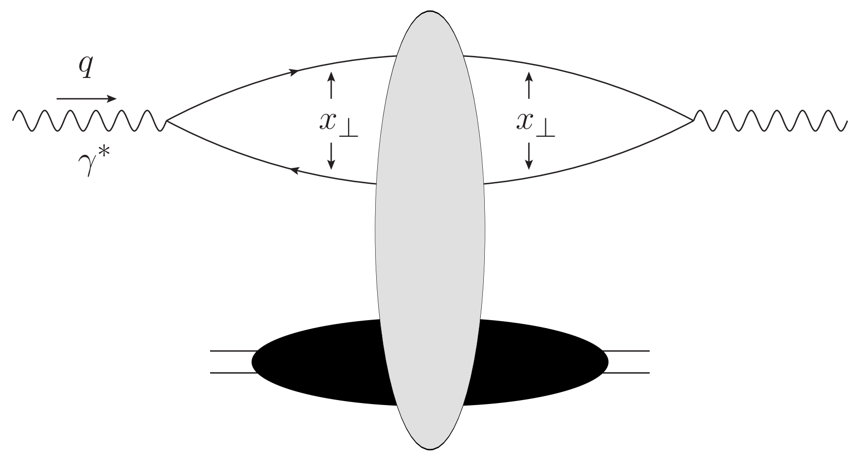


Transition to saturation region is characterized by the saturation scale

$$Q_S^2 \sim A^{1/3} \left(\frac{1}{x}\right)^{\lambda}$$

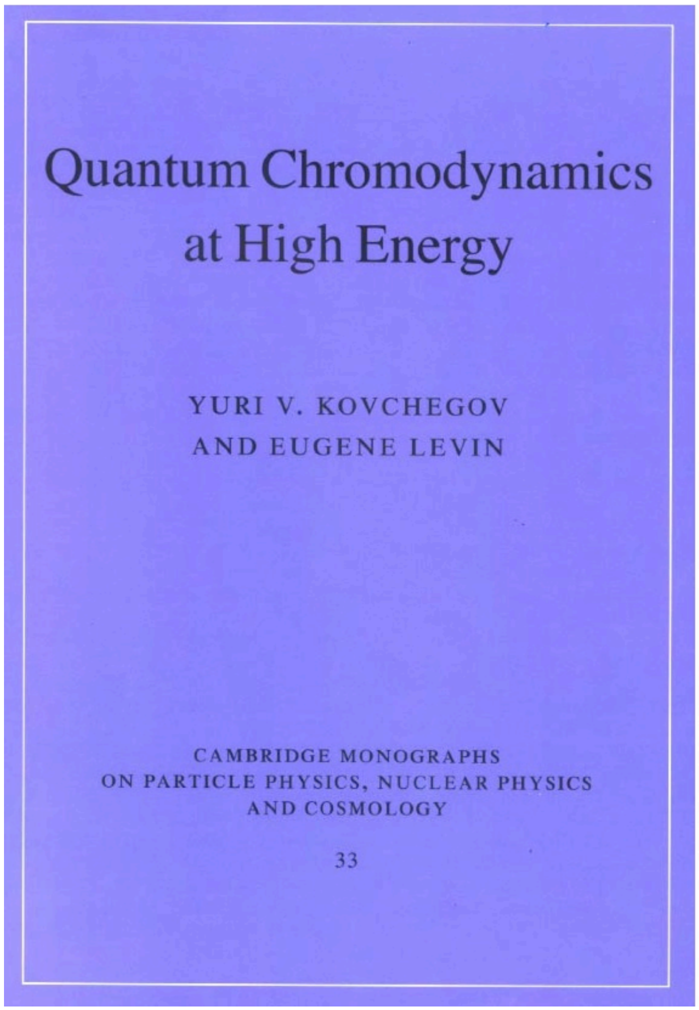
Dipole Amplitude

- Dipole scattering amplitude is a universal degree of freedom in saturation physics.
- It describes the total DIS cross section and structure functions:



- It also describes single inclusive quark and gluon production cross section in DIS and in p+A collisions.
- Works for diffraction in DIS: will show this next.
- For correlations need also quadrupoles. (J.Jalilian-Marian, Yu.K. '04; Dominguez et al '11)

A reference



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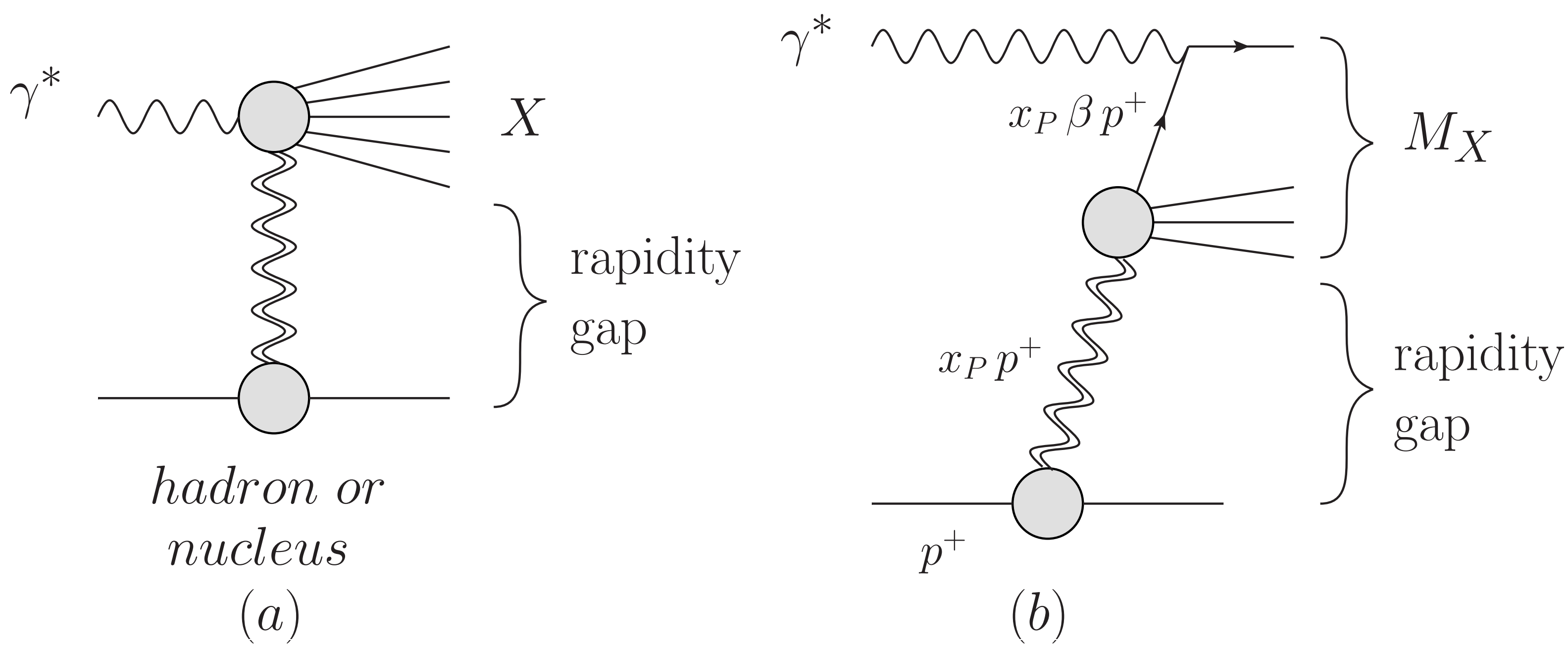
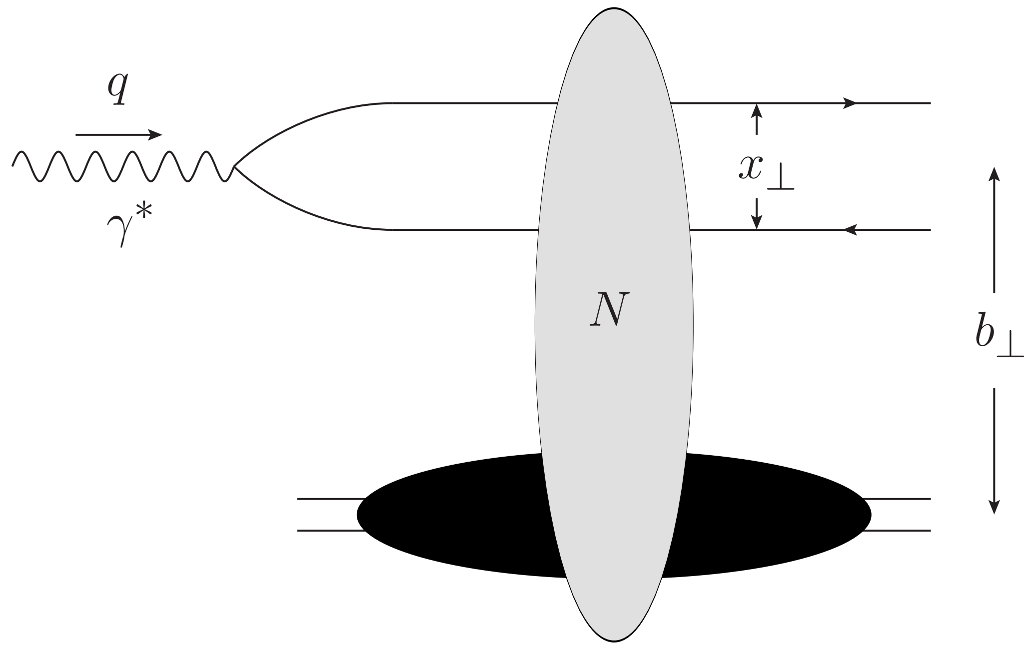
Low-mass diffraction in DIS

Diffraction terminology



Quasi-elastic DIS

Consider the case when nothing but the quark-antiquark pair is produced:

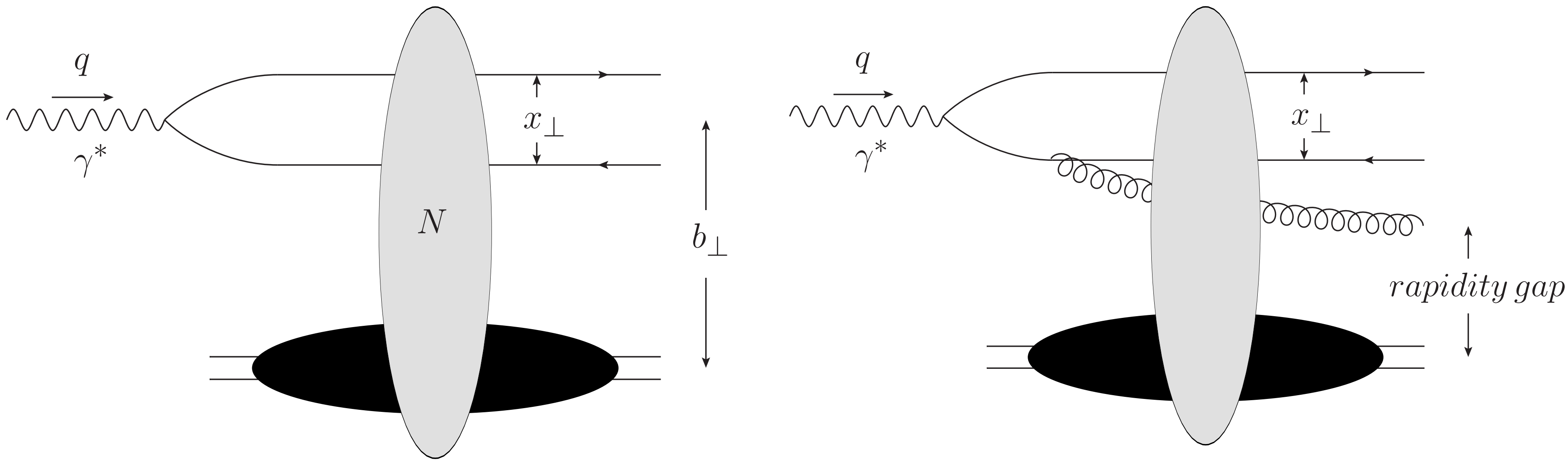


The quasi-elastic cross section is then

$$\sigma_{el}^{\gamma^* A} = \int \frac{d^2 x_\perp}{4\,\pi}\, d^2 b_\perp \int\limits_0^1 \frac{dz}{z\,(1-z)}\, |\Psi^{\gamma^* \to q\bar{q}}(\vec{x}_\perp, z)|^2 \; N^2(\vec{x}_\perp, \vec{b}_\perp, Y)$$

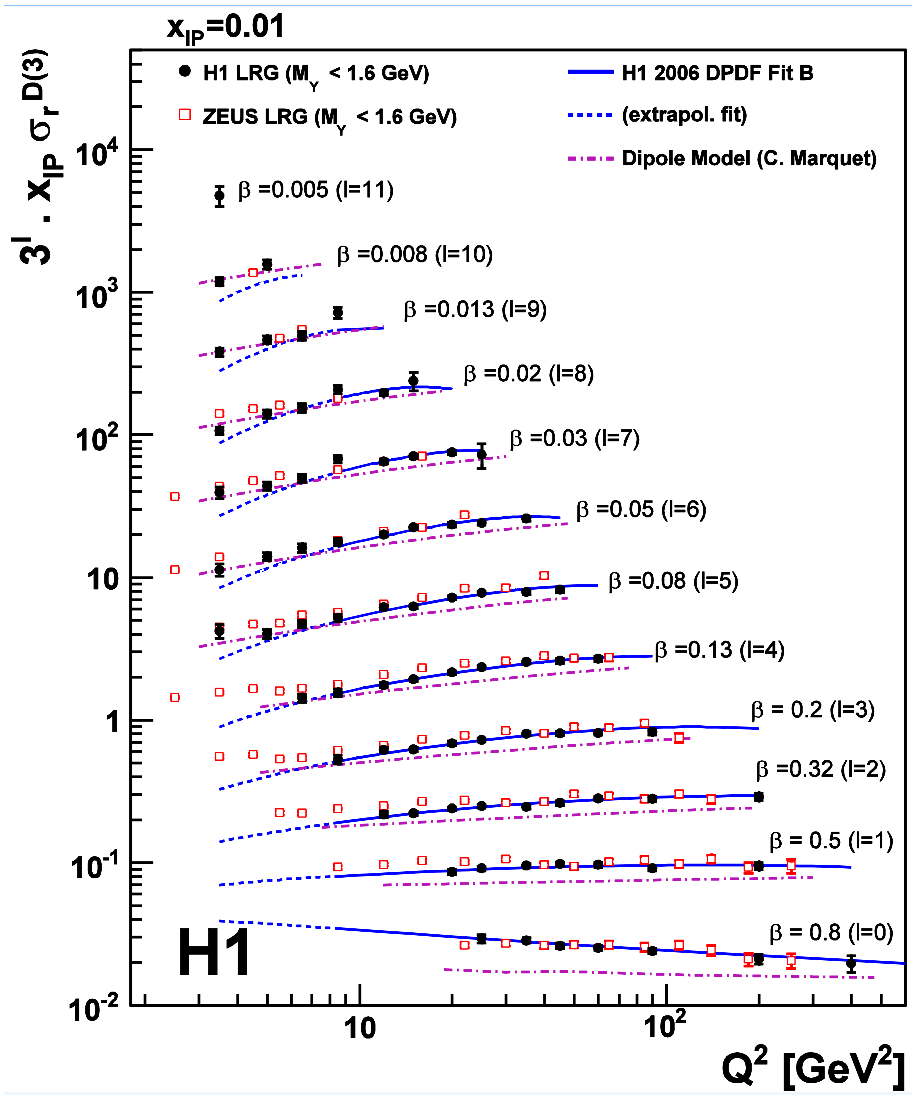
Buchmuller et al '97, McLerran and Yu.K. '99

Low-mass diffraction



- To describe processes with a larger invariant mass M_X of the produced system, need to include higher Fock states, like the q-qbar-gluon one shown here.
- Apparently this is enough to roughly describe HERA diffraction data.

HERA data for reduced diffractive cross section



(from the talk by K. Daum at DIS 2012)

"C. Marquet" = Kowalski, Lappi, Marquet, Venugopalan '08

based on IP-sat dipole model

The CGC fit was trained on older data, it does not do that great now, but there is room for theoretical improvements + one of the few that can make nuclear predictions

Diffraction on a black disk

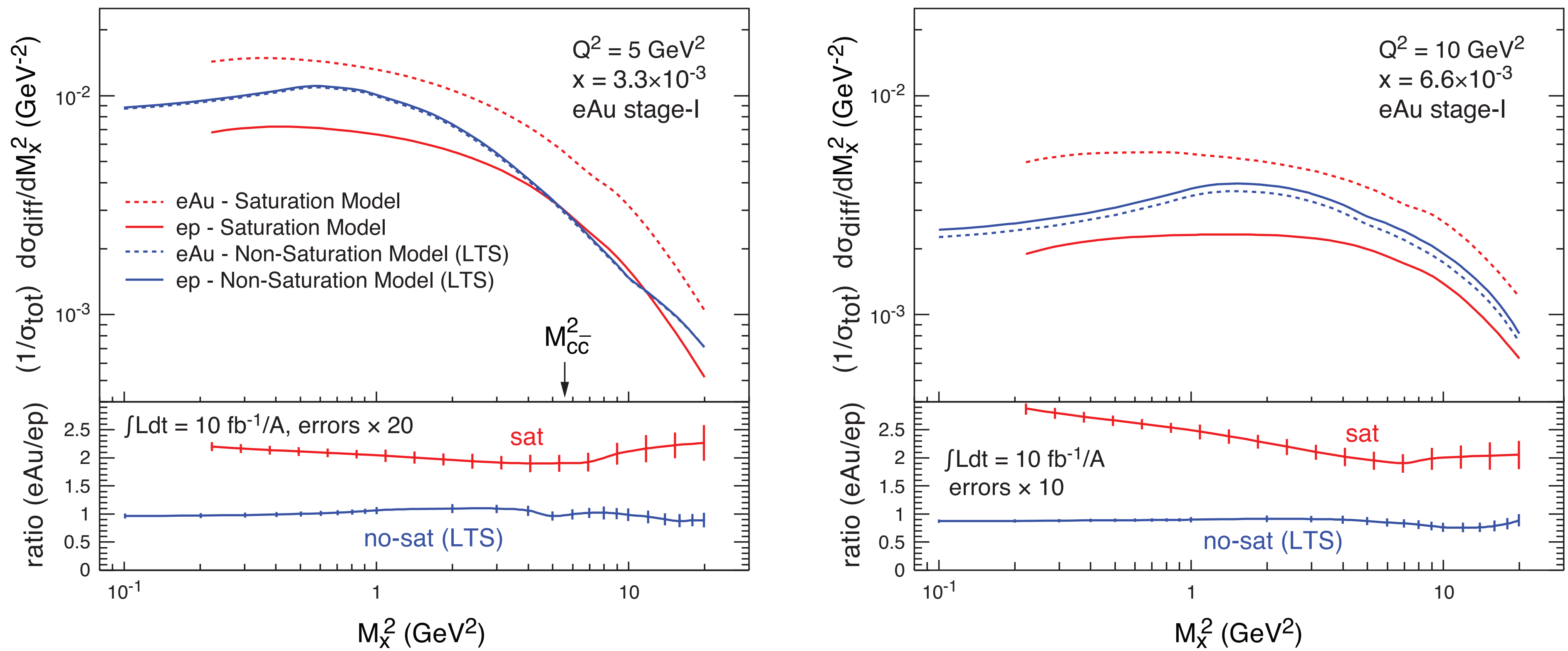
- For low Q^2 (large dipole sizes) the black disk limit is reached with N=1
- Diffraction (elastic scattering) becomes a half of the total cross section

$$\frac{\sigma_{el}^{q\bar{q}A}}{\sigma_{tot}^{q\bar{q}A}} = \frac{\int d^2b\, N^2}{2\, \int d^2b\, N} \longrightarrow \frac{1}{2}$$

- Large fraction of diffractive events in DIS is a signature of reaching the black disk limit!

Diffractive over total cross sections

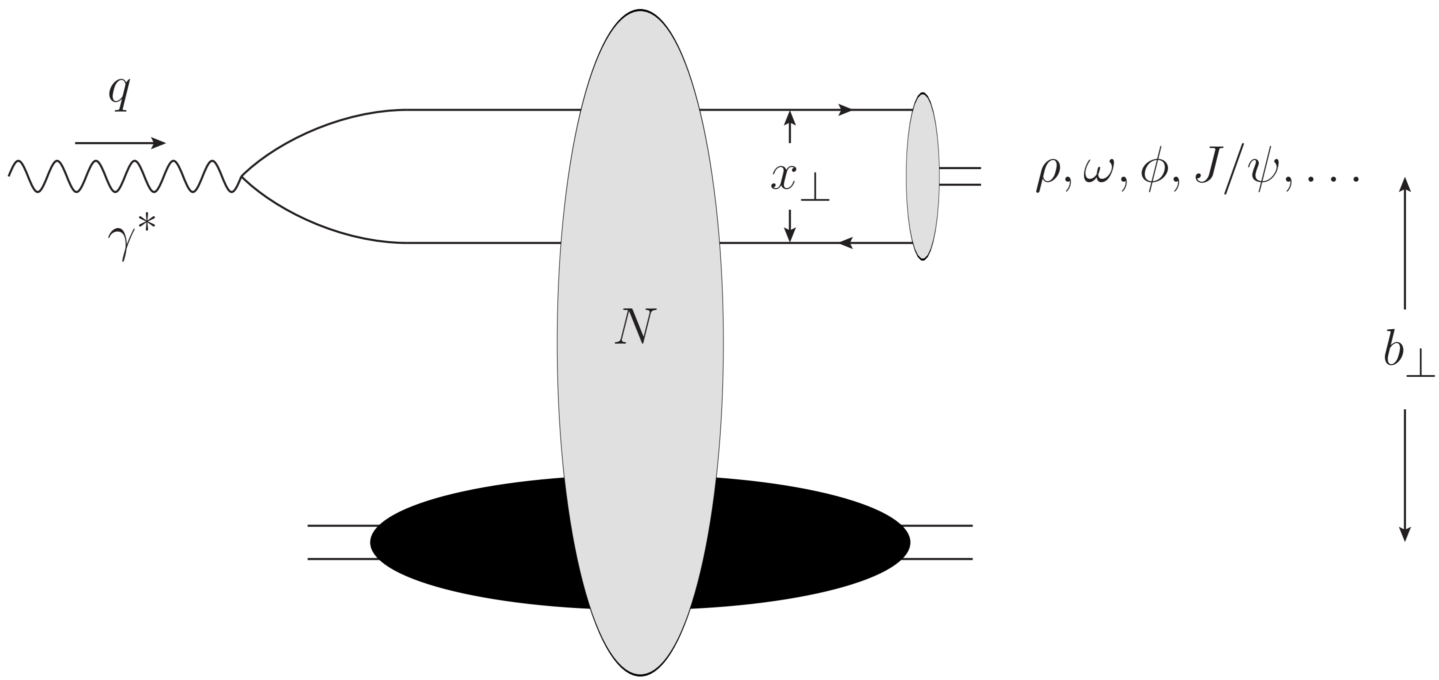
- Here's an EIC stage-I measurement which may distinguish saturation from non-saturation approaches:



sat = Kowalski et al '08, plots generated by Marquet
no-sat = Leading Twist Shadowing (LTS), Frankfurt, Guzey, Strikman '04, plots by Guzey

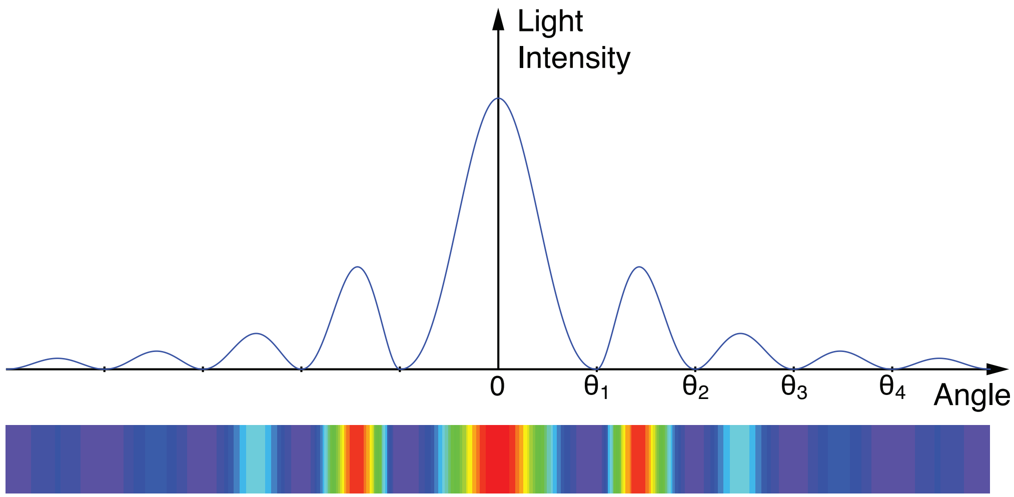
Exclusive Vector Meson Production

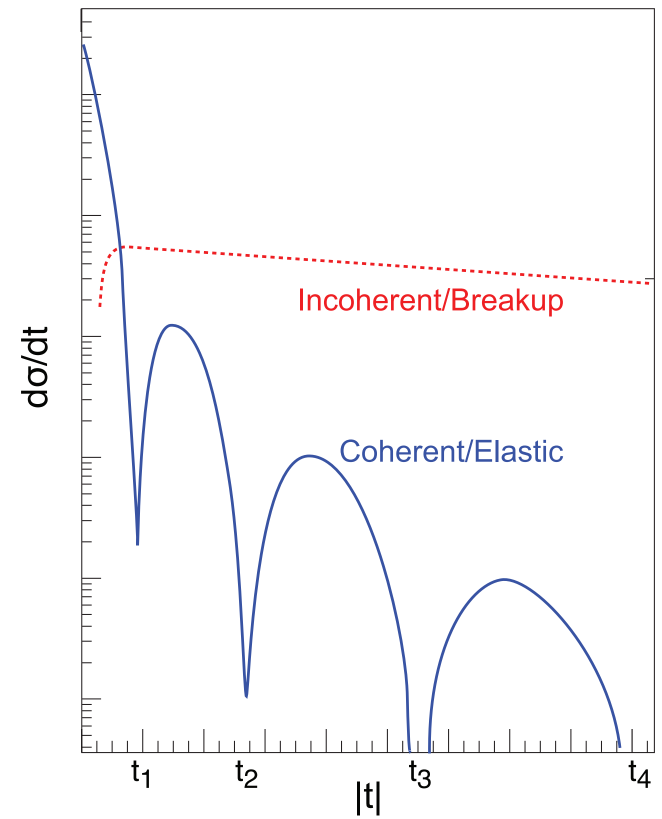
- Another important diffractive process which can be measured at EIC is exclusive vector meson production:



Optical Analogy

Diffraction in high energy scattering is not very different from diffraction in optics: both have diffractive maxima and minima:

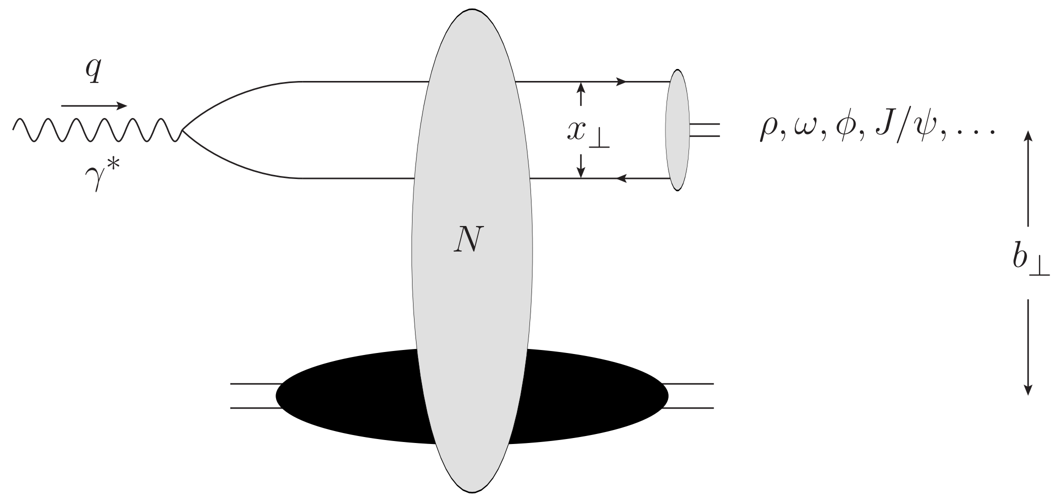




Coherent: target stays intact;

Incoherent: target nucleus breaks up, but nucleons are intact.

Exclusive VM Theory



- Differential exclusive VM production cross section is

$$\frac{d\sigma^{\gamma^*+A\to V+A}}{dt} = \frac{1}{4\pi} \left| \int d^2b \, e^{-i\,\vec{q}_\perp \cdot \vec{b}_\perp} \, T^{q\bar{q}A}(\hat{s}, \vec{b}_\perp) \right|^2$$

$$t = -\vec{q}_\perp^{\,2}$$

- the T-matrix is related to the dipole amplitude N:

$$T^{q\bar{q}A}(\hat{s}, \vec{b}_\perp) = i \int \frac{d^2x_\perp}{4\,\pi} \int_0^1 \frac{dz}{z\,(1-z)} \, \Psi^{\gamma^*\to q\bar{q}}(\vec{x}_\perp, z) \; N(\vec{x}_\perp, \vec{b}_\perp, Y)\, \Psi^V(\vec{x}_\perp, z)^*$$

Brodsky et al ‘94, Ryskin ‘93

Impact Parameter Dependence

- Using exclusive VM production one can study the b-dependence of the T-matrix since inverting the above formula one gets (Munier, Stasto, Mueller '01)

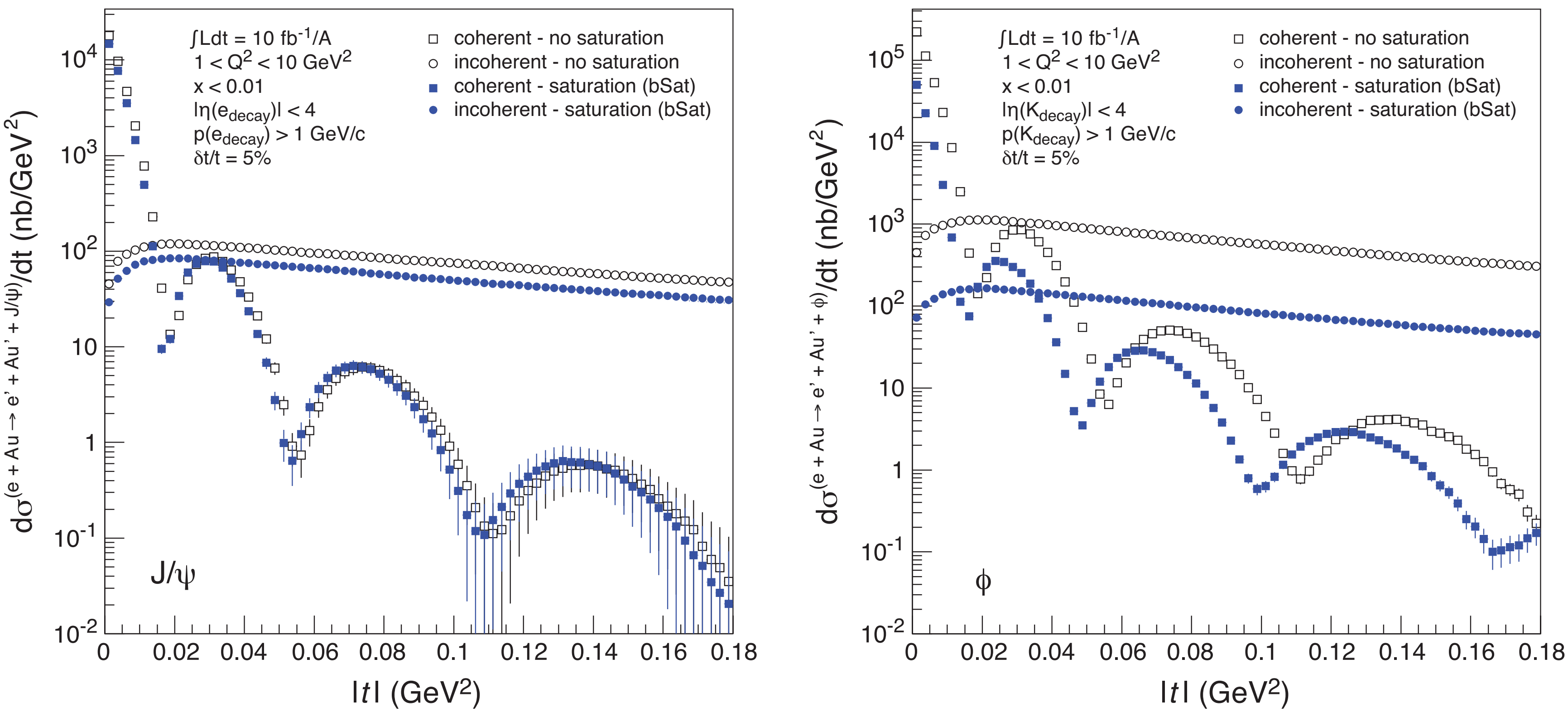
$$T^{q\bar{q}A}(\hat{s}, \vec{b}_\perp) = \frac{i}{2\,\pi^{3/2}} \int d^2q\, e^{i\,\vec{q}_\perp \cdot \vec{b}_\perp} \sqrt{\frac{d\sigma^{\gamma^*+A\to V+A}}{dt}}$$

- However, to find N one needs to de-convolute the wave functions…

$$T^{q\bar{q}A}(\hat{s}, \vec{b}_\perp) = i \int \frac{d^2x_\perp}{4\,\pi} \int_0^1 \frac{dz}{z\,(1-z)}\; \Psi^{\gamma^*\to q\bar{q}}(\vec{x}_\perp, z)\; N(\vec{x}_\perp, \vec{b}_\perp, Y)\, \Psi^V(\vec{x}_\perp, z)^*$$

but hopefully VM wf is localized in $\vec{x}_\perp$ making de-convolution easier.

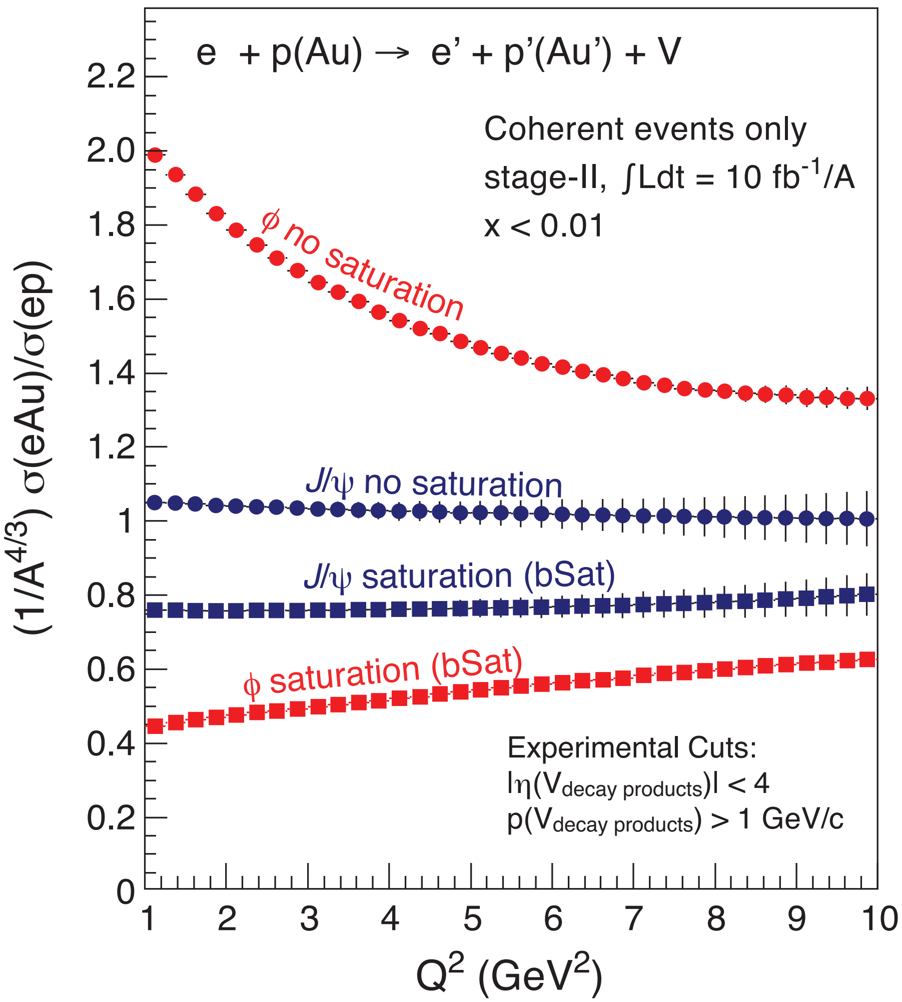
Exclusive VM Production as a Probe of Saturation



Plots by T. Toll and T. Ullrich using the Sartre even generator (b-Sat (=GBW+b-dep+DGLAP) + WS + MC).

- J/psi is smaller, less sensitive to saturation effects
- Phi meson is larger, more sensitive to saturation effects
- EIC stage-II measurement (most likely)

Exclusive VM Production as a Probe of Saturation



There is also a clear difference in the integrated over t cross sections as functions of Q^2.

Plots by T. Toll and T. Ullrich using the Sartre even generator (b-Sat(=GBW+b-dep+DGLAP) + WS + MC).

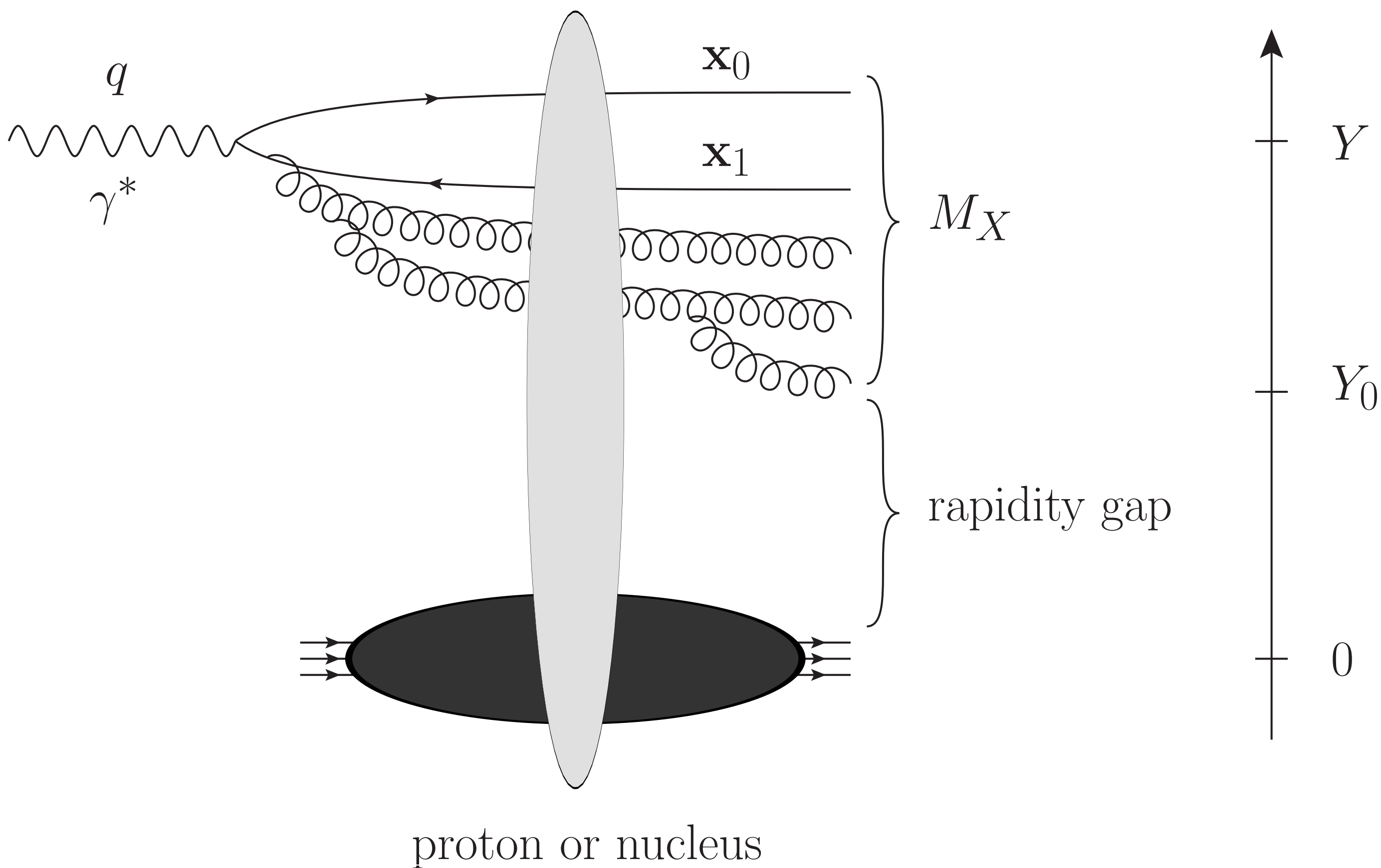
Low-mass diffraction: Conclusions

- The theory is well-developed.
- Diffractive scattering is a sensitive test of saturation physics at EIC. Saturation physics seems to be consistent with diffractive HERA data.
- We can use diffractive processes to discover saturation at EIC and test the theoretical framework presented above.
- Can also determine the b-distribution of strong small-x gluon fields in the target nucleus.

High-mass diffraction in DIS

The process

- At high M_X, in addition to the qqbar and qqbarG Fock states, one may have many more gluons produced:



What one has to calculate

- In the leading ln M_X^2 approximation we need to resum all soft gluon emissions with $y>Y_0$ in the initial and final states.

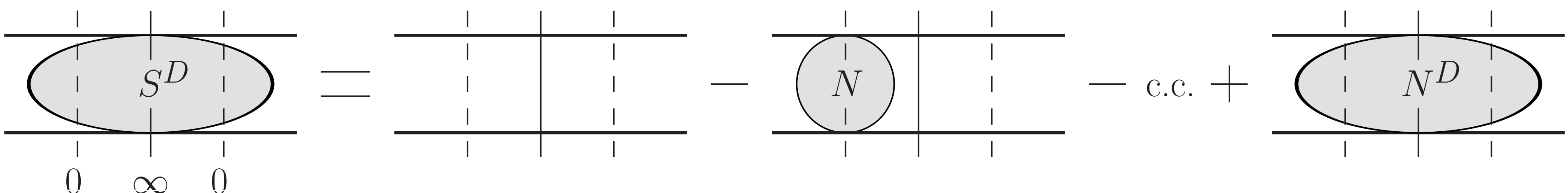
- The single diffractive cross section is

$$M_X^2 \, \frac{d\sigma_{diff}^{\gamma^* A}}{dM_X^2} = - \int d^2 x_0 \, d^2 x_1 \int_0^1 dz \, |\Psi^{\gamma^* \to q\bar{q}}(x_{01}, z)|^2 \, \frac{\partial N^D_{\mathbf{x}_0, \mathbf{x}_1}(Y, Y_0)}{\partial Y_0}$$

- N^D (Y, Y_0) is the diffractive cross section per unit impact parameter with the rapidity gap greater than or equal to Y_0.

Diffractive S-matrix

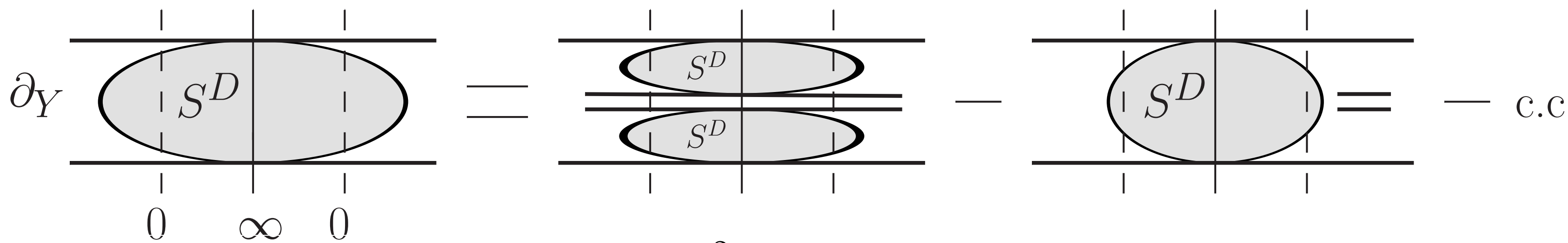
- To calculate N^D we first define a new quantity – the diffractive S-matrix S^D: it includes N^D along with all the non-interaction terms on either side (or both sides) of the final state cut (rapidity gap is still present between 0 and Y_0):



$$S^D_{\mathbf{x}_0,\mathbf{x}_1}(Y, Y_0) = 1 - 2\, N_{\mathbf{x}_0,\mathbf{x}_1}(Y) + N^D_{\mathbf{x}_0,\mathbf{x}_1}(Y, Y_0)$$

Nonlinear Equation for Diffraction

- For $Y>Y_0$, S^D obeys the following evolution equation in the large-N_c limit, which is just the BK equation for the S-matrix:



$$\partial_Y S^D_{\mathbf{x}_0,\mathbf{x}_1}(Y,Y_0) = \frac{\alpha_s N_c}{2\pi^2} \int d^2x_2 \frac{x_{10}^2}{x_{20}^2 x_{21}^2} \left[S^D_{\mathbf{x}_0,\mathbf{x}_2}(Y,Y_0)\, S^D_{\mathbf{x}_2,\mathbf{x}_1}(Y,Y_0) - S^D_{\mathbf{x}_0,\mathbf{x}_1}(Y,Y_0)\right]$$

Levin, Yu.K. '00 (large-N_c); Hentschinski, Weigert, Schafer '06 (all-N_c); this derivation is similar to Hatta et al '06.

- The initial condition is $S^D_{\mathbf{x}_0,\mathbf{x}_1}(Y=Y_0,Y_0) = [1 - N_{\mathbf{x}_0,\mathbf{x}_1}(Y_0)]^2$
- This results from scattering being purely elastic when $Y=Y_0$:

$$N^D_{\mathbf{x}_0,\mathbf{x}_1}(Y=Y_0,Y_0) = [N_{\mathbf{x}_0,\mathbf{x}_1}(Y_0)]^2$$

Nonlinear Equation for Diffraction

- The equation for N^D reads

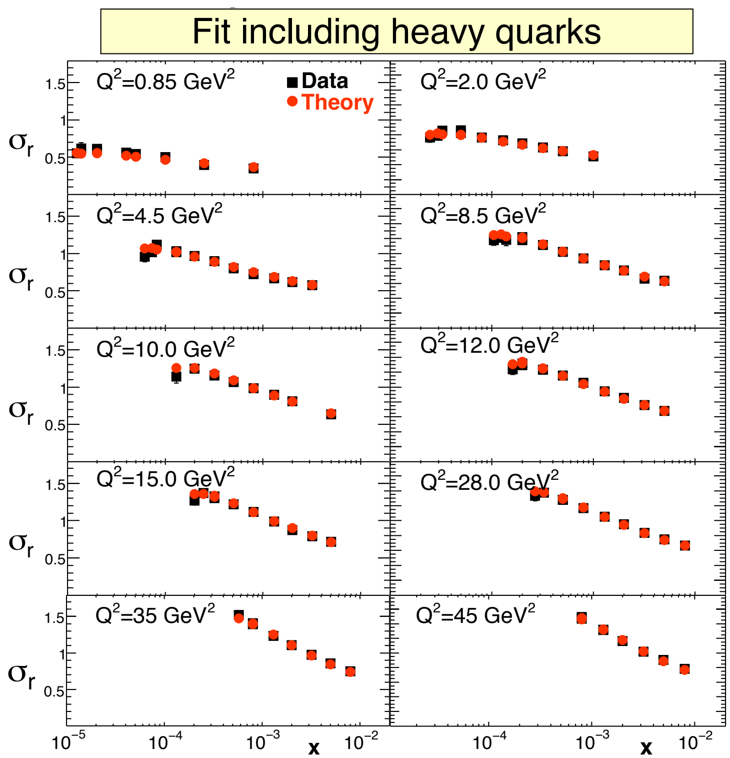
Levin, Yu.K. '00

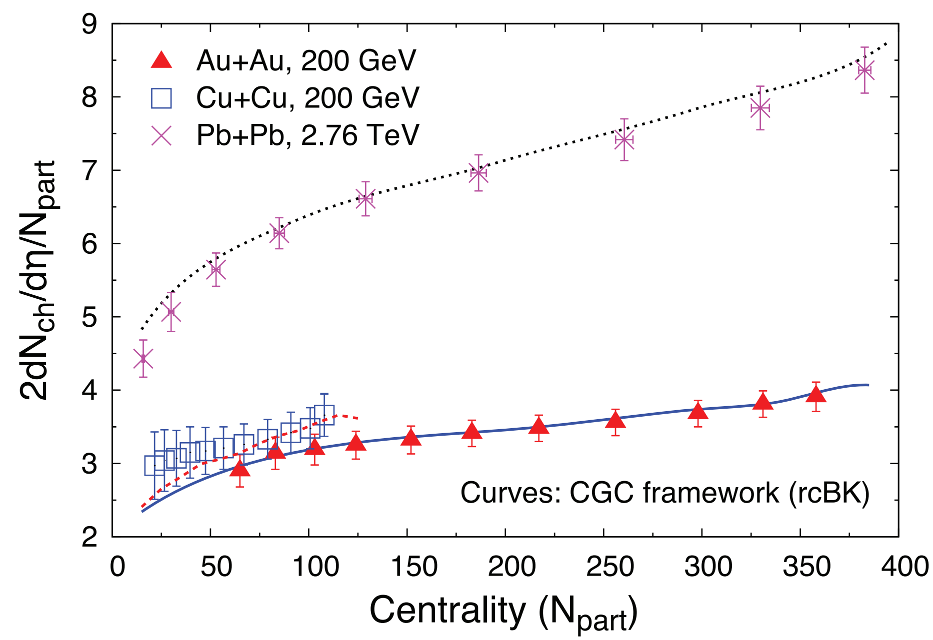
$$\partial_Y N^D_{\mathbf{x}_0,\mathbf{x}_1}(Y,Y_0) = \frac{\alpha_s N_c}{2\pi^2} \int d^2x_2 \frac{x_{10}^2}{x_{20}^2 x_{21}^2} \Big[N^D_{\mathbf{x}_0,\mathbf{x}_2}(Y,Y_0) + N^D_{\mathbf{x}_2,\mathbf{x}_1}(Y,Y_0) - N^D_{\mathbf{x}_0,\mathbf{x}_1}(Y,Y_0)$$
$$+ N^D_{\mathbf{x}_0,\mathbf{x}_2}(Y,Y_0) N^D_{\mathbf{x}_2,\mathbf{x}_1}(Y,Y_0) - 2 N_{\mathbf{x}_0,\mathbf{x}_2}(Y) N^D_{\mathbf{x}_2,\mathbf{x}_1}(Y,Y_0) - 2 N^D_{\mathbf{x}_0,\mathbf{x}_2}(Y,Y_0) N_{\mathbf{x}_2,\mathbf{x}_1}(Y)$$
$$+ 2 N_{\mathbf{x}_0,\mathbf{x}_2}(Y) N_{\mathbf{x}_2,\mathbf{x}_1}(Y) \Big]$$

- The initial condition is $N^D_{\mathbf{x}_0,\mathbf{x}_1}(Y = Y_0, Y_0) = [N_{\mathbf{x}_0,\mathbf{x}_1}(Y_0)]^2$ where $N(Y_0)$ is found from the BK equation.

What about the running coupling?

- rcBK has been very successful in describing the DIS HERA data (Albacete et al, 2011) and heavy ion collisions (Albacete and Dumitru, ’10):





LHC line is a prediction!

- Seems like to do serious phenomenology one needs running coupling corrections for diffractive evolution.

Nonlinear Evolution for Diffraction with Running Coupling Corrections

- The running-coupling evolution (BLM prescription) for diffraction reads (Yu. K. '11):

$$\partial_Y S^D_{\mathbf{x}_0,\mathbf{x}_1}(Y,Y_0) = \int d^2x_2\, K(\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2)\ \left[S^D_{\mathbf{x}_0,\mathbf{x}_2}(Y,Y_0)\, S^D_{\mathbf{x}_2,\mathbf{x}_1}(Y,Y_0) - S^D_{\mathbf{x}_0,\mathbf{x}_1}(Y,Y_0)\right]$$

- The rc-kernel is in Balitsky prescription given by (cf. rcBK)

$$K^{Bal}_{rc}(\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2) = \frac{N_c\,\alpha_s(x_{10}^2)}{2\pi^2}\left[\frac{x_{10}^2}{x_{20}^2\,x_{21}^2} + \frac{1}{x_{20}^2}\left(\frac{\alpha_s(x_{20}^2)}{\alpha_s(x_{21}^2)} - 1\right) + \frac{1}{x_{21}^2}\left(\frac{\alpha_s(x_{21}^2)}{\alpha_s(x_{20}^2)} - 1\right)\right]$$

- In the KW prescription it is

$$K^{KW}_{rc}(\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2) = \frac{N_c}{2\pi^2}\left[\alpha_s(x_{20}^2)\frac{1}{x_{20}^2} - 2\,\frac{\alpha_s(x_{20}^2)\,\alpha_s(x_{21}^2)}{\alpha_s(R^2)}\ \frac{\mathbf{x}_{20}\cdot\mathbf{x}_{21}}{x_{20}^2\,x_{21}^2} + \alpha_s(x_{21}^2)\frac{1}{x_{21}^2}\right]$$

with

$$R^2 = x_{20}\,x_{21}\left(\frac{x_{21}}{x_{20}}\right)^{\frac{x_{20}^2+x_{21}^2}{x_{20}^2-x_{21}^2} - 2\,\frac{x_{20}^2\,x_{21}^2}{\mathbf{x}_{20}\cdot\mathbf{x}_{21}}\,\frac{1}{x_{20}^2-x_{21}^2}}$$

Conclusions

- Low- and high-mass diffraction theory is well-developed in the saturation framework: we know how to include multiple GM/MV rescatterings, nonlinear BK/JIMWLK evolution, and rc corrections.

- HERA diffractive cross section in e+p can be reasonably well described in the saturation picture.

- It appears possible for EIC e+A diffractive data to clearly differentiate between saturation and non-saturation predictions, hopefully completing the discovery of the saturation phenomena.

Backup Slides

A-scaling

- In the linear regime N ~ $A^{1/3}$ such that $\sigma_{diff}^{\gamma^* A} \sim A^{4/3}$

  while $\sigma_{tot}^{\gamma^* A} \sim A$

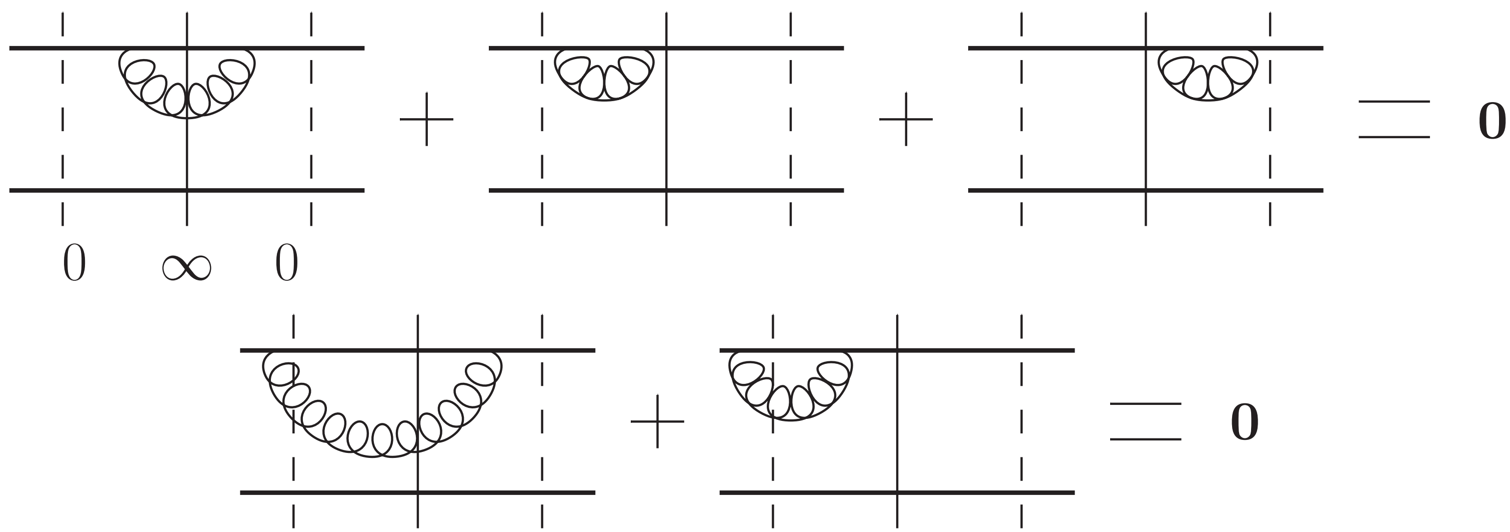
  giving a <u>small</u> but growing ratio $$\boxed{\frac{\sigma_{diff}^{\gamma^* A}}{\sigma_{tot}^{\gamma^* A}} \sim A^{1/3}}$$

- In the saturation regime N = 1 and the ratio is <u>large</u>, but A-independent

$$\boxed{\frac{\sigma_{diff}^{\gamma^* A}}{\sigma_{tot}^{\gamma^* A}} \sim A^0}$$

Cancelation of final state interactions

- The equation works due to the following cancellations of final state interactions (Z. Chen, A. Mueller '95) for gluons with $y>Y_0$ (those gluons have no final state constraints):



- Only initial-state emission remain, both in the amplitude and in the cc amplitude.

Main Principle

To set the scale of the coupling constant we will first calculate the $\alpha_S N_f$ (quark loops) corrections to LO gluon production cross section to <u>all orders</u>.

We then will complete N_f to the full QCD beta-function

$$\beta_2 = \frac{11 N_C - 2 N_f}{12 \pi}$$

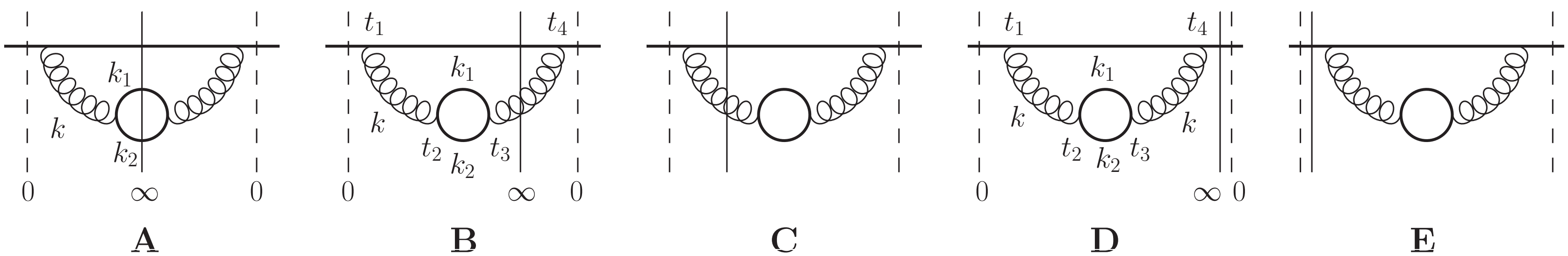
by replacing

$$N_f \to -6 \pi \beta_2$$

(Brodsky, Lepage, Mackenzie ' 83 – BLM prescription) .

Running Coupling Corrections

- …are straightforward to include using BLM prescription.
- Late-time cancellations apply to the running coupling corrections as well. Here's one example of cancellations:



$$A + B + C + D + E = 0$$

- The rc corrections are the same as for rcBK. as calculated by Balitsky '06, Weigert and Yu.K. '06, and Gardi et al '06.
- Since S^D already satisfies BK evolution, we can simply use the rcBK kernel to construct the diffractive evolution with running coupling.

Initial condition

- The initial condition is still set by

$$S^D_{\mathbf{x}_0,\mathbf{x}_1}(Y=Y_0,Y_0) = \left[1 - N_{\mathbf{x}_0,\mathbf{x}_1}(Y_0)\right]^2$$

with the amplitude N now found from rcBK equation.

- The rc-evolution for N^D is (Yu.K. '11)

$$\partial_Y N^D_{\mathbf{x}_0,\mathbf{x}_1}(Y,Y_0) \; = \int d^2x_2\, K(\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2) \Big[N^D_{\mathbf{x}_0,\mathbf{x}_2}(Y,Y_0) + N^D_{\mathbf{x}_2,\mathbf{x}_1}(Y,Y_0) - N^D_{\mathbf{x}_0,\mathbf{x}_1}(Y,Y_0)$$
$$+ N^D_{\mathbf{x}_0,\mathbf{x}_2}(Y,Y_0)\, N^D_{\mathbf{x}_2,\mathbf{x}_1}(Y,Y_0) - 2\, N_{\mathbf{x}_0,\mathbf{x}_2}(Y)\, N^D_{\mathbf{x}_2,\mathbf{x}_1}(Y,Y_0) - 2\, N^D_{\mathbf{x}_0,\mathbf{x}_2}(Y,Y_0)\, N_{\mathbf{x}_2,\mathbf{x}_1}(Y)$$
$$+ 2\, N_{\mathbf{x}_0,\mathbf{x}_2}(Y)\, N_{\mathbf{x}_2,\mathbf{x}_1}(Y) \Big]$$