A posteriori inclusion of parton density functions in NLO QCD calculations at hadron colliders: The APPLGRID Project

Pavel Starovoitov
DESY, Hamburg, Germany

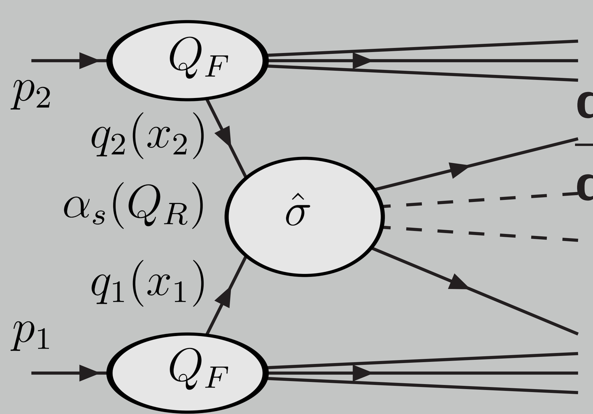
Motivation

The calculation of cross-sections at Next-to-Leading Order (NLO) in QCD involves numerical Monte Carlo integration over the phase space of the final state partons in order to cancel the infra-red and collinear divergences. Any cross section with a different choice for the proton parton density functions (PDFs) or a different choice of factorisation or renormalisation scale requires the complete calculation to be performed again.

- Coupling constant and PDFs are non-perturbative inputs in formula
- Matrix elements are independent from PDF functions due to factorization theorem $\longrightarrow$ we can split calculation into two parts
  - Step 1 : Store perturbative coefficients on grids (long run: $\sim$ days).
  - Step 2: Convolute grid with PDF's ($\sim$ 10–100 ms).

APPLgrid method

Based on Factorisation Theorem



$$\frac{d\sigma}{dX} \sim \sum_{(i,j,p)} \int d\Gamma \; \alpha_s^p(Q_R^2)\, q_i(x_1, Q_F^2) q_j(x_2, Q_F^2) \times \frac{d\hat{\sigma}^{ij}_{(p)}}{dX}(x_1, x_2, Q_F^2, Q_R^2;\; S)$$

The Goal:

$$d\hat{\sigma}^{ij}_{(p)}/dX \xrightarrow{\text{APPLgrid}} w^{(p)(l)}(x_1^m, x_2^n, Q^{2k}) \;\text{(3D-grid)}\; (Q_R^2 \equiv Q_F^2)$$

Step 1

- Phasespace binning
  - $(x_1, x_2, Q^2) \xrightarrow{N_x, N_{Q^2}} \left[x_1^m, x_1^n, Q^{2k}\right]$
  - variable transformation

$$y(x) = \ln\frac{1}{x} + a(1-x); \quad \tau(Q^2) = \ln\left(\ln\frac{Q^2}{\Lambda^2}\right)$$

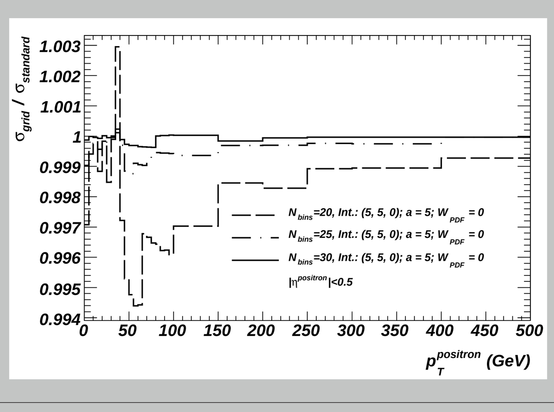
facilitates fine binning in regions of the phase-space where PDFs are quickly changing

- Interpolation (to reduce the number of bins)
- Initial flavour decomposition, $N_{flavours} \times N_{flavours} \to \mathcal{L}$, exploiting symmetry of underlying production process : $q_i(x_1, Q_F^2) q_j(x_2, Q_F^2) \to F^{(l)}(x_1, x_2, Q^2)$

Step 2

Convolute grid with PDF's .

- integral $\longrightarrow$ sum
- any coupling, pdf



The result

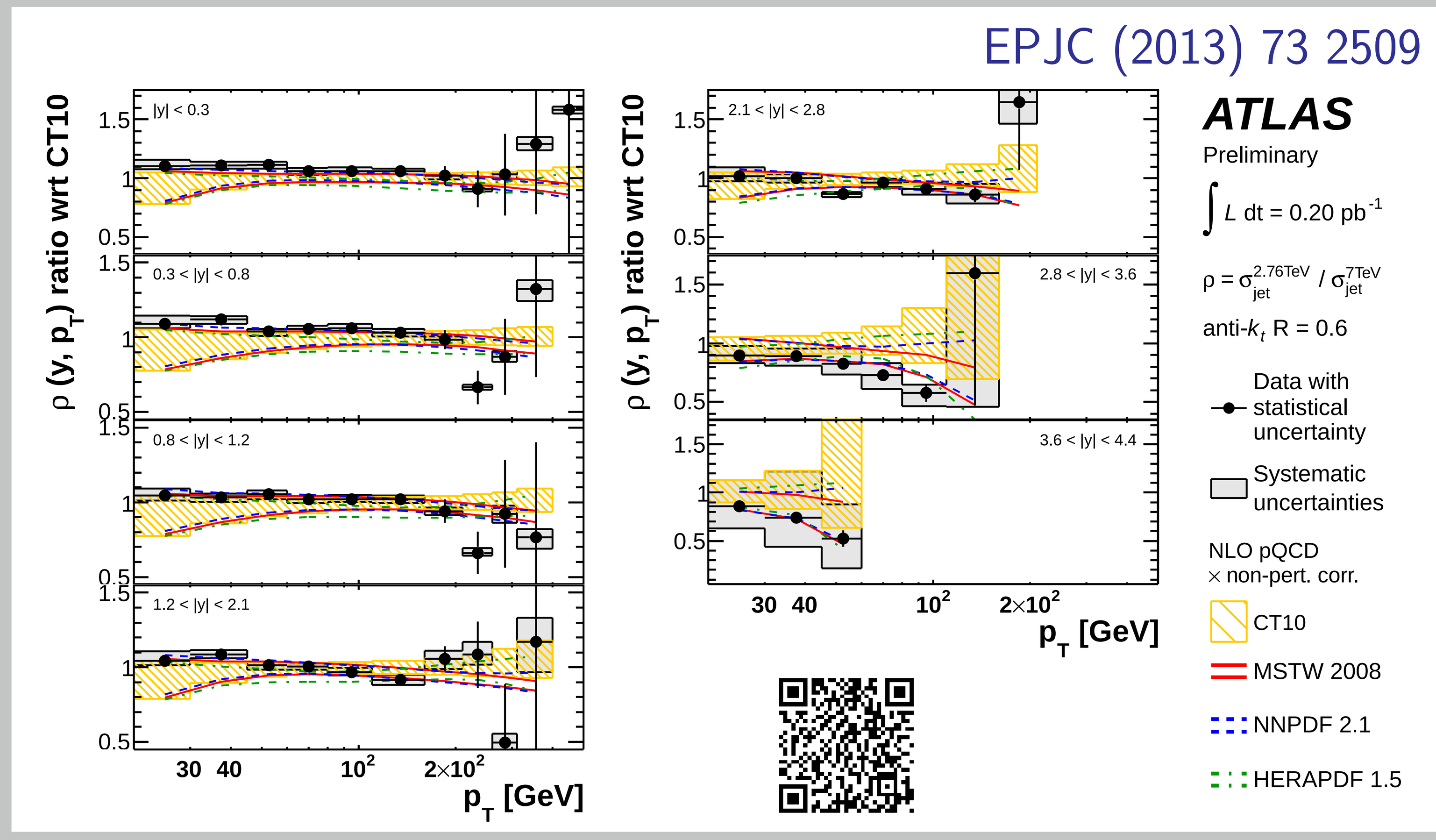
$$\sum_p \sum_{l=0}^{L} \sum_{m,n,k} w^{(p)(l)}{}_{m,n,k} \left(\frac{\alpha_s(Q_k^2)}{2\pi}\right)^{p_l} F^{(l)}(x_{1m}, x_{2n}, Q_k^2) \to \frac{d\sigma}{dX}$$

Production processes implemented

Calculations are performed via the interface to Monte Carlo calculators

- NLOJET++ : jet production in **ep** and $pp(\bar{p})$ collisions
- MCFM
  - $W^\pm, Z^0$
  - $W^\pm$+charm, $W^\pm/Z^0/\gamma$+jet
  - $Q\bar{Q}(c\bar{c}, b\bar{b}, t\bar{t})$
  - $t\bar{t} \to b\ell^+\nu_\ell + b\ell^-\bar{\nu}_\ell$, $t\bar{t} \to b\ell^+\nu_\ell + bq\bar{q}$, $t\bar{t} \to bq\bar{q} + b\ell^-\bar{\nu}_\ell$

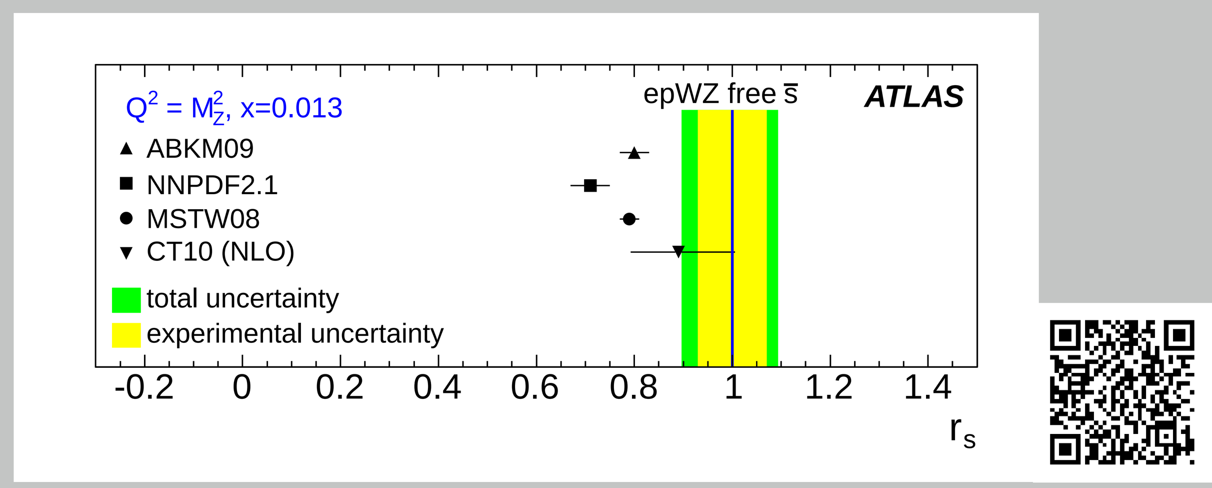
Use-case : Data unterpretation



ATLAS measurement of ratio of the inclusive jet cross-section at $\sqrt{s} = 2.76$ TeV to the one at $\sqrt{s} = 7$ TeV, shown as double ratio to the theoretical prediction calculated with the CT10 PDFs as a function of the jet p_T in bins of jet rapidity, for anti-kt jets with R=0.6. The theory uncertainties are calculated using APPLgrid interface to NLOJET++.

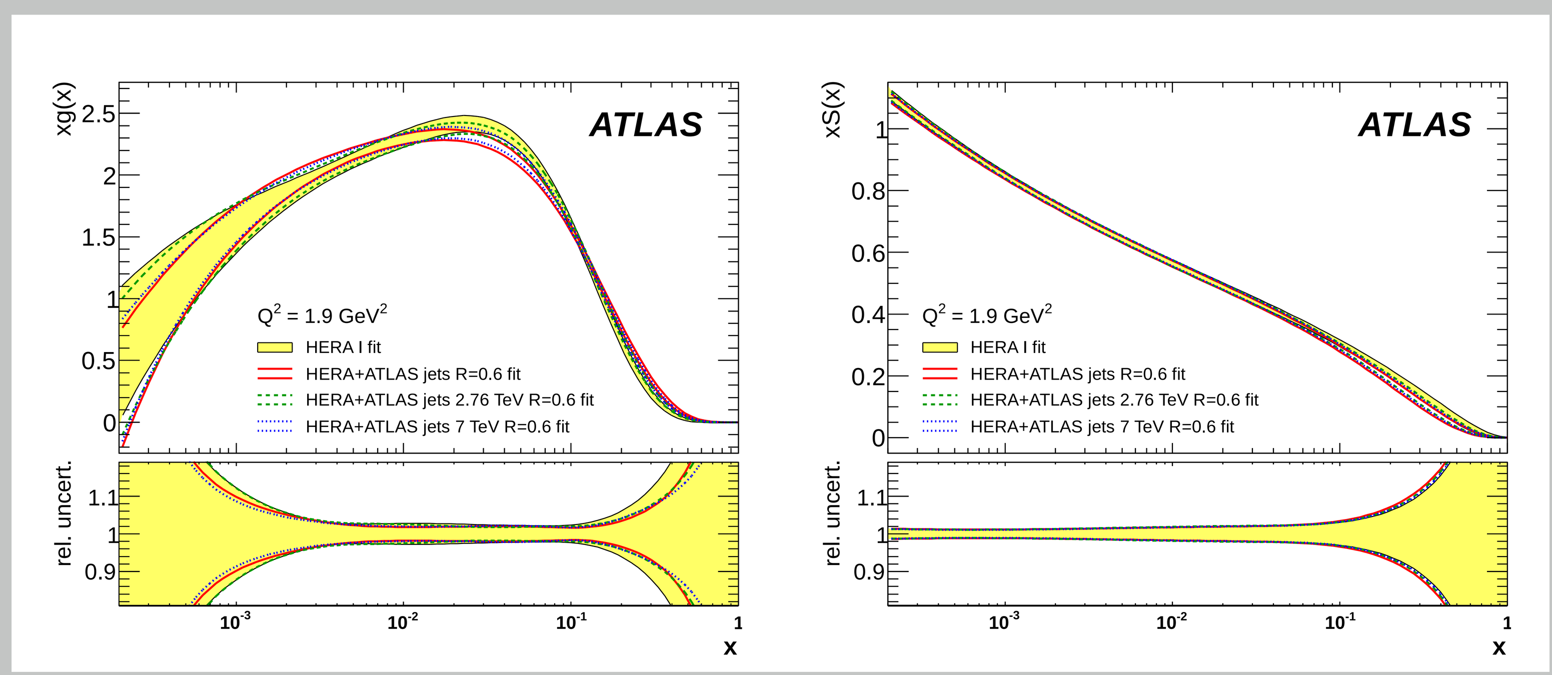
Use-case : PDF fits

Phys.Rev.Lett. 109 (2012) 012001



Determination of the strange quark density of the proton from ATLAS measurements of **W** and **Z** cross sections

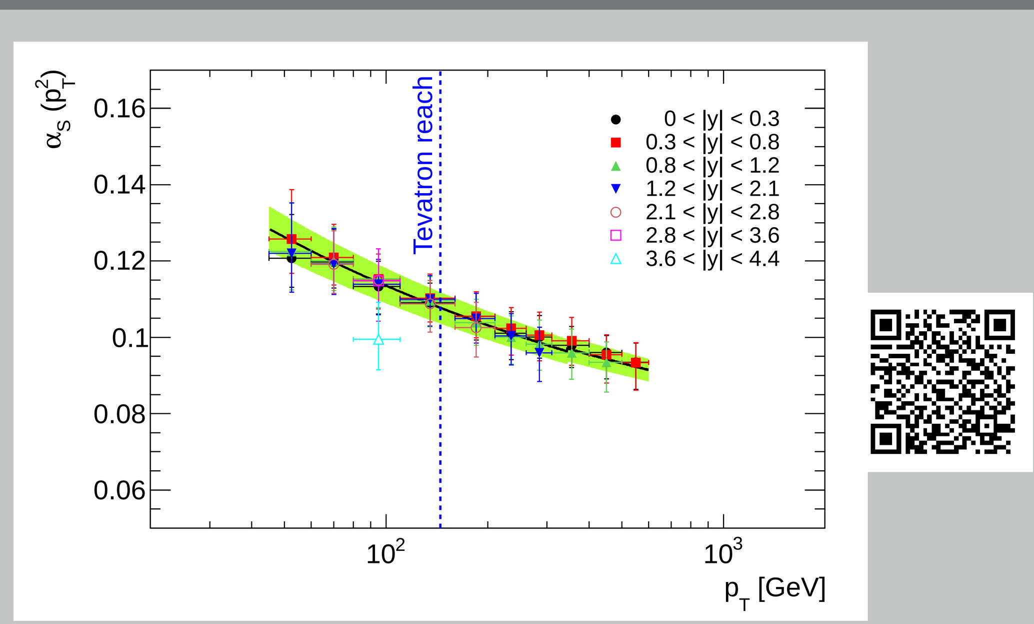
EPJC (2013) 73 2509



Momentum distributions of the gluon xg(x) together with their relative experimental uncertainty as a function of x for $Q^2 = 1.9\text{GeV}^2$. Fit is performed witin HERAFITTER framework.

Use-case : Strong coupling determination

EPJC (2012) 72 2041



Determination of strong coupling from ATLAS inclusive jet measurements

Summary

- APPLGrid is an open project, source code is available as HEPforge package:https://projects.hepforge.org/applgrid Eur.Phys.J. C66 (2010) 503-524
- A list of QCD and electroweak processes can be studied
  - Jet production cross sections using NLOJET++
  - Electroweak observables using MCFM
- Efficient a posteriori evaluation of uncertainties from renormalisation and factorisation scale variations, strong coupling measurement and PDFs error sets. A posteriori centre-of-mass energy rescaling
- Allows rigorous inclusion of NLO cross-sections in a fit.