

Kaon Freeze-out Dynamics in $\sqrt{s_{NN}}=200$ GeV Au+Au Collisions at RHIC*

Michal Šumbera

sumbera@ujf.cas.cz



Nuclear Physics Institute
Czech Academy of Sciences

for the



collaboration

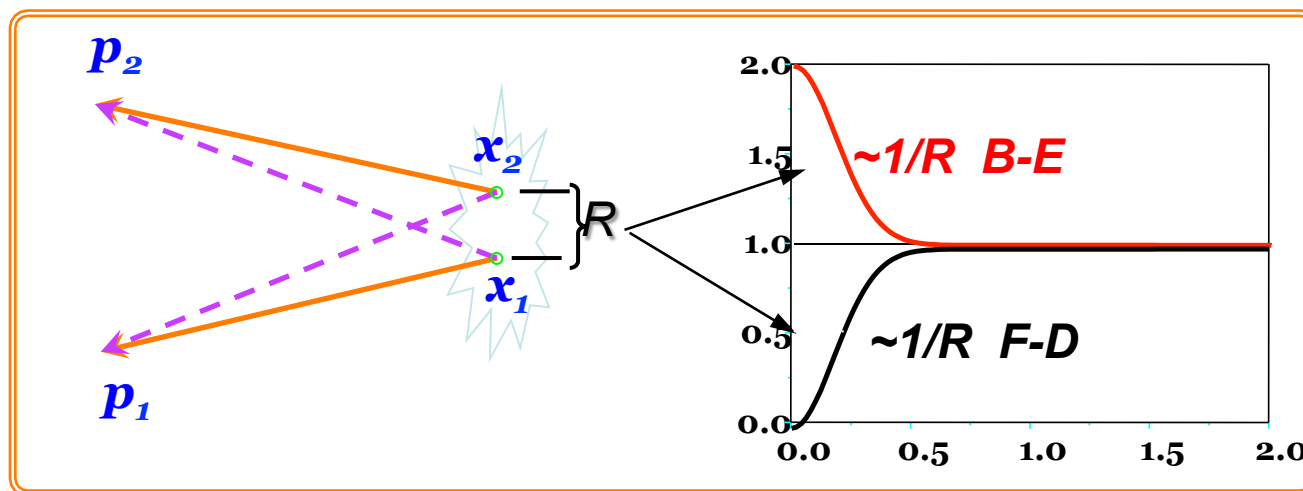
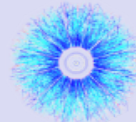
*) [arXiv:1302.3168 \[nucl-ex\]](https://arxiv.org/abs/1302.3168) accepted in Phys. Rev. C

XLIII International Symposium on Multiparticle Dynamics

September 15-20, 2013 Illinois Institute of Technology, Chicago, IL



Correlation femtoscopy in a nutshell (1/3)



$$C_{\vec{P}}^{ab}(\vec{q}) = \frac{d^6 N^{ab} / (dp_a^3 dp_b^3)}{(d^3 N^a / dp_a^3)(d^3 N^b / dp_b^3)}$$

$$\vec{P} = \vec{p}_a + \vec{p}_b$$

$$\vec{q} = \frac{1}{2}(\vec{p}_a - \vec{p}_b)$$

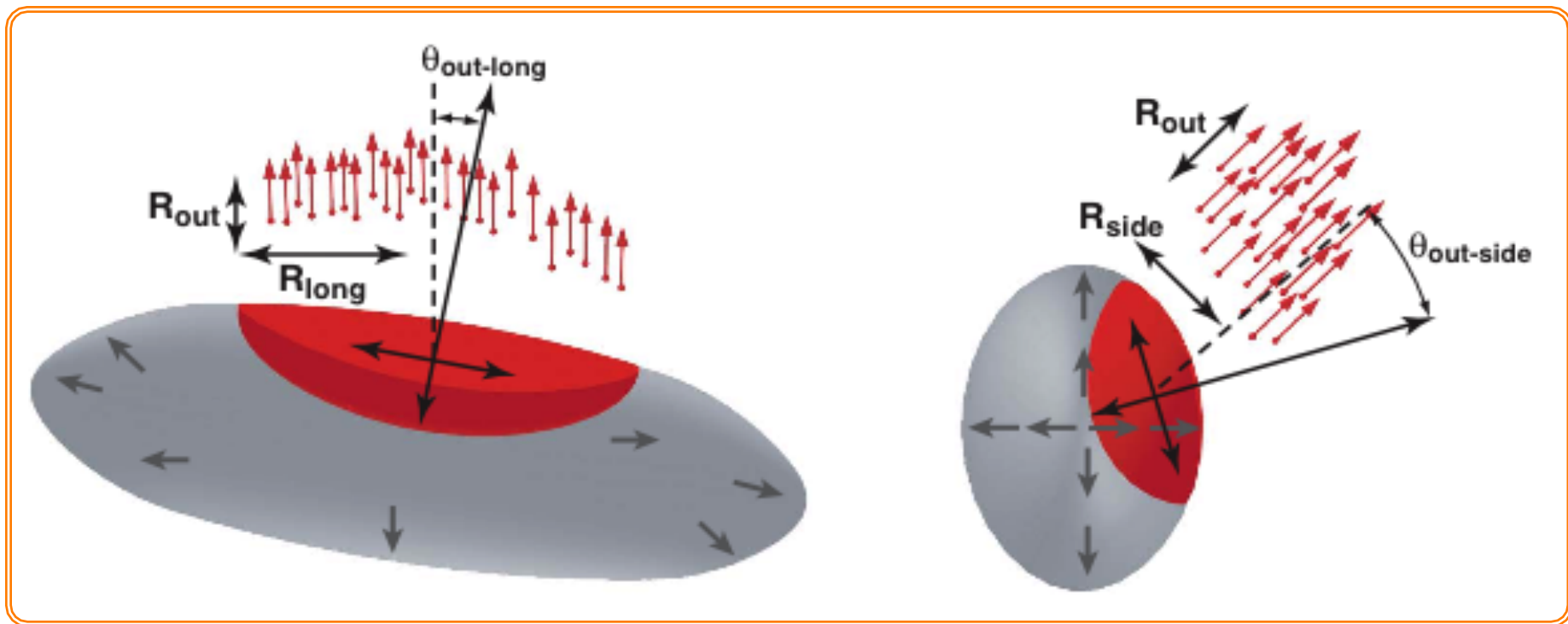
Correlation function of two identical bosons/fermions at small momentum difference q shows effect of quantum statistics.

Height/depth of the B-E/F-D bump λ is related to the fraction ($\lambda^{1/2}$) of particles participating in the enhancement.

Its width scales with the emission radius as R^{-1} .

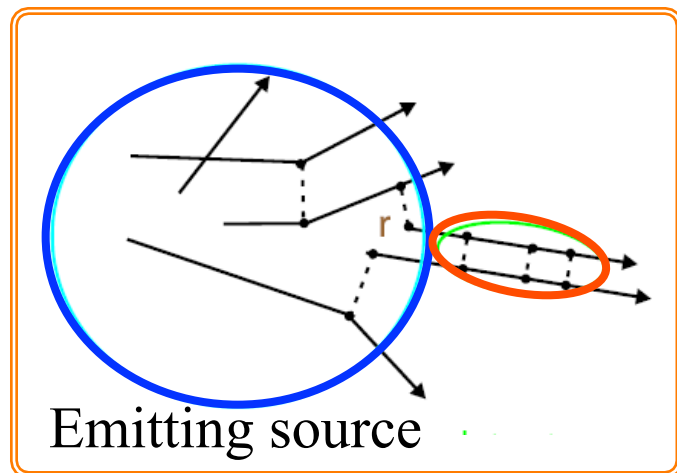
Correlation femtoscopy in a nutshell (2/3)

The correlation is determined by the size of **region** from which particles with roughly the same velocity are emitted



⇒ Femtoscopy measures size, shape, and orientation of **homogeneity regions**

Correlation femtoscopy in a nutshell (3/3)



1D Koonin-Pratt equation

$$C(q) - 1 = 4\pi \int K(q, r) S(r) r^2 dr$$

Encodes FSI

Correlation
function

Source function
(Distribution of pair
separations in the
pair rest frame)

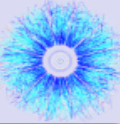
Kernel $K(q, r)$ is independent of freeze-out conditions

$S(r)$ is often assumed to be Gaussian \Rightarrow HBT radii

Other option: Inversion of linear integral equation to
obtain source function

\Rightarrow Model-independent analysis of emission shape
(goes beyond Gaussian shape assumption)

Source Imaging



Technique devised by
 D. Brown and P. Danielewicz
 PLB398:252, 1997
 PRC57:2474, 1998

Geometric information from imaging.

$$R(q) = \int K(q, r) S(r) r^2 dr$$

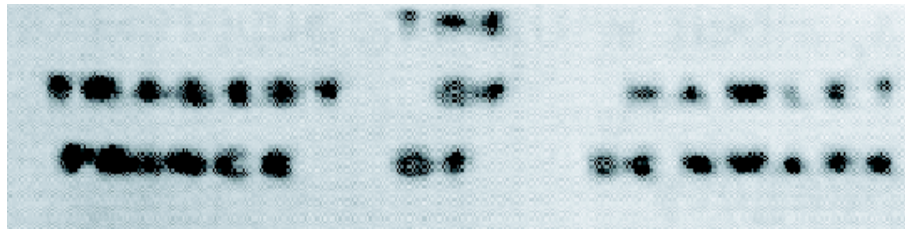
General task:

From data w/errors, $R(q)$, determine the source $S(r)$.

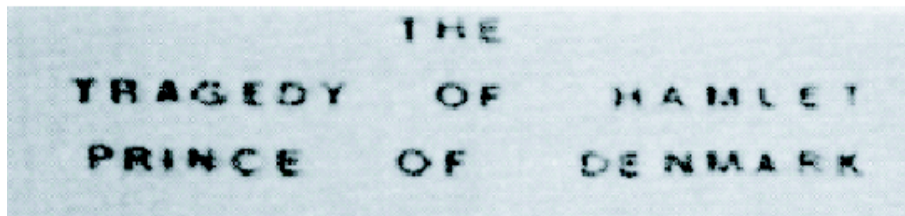
Requires inversion of the kernel K .

Optical recognition: K - blurring function, max entropy method

R :

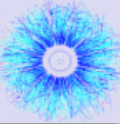


S :



Any determination of source characteristics from data, unaided by reaction theory, is an imaging.

Inversion procedure



$$R(q) \equiv C(q) - 1 = 4\pi \int dr r^2 K(q, r) S(r)$$

$$K(q, r) = \frac{1}{2} \int d\cos\theta_{\vec{q}, \vec{r}} \left[|\phi(\vec{q}, \vec{r})|^2 - 1 \right]$$

Freeze-out occurs after the last scattering. \Rightarrow Only Coulomb & quantum statistics effects included in the kernel.

Expand into B-spline basis

$$S(r) = \sum_j S_j \cdot B_j(r)$$

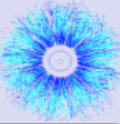
$$C^{Th}(q_i) = \sum_j K_{ij} \cdot S_j$$

$$K_{ij} = \int dr \cdot K(q_i, r) B_j(r)$$

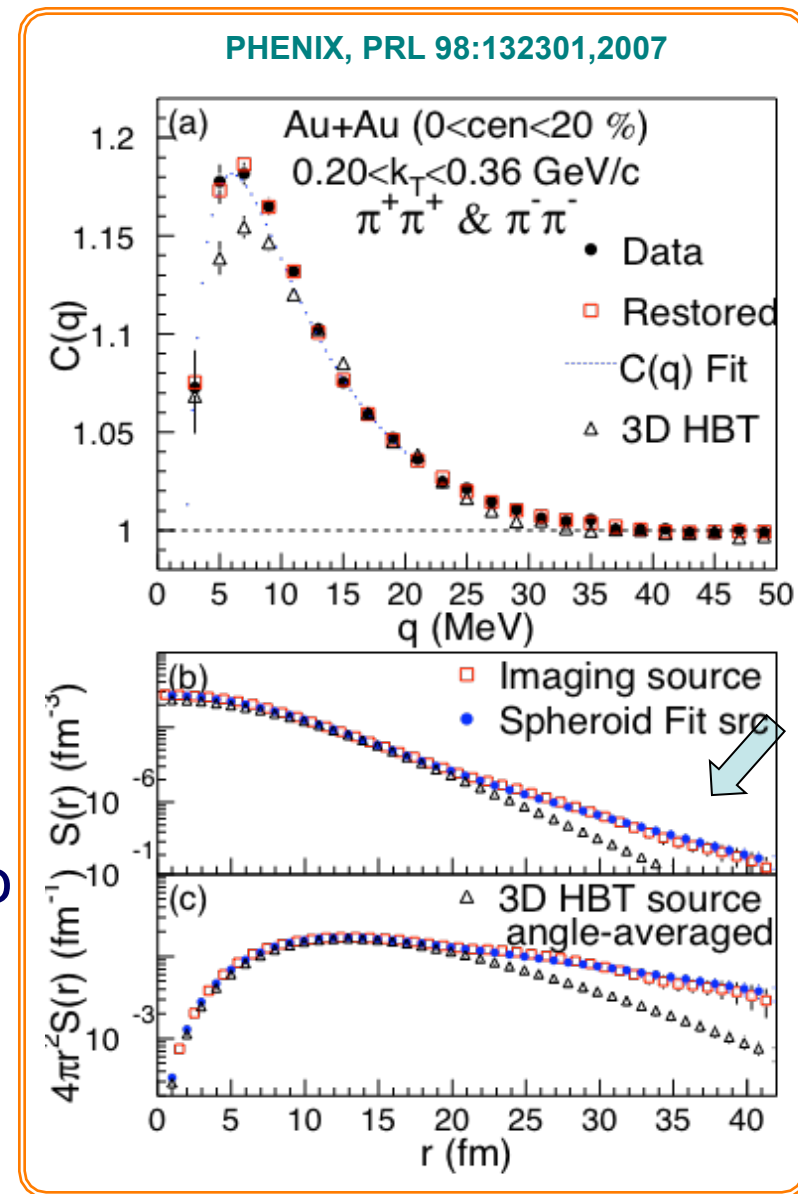
Vary S_j to minimize χ^2

$$\chi^2 = \frac{\left(C^{Expt}(q_i) - \sum_j K_{ij} \cdot S_j \right)^2}{\left(\Delta C^{Expt}(q_i) \right)^2}$$

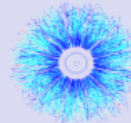
Why Kaons?



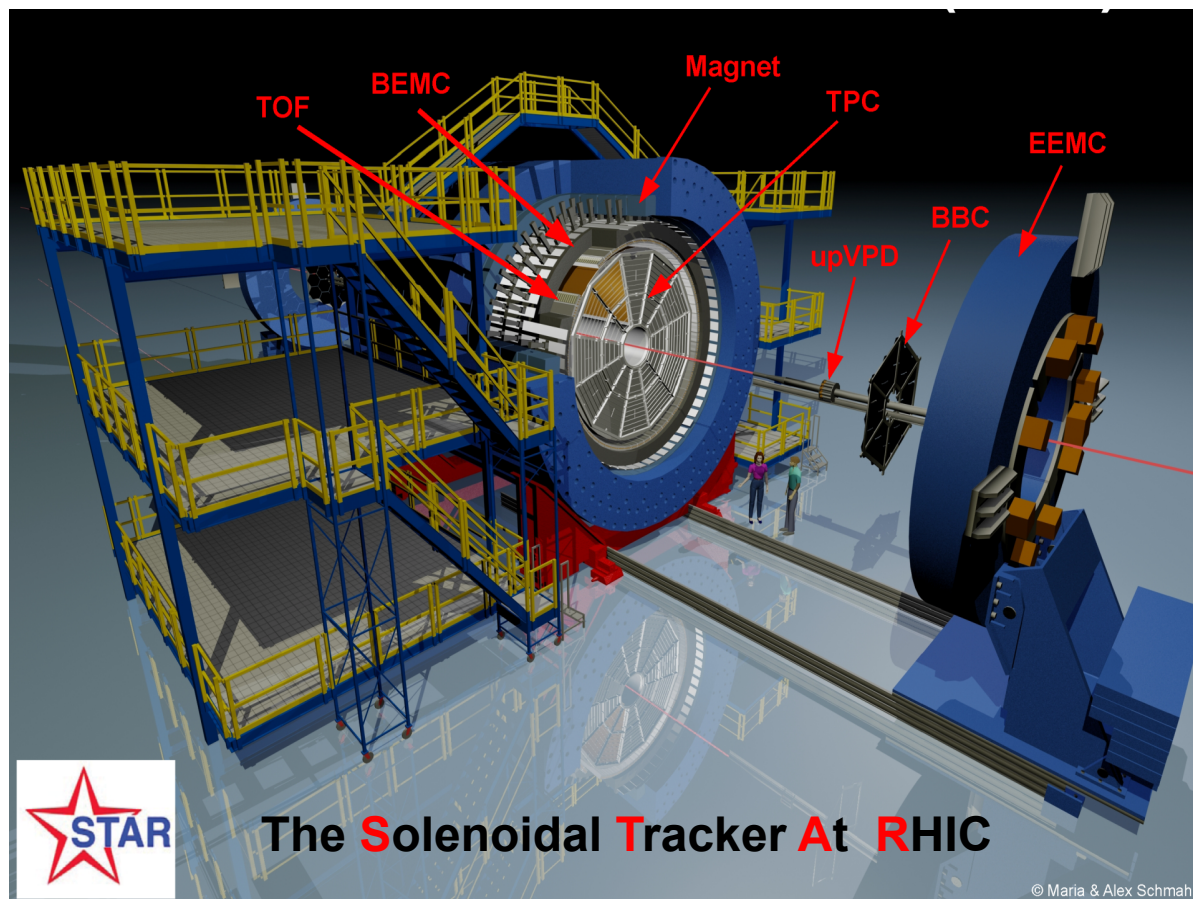
- Pion source shows a **heavy, non-Gaussian tail**
- Interpretation is problematic
 - Tail attributed to decays of long-lived resonances, non-zero emission duration etc.
- Kaons: cleaner probe
 - less contribution from resonances
- PHENIX 1D kaon result shows also a long non-Gaussian tail



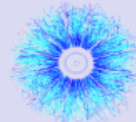
The STAR Experiment



- **Time Projection Chamber**
 - ID via energy loss (dE/dx)
 - Momentum (p)
- Full azimuth coverage
- Uniform acceptance for different energies and particles



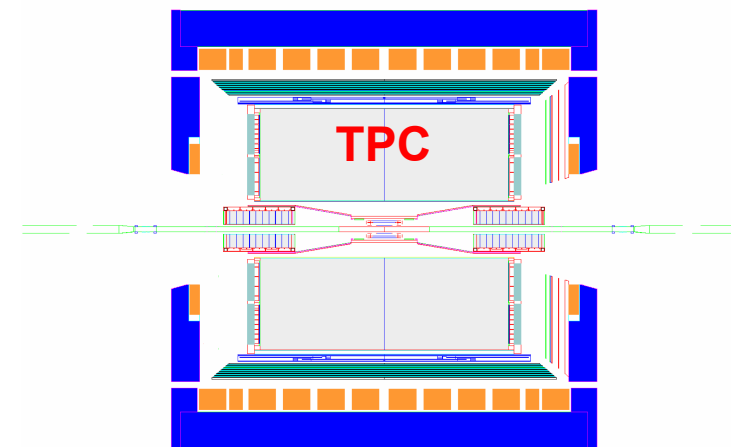
Kaon femtoscopy analyses



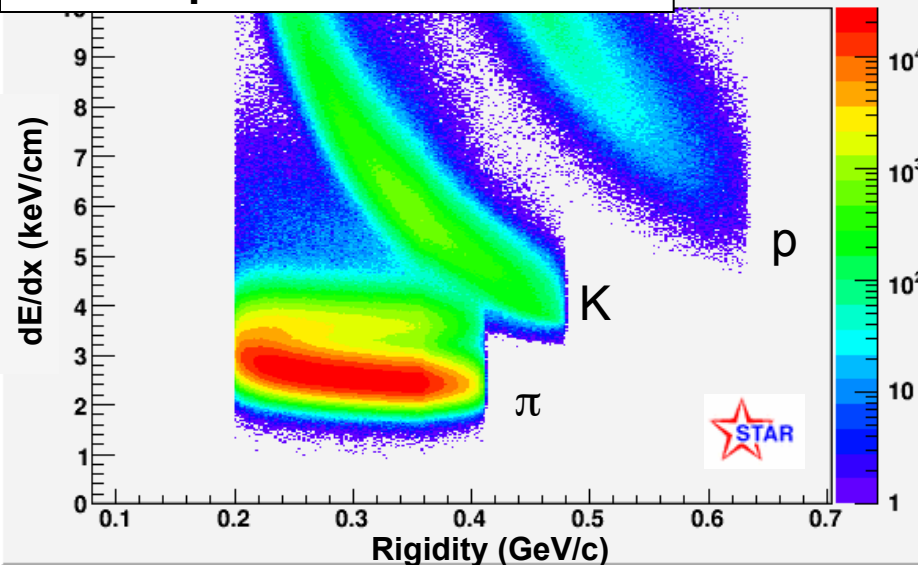
Au+Au @ $\sqrt{s_{NN}}=200$ GeV

Mid-rapidity $|y|<0.5$

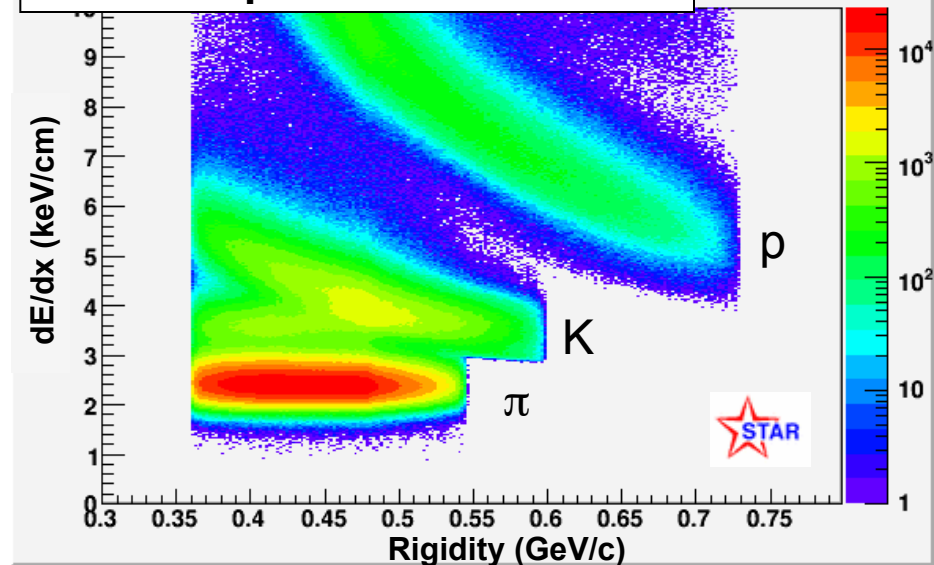
1. Source shape: 20% most central
Run 4: 4.6 Mevts, Run 7: 16 Mevts
2. m_T -dependence: 30% most central
Run 4: 6.6 Mevts



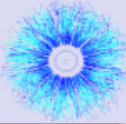
$0.2 < k_T < 0.36$ GeV/c



$0.36 < k_T < 0.48$ GeV/c



PID cut applied



1. Source shape analysis

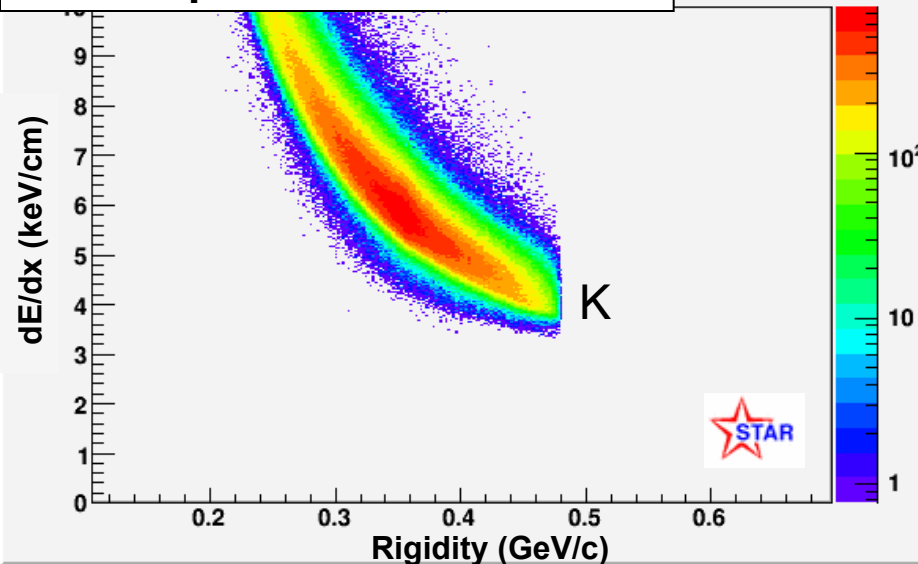
- dE/dx : $n\sigma(\text{Kaon}) < 2.0$ and $n\sigma(\text{Pion}) > 3.0$ and $n\sigma(\text{electron}) > 2.0$
 $n\sigma(X)$: deviation of the candidate dE/dx from the normalized distribution of particle type X at a given momentum
- $0.2 < p_T < 0.4 \text{ GeV}/c$

2. m_T -dependent analysis

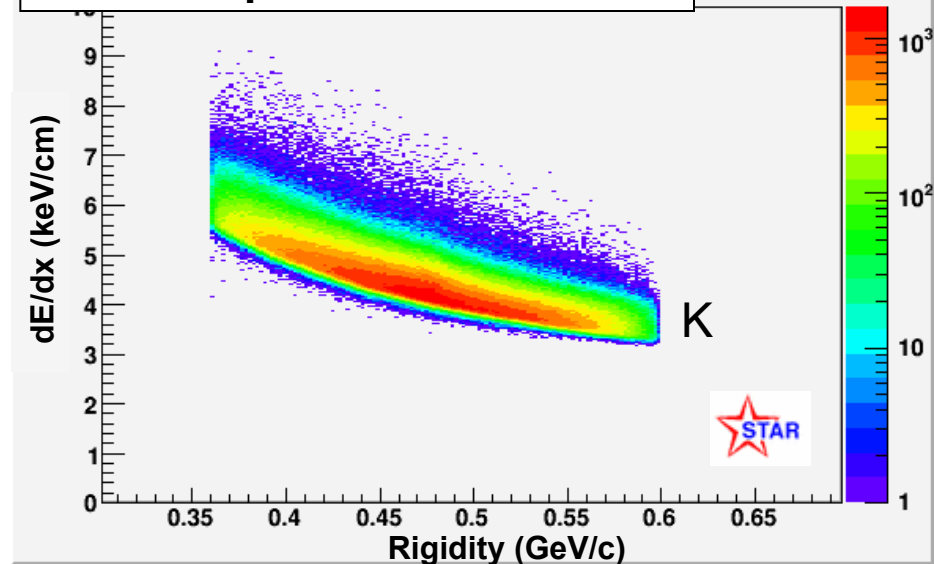
$$-1.5 < n\sigma(\text{Kaon}) < 2.0$$

$$-0.5 < n\sigma(\text{Kaon}) < 2.0$$

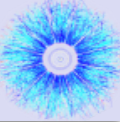
$0.2 < k_T < 0.36 \text{ GeV}/c$



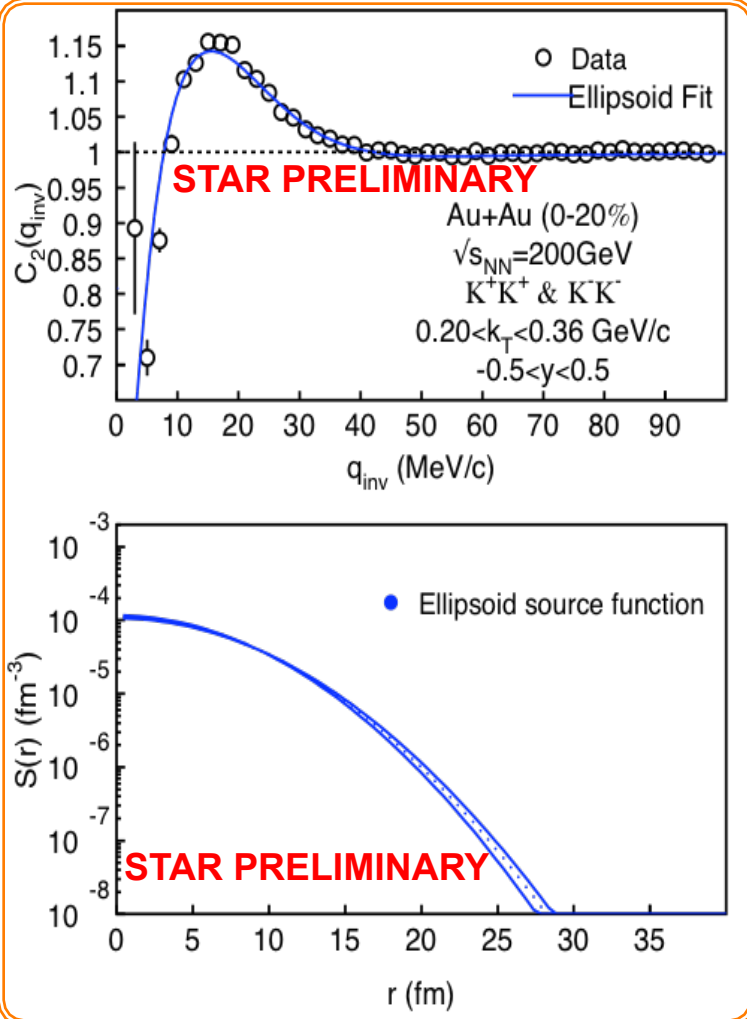
$0.36 < k_T < 0.48 \text{ GeV}/c$



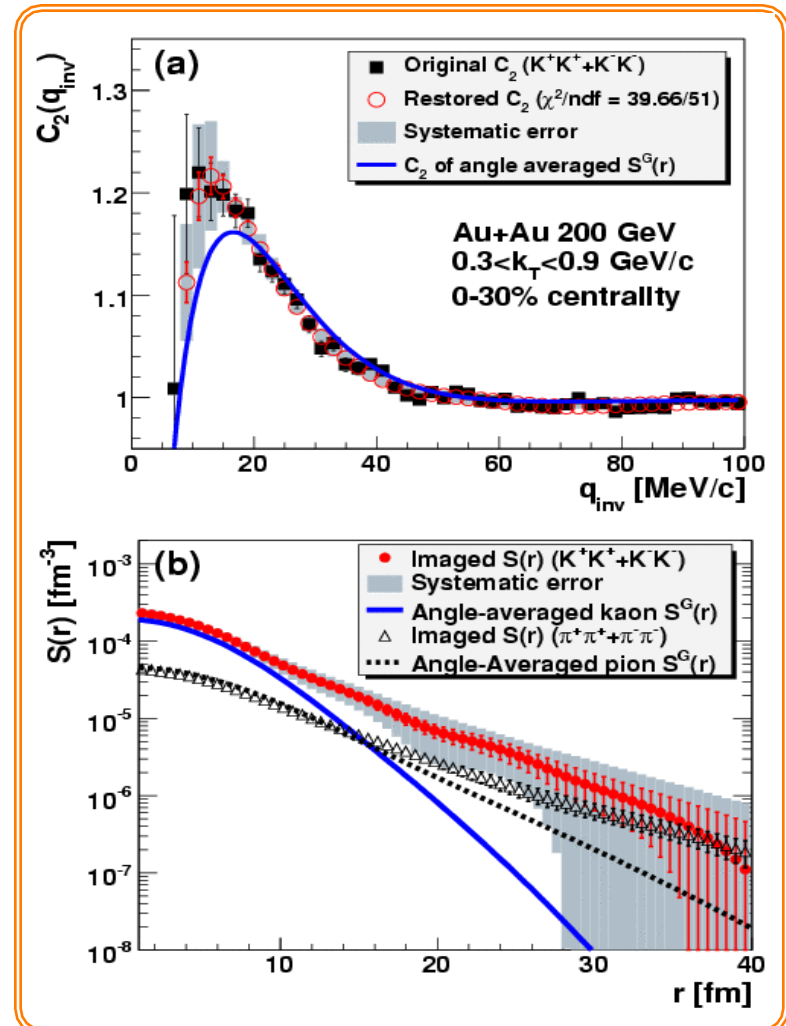
1D analysis result



34M+83M=117M (K^+K^+ & K^-K^-) pairs

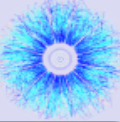


PHENIX, PRL 103:,142301, 2009



STAR data well described by a single Gaussian. Contrary to PHENIX no non-gaussian tails observed. May be due to a different k_T -range: STAR bin is 4x narrower.

3D source shapes



Expansion of $R(\mathbf{q})$ and $S(\mathbf{r})$ in Cartesian Harmonic basis

Danielewicz and Pratt, Phys.Lett. B618:60, 2005

$$R(\mathbf{q}) = \sum_l \sum_{\alpha_1 \dots \alpha_l} R_{\alpha_1 \dots \alpha_l}^l(q) A_{\alpha_1 \dots \alpha_l}^l(\Omega_q) \quad (1)$$

$$S(\mathbf{r}) = \sum_l \sum_{\alpha_1 \dots \alpha_l} S_{\alpha_1 \dots \alpha_l}^l(r) A_{\alpha_1 \dots \alpha_l}^l(\Omega_q) \quad (2)$$

$\alpha_i = \mathbf{x}, \mathbf{y}$ or \mathbf{z}

$\mathbf{x} = \text{out-direction}$

$\mathbf{y} = \text{side-direction}$

$\mathbf{z} = \text{long-direction}$

3D Koonin-Pratt:

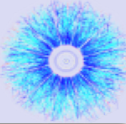
$$R(\mathbf{q}) = C(\mathbf{q}) - 1 = 4\pi \int dr^3 K(\mathbf{q}, \mathbf{r}) S(\mathbf{r}) \quad (3)$$

$$\text{Plug (1) and (2) into (3)} \Rightarrow R_{\alpha_1 \dots \alpha_l}^l(q) = 4\pi \int dr^3 K_l(q, r) S_{\alpha_1 \dots \alpha_l}^l(r) \quad (4)$$

$$\text{Invert (1)} \Rightarrow R_{\alpha_1 \dots \alpha_l}^l(q) = \frac{(2l+1)!!}{l!} \int \frac{d\Omega_q}{4\pi} A_{\alpha_1 \dots \alpha_l}^l(\Omega_q) R(\mathbf{q})$$

$$\text{Invert (2)} \Rightarrow S_{\alpha_1 \dots \alpha_l}^l = \frac{(2l+1)!!}{l!} \int \frac{d\Omega_q}{4\pi} A_{\alpha_1 \dots \alpha_l}^l(\Omega_q) S(\mathbf{q})$$

Shape analysis



- $\ell=0$ moment agrees 1D $C(q)$
Higher moments relatively small
- Trial function form for $S(r)$:
4-parameter ellipsoid (3D Gauss)

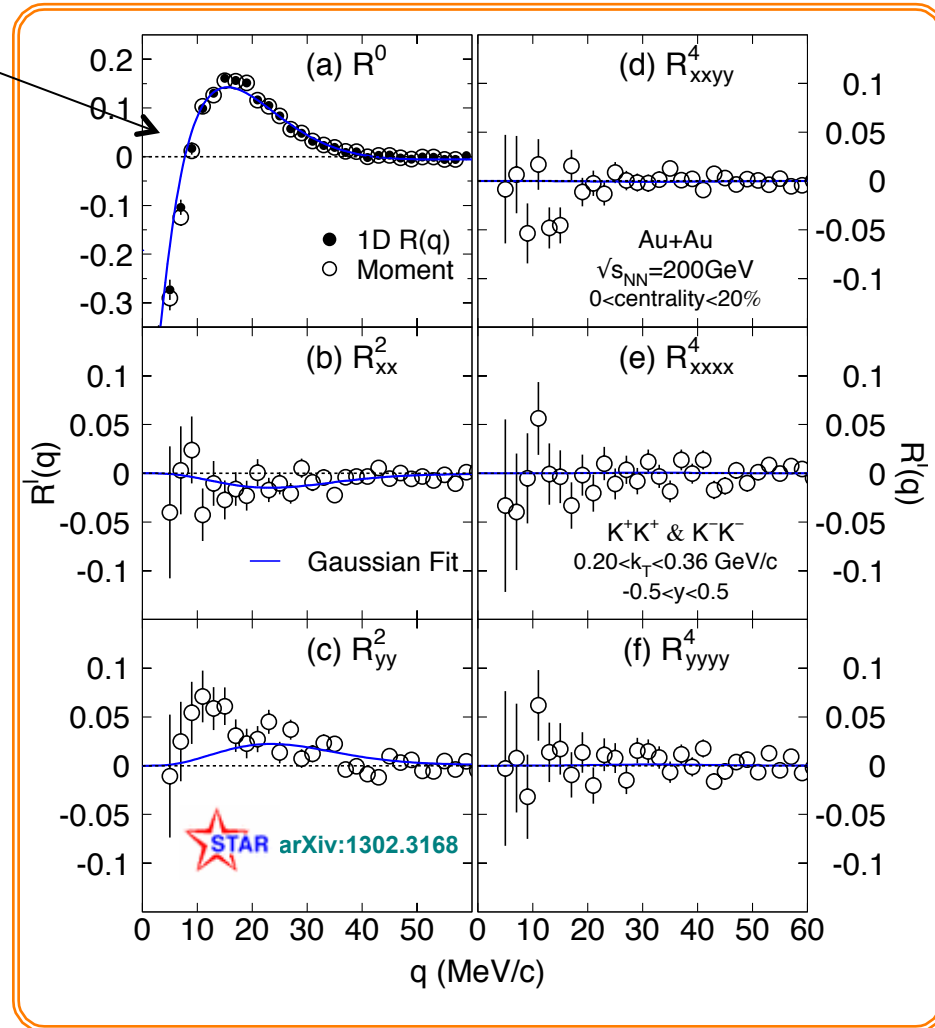
$$S^G(x, y, z) \equiv \frac{\lambda}{(2\sqrt{\pi})^3 r_x r_y r_z} \exp\left[-\left(\frac{x^2}{4r_x^2} + \frac{y^2}{4r_y^2} + \frac{z^2}{4r_z^2}\right)\right]$$

- Fit to $C(q)$: technically a simultaneous fit on 6 independent moments

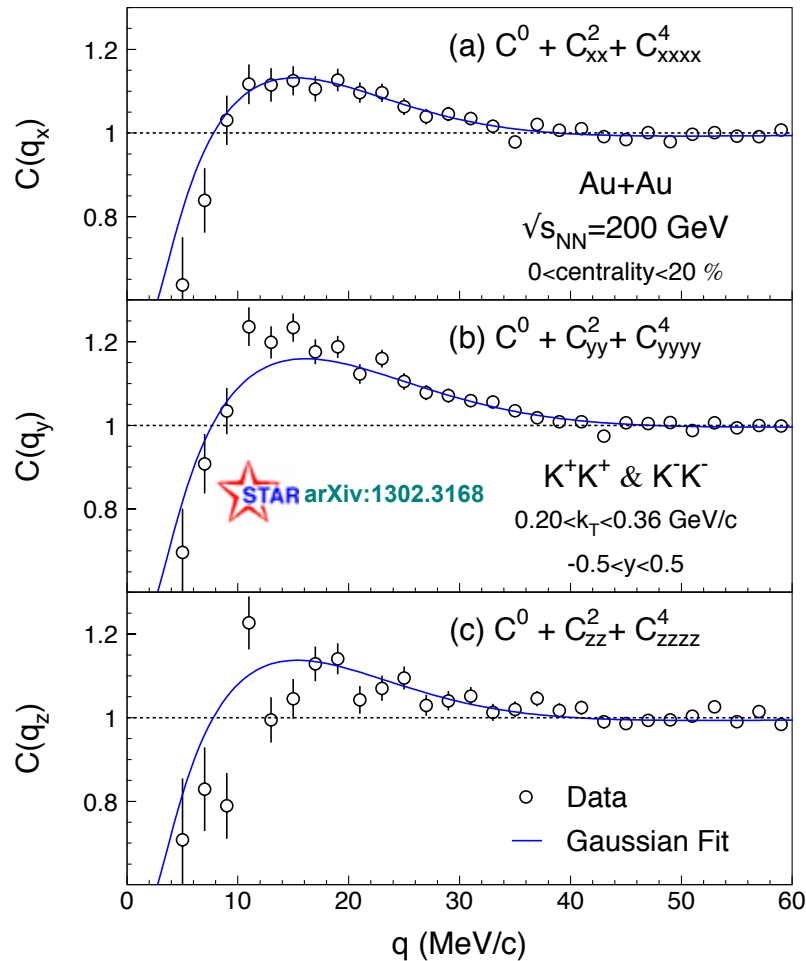
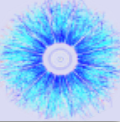
$$R_{\alpha_1 \dots \alpha_\ell}^{\ell}, \quad 0 \leq \ell \leq 4$$

- Result: statistically good fit

Run4+Run7	$\lambda = 0.48 \pm 0.01$
200 GeV Au+Au	$r_x = (4.8 \pm 0.1) \text{ fm}$
Centrality <20%	$r_y = (4.3 \pm 0.1) \text{ fm}$
$0.2 < k_T < 0.36 \text{ GeV}/c$	$r_z = (4.7 \pm 0.1) \text{ fm}$



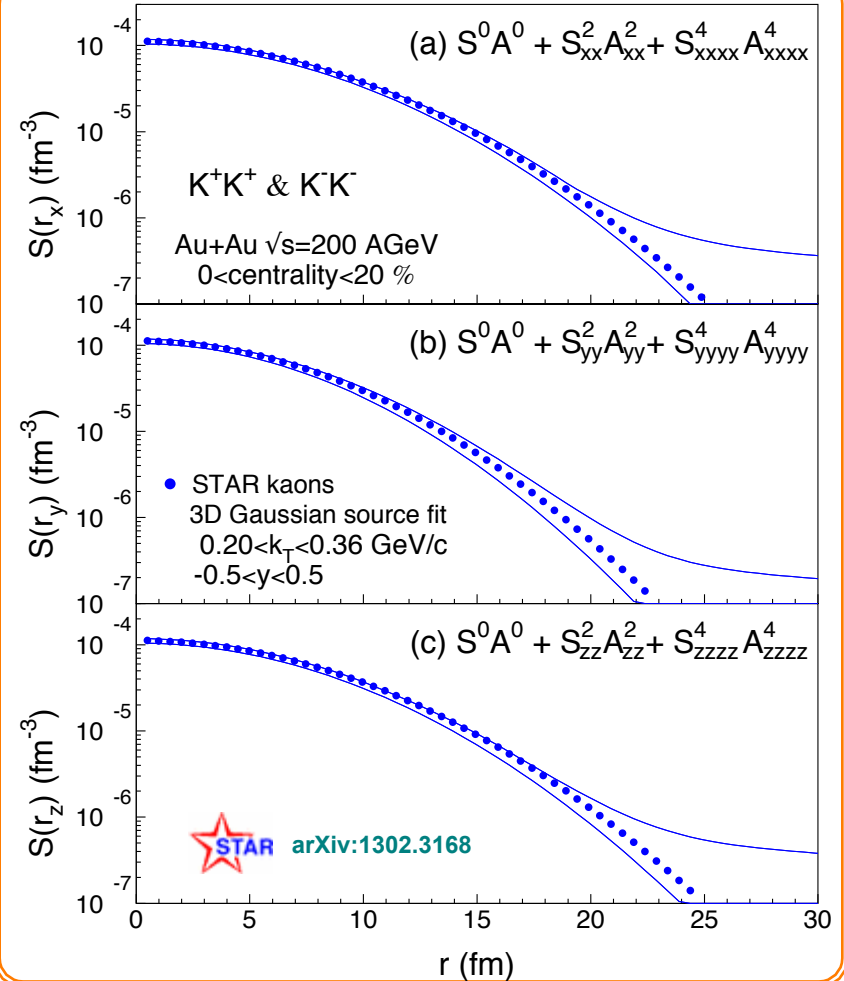
Correlation profiles and source



$$C(q_x) \equiv C(q_x, 0, 0)$$

$$C(q_y) \equiv C(0, q_y, 0)$$

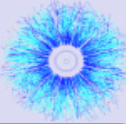
$$C(q_z) \equiv C(0, 0, q_z)$$



Gaussian source fit with error band

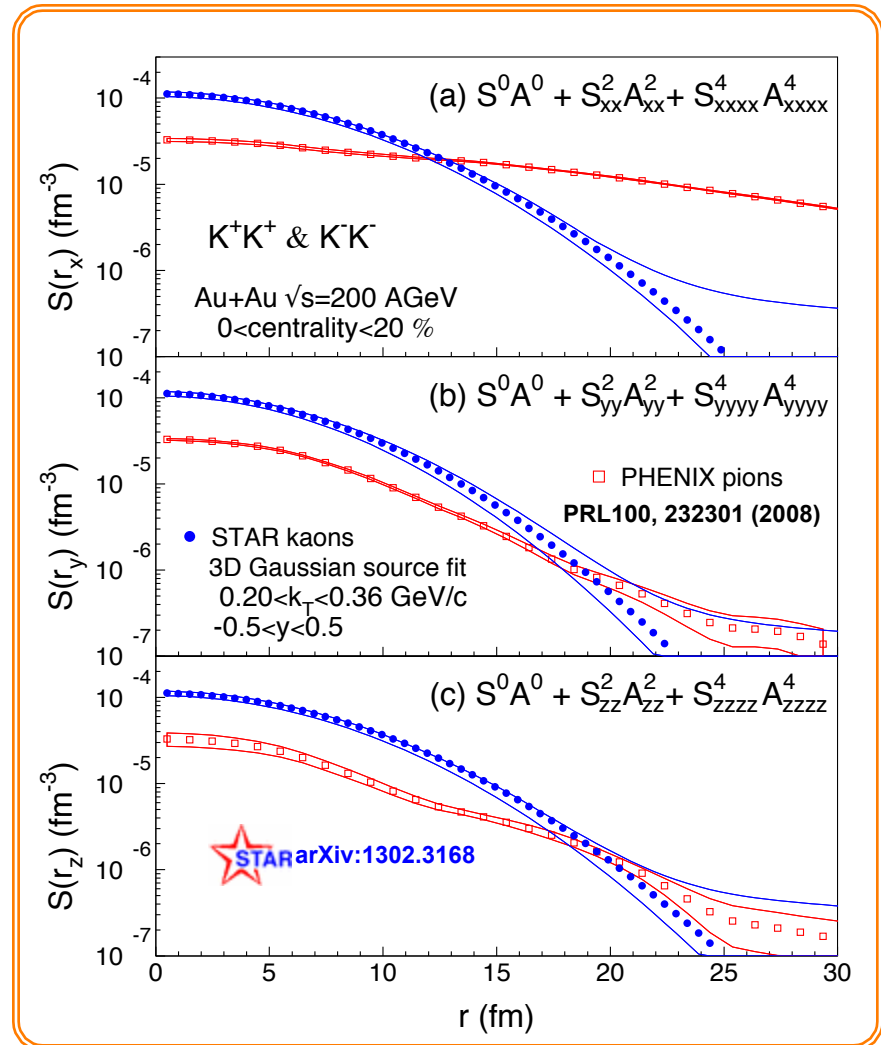
N.B.: Low statistics shows up as systematic uncertainty on shape assumption

Source: Data comparison



kaon vs. pion: different shape

- Long pion tail caused by resonances and/or emission duration?
- Sign of different freeze-out dynamics?



Source: Model comparison

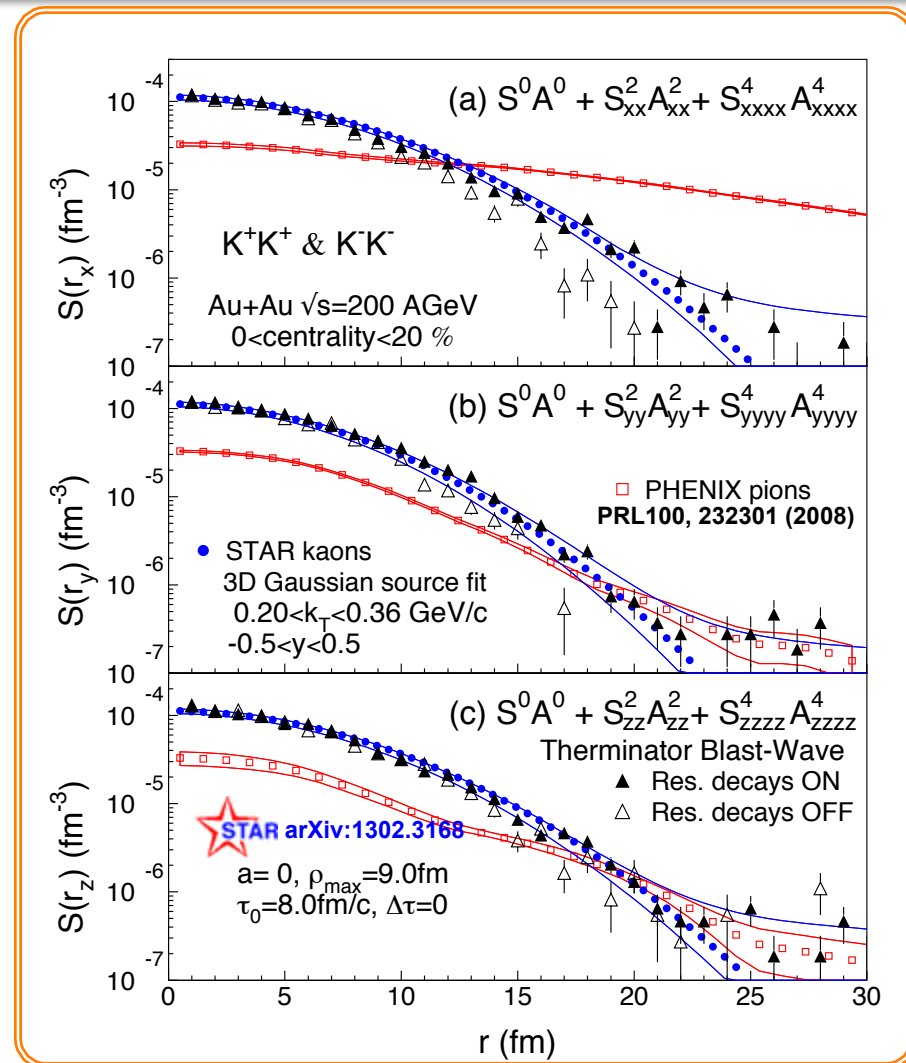


Therminator

- Blast-wave model (STAR tune):
 - Expansion: $v_t(\rho) = (\rho/\rho_{\max}) / (\rho/\rho_{\max} + v_t)$
 - Freeze-out occurs at $\tau = \tau_0 + ap$.
 - Finite emission duration $\Delta\tau$
- Kaons: Instant freeze-out ($\Delta\tau = 0$, compare to $\Delta\tau \sim 2$ fm/c of pions) at $\tau_0 = 0.8$ fm/c
- Resonances are needed for proper description

Hydrokinetic model

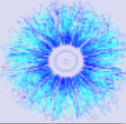
- Hybrid model
 - Glauber initial+Hydro+UrQMD
- Consistent in “side”
- Slightly more tail ($r > 15$ fm) in “out” and “long”



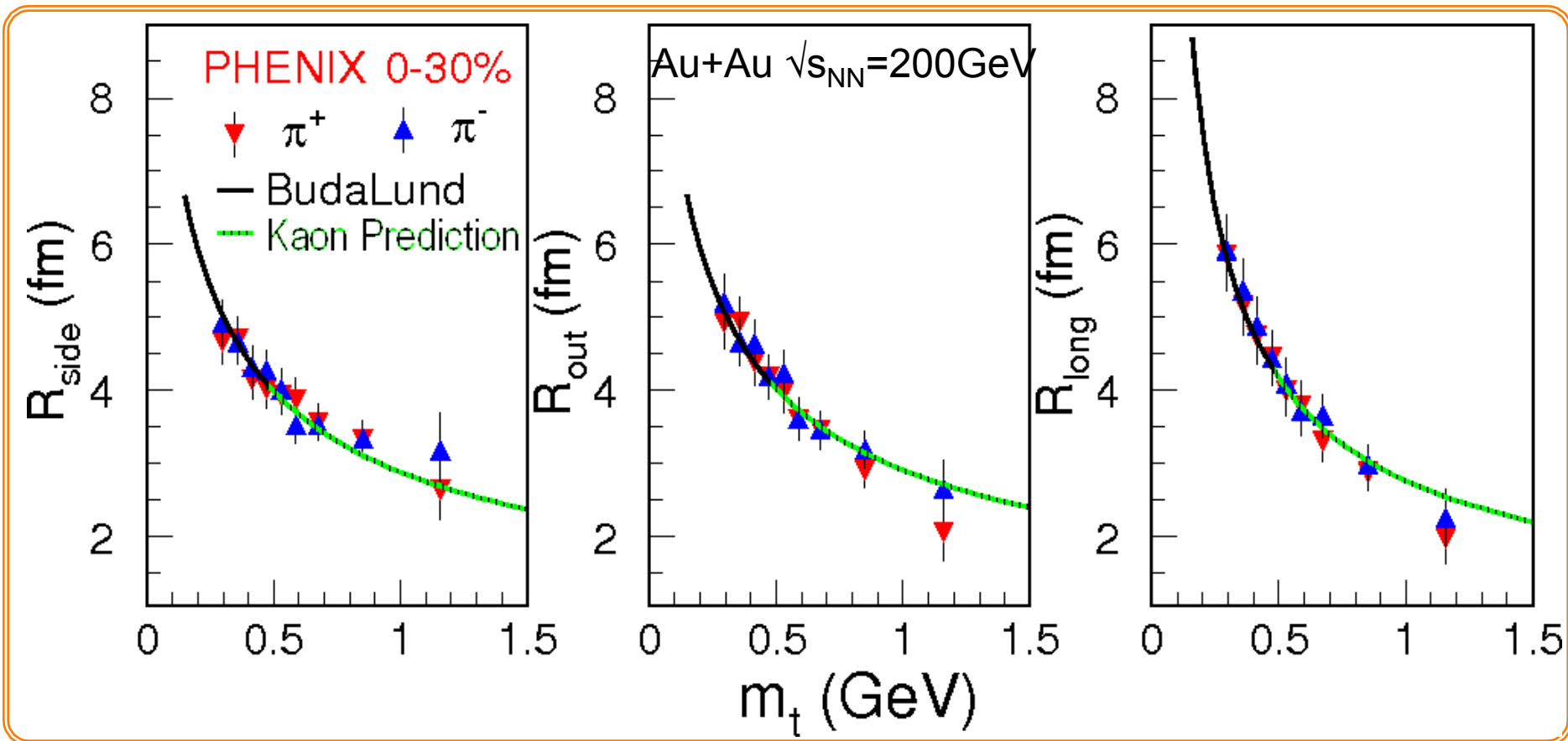
Therminator: Kisiel, Taluc, Broniowski, Florkowski,
 Comput. Phys. Commun. 174 (2006) 669.

HKM: PRC81, 054903 (2010)
 data from Shapoval, Sinyukov, private communication

RHIC pion radii and perfect fluid hydrodynamics



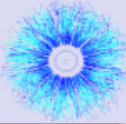
M. Csanád and T. Csörgő: arXiv:0800.0801[nucl-th]



Excellent description of PHENIX pion data (PRL 93:152302, 2004) using exact solutions of perfect fluid hydrodynamics (Buda-Lund).

Ideal hydro has inherent m_T -scaling \Rightarrow predicts kaon radii m_T -dependence

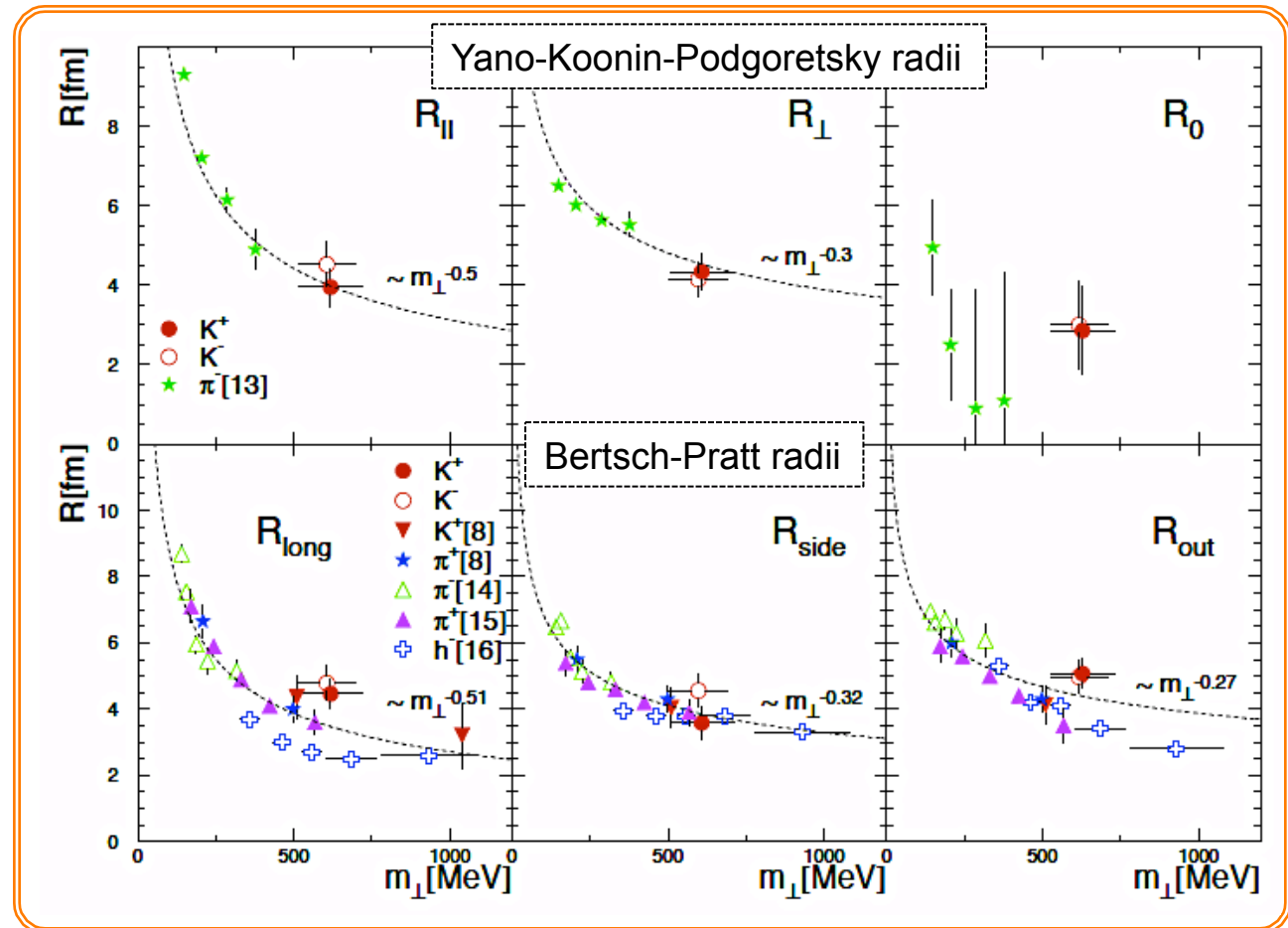
SPS results on pions and kaons



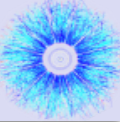
S.V. Afanasiev *et al.* (NA49 Coll.): Phys. Lett B557(2003)157

- “The kaon radii are fully consistent with pions and the hydrodynamic expansion model.”

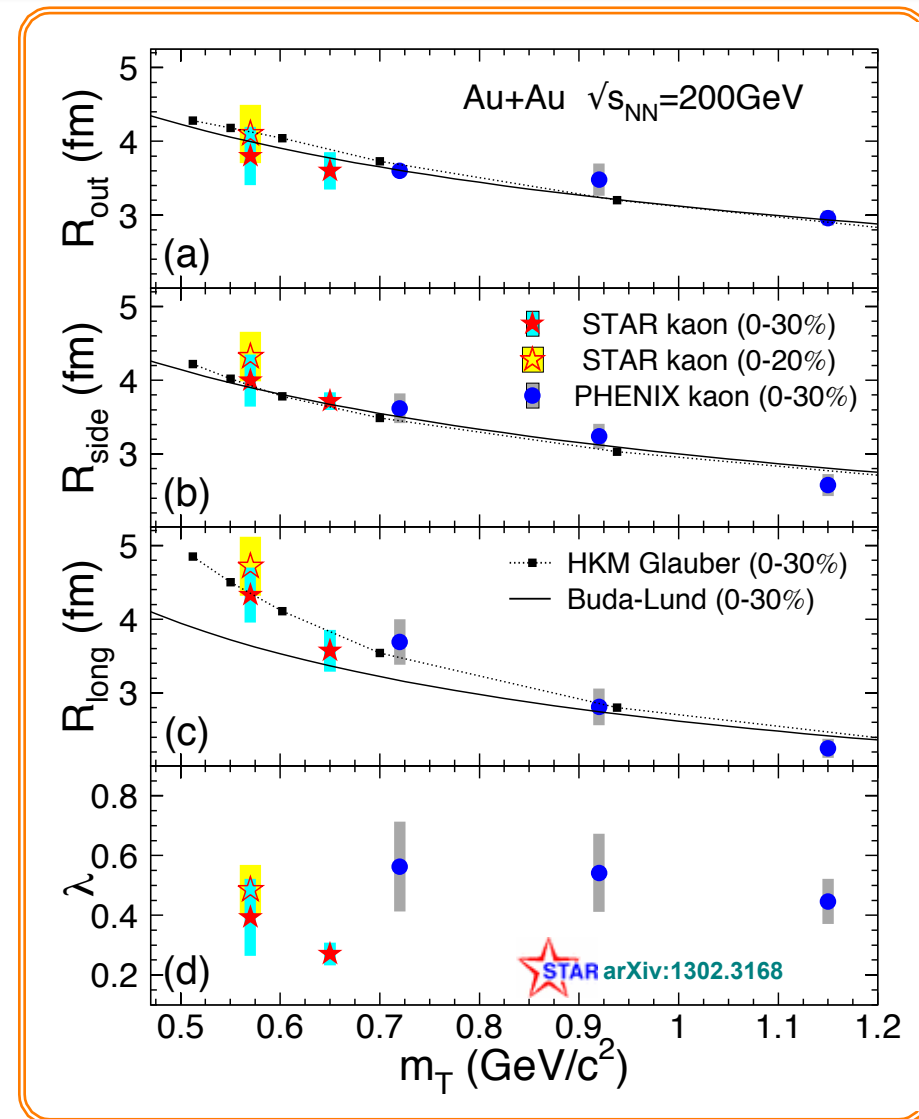
- “Pions and kaons seem to decouple simultaneously.”



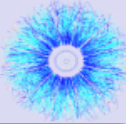
Kaon RHIC result



- Radii: rising trend at low m_T
 - Strongest in “long”
- Buda-Lund model
 - Perfect hydrodynamics, inherent m_T -scaling
 - Works perfectly for pions
 - Deviates from kaons in the “long” direction in the lowest m_T bin
- HKM (Hydro-kinetic model)
 - Describes all trends
 - Some deviation in the “out” direction
 - Note the different centrality definition

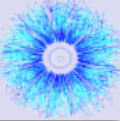


Summary



- First model-independent extraction of kaon 3D source shape presented
- No significant non-Gaussian tail is observed in RHIC $\sqrt{s_{NN}}=200$ GeV central Au+Au data
- Model comparison indicates that kaons and pions may be subject to different dynamics
- The m_T -dependence of the Gaussian radii indicates that m_T -scaling is broken in the “long” direction

Thank You!

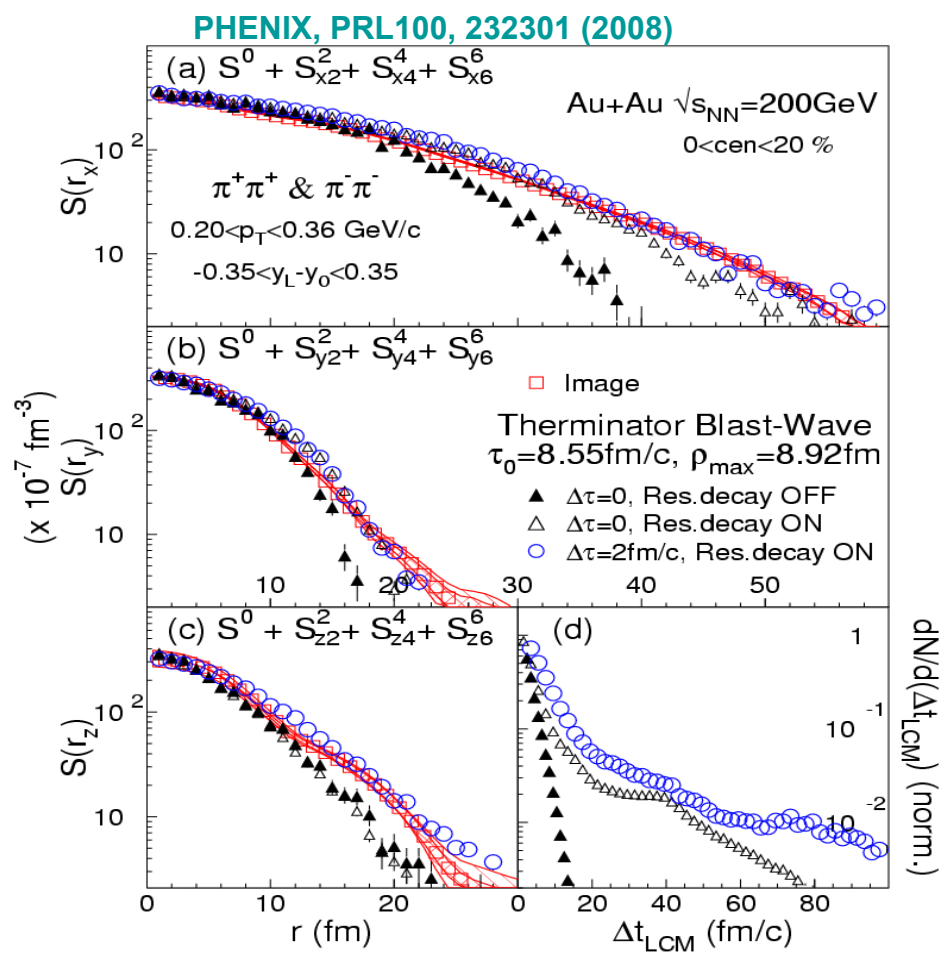
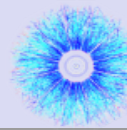


Argonne National Laboratory, Argonne, Illinois 60439
 Brookhaven National Laboratory, Upton, New York 11973
 University of California, Berkeley, California 94720
 University of California, Davis, California 95616
 University of California, Los Angeles, California 90095
 Universidade Estadual de Campinas, Sao Paulo, Brazil
 University of Illinois at Chicago, Chicago, Illinois 60607
 Creighton University, Omaha, Nebraska 68178
 Czech Technical University in Prague, FNSPE, Prague, 115 19,
 Czech Republic
 Nuclear Physics Institute AS CR, 250 68 Řež/Prague, Czech
 Republic
 University of Frankfurt, Frankfurt, Germany
 Institute of Physics, Bhubaneswar 751005, India
 Indian Institute of Technology, Mumbai, India
 Indiana University, Bloomington, Indiana 47408
 Alikhanov Institute for Theoretical and Experimental Physics,
 Moscow, Russia
 University of Jammu, Jammu 180001, India
 Joint Institute for Nuclear Research, Dubna, 141 980, Russia
 Kent State University, Kent, Ohio 44242
 University of Kentucky, Lexington, Kentucky, 40506-0055
 Institute of Modern Physics, Lanzhou, China
 Lawrence Berkeley National Laboratory, Berkeley, California
 94720
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 Max-Planck-Institut für Physik, Munich, Germany
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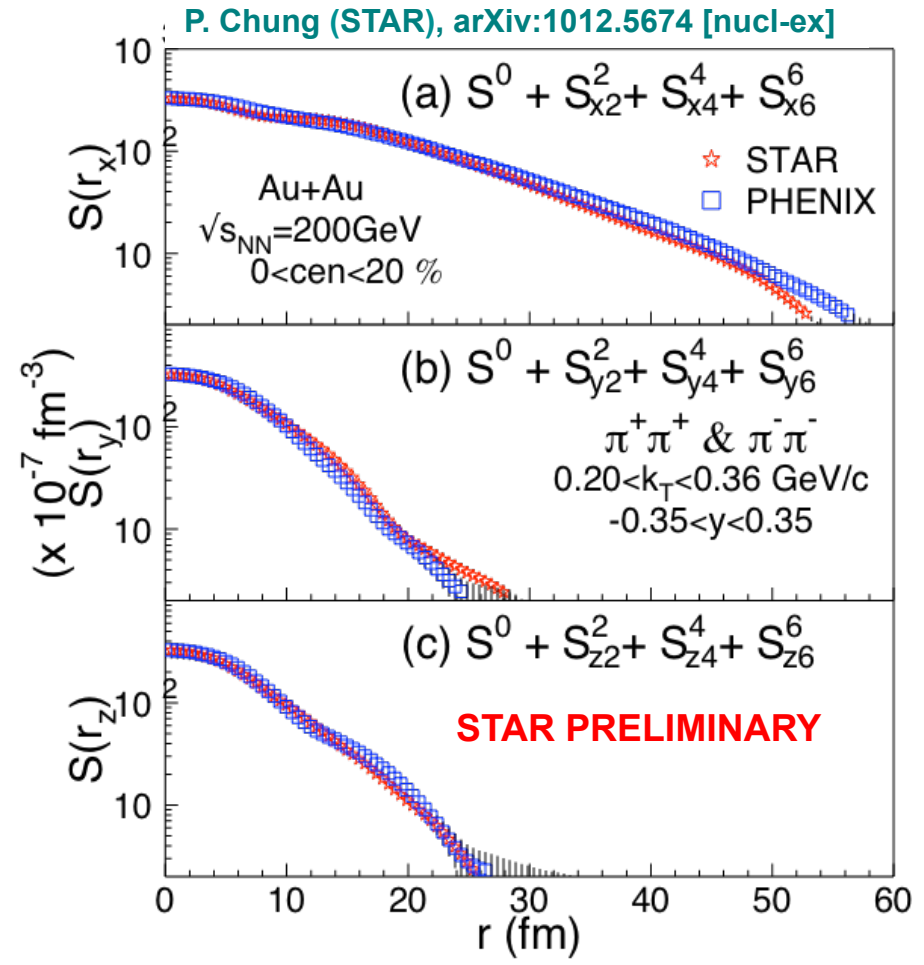
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 16802
 Institute of High Energy Physics, Protvino, Russia
 Purdue University, West Lafayette, Indiana 47907
 Pusan National University, Pusan, Republic of Korea
 University of Rajasthan, Jaipur 302004, India
 Rice University, Houston, Texas 77251
 Universidade de Sao Paulo, Sao Paulo, Brazil
 University of Science & Technology of China, Hefei 230026, China
 Shandong University, Jinan, Shandong 250100, China
 Shanghai Institute of Applied Physics, Shanghai 201800, China
 SUBATECH, Nantes, France
 Texas A&M University, College Station, Texas 77843
 University of Texas, Austin, Texas 78712
 University of Houston, Houston, TX, 77204
 Tsinghua University, Beijing 100084, China
 United States Naval Academy, Annapolis, MD 21402
 Valparaiso University, Valparaiso, Indiana 46383
 Variable Energy Cyclotron Centre, Kolkata 700064, India
 Warsaw University of Technology, Warsaw, Poland
 University of Washington, Seattle, Washington 98195
 Wayne State University, Detroit, Michigan 48201
 Institute of Particle Physics, CCNU (HZNU), Wuhan 430079, China
 Yale University, New Haven, Connecticut 06520
 University of Zagreb, Zagreb, HR-10002, Croatia

STAR Collaboration

3D pions, PHENIX and STAR

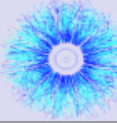


Elongated source in “out” direction
 Therminator Blast Wave model suggests non-zero emission duration

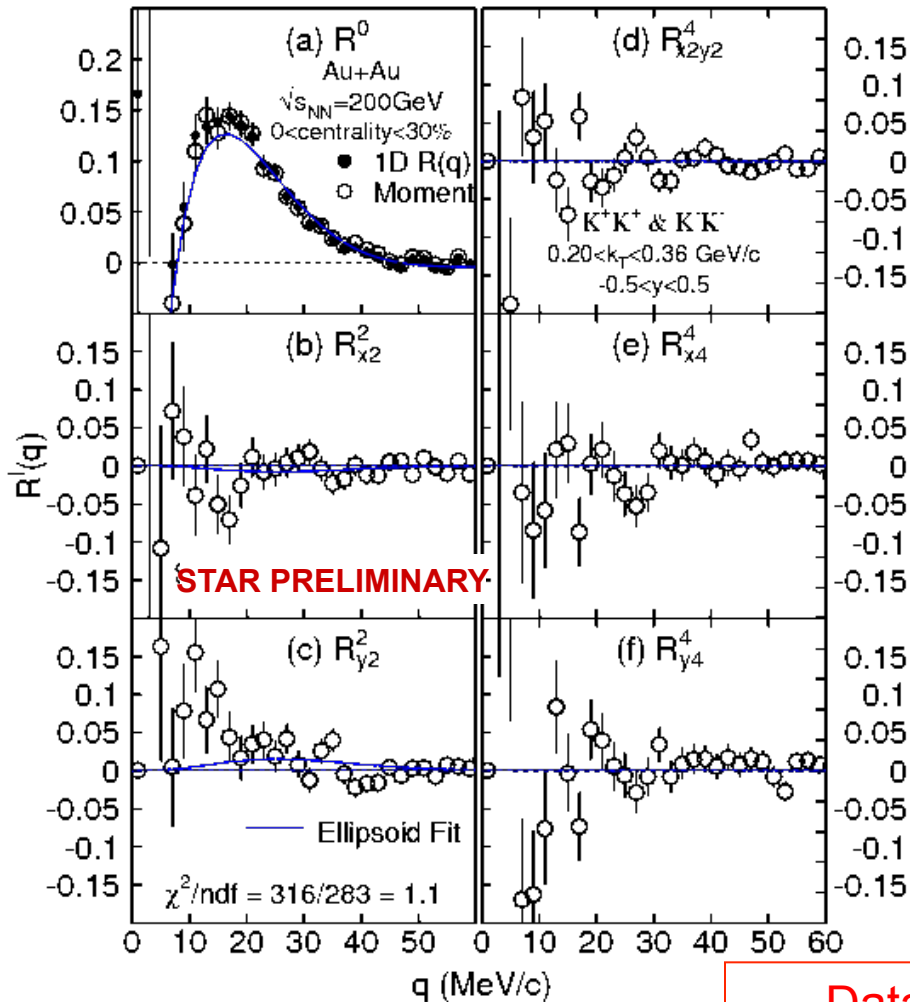


Very good agreement of PHENIX and STAR 3D pion source images

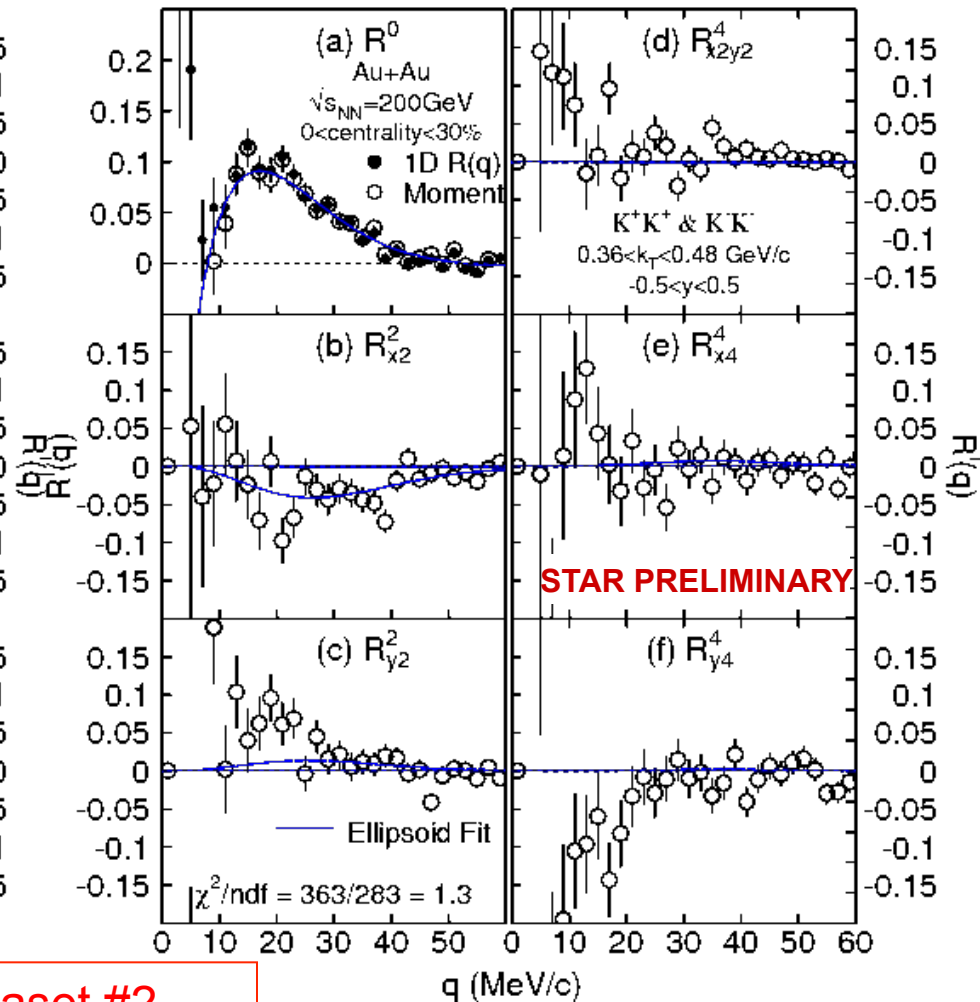
Fit to correlation moments



0.2 < kT < 0.36 GeV/c

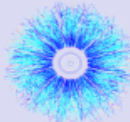


0.36 < kT < 0.48 GeV/c



Dataset #2
Run4 Cent < 30%

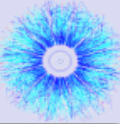
Source parameters



Year	2004+2007		2004
Centrality	0–20%		0–30%
k_T [GeV/ c]	0.2–0.36		0.36–0.48
R_x [fm]	$4.8 \pm 0.1 \pm 0.2$	$4.3 \pm 0.1 \pm 0.4$	$4.5 \pm 0.2 \pm 0.3$
R_y [fm]	$4.3 \pm 0.1 \pm 0.1$	$4.0 \pm 0.1 \pm 0.3$	$3.7 \pm 0.1 \pm 0.1$
R_z [fm]	$4.7 \pm 0.1 \pm 0.2$	$4.3 \pm 0.2 \pm 0.4$	$3.6 \pm 0.2 \pm 0.3$
λ	$0.49 \pm 0.02 \pm 0.05$	$0.39 \pm 0.01 \pm 0.09$	$0.27 \pm 0.01 \pm 0.04$
χ^2 / ndf	497/289	316/283	367/283

TABLE I. Parameters obtained from the 3-D Gaussian source function fits for the different datasets. The first errors are statistical, the second errors are systematic.

Cartesian harmonics basis



- Based on the products of unit vector components, $n_{\alpha_1} n_{\alpha_2}, \dots, n_{\alpha_\ell}$. Unlike the spherical harmonics **they are real**.
- Due to the normalization identity $n_x^2 + n_y^2 + n_z^2 = 1$, at a given $\ell \geq 2$, the different component products **are not linearly independent** as functions of spherical angle.
- At a given ℓ , the products are spanned by spherical harmonics of rank $\ell' \leq \ell$, with ℓ' of the same evenness as ℓ .

$$\mathcal{A}_x^{(1)} = n_x$$

$$\mathcal{A}_{xx}^{(2)} = n_x^2 - 1/3$$

$$\mathcal{A}_{xy}^{(2)} = n_x n_y$$

$$\mathcal{A}_{xxx}^{(3)} = n_x^3 - (3/5)n_x$$

$$\mathcal{A}_{xxy}^{(3)} = n_x^2 n_y - (1/5)n_y$$

$$\mathcal{A}_{xyz}^{(3)} = n_x n_y n_z$$

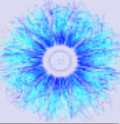
$$\mathcal{A}_{xxxx}^{(4)} = n_x^4 - (6/7)n_x^2 + 3/35$$

$$\mathcal{A}_{xxxy}^{(4)} = n_x^3 n_y - (3/7)n_x n_y$$

$$\mathcal{A}_{xxyy}^{(4)} = n_x^2 n_y^2 - (1/7)n_x^2 - (1/7)n_y^2 + 1/35$$

$$\mathcal{A}_{xxyz}^{(4)} = n_x^2 n_y n_z - (1/7)n_y n_z$$

Spherical Harmonics basis



$$\mathcal{R}_{\ell m}(q) = (4\pi)^{-1/2} \int d\Omega_{\mathbf{q}} Y_{\ell m}^*(\Omega_{\mathbf{q}}) \mathcal{R}(\mathbf{q}),$$
$$\mathcal{S}_{\ell m}(r) = (4\pi)^{-1/2} \int d\Omega_{\mathbf{r}} Y_{\ell m}^*(\Omega_{\mathbf{r}}) \mathcal{S}(\mathbf{r}).$$

- Disadvantage: connection between the geometric features of the real source function $\mathcal{S}(\mathbf{r})$ and the complex valued projections $\mathcal{S}_{\ell m}(r)$ is not transparent.
- $Y_{\ell m}$ harmonics are convenient for analyzing quantum angular momentum, but are clumsy for expressing anisotropies of real-valued functions.