



# Theory Overview

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# Jet Modification in heavy-ion collisions: Theory Overview

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Wayne State University



# Outline

- A bit of history
- The pQCD paradigm, lost and found
- The underlying physics
- Outstanding challenges
- Future calculations



From my last ISMD 2007



From my last ISMD 2007



**Duke Physics**

***The Study of Dense Matter through  
Perturbative Jet Modification***

***A. Majumder  
Duke University***

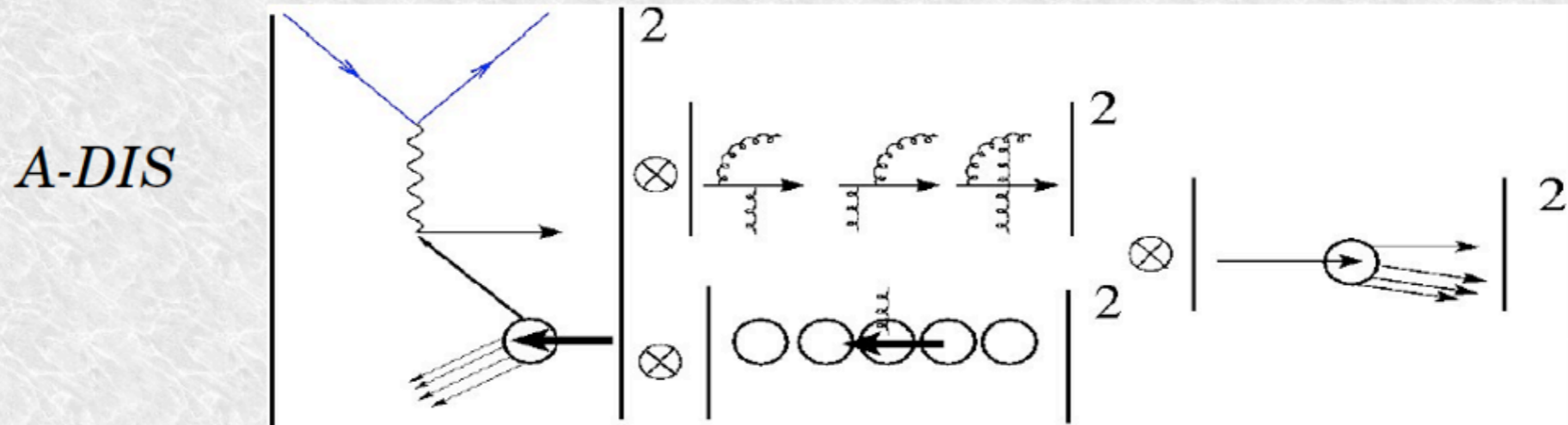
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***International Symposium on Multi-particle Dynamics 2007,  
LBNL, Aug 3-9***

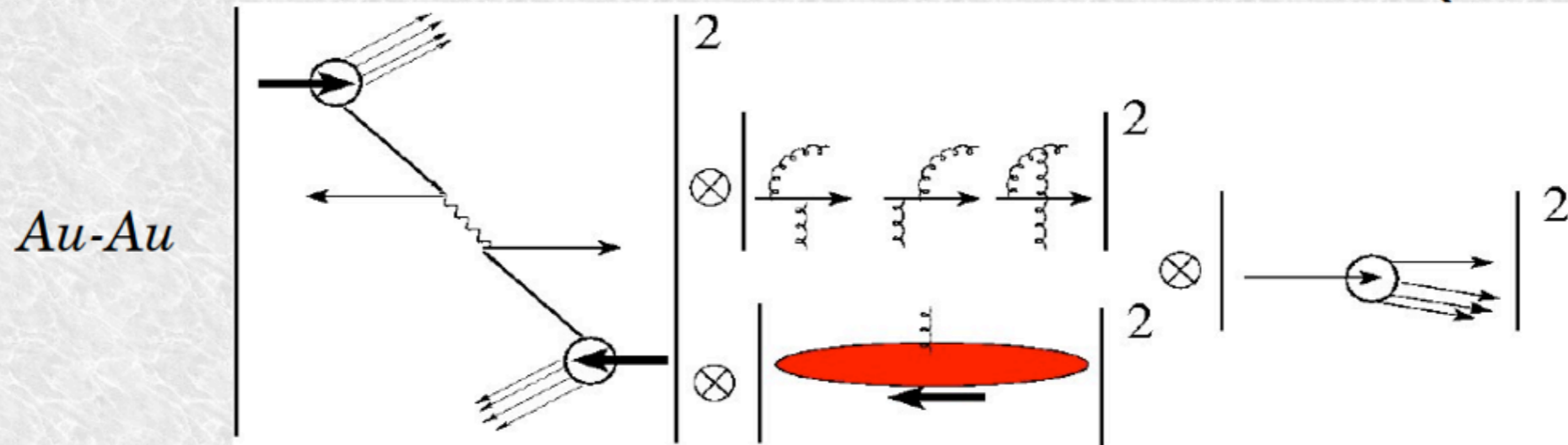


From my last ISMD 2007

## ***E-loss kernel same: factorization***



$$A F(p|p_h) d\sigma_{e^-+q \rightarrow q+e^-} \quad L \int dt \langle F^{\mu\alpha}(t) v_\alpha F_\mu^\beta(0) v_\beta \rangle \quad \sigma_T \propto \frac{A^{1/3} \mu_H^2}{Q^2}$$



Guo, Wang, *NPA* 696:78, 2001. Majumder, *EPJC* 43, 259, 2005.



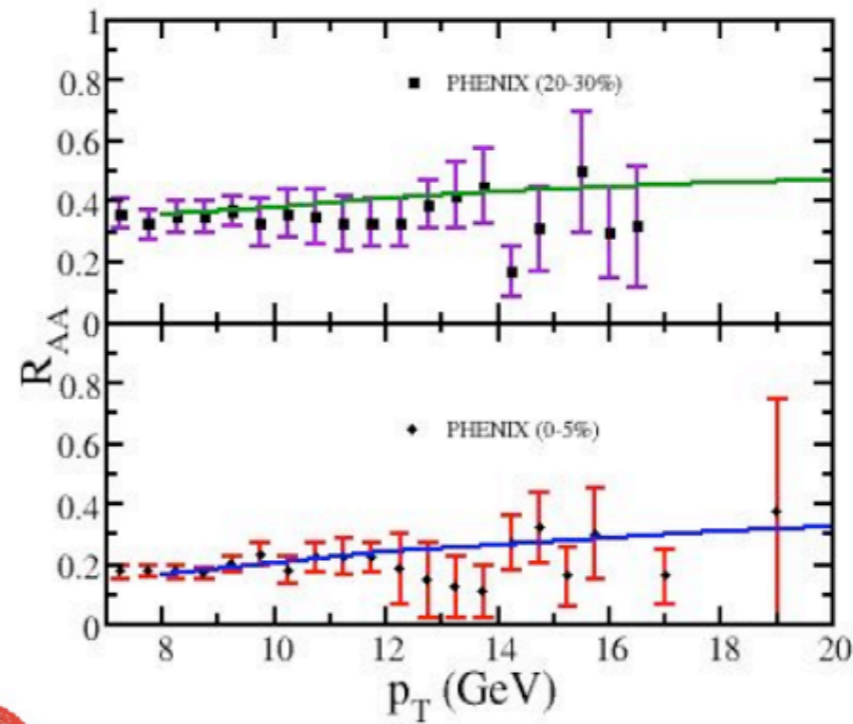
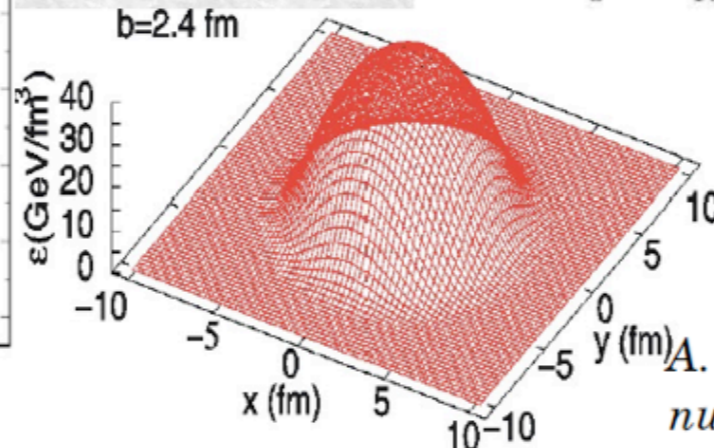
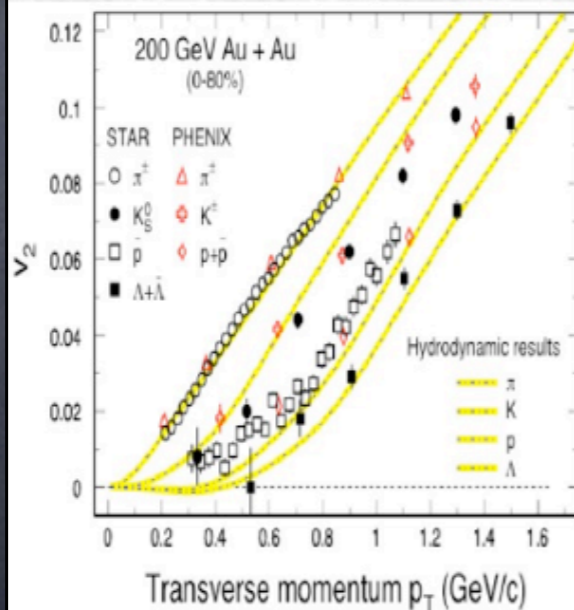
# From my last ISMD 2007

## An example: the space-time profile

$$\hat{q}^{\mu\nu} = \hat{q}_0(f) \delta^{\mu\nu} \frac{\gamma_{\perp}(x, y, z, t) T^3(x, y, z, t)}{T_0^3(x, y, z, t)} \quad \hat{q}_0(\text{quarks}) = 1.3 \text{ GeV}^2 / \text{fm}$$

Test of the hydro model!

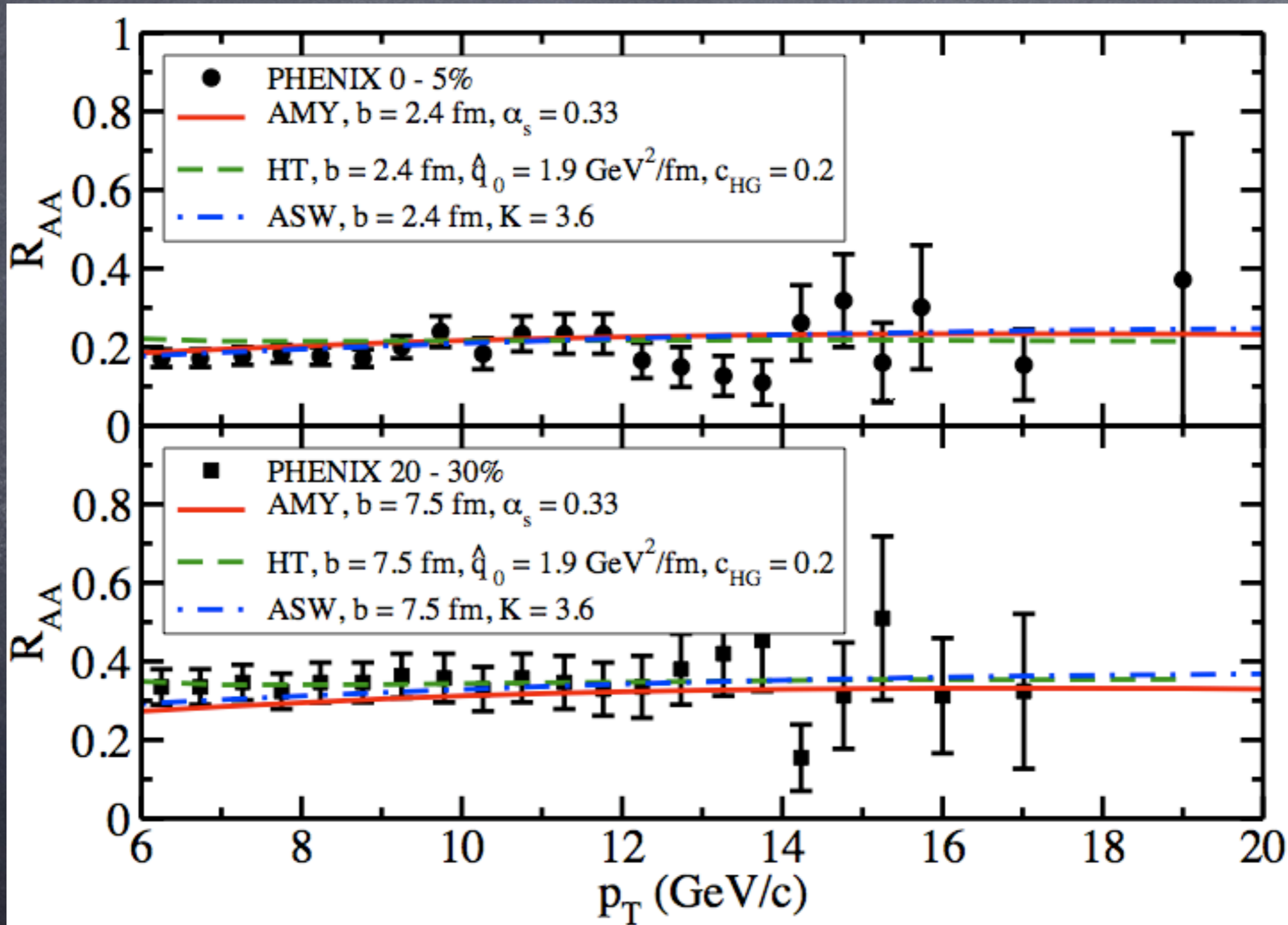
Use the same hydrodynamic model that predicted the soft spectra



A. Majumder, C. Nonaka, S. Bass, nucl-th/0703019



But there were multiple formalisms that were indistinguishable!



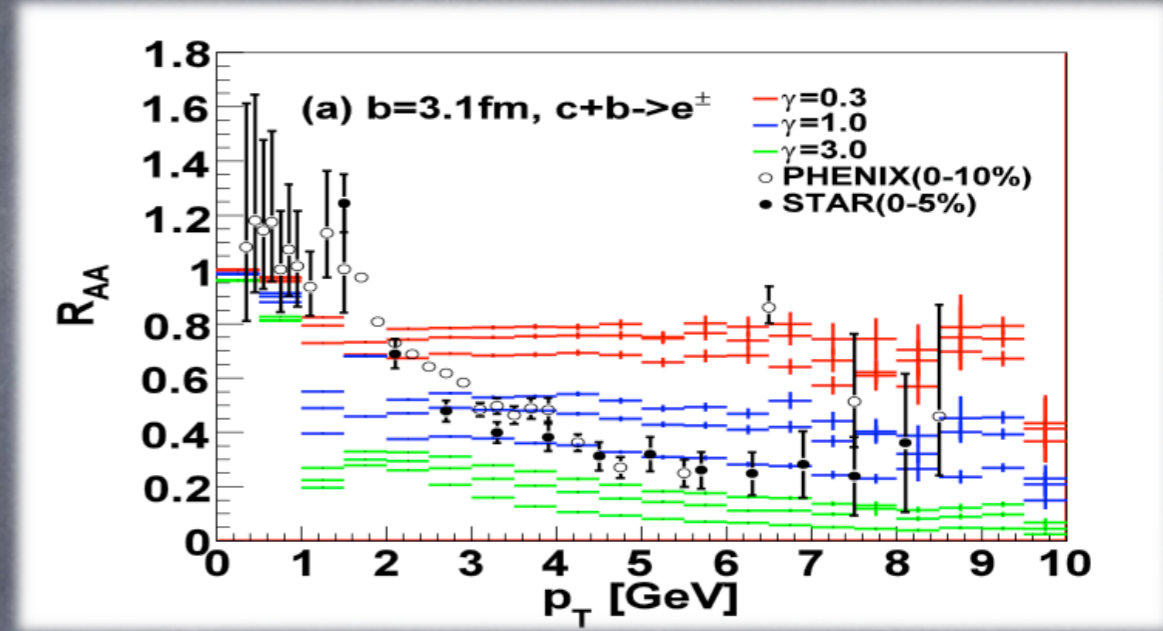
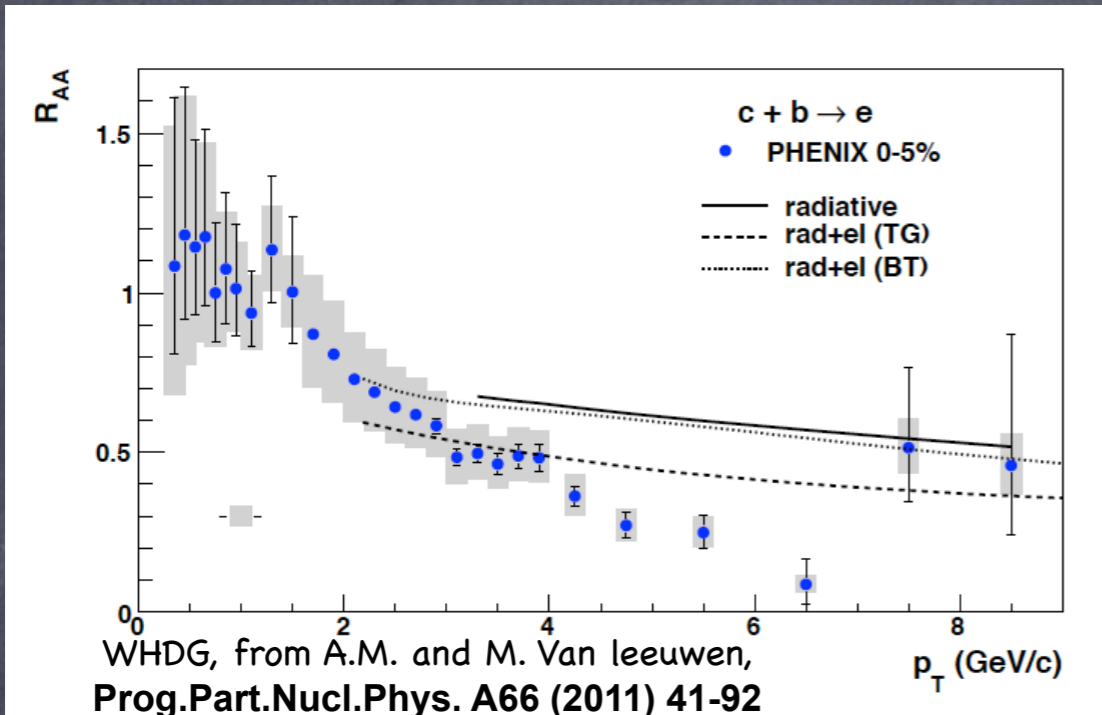


But there were multiple formalisms that were indistinguishable!

$\hat{q}(\vec{r}, \tau)$ scales as	ASW $\hat{q}_0$	HT $\hat{q}_0$	AMY $\hat{q}_0$
$T(\vec{r}, \tau)$	10 GeV <sup>2</sup> /fm	2.3 GeV <sup>2</sup> /fm	4.1 GeV <sup>2</sup> /fm
$\epsilon^{3/4}(\vec{r}, \tau)$	18.5 GeV <sup>2</sup> /fm	4.5 GeV <sup>2</sup> /fm	
$s(\vec{r}, \tau)$		4.3 GeV <sup>2</sup> /fm	

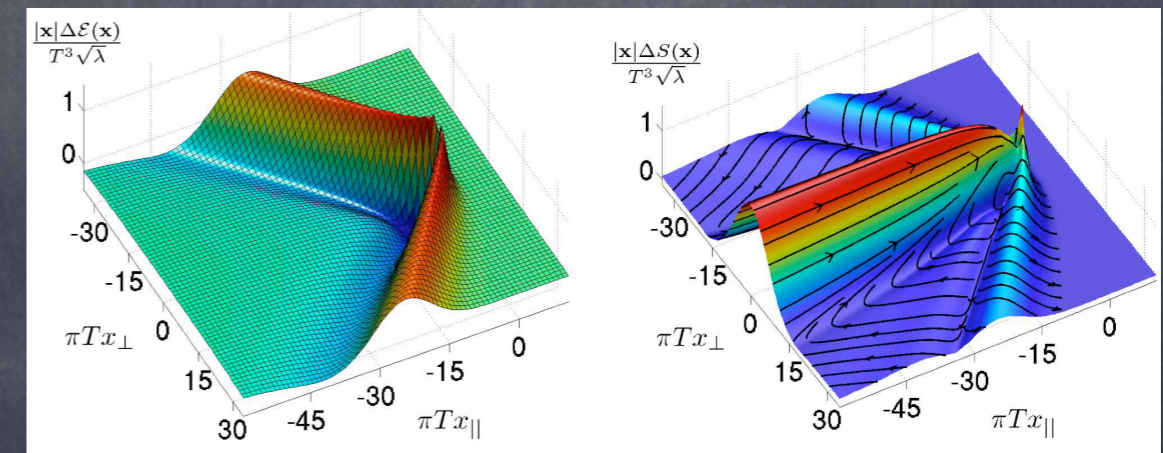
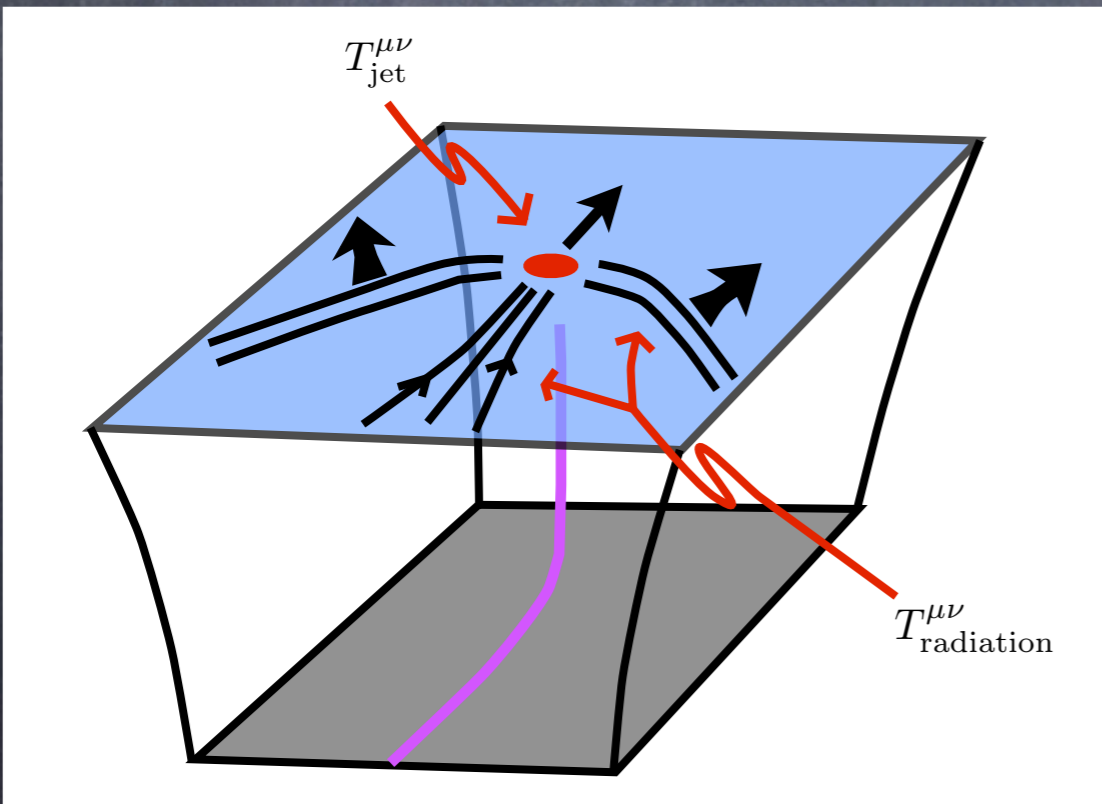


# pQCD lost!



Akamatsu, Hatsuda Hirano 2008

$$\Delta \vec{p} = -\gamma \frac{T^2}{M} \vec{p} \Delta t + \vec{\xi}(t)$$



Chesler and Yaffe



# The LHC and the return of pQCD

## Jets @ LHC

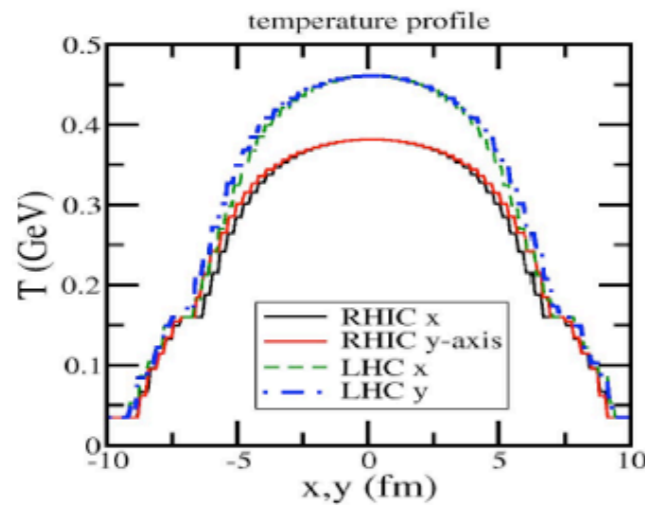
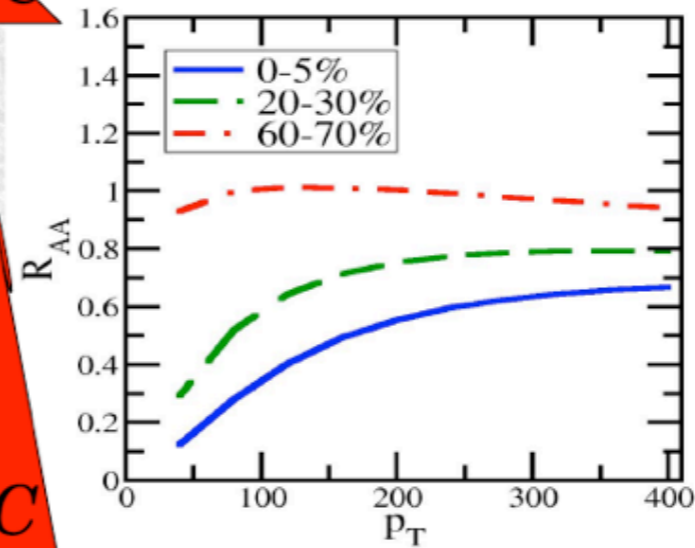
*How to deal with denser medium,*

*Medium may be denser overall*

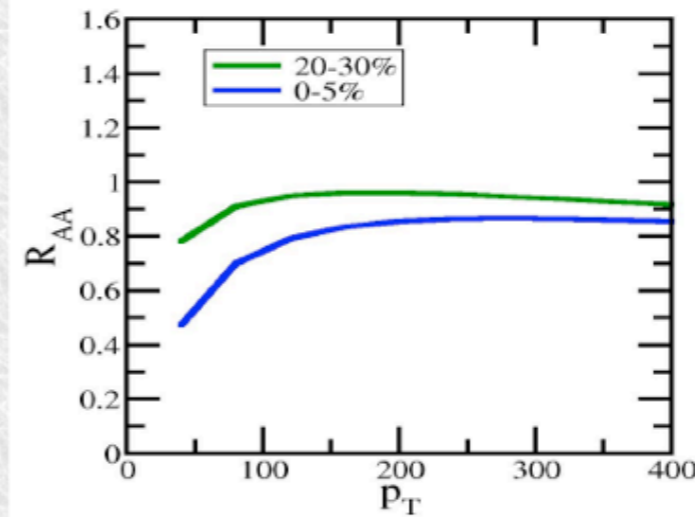
*Space time dist. may be different*

*Jet correlations will tell the difference*

@ RHIC

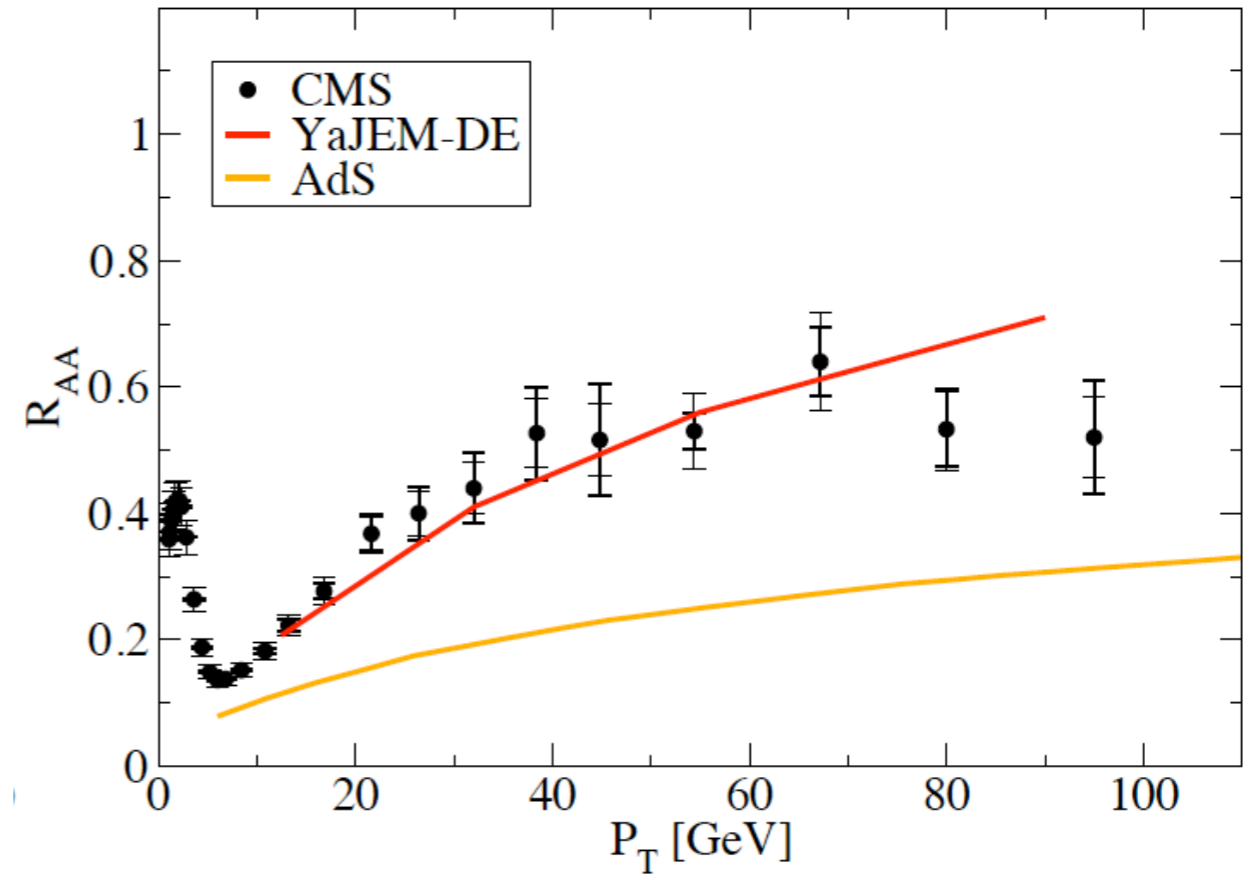
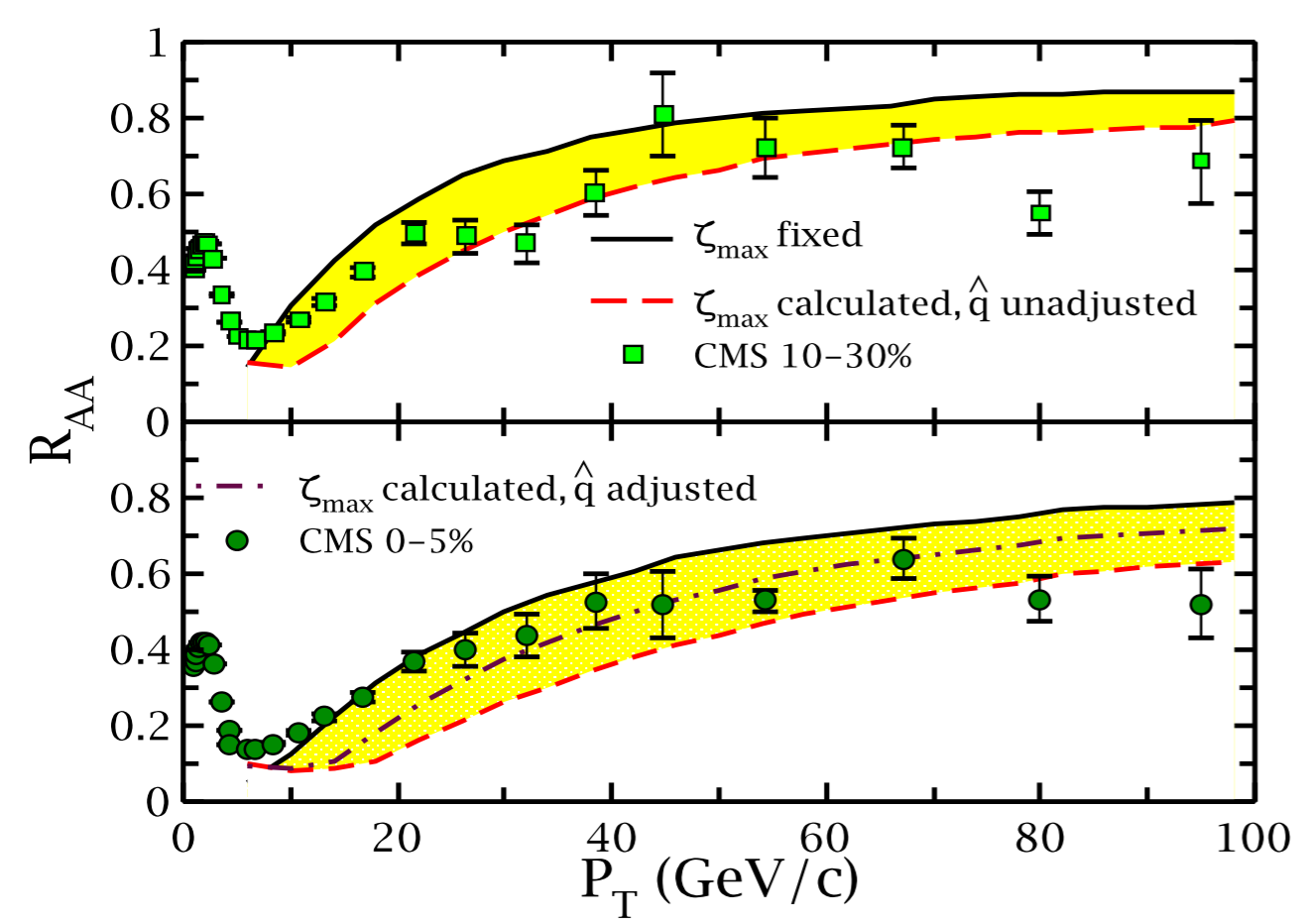


*Density bunched up in the middle*





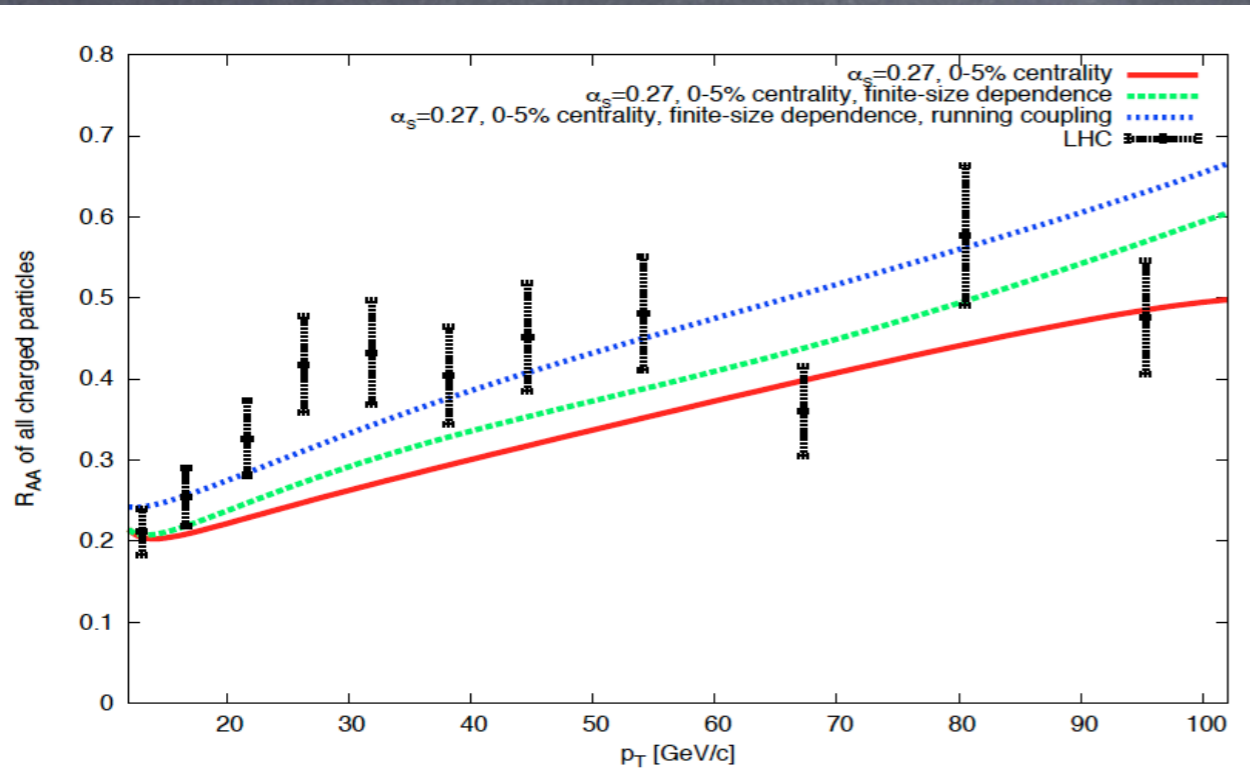
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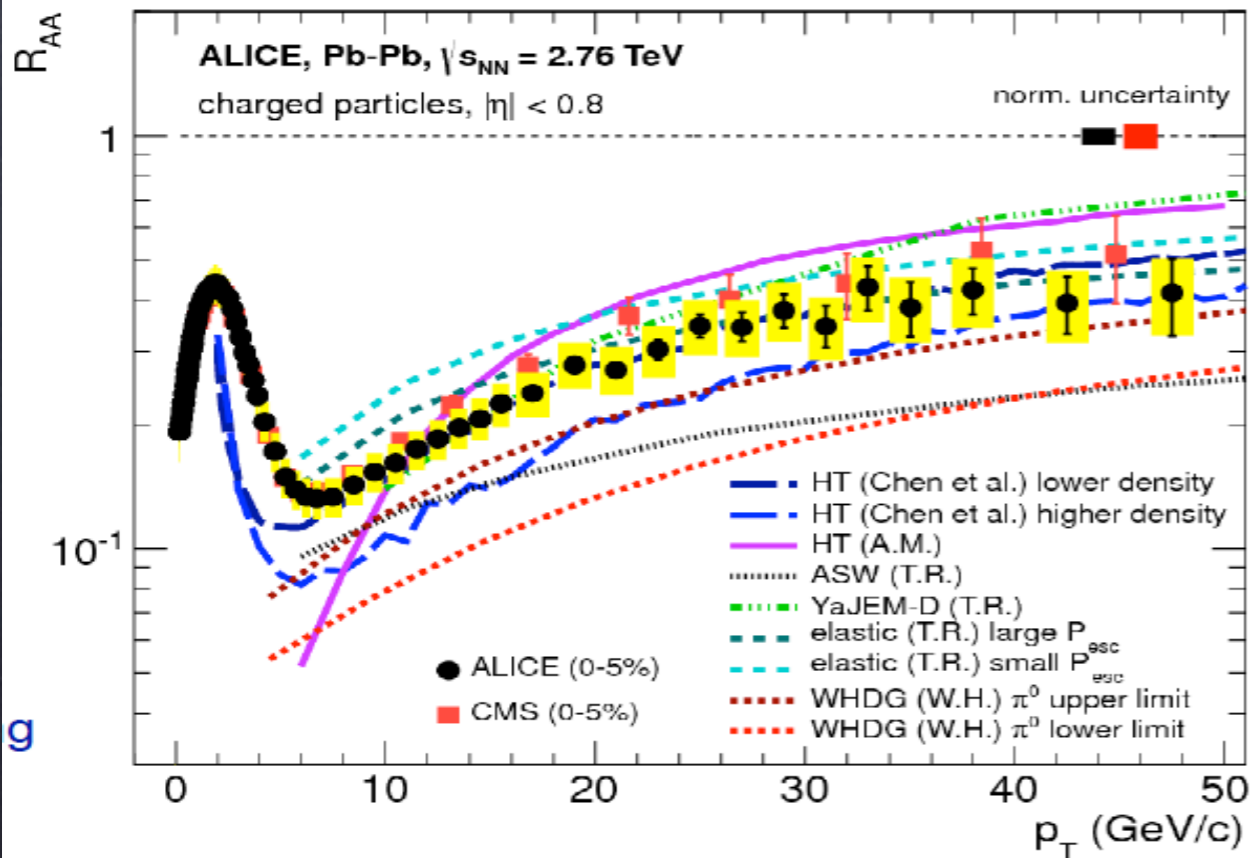
strong coupling energy loss ruled out



# LHC also makes it hard for other unfactorized pQCD approaches



AMY: ignores  $\alpha_s$  running, ignores initial virtuality



ASW: Strictly Eikonal ruled out

GLV: ignores  $\alpha_s$  running



What about that  $\hat{q}$  ?

$\hat{q}(\vec{r}, \tau)$ scales as	ASW $\hat{q}_0$	HT $\hat{q}_0$	AMY $\hat{q}_0$
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Ruled out  
by LHC

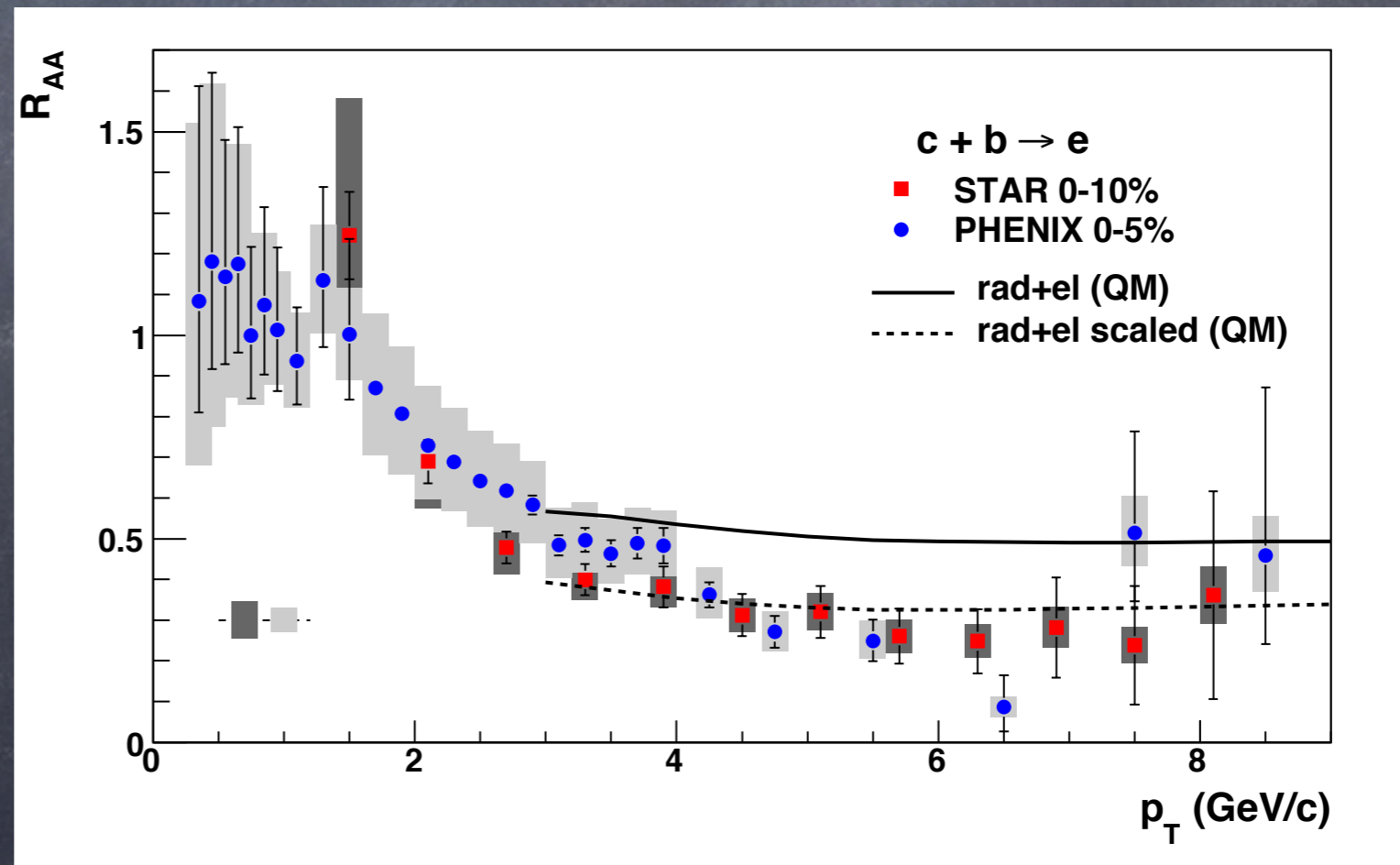


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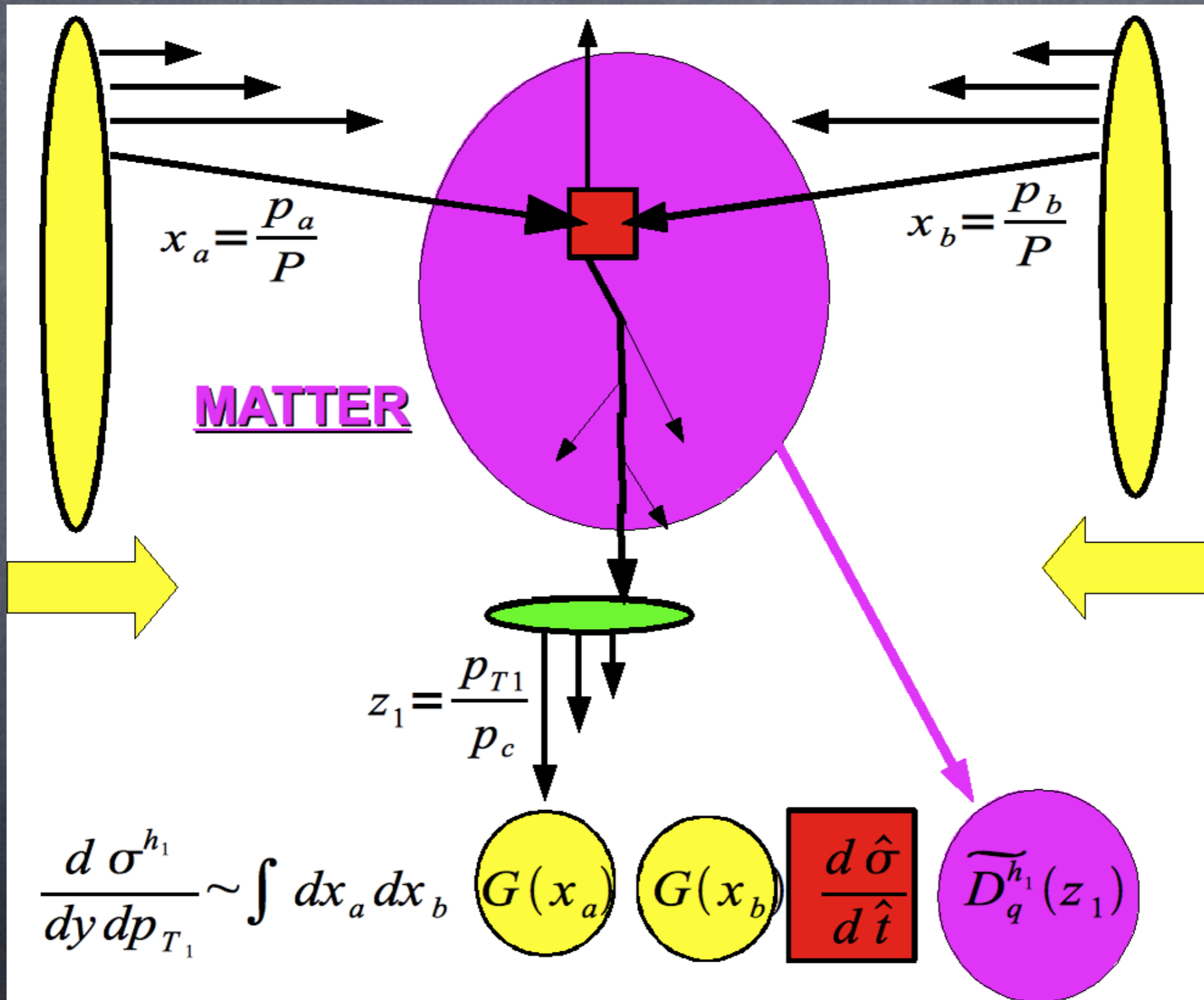
Ruled out  
by LHC

Drop the  
requirement that  
the medium can be  
described by LO  
pQCD





# The requirements for a successful pQCD formalism





# What goes into this calculation

Jet scale assumed much harder than medium scale  
(factorization of jet from soft matrix element)

Multiple scatterings resummed in single gluon emission

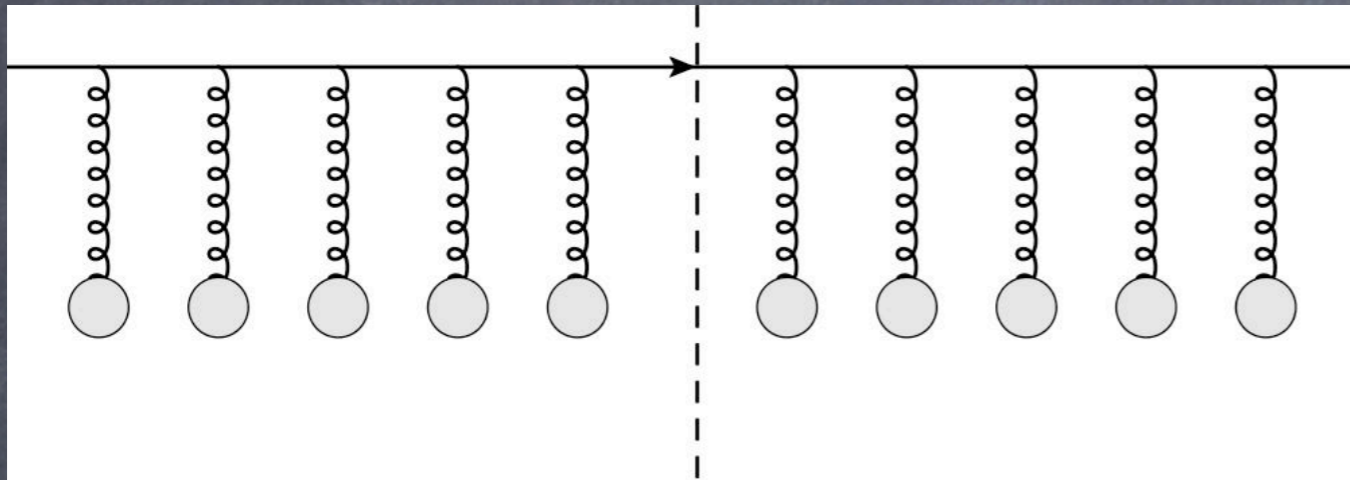
Expansion in powers of  $\Lambda^2/Q^2$

DGLAP  $k_T^2$  systematics assumed for multiple emissions

Fluid dynamical simulation of medium and trans. coeffs.

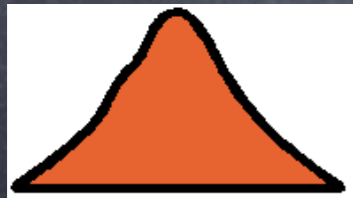


# Multiple scattering induced transverse broadening



$$q^- \rightarrow \infty$$

Assuming independent scattering of nucleons gives a diff. equation  
 These cannot be soft, they must have transverse momentum, Glauber gluons.



$$\frac{\partial f(p_{\perp}, t)}{\partial t} = \nabla_{p_{\perp}} \cdot D \cdot \nabla_{p_{\perp}} f(p_{\perp}, t)$$



$$\langle p_{\perp}^2 \rangle = 4Dt$$

$$\hat{q} = \frac{p_{\perp}^2}{t} = \frac{2\pi^2 \alpha_s C_R}{N_c^2 - 1} \int d\tilde{t} \langle F^{\mu\alpha}(\tilde{t}) v_{\alpha} F_{\mu}^{\beta}(0) v_{\beta} \rangle$$



# Longitudinal drag and diffusion

A close to on shell  
parton has a 3-D  
distribution

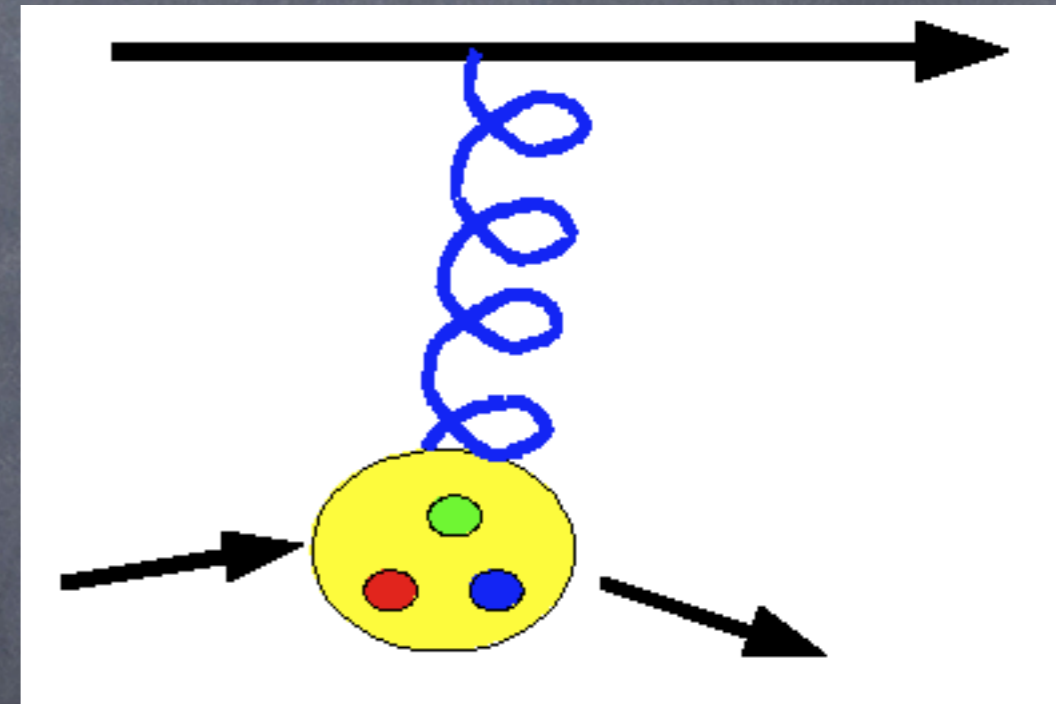
$$p^+ = \frac{p_{\perp}^2}{2p^-}$$

$$f(\vec{p}) \equiv \delta^2(p_{\perp}^2) \delta(p^- - q^- + k^-)$$

Using the same analysis, we  
get a drag. and diff. term

$$\frac{\partial f(p^-, L^-)}{\partial L^-} = c_1 \frac{\partial f}{\partial p^-} + c_2 \frac{\partial^2 f}{\partial^2 l^-}$$

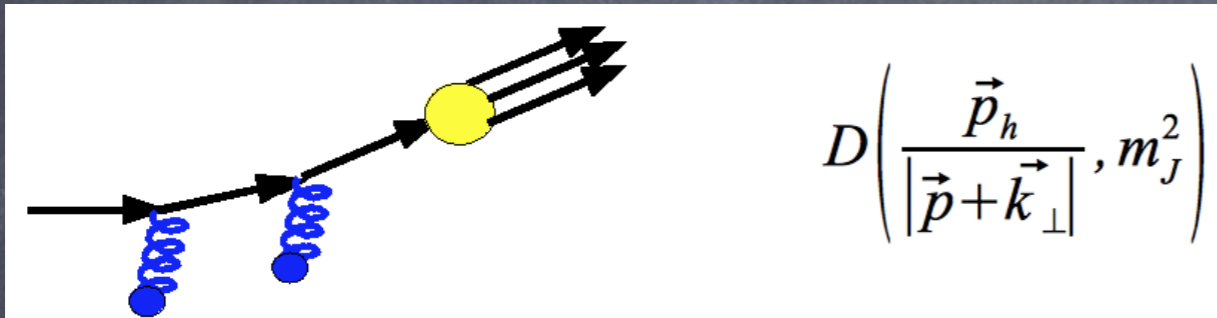
$c_1$  is  $dE/dL$ , calculate in a  
deconfined quasi-particle medium.





There are a bunch of medium properties which modify the parton and frag. func.

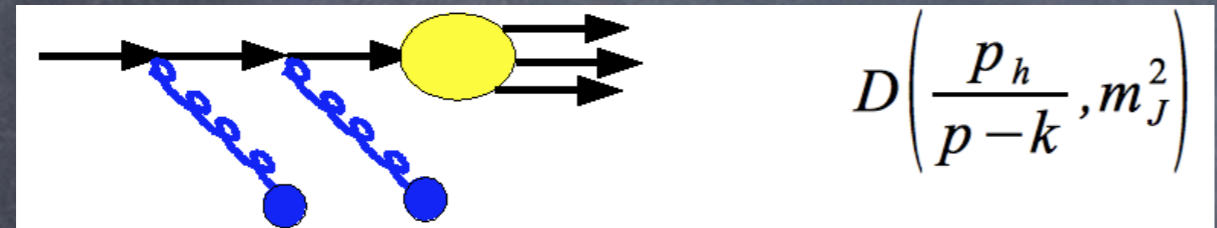
$$\hat{q}, \hat{e} = dE/dL \text{ and } \hat{f} = dN/dL$$



$$D\left(\frac{\vec{p}_h}{|\vec{p} + \vec{k}_\perp|}, m_J^2\right)$$

$$\hat{q} = \frac{\langle p_T^2 \rangle_L}{L}$$

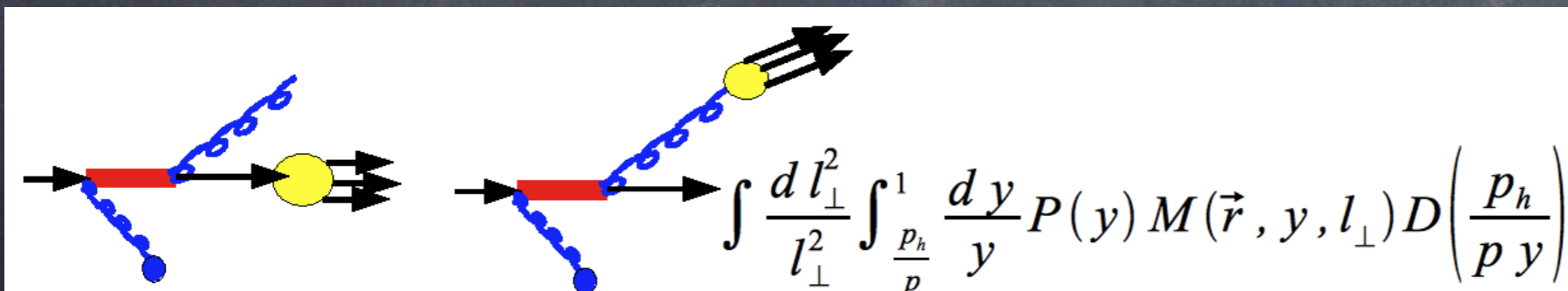
Transverse momentum diffusion rate



$$D\left(\frac{p_h}{p - k}, m_J^2\right)$$

$$\hat{e} = \frac{\langle \Delta E \rangle_L}{L}$$

Elastic energy loss rate also diffusion rate  $e_2$



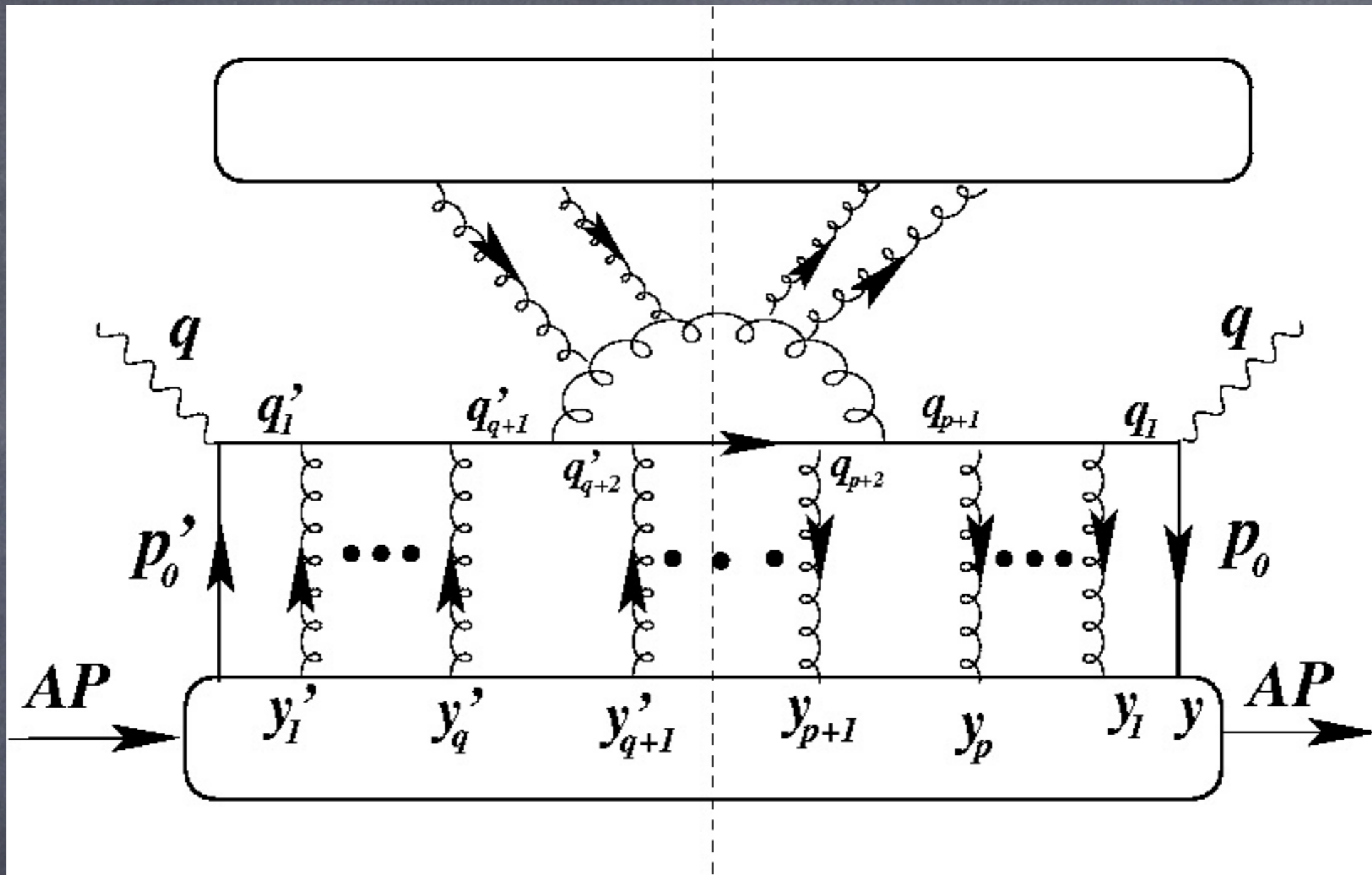
$$\int \frac{d l_\perp^2}{l_\perp^2} \int_0^1 \frac{d y}{y} P(y) M(\vec{r}, y, l_\perp) D\left(\frac{p_h}{p y}\right)$$

Gluon radiation is sensitive to all these transport coefficients

And a bunch of off diagonal and higher order transport coefficients



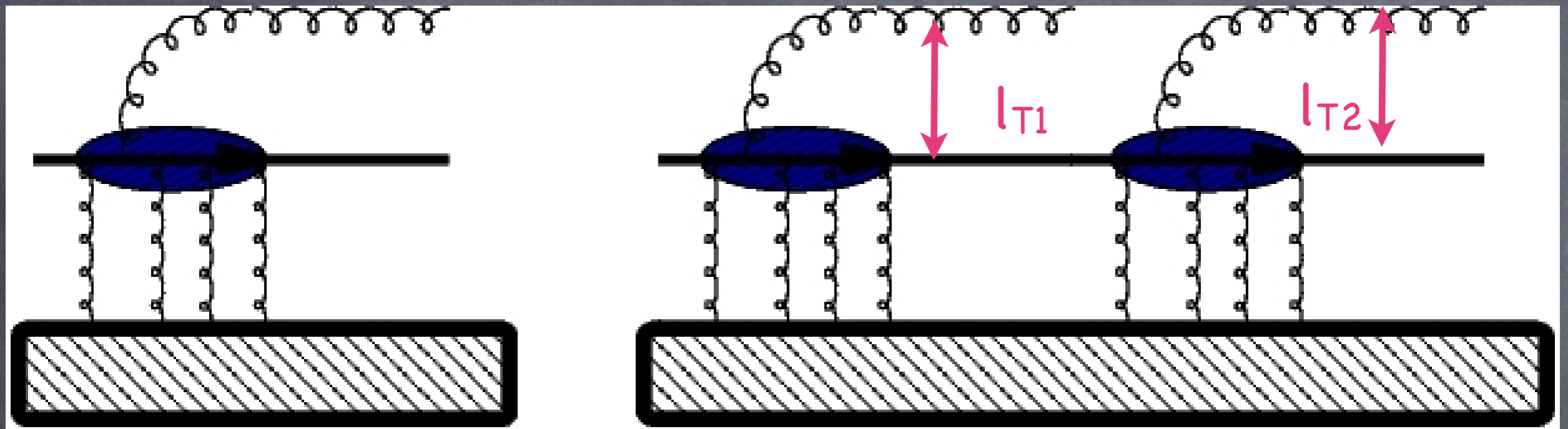
# The single gluon emission kernel



Calculate 1 gluon emission with quark & gluon N-scattering with transverse broadening and elastic loss built in. Finally solved analytically, in large  $Q^2$  limit.



# Need to repeat the kernel



What is the relation between subsequent radiations ?  
 In the large  $Q^2$  we can argue that there should be ordering of  $l_{\tau}$ .

$$\text{if } \hat{q}L < Q^2$$

$$\text{then } \frac{dQ^2}{Q^2} \left[ 1 + c_1 \frac{\hat{q}L}{Q^2} \right] \leq \frac{dQ^2}{Q^2} [1 + c_1]$$

However, at lower  $Q^2$ , possible anti-ordering

Coherence effects and broadening in medium-induced QCD radiation off a massive  $q\bar{q}$  antenna

Néstor Armesto, Hao Ma, Yacine Mehtar-Tani, Carlos A. Salgado, Konrad Tywoniuk

JHEP 1201 (2012) 109



# Analytical calculations always have approximations

$$\frac{\partial D_q^h(z, \mu^2)}{\partial \log(\mu^2)} = \frac{\alpha_s}{2\pi} \int_z^1 \frac{dy}{y} P_{q \rightarrow i}(y) D_i^h\left(\frac{z}{y}, \mu^2\right)$$

+

$$\begin{aligned} \frac{\partial D_q^{h^2}(z, M^2, q^-)|_{\zeta_i}^{\zeta_f}}{\partial \log(M^2)} &= \frac{\alpha_s}{2\pi} \int_z^1 \frac{dy}{y} \frac{\tilde{P}_{q \rightarrow i}(y)}{M^2} \int_{\zeta_i}^{\zeta_f} d\zeta \frac{2\pi\alpha_s}{N_c} \\ &\times \rho_g(\zeta) \left[ 2 - 2 \cos \left\{ \frac{M^2(\zeta - \zeta_i)}{2q^- y(1-y)} \right\} \right] \\ &\times D_q^{h^1}\left(\frac{z}{y}, M^2, q^- y\right) \Big|_{\zeta}^{\zeta_f} \end{aligned}$$

Thus you need a grid in  $z$ ,  $q^-$ , and  $\zeta$

Really hard numerically, so far grid in  $z$ ,  $q^-$ , and in  $z, \zeta$

To go beyond this would require a MC Evt. Gen.



A DGLAP formalism requires an upper scale  
and a lower scale

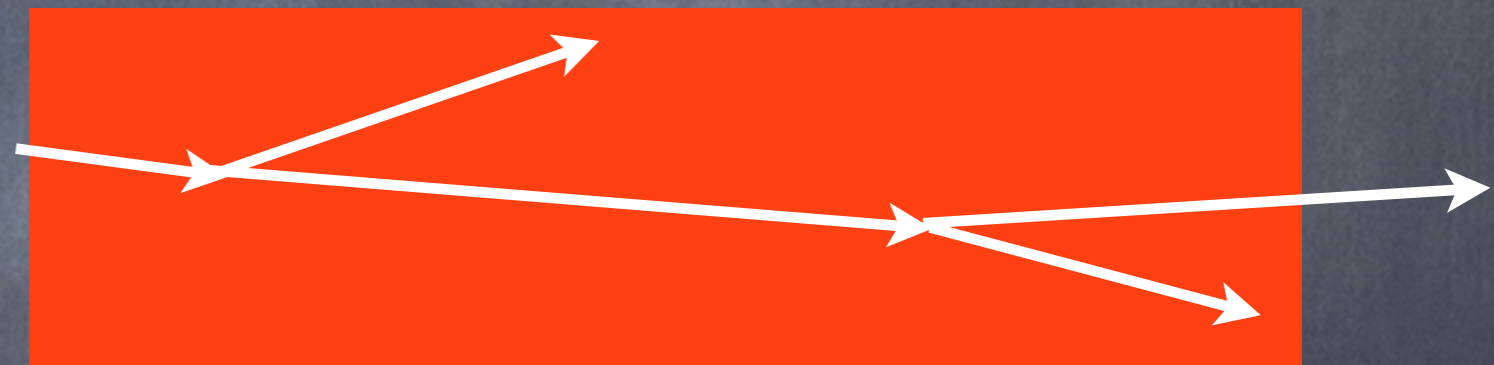
Upper scale is  $p_T^2$ , same as in vacuum

What is the lower scale?

what is the virtuality of a parton on exit ?

Natural choice

$$Q_{\min}^2 = E/L$$



Realistically, this should be done for each path

In reality we average kernel over many paths

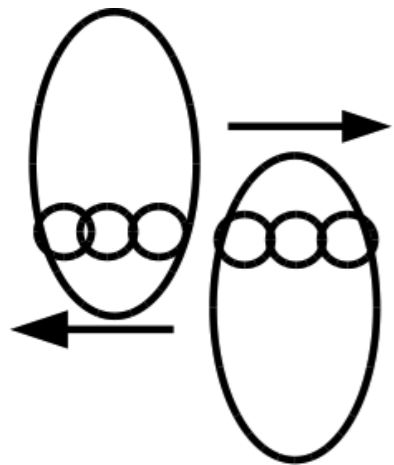
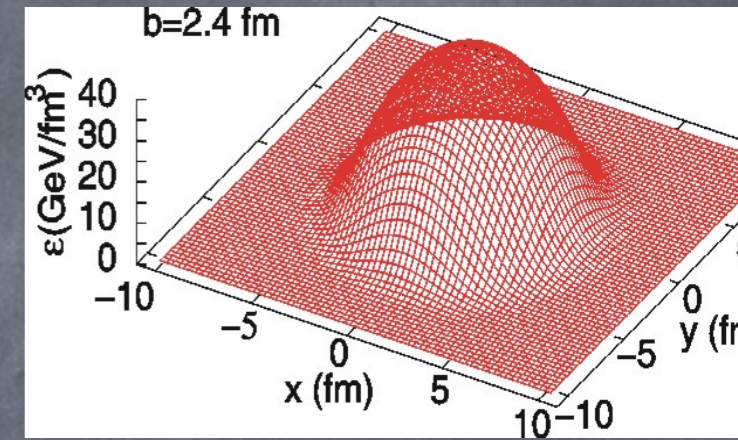
and calculate a mean distance based on the maximum length  
that the jet can travel in the representative brick



# Bulk medium described by viscous fluid dynamics

Medium evolves hydro-dynamically as the jet moves through it

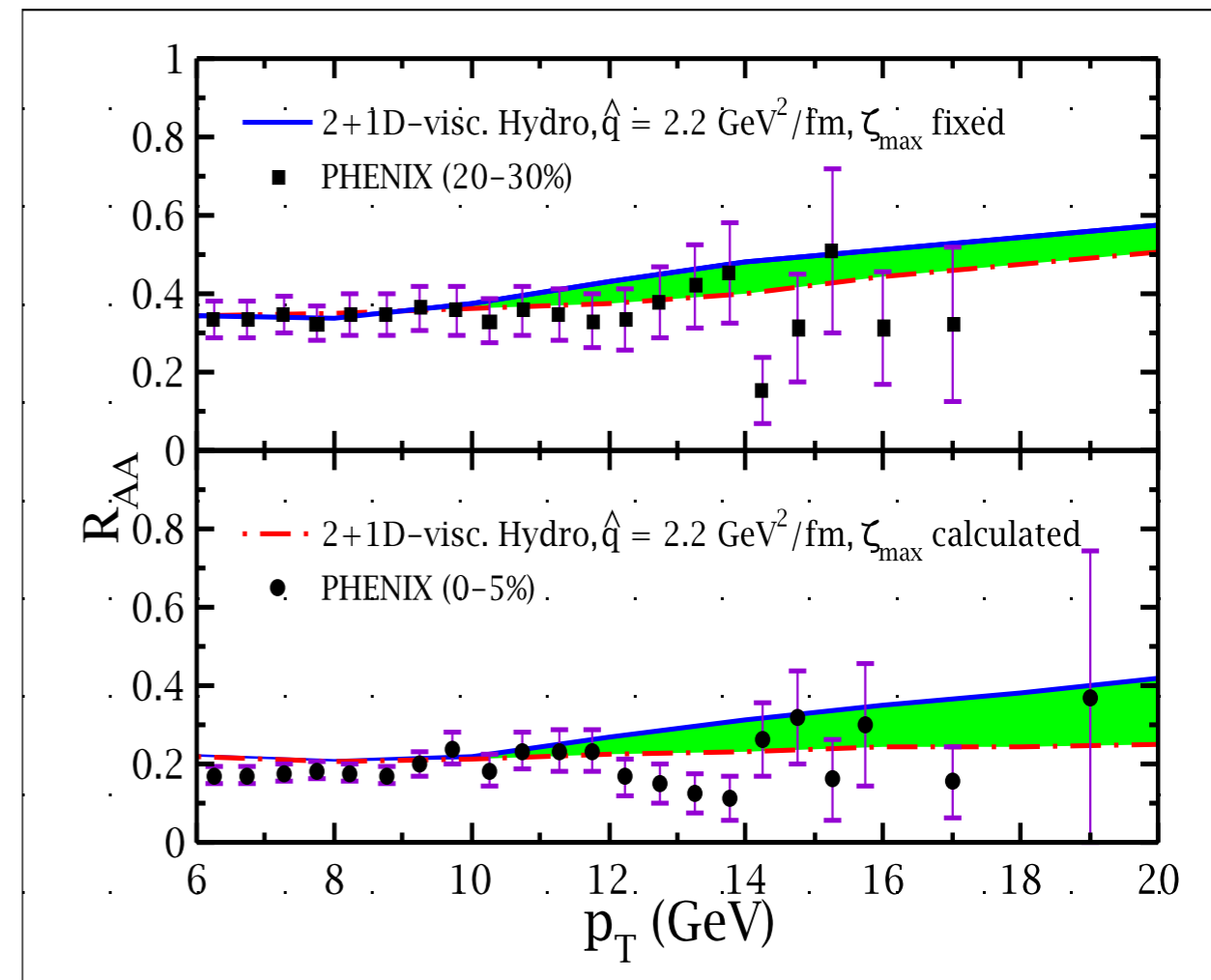
Fit the  $q$  for the initial  $T$  in the hydro in central coll.



$$\hat{q}(\vec{r}, t) = \hat{q}_0 \frac{s(\vec{r}, t)}{s_0}$$

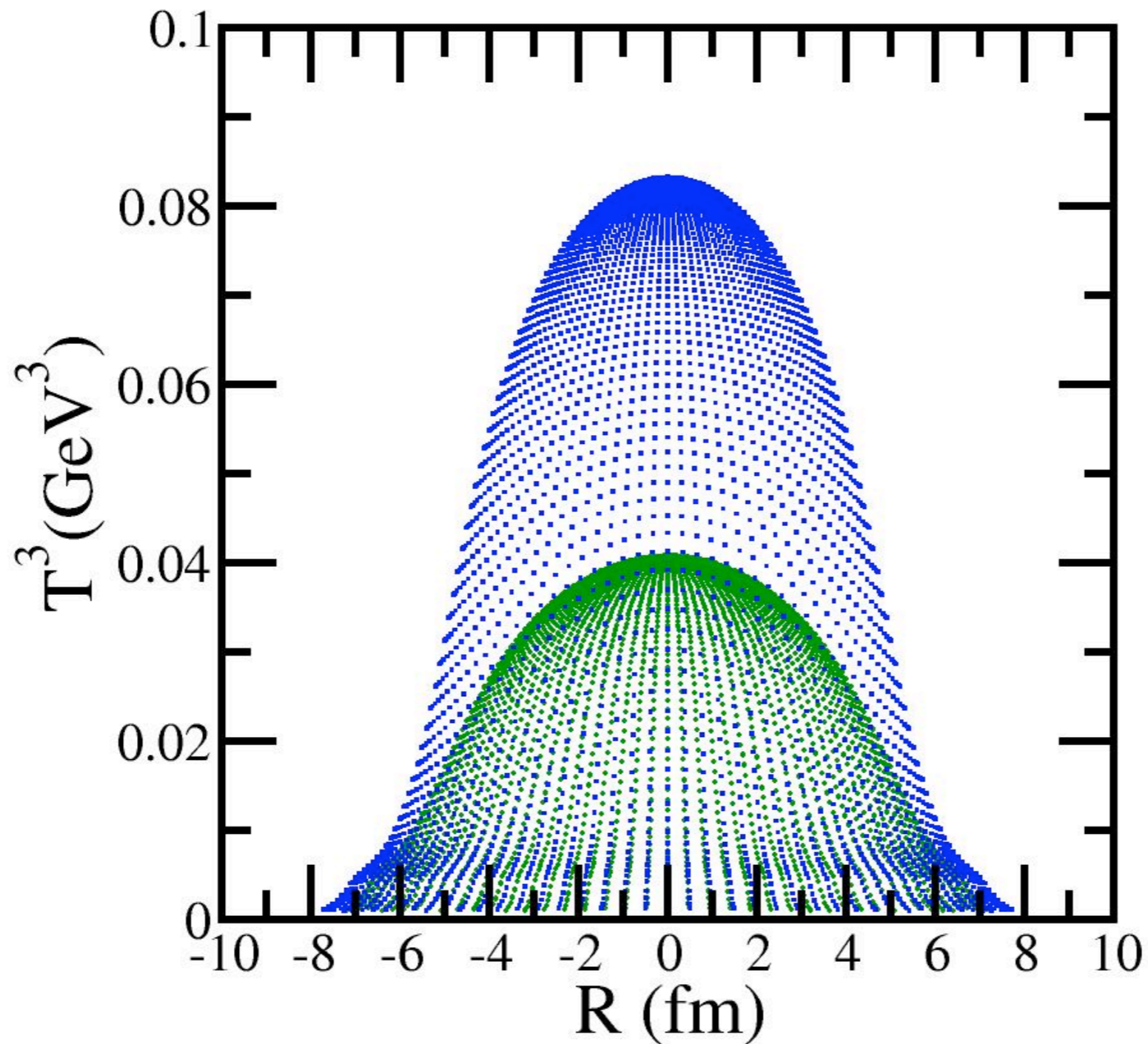
$$s_0 = s(T_0)$$

$$R_{AA} \sim \frac{\frac{dN_{AA}}{dp_T dy}}{N_{bin} \frac{dN_{pp}}{dp_T dy}}$$



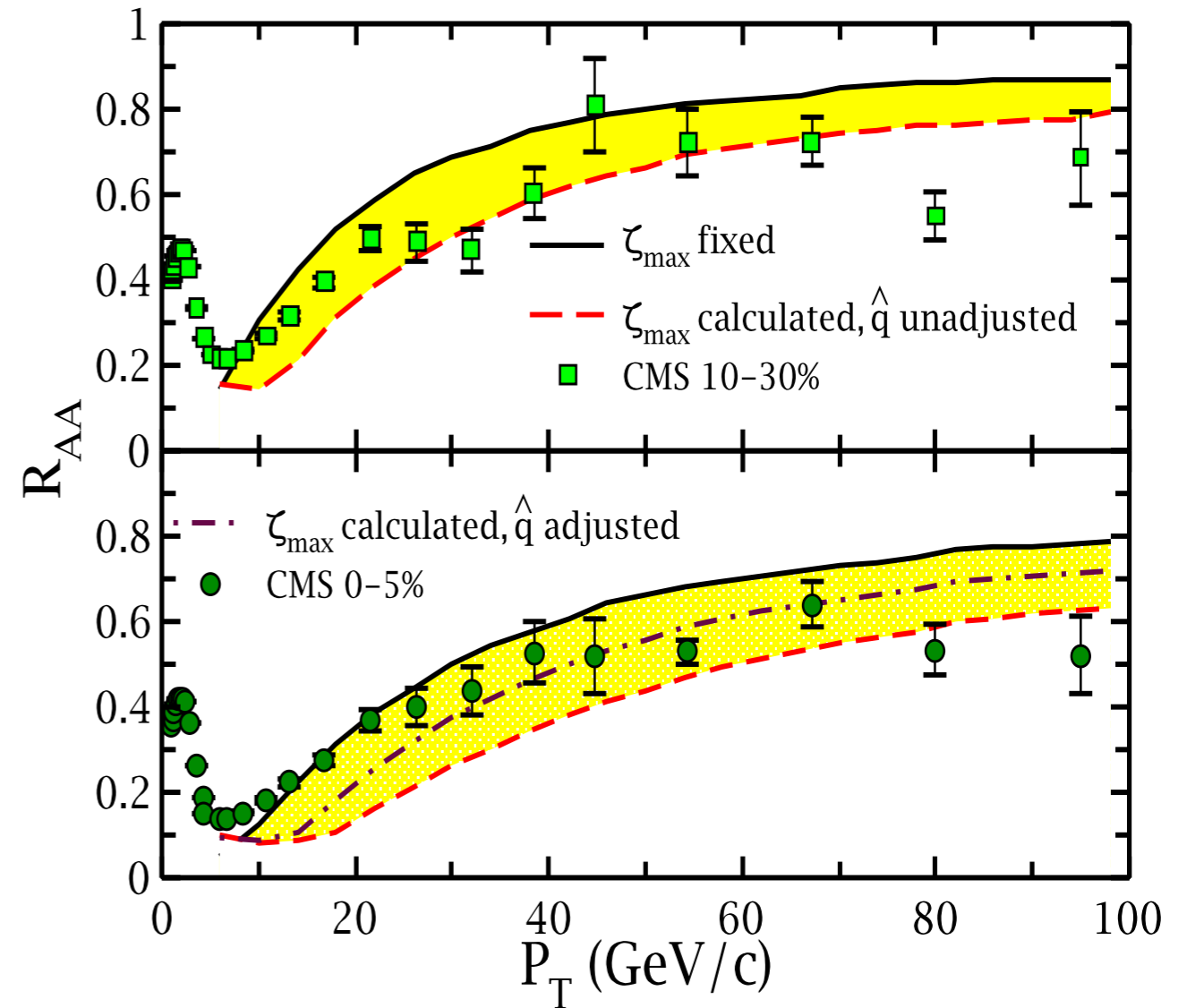
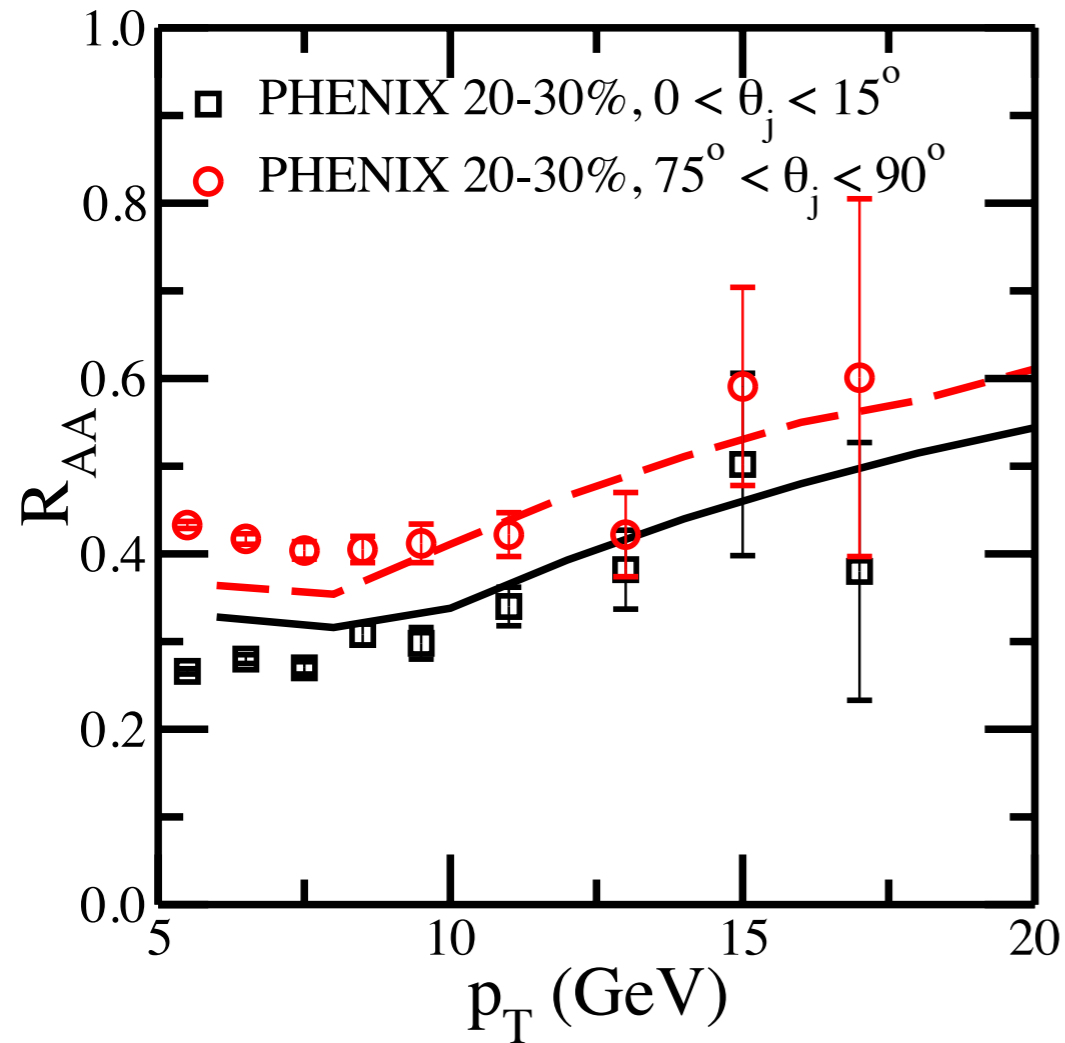


Note: no refitting between RHIC and LHC.





# Versus reaction plane, versus energy



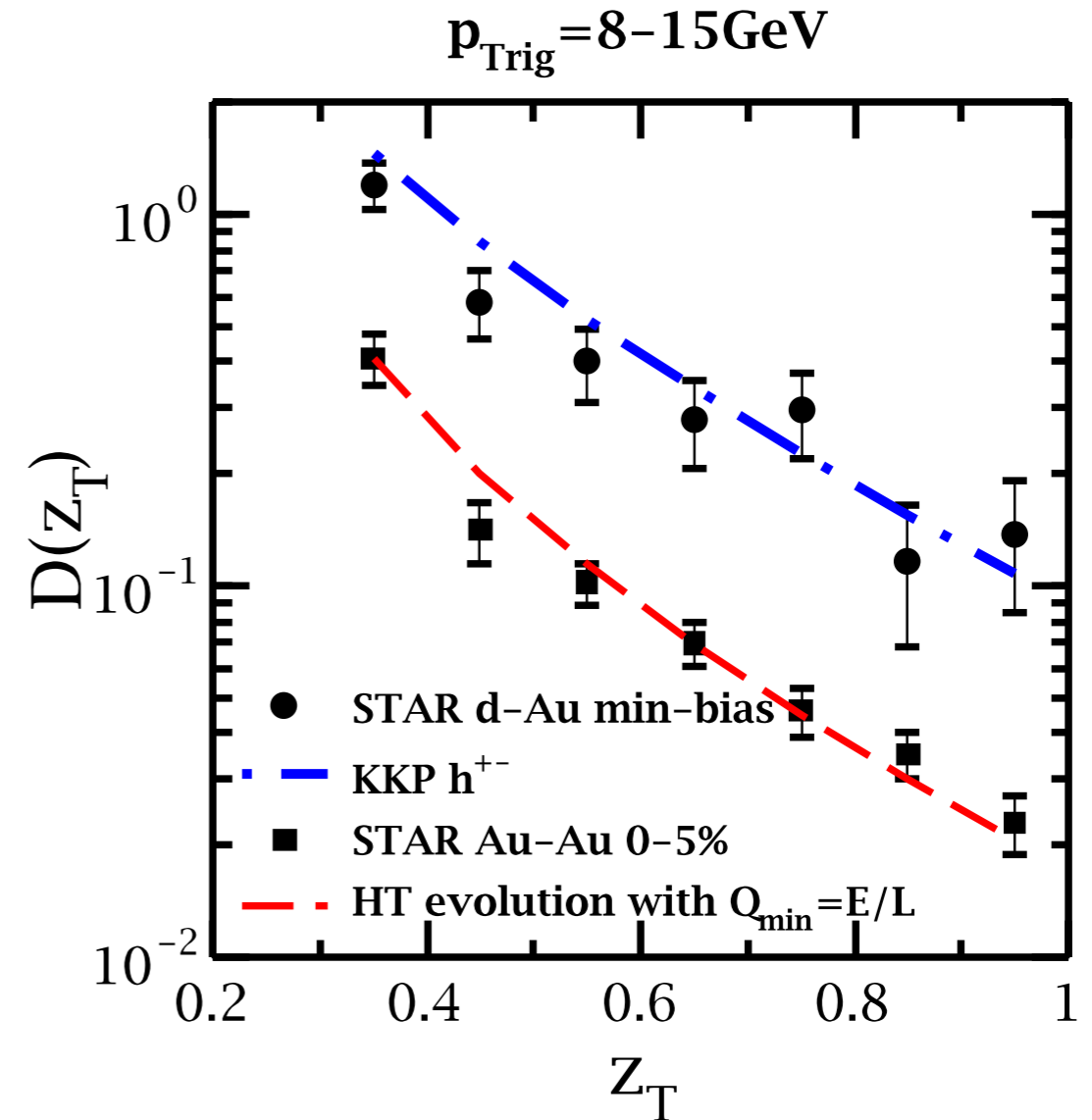
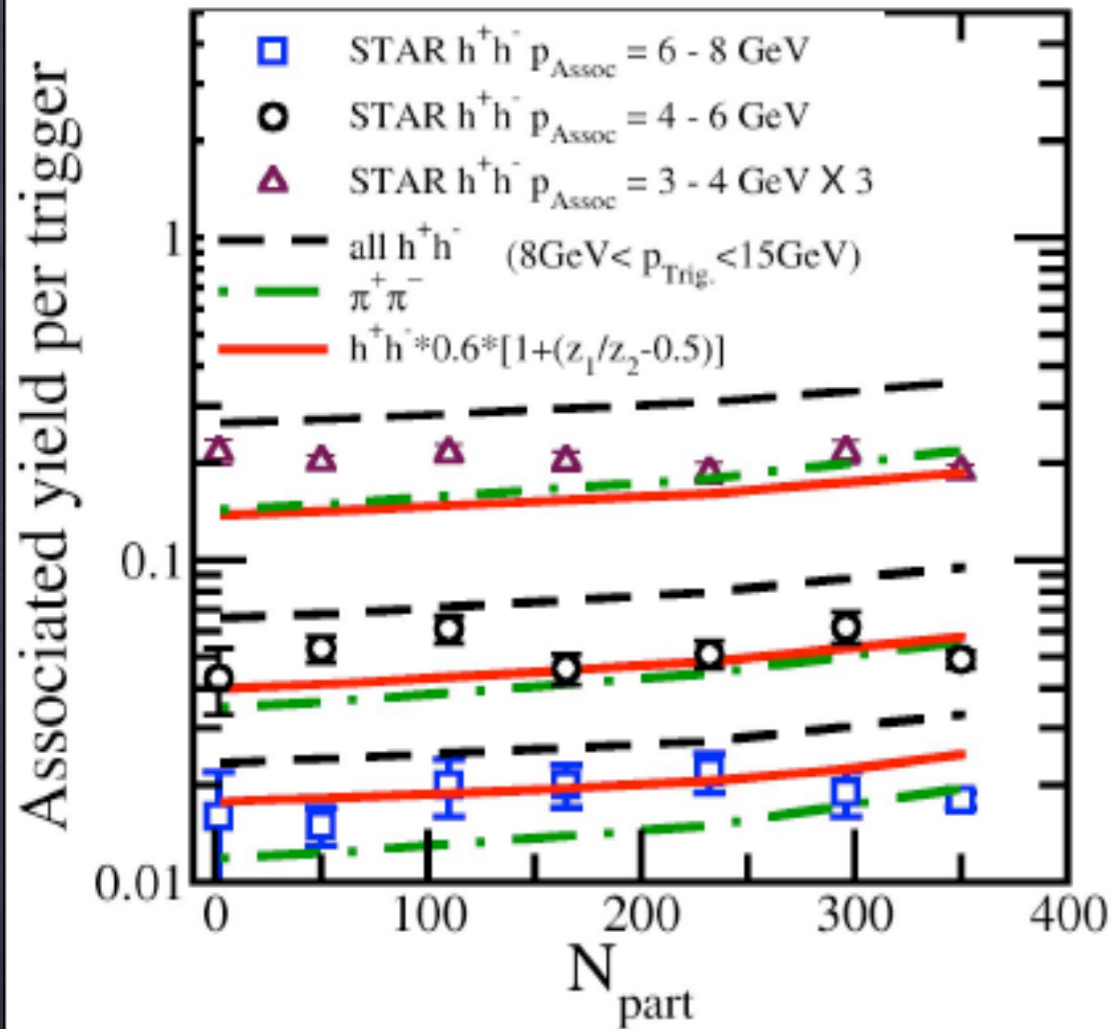
Reasonable agreement with data

Several improvements can be made from this point



# Completely consistent predictions for Dihadrons

A. Majumder, et. al., nucl-th/0412061



These are parameter free calculations  
 The near side involves a new non-perturbative object  
 the dihadron fragmentation function



# From factorized analytical approaches to event generators



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Looks at full jet, so less sensitive to fragmentation



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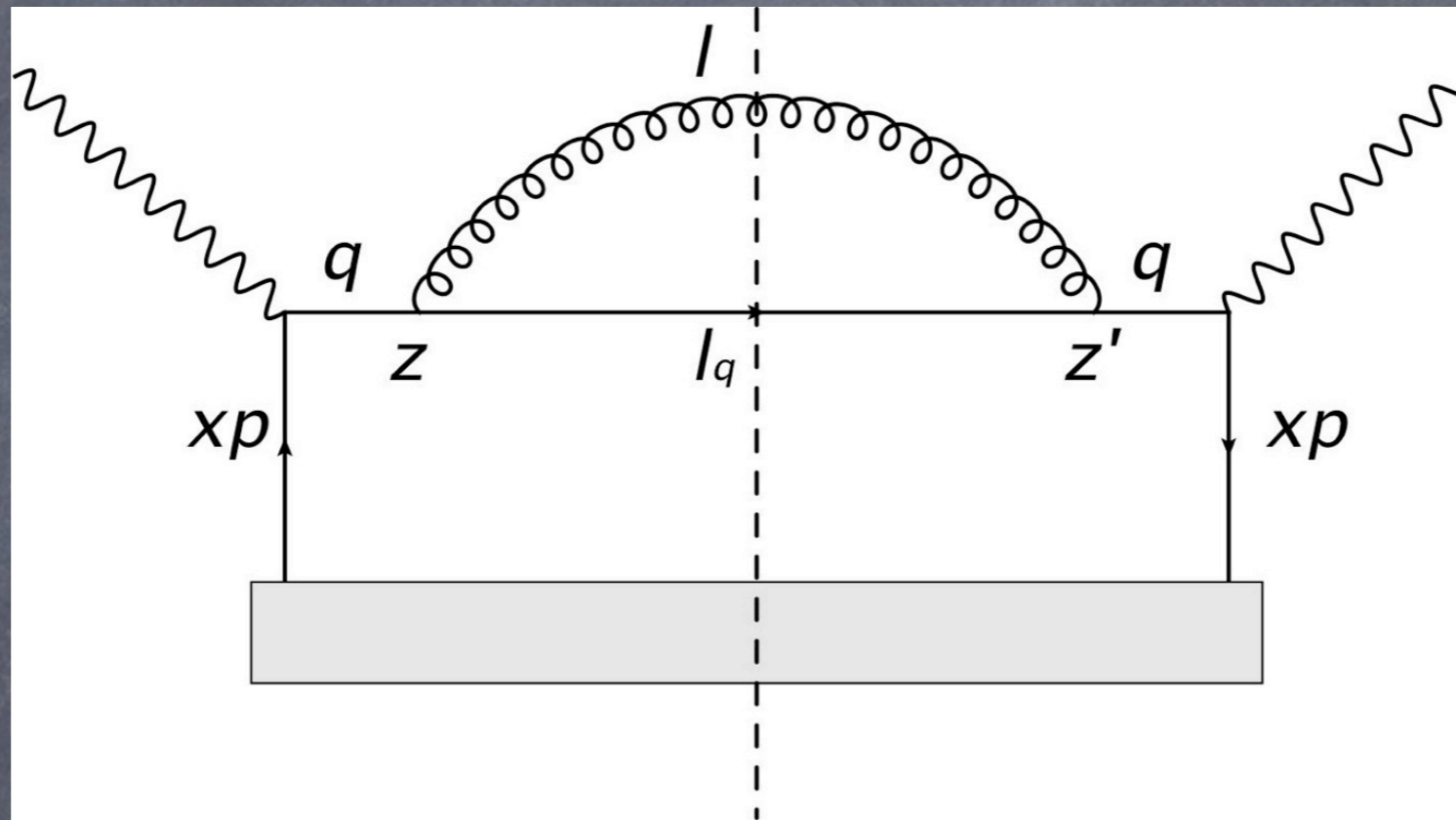
Rigorously calculating this requires more non-perturbative transport coefficients



Main problem: Introducing distance into a DGLAP shower

No space-time in the usual Monte-Carlo showers

$$\bar{z} = \frac{z + z'}{2}$$



$$\delta z = z - z'$$

what is the role of  $z$  and  $z'$  ?

$$\int_0^\infty d^4 \bar{z} \exp [i(\delta q) \bar{z}] \int d^4 \delta z \exp [i\delta z (l + l_q - q)]$$

$\delta q$  is the uncertainty in  $q$ ,



# How much uncertainty can there be ?

To be sensible:  $\delta q \ll q$

we assume a Gaussian distribution around  $q^+$

And try different functional forms of the width

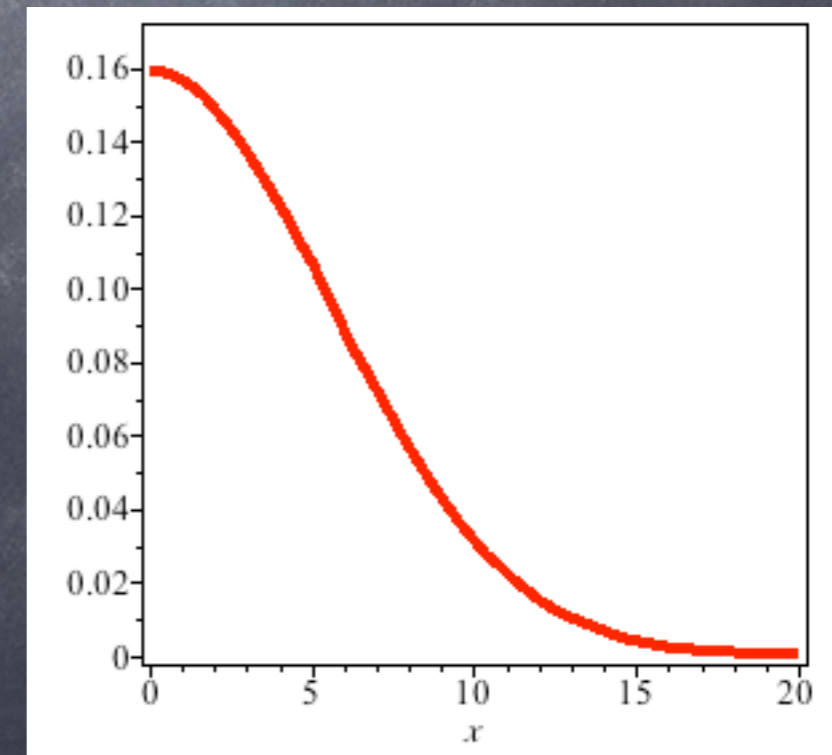
We set the form by insisting  $\langle \tau \rangle = 2q^+ / (Q^2)$

to obtain the  $z^-$  distribution only need to assume a  $\delta q^+$  distribution

$$\rho(\delta q^+) = \frac{e^{-\frac{(\delta q^+)^2}{2[2(q^+)^2/\pi]}}}{\sqrt{2\pi[2(q^+)^2/\pi]}}$$

FT gives  
the following  
distribution in  
distance

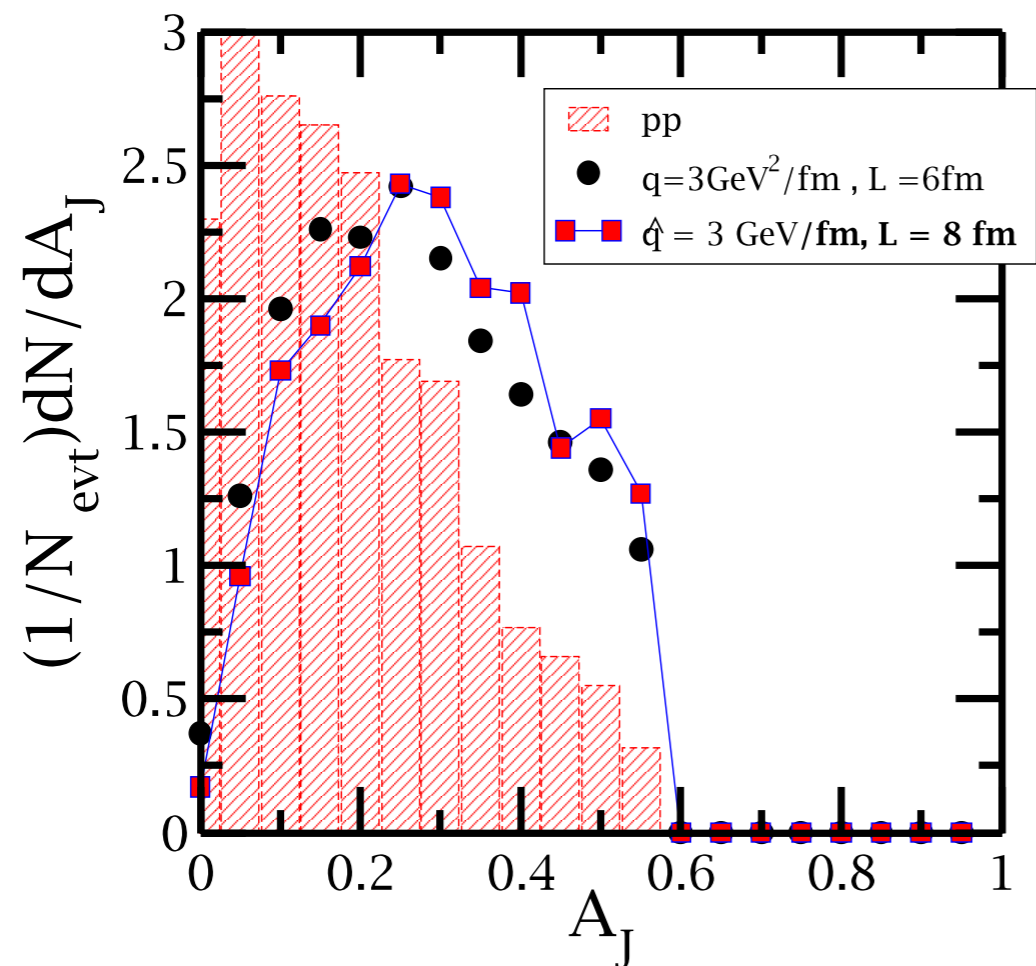
A normalized Gaussian with  
a variance  $2q^+/\pi$



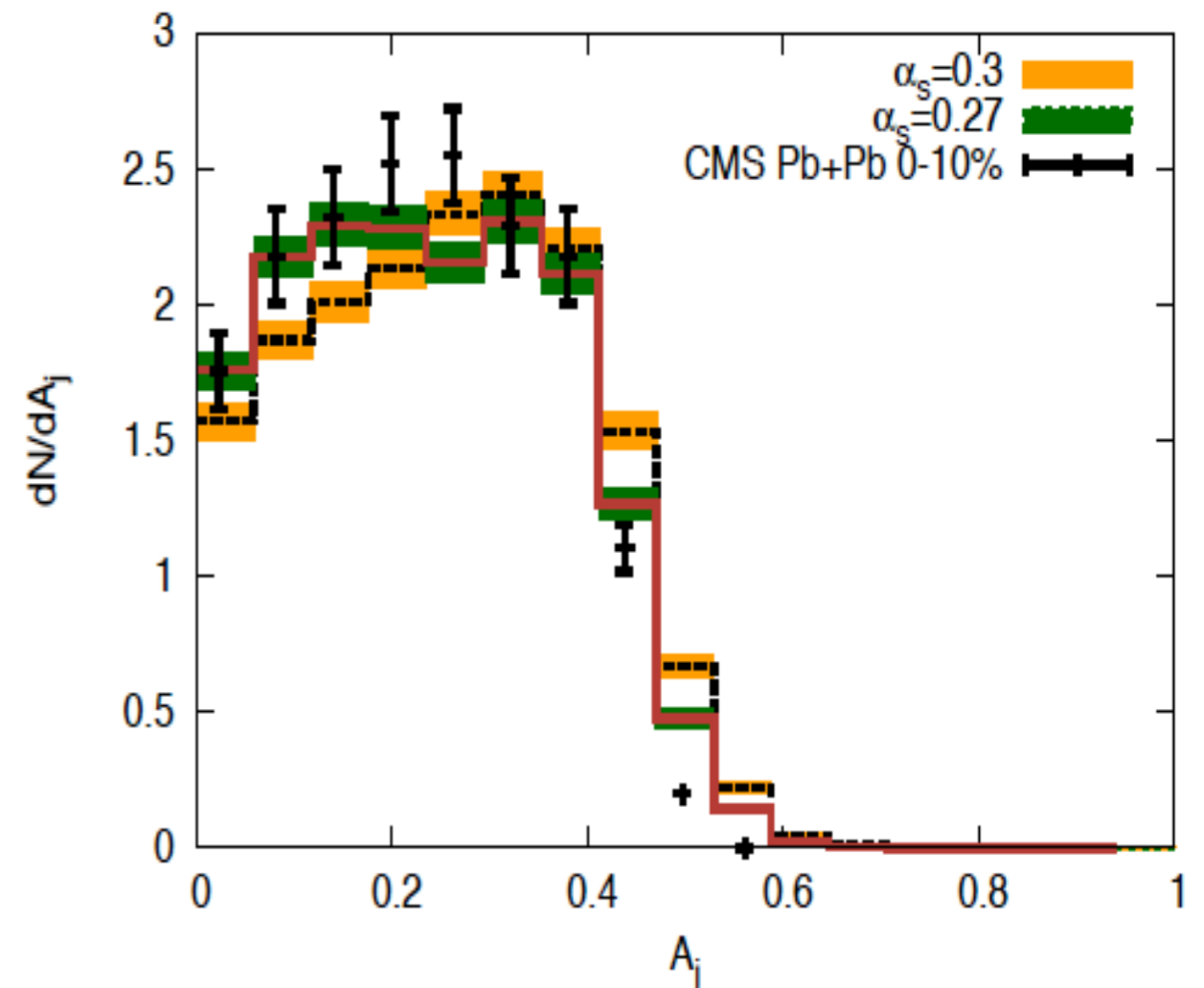


# Observables 1. $A_J$

If you ignore  $R_{AA}$  this is not hard



Higher Twist in box

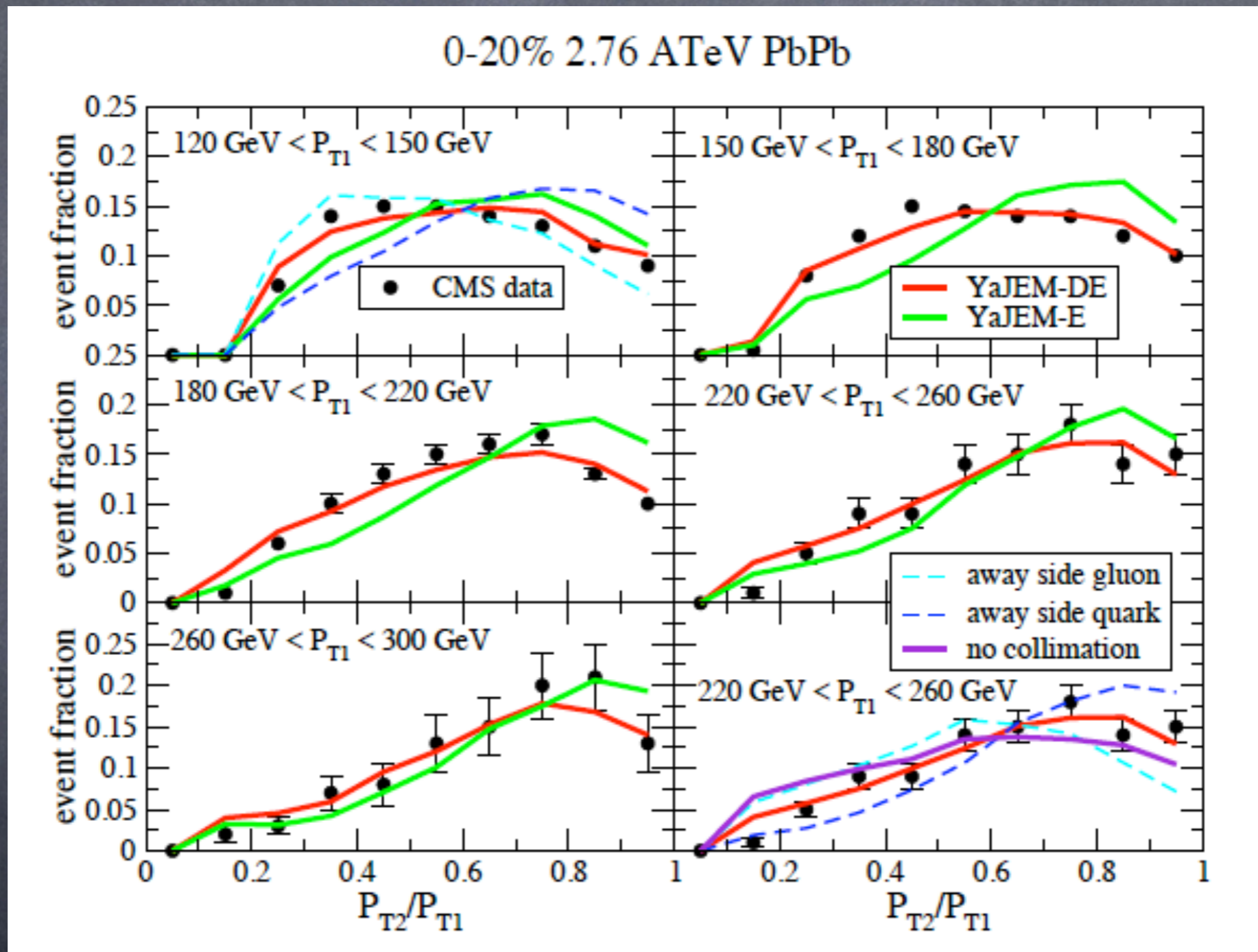


MARTINI without  $R_{AA}$



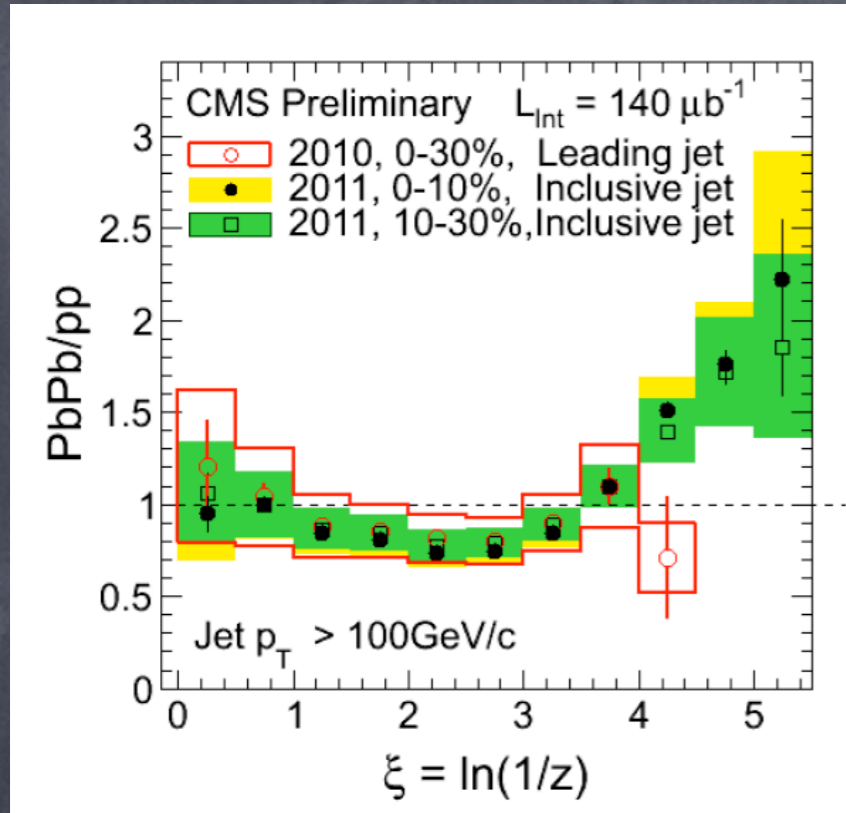
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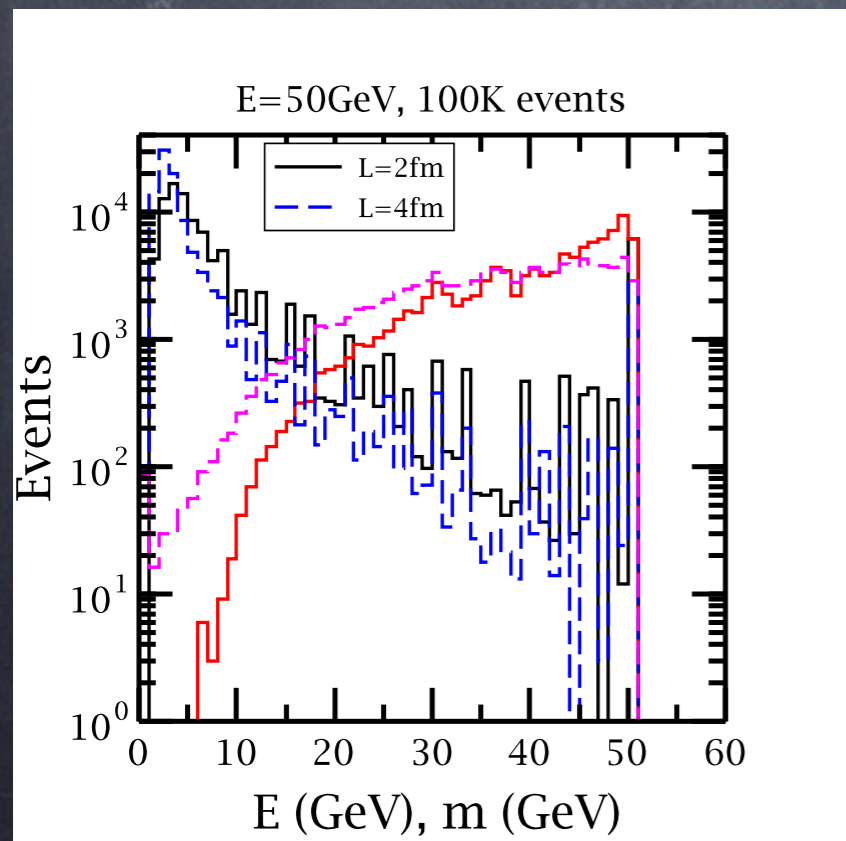
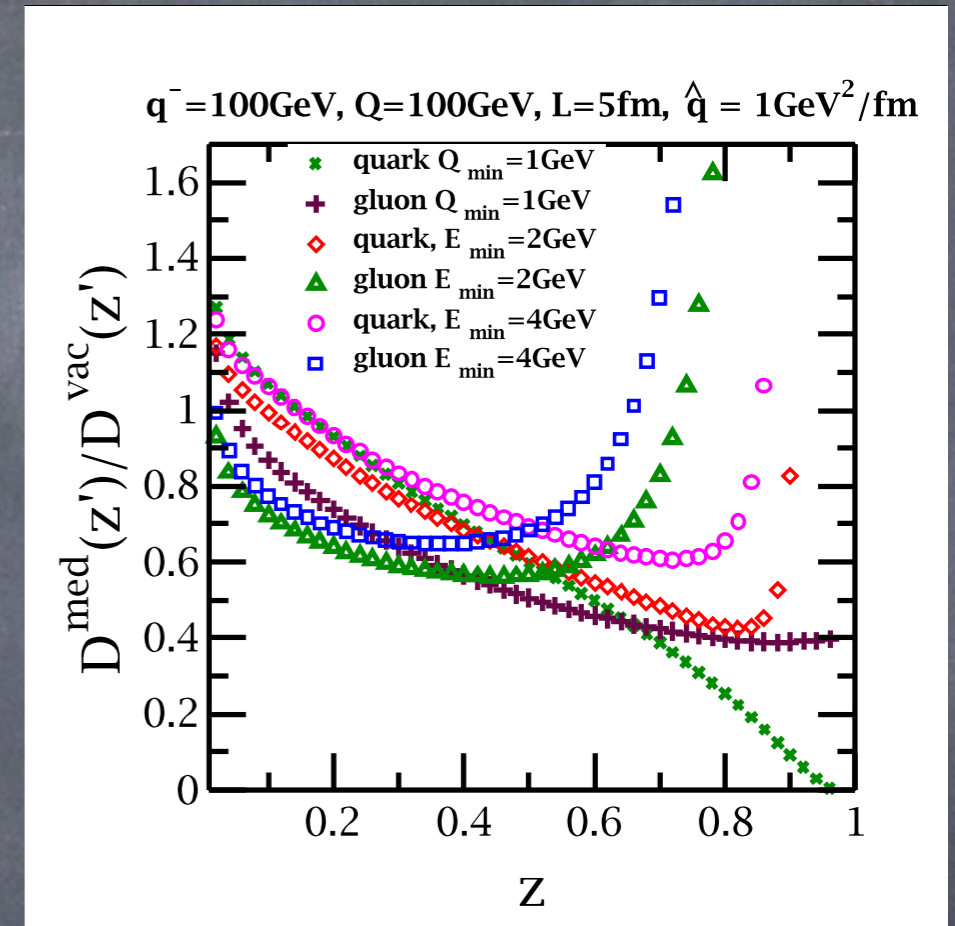


# Observable 2: Fragmentation function!

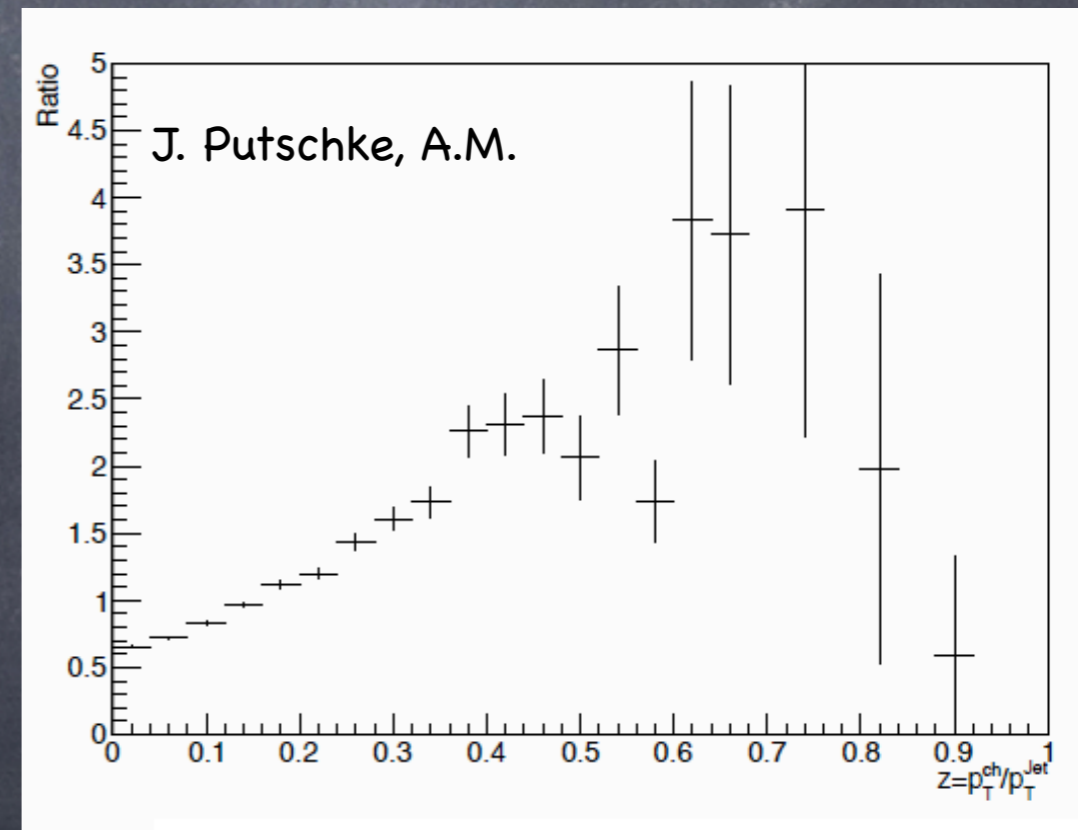


lost energy  $\rightarrow$

loss of virtuality



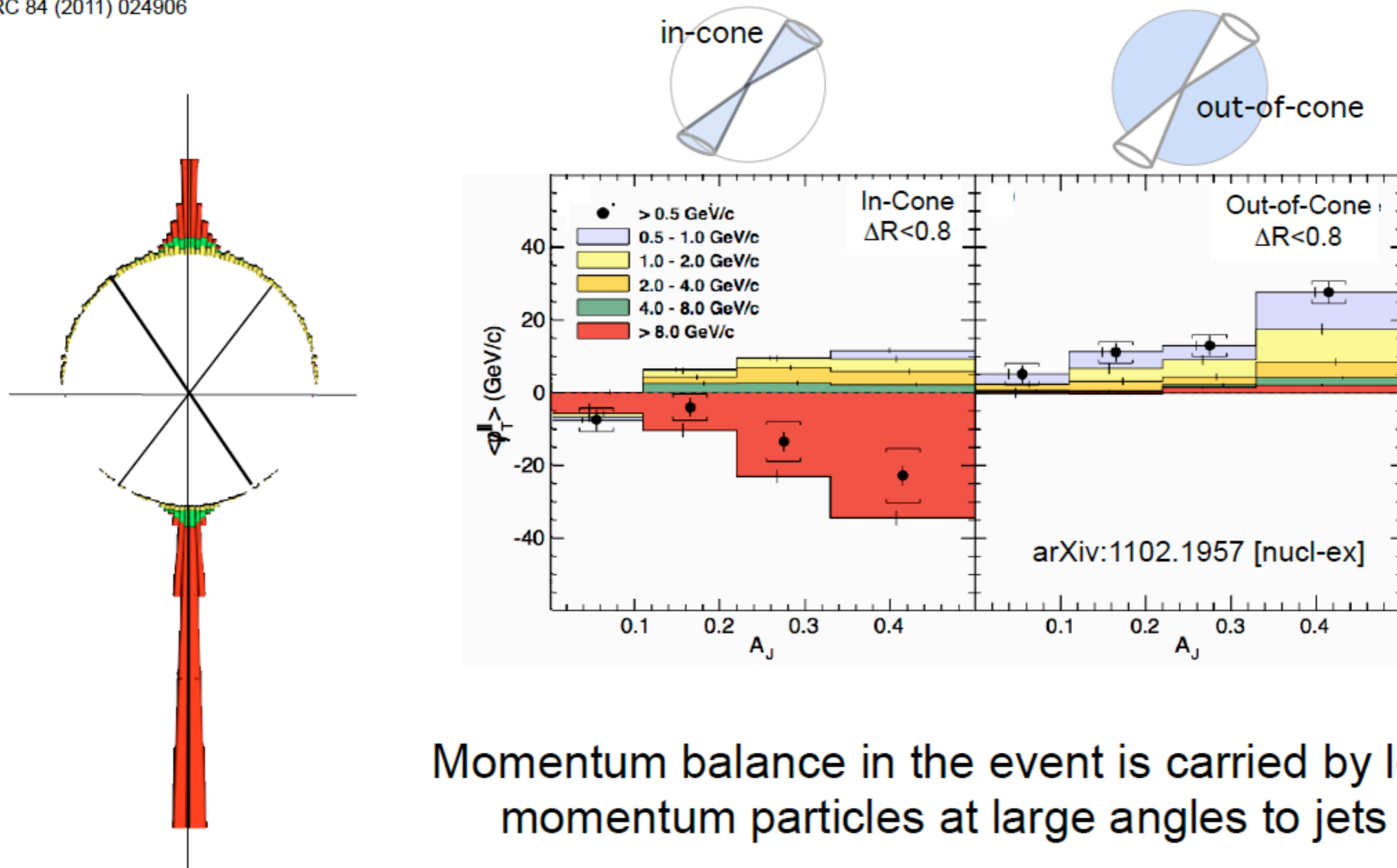
ratio of fragmentation functions with different virtuality





# Observable 3. Appearance of lost Energy

PRC 84 (2011) 024906

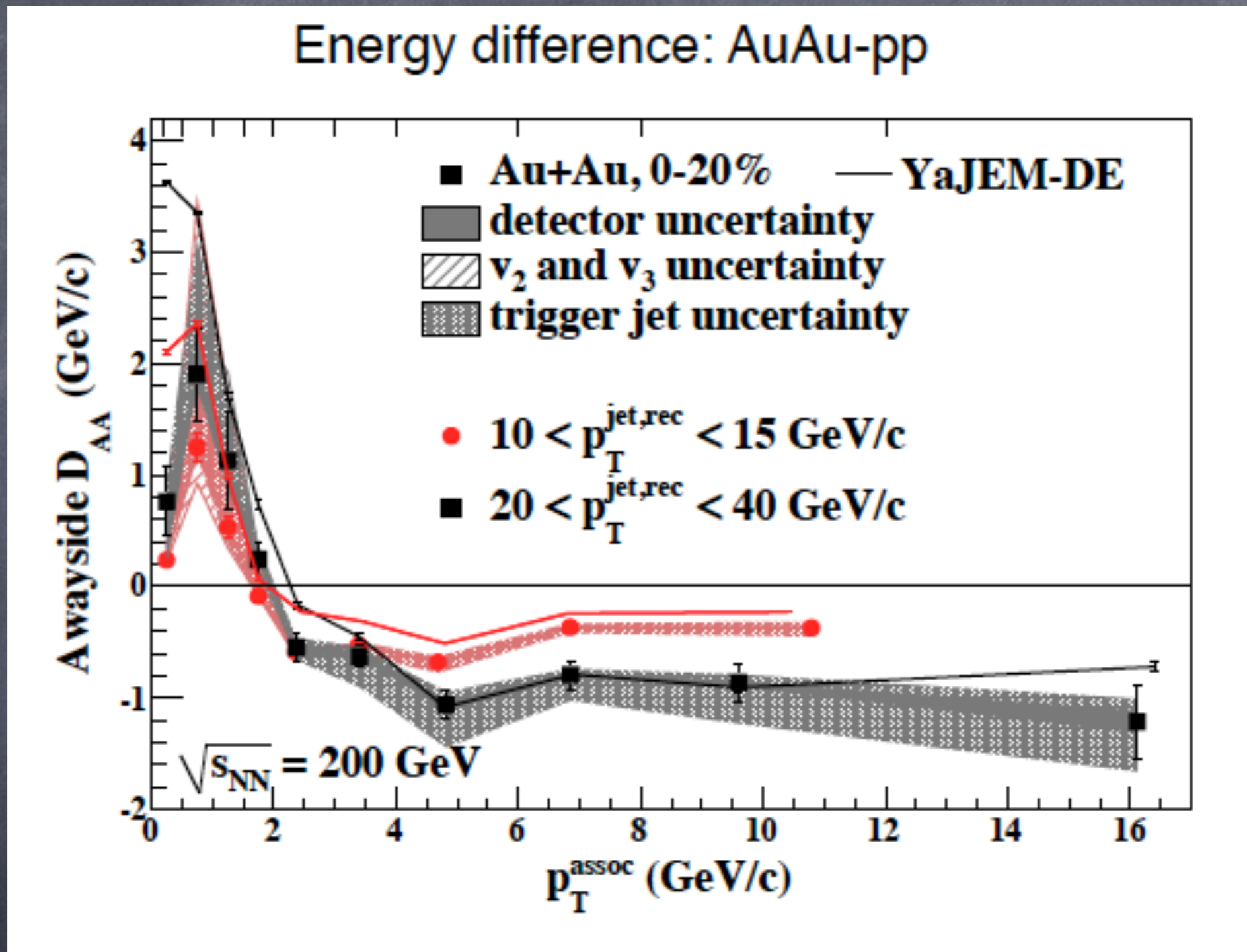


Momentum balance in the event is carried by low momentum particles at large angles to jets

$$\cancel{p}_T^{\parallel} = \sum_{\text{Tracks}} -p_T^{\text{Track}} \cos(\phi_{\text{Track}} - \phi_{\text{Leading Jet}})$$

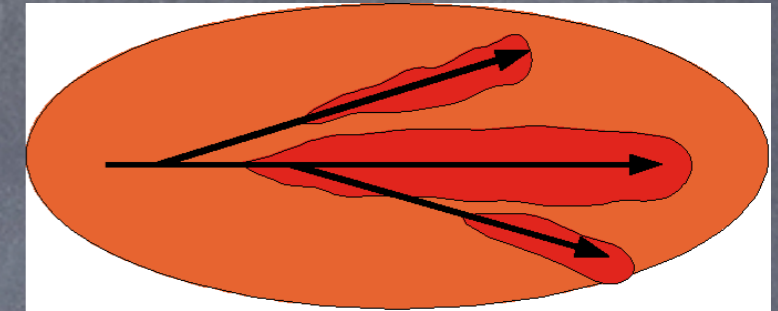
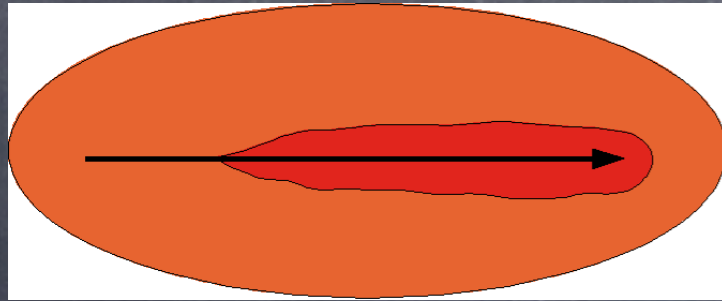


# Observable 3. Appearance of lost Energy



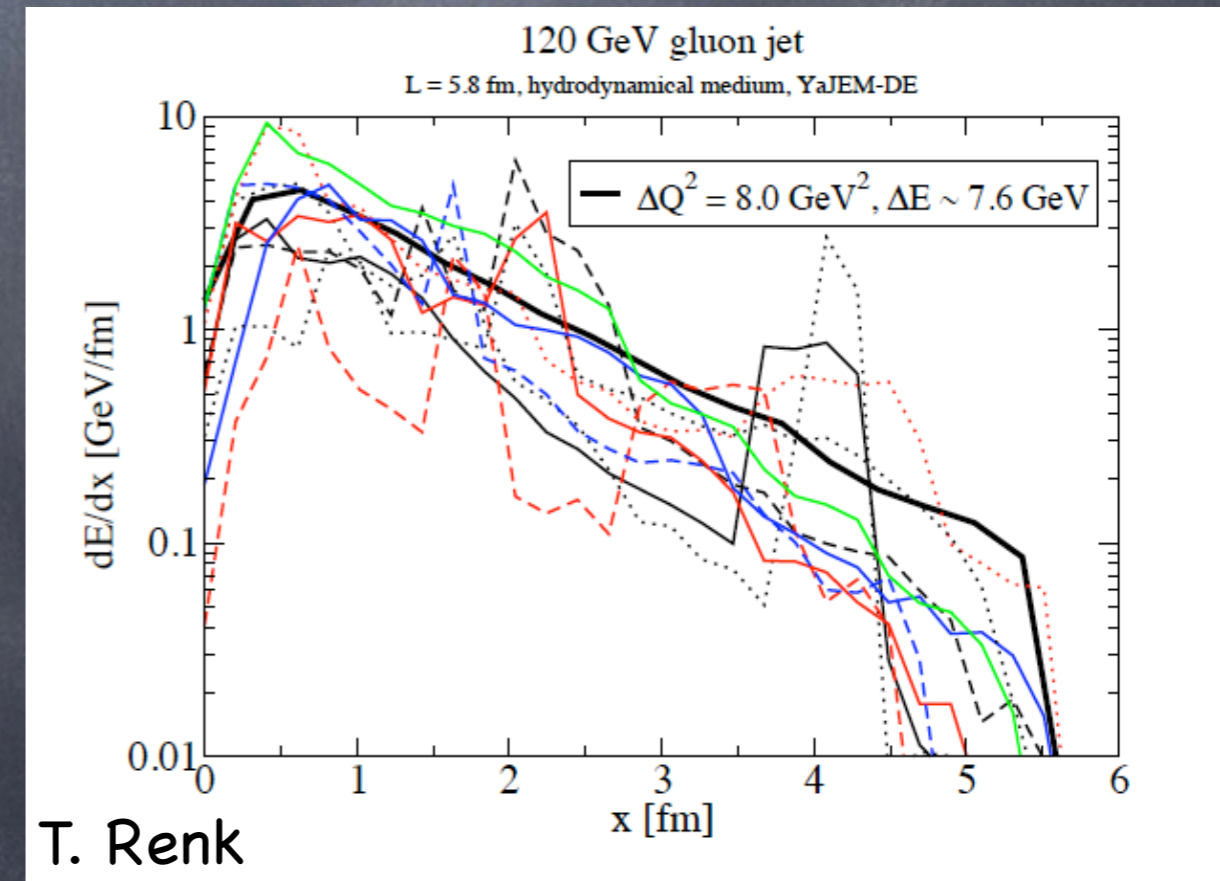
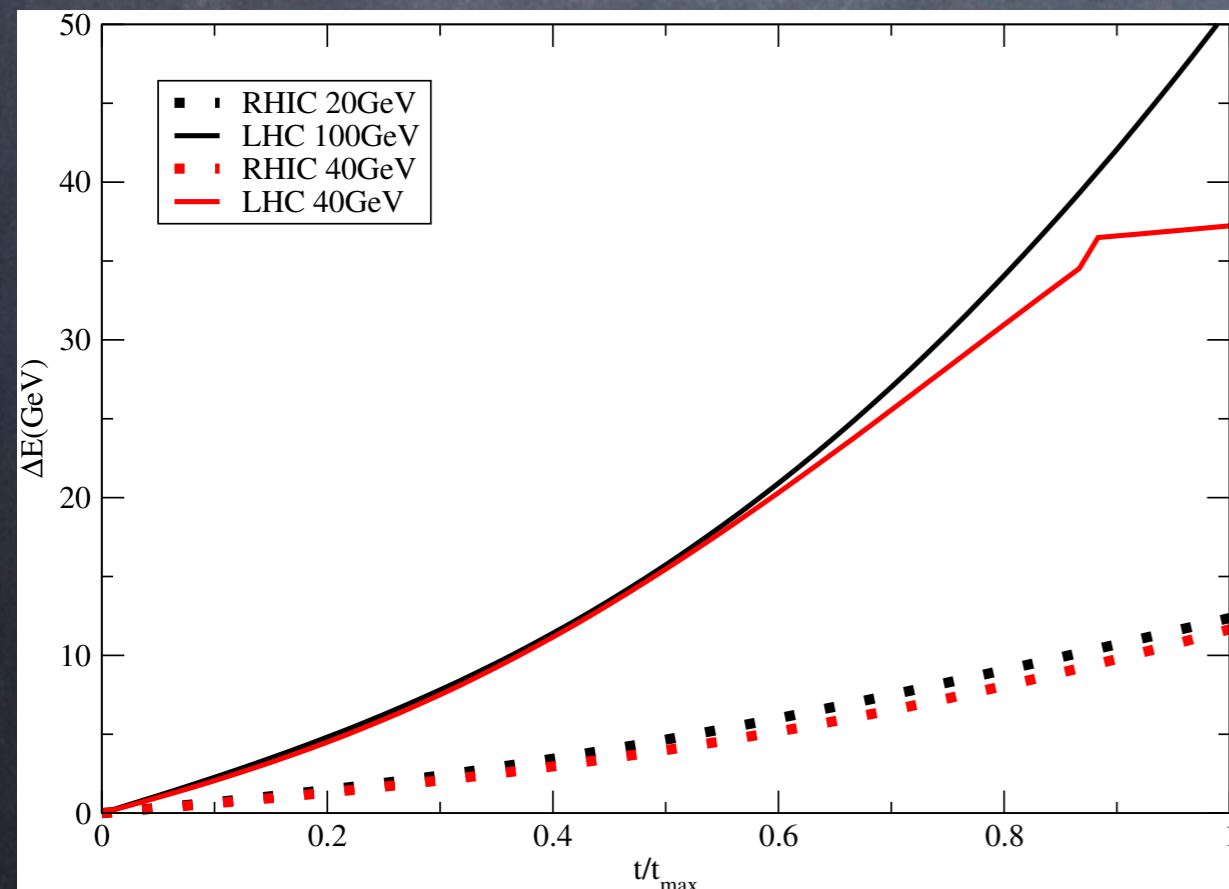


# To understand this need to know how jets deposit energy into a medium



Rate of energy deposition greater at LHC  
large part of the jet escapes the medium

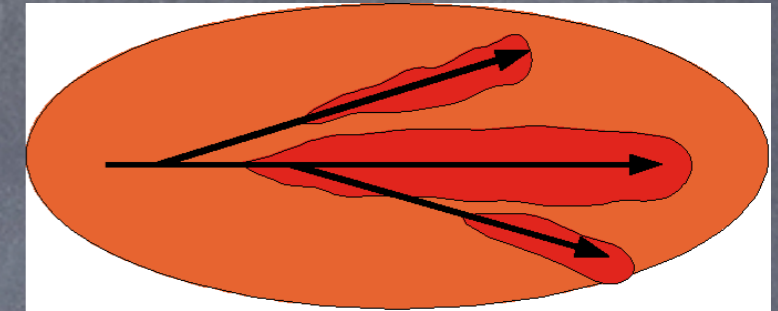
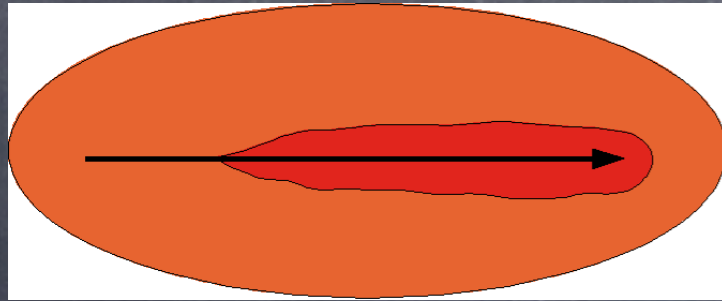
Medium dissipates in time,  
so early energy loss is important



T. Renk

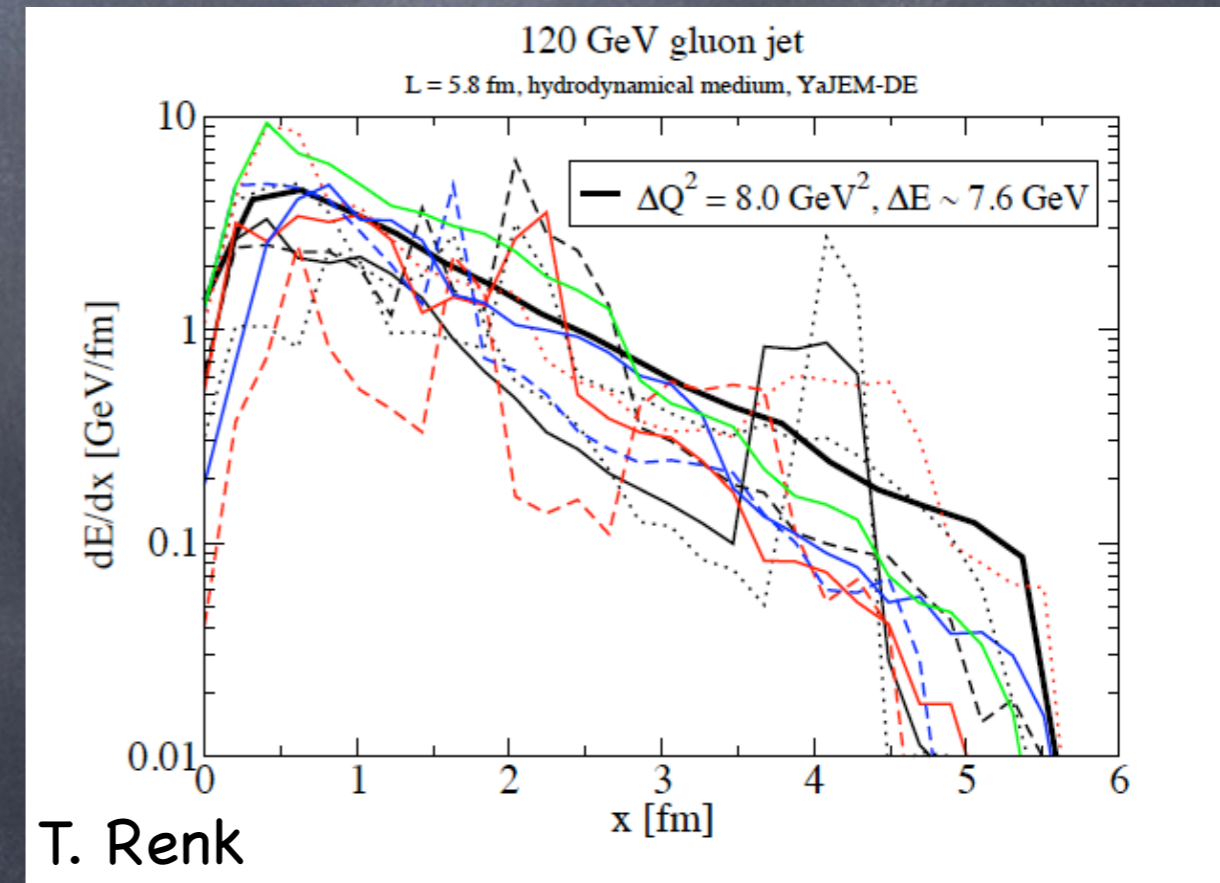
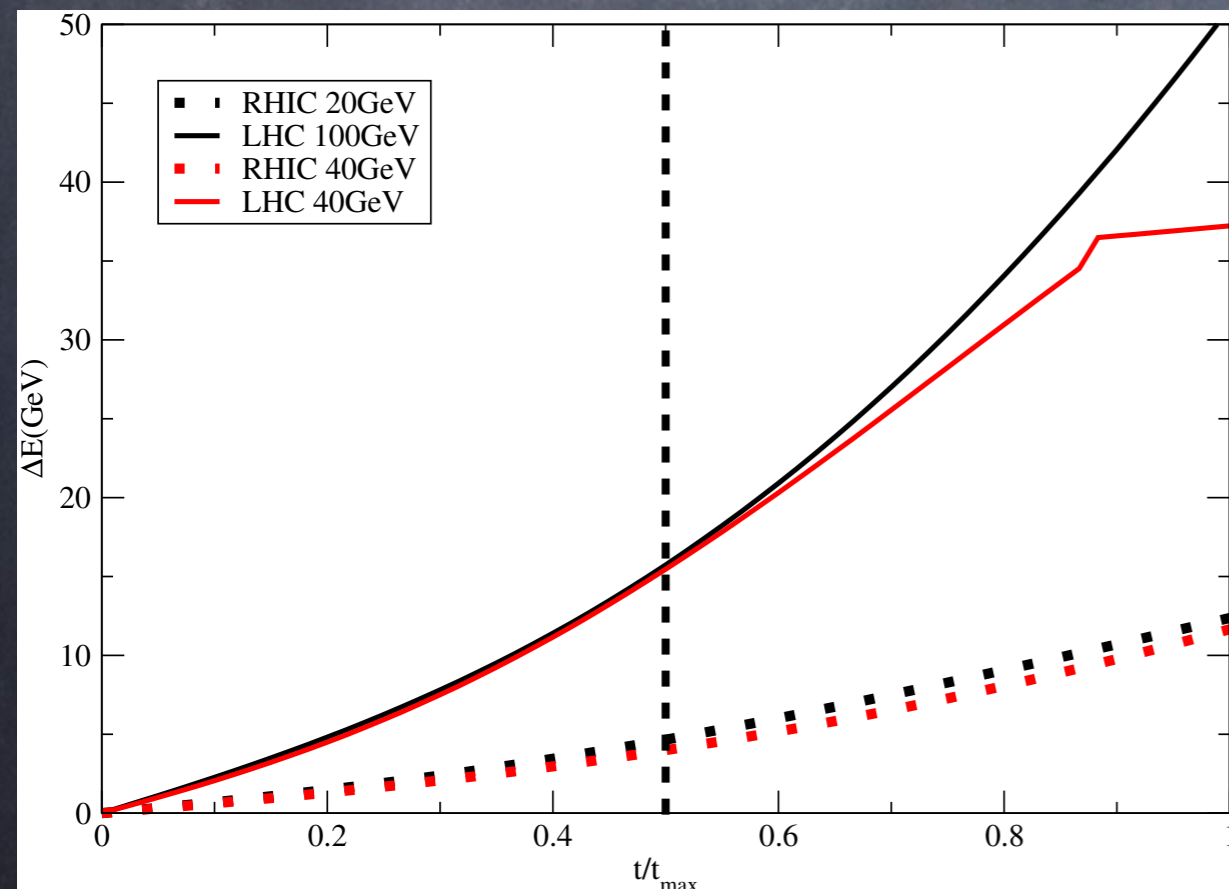


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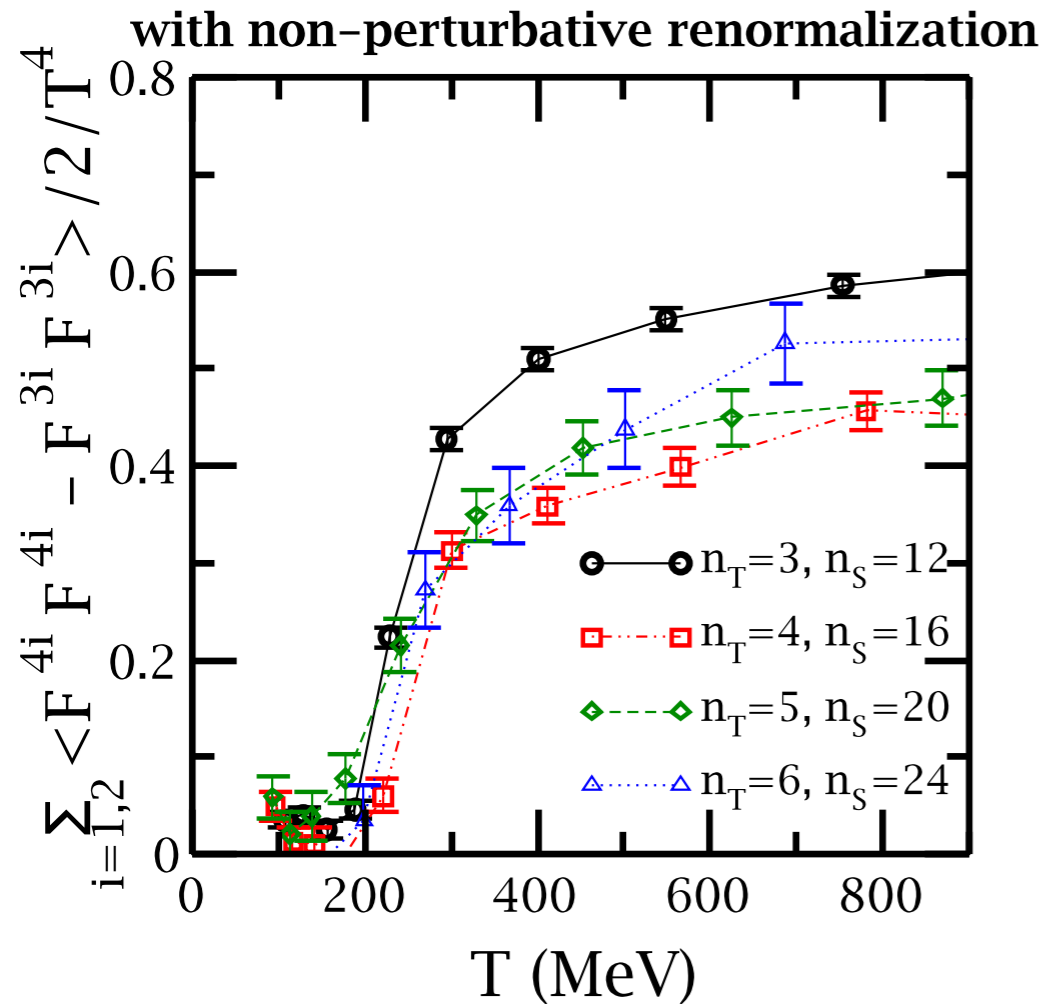


T. Renk

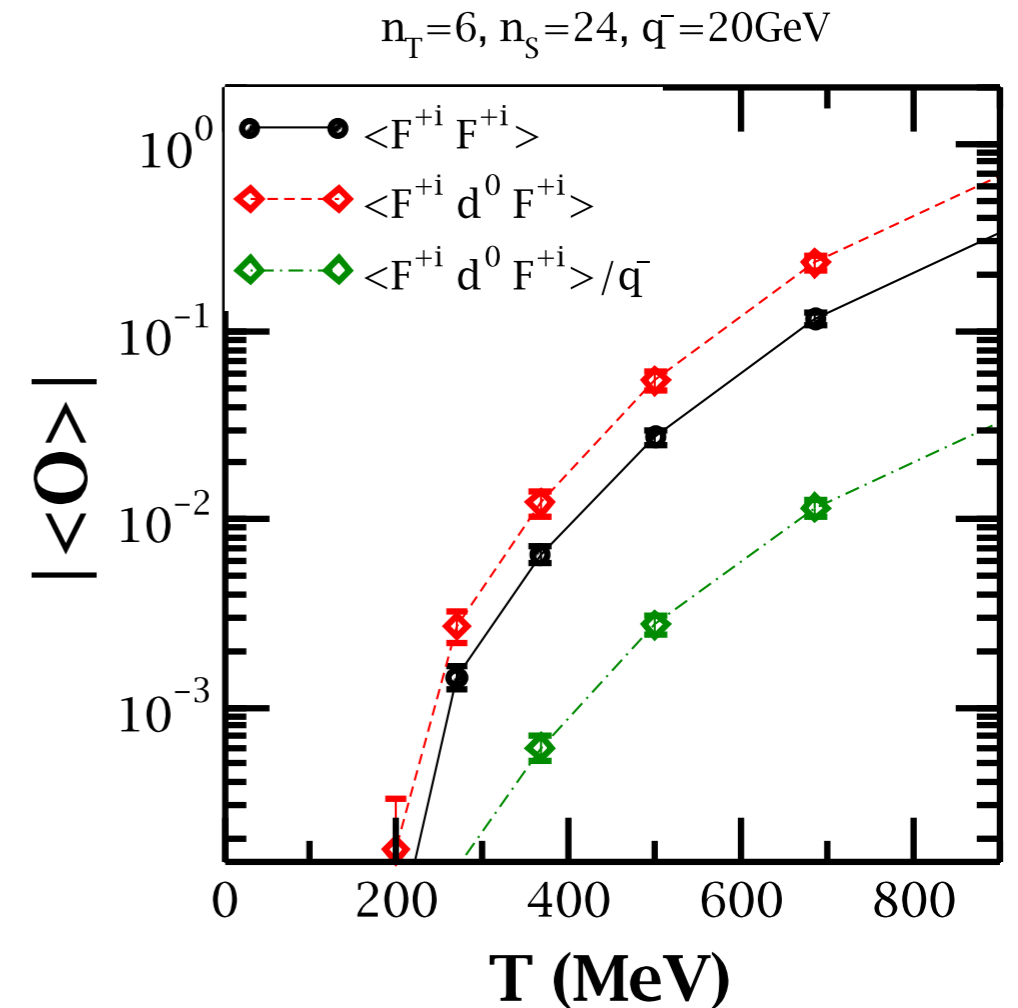


# Getting ahead of the experiment

## Calculating $\hat{q}$ on the lattice



AM, PRC 87 034905



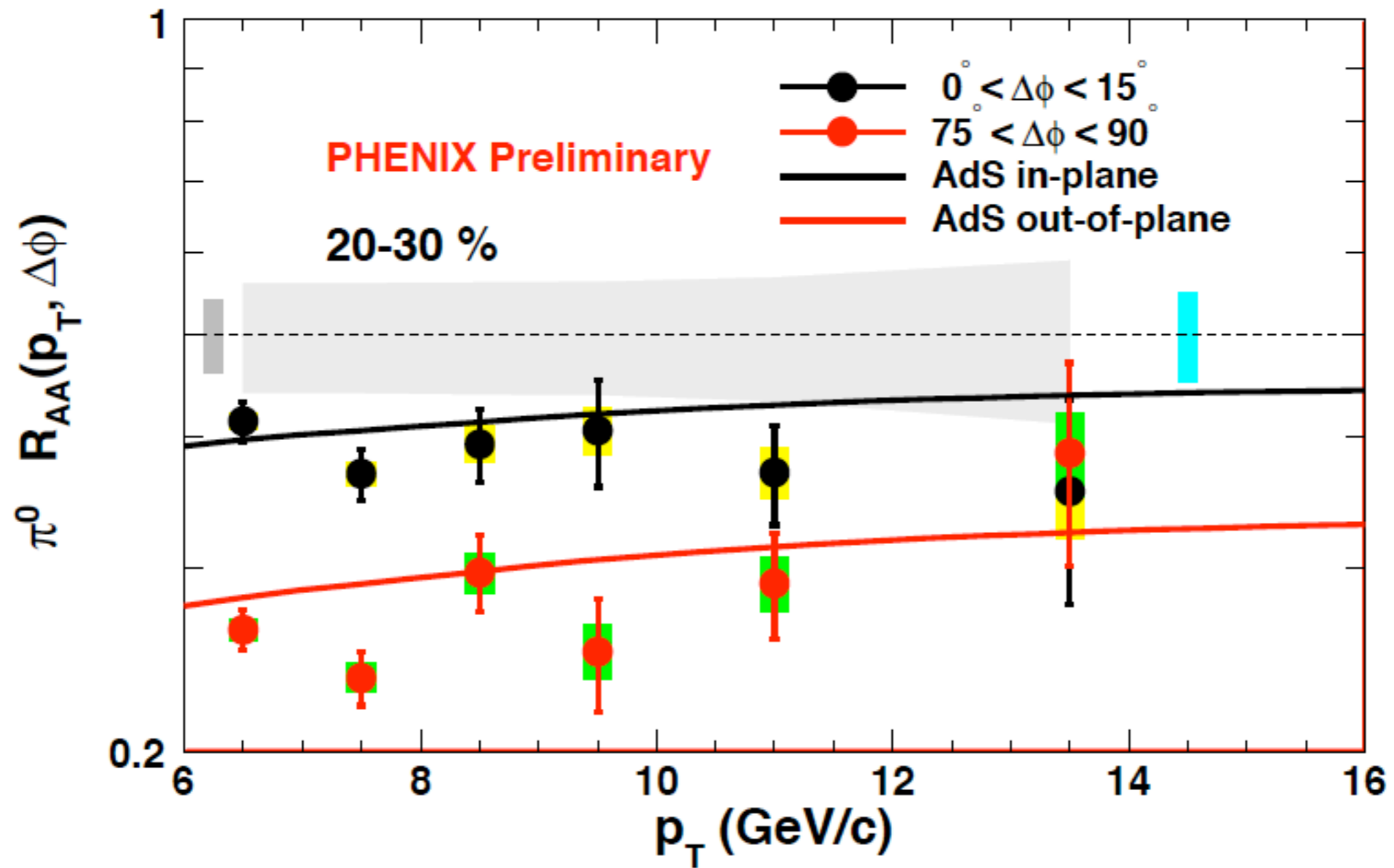
Future calculations will have T dependent  $\hat{q}$  input from lattice



# Conclusions

- I have ignored  $\gamma$ -h and  $\gamma$ -jet, lack of space
- There is now a clear theory of pQCD based jet modification (Jet coupled weakly to a strongly coupled medium)
- Have a series of transport coefficients from few h data
- Sensitivity to new transport coefficient from new jet data
- Lots of work to be done in resolving the intricate details of comparing and tuning event generators to data.



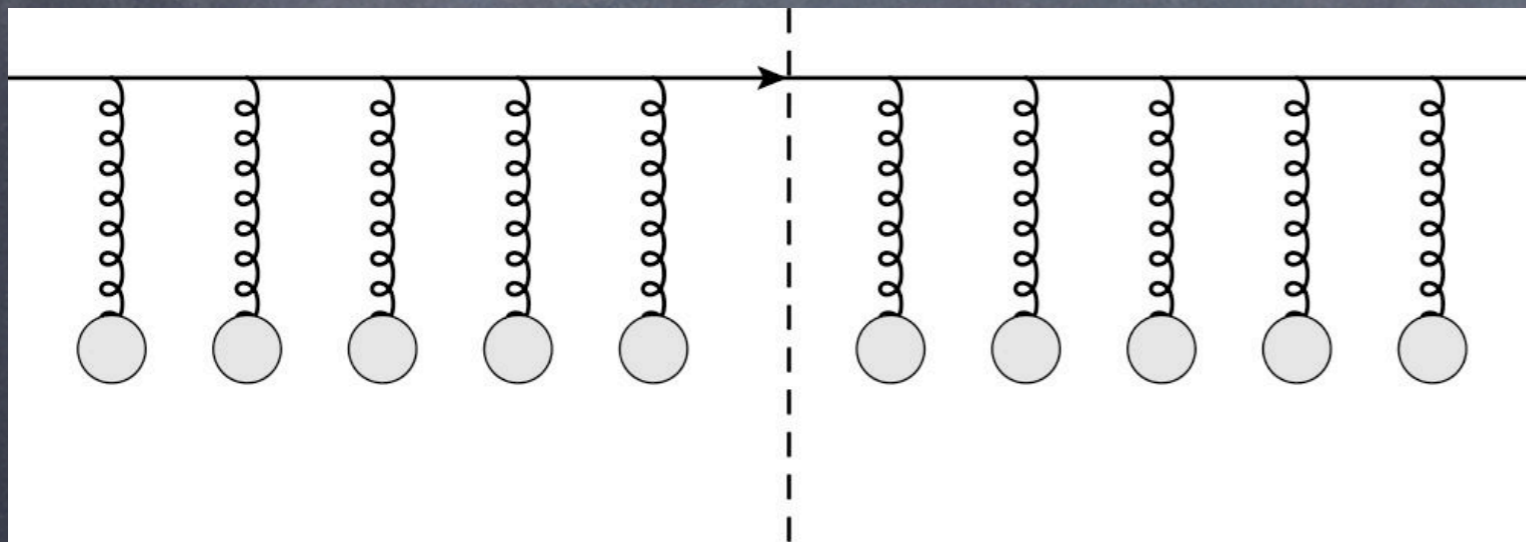




# How the medium affects the parton.

A parton in a jet shower, has momentum components

$$q = (q^-, q^+, q_\perp) = (1, \lambda^2, \lambda)Q, \quad Q: \text{Hard scale}, \quad \lambda \ll 1, \quad \lambda Q \gg \Lambda_{\text{QCD}}$$



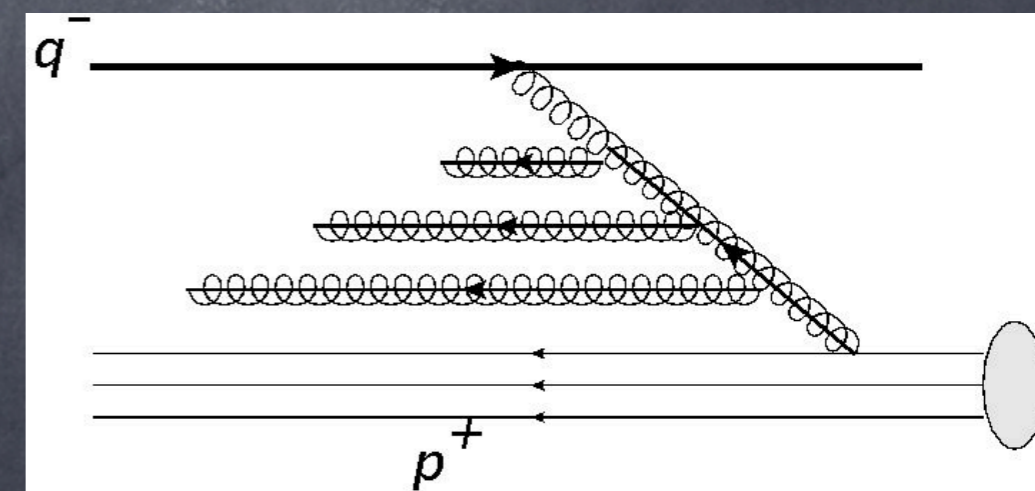
$$p^+ = \frac{p^0 + p_z}{\sqrt{2}}$$

$$p^- = \frac{p^0 - p_z}{\sqrt{2}}$$

hence, gluons have

$$k_\perp \sim \lambda Q, \quad k^+ \sim \lambda^2 Q$$

could also have  $k^- \sim \lambda Q$





## The Basic steps:

- 1) Write down the general structure in position space.
- 2) Fourier transpose all propagators to momentum space
- 3) Assume all  $k^-$  are  $\ll q^-$ , integrate. out the  $k^-$  .
- 4) Do as many  $k^+$  integrals, this time-orders the locations
- 5) There will always be one propagator not on shell
- 6) Expand in  $k_T^2/l_T^2$  and keep the leading term.



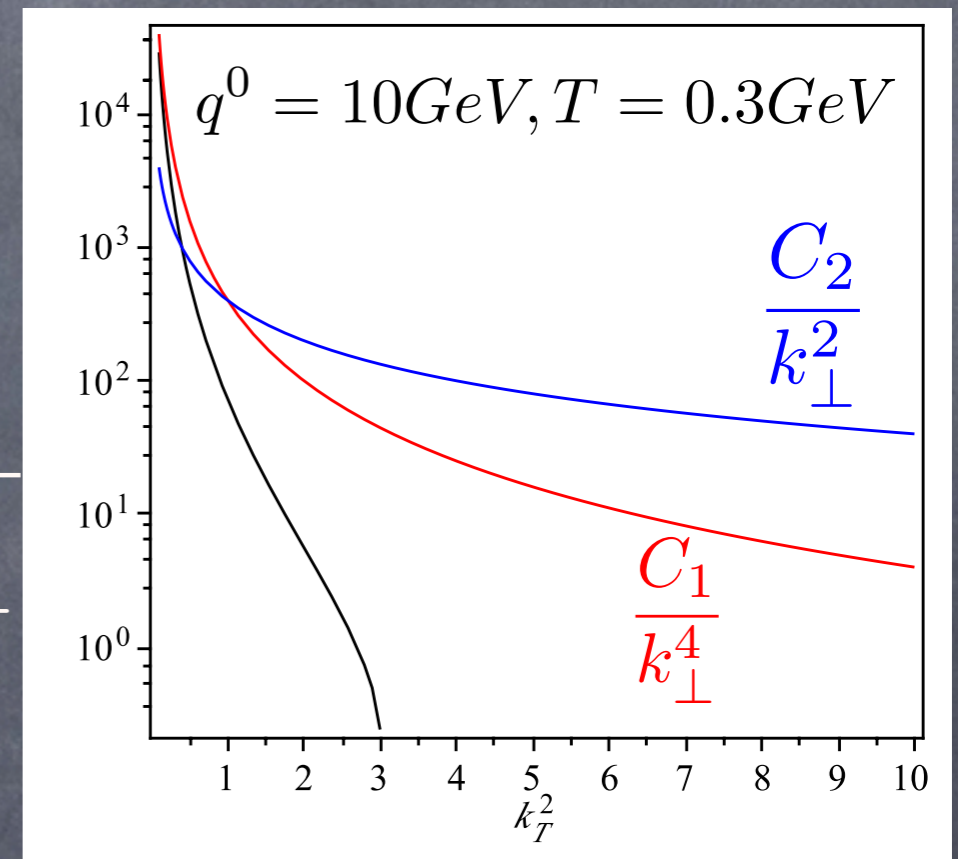
# Gaussian distribution/temperature dependence/fit parameter !!!

Multiple scattering off any distribution samples a Gaussian

$$\hat{q} \sim T^3, s, \epsilon^{3/4}$$

is basically a model

$$\frac{d\sigma}{dk_{\perp}^2}$$



Ultimately you have to fit the normalization to 1 data point at one centrality, one value of  $p_{\text{T}}$ , one HIC energy

“So, its not really first principles!”, S.S. Gubser