# Measurement of anisotropic radial flow in relativistic heavy ion collisions

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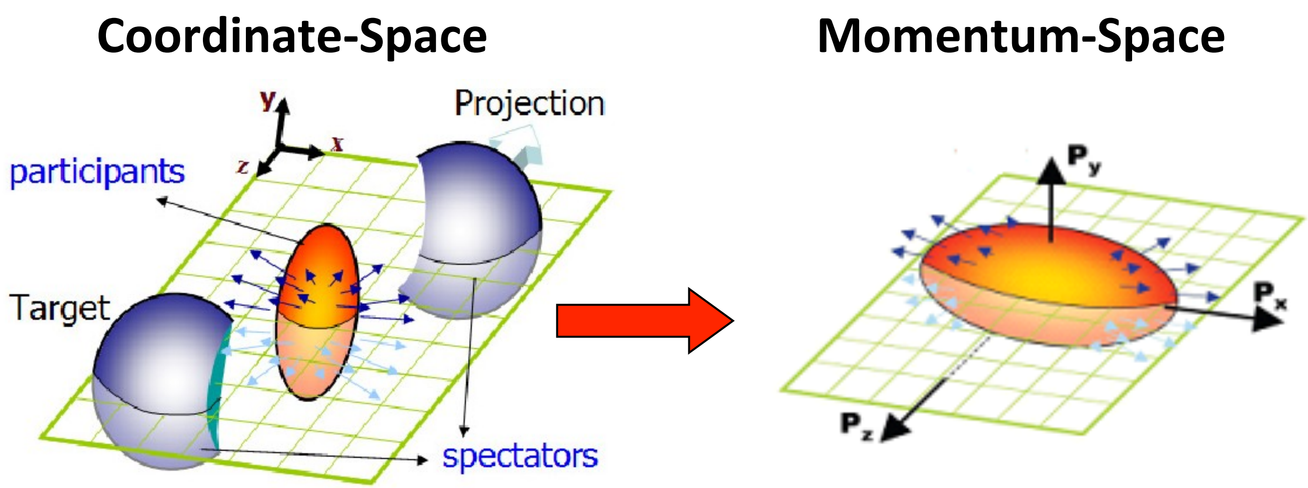
Li Lin, Li Na, Wu Yuanfang, CPC, 36, 423(2012); J. Phys. G40, 075104(2013)

# 1. Motivation

◆ **Elliptic flow:**




**Azimuthal multiplicity distribution:**

$$\frac{dN}{d\phi} \propto 1 + \sum_{n=1}^{\infty} 2v_n(N)\cos\left(n(\phi - \psi_r)\right) \qquad \phi = \tan^{-1}\left(\frac{p_y}{p_x}\right)$$

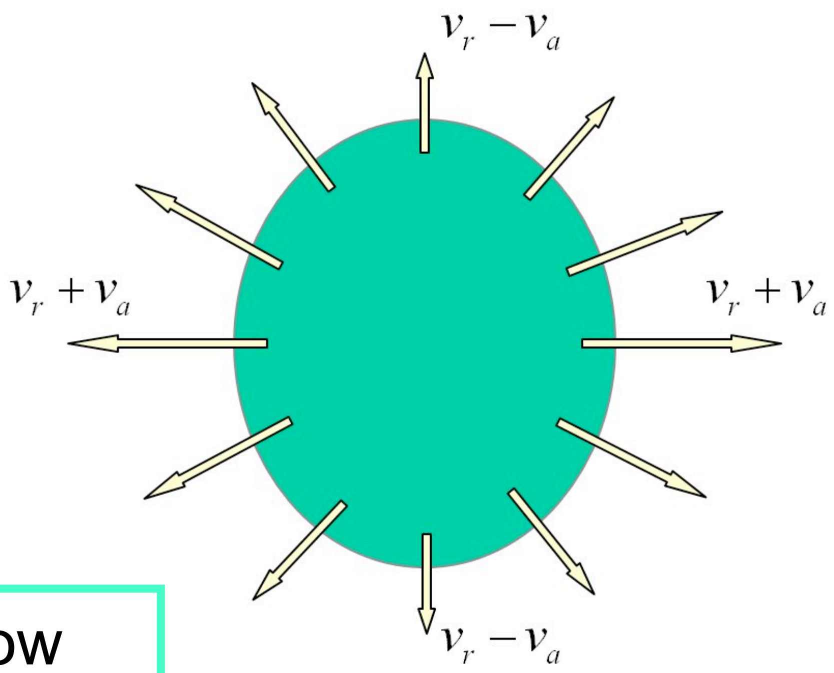
**Define elliptic flow:** $v_2(N) = \langle \cos(2(\phi - \psi_r)) \rangle$

➢ **It measures the anisotropy of azimuthal multiplicity distribution !**

# ◆ Radial expansion

3 velocities:

- Average radial expansion (isotropic) velocity ($v_r$)
- Anisotropic velocity ($v_a$ )
- Thermal velocity




Radial expansion + Elliptic flow

⇓

Particle mass splitting of differential elliptic flow at low transverse momentum region.

Sergei A. Voloshin, Arthur M. Poskanzer, and Raimond Snellings arXiv: 0809.2949

➢ Radial velocities are input parameters of hydrodynamic calculations !

# ◆ Generalized Blast-wave parameterization

## Cooper-Frye formula:

E. Schnedermann, et.al, PRC 48, 2462(1993);
W.Broniowski et.al, PRL 87 272302(2001);
P. Huovinen, et.al, PLB 503, 58(2001).

$$E\frac{d^3N}{d^3p_t} \propto \frac{1}{(2\pi)^3}\int_{\Sigma_f} p^{\mu} d\sigma_{u}(x) f(x,p)$$

$f(x,p)$ : Boltzman, or Tsallis statistics

$$\frac{dN}{p_t dp_t d\phi} \propto \int_0^{2\pi} d\phi_s \int_{-y_b}^{y_b} dy e^{\sqrt{{y_b}^2-y^2}} \cosh y \int_0^R m_t r dr [1+\frac{q-1}{T}(m_t \cosh y \cosh\rho - p_t \sinh\rho \cos(\phi_b-\phi))]^{-\frac{1}{q-1}}$$

**Radial flow:** $\rho = \tilde{r}(\rho_0 + \rho_2 \cos(\phi_m))$

$\rho_0$: the isotropic radial flow rapidity; $\rho_2$: the anisotropic radial flow rapidity

$\rho_0$ and, $\rho_2$ are determined by fitting:

(1) Transverse momentum spectrum

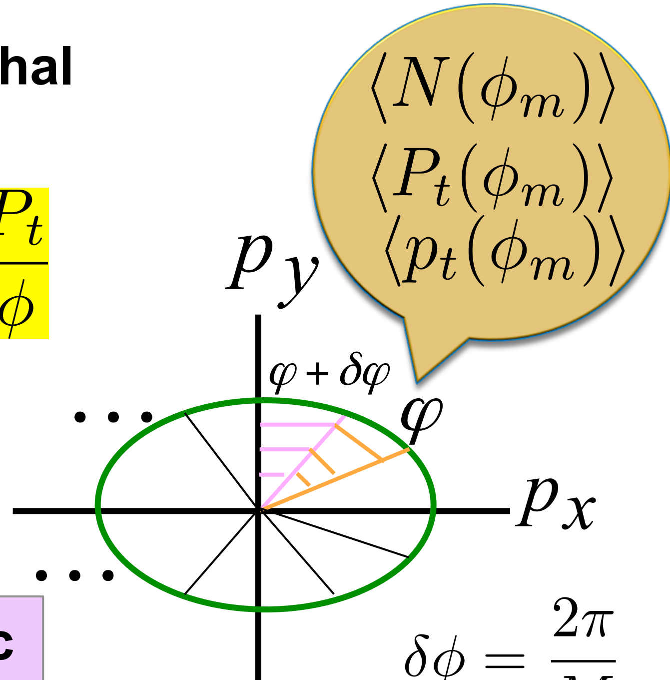
(2) Elliptic flow

➢ **Such extracted parameters are model dependent !**

# 2. Measure of radial expansion

- **Total transverse momentum** **in a given azimuthal angle bin:**

$$\langle P_t(\phi_m)\rangle = \frac{1}{N_{\text{event}}}\sum_{j=1}^{N_{\text{event}}}\left(\sum_{i=1}^{N_m}(p^j_{t,i}(\phi_m))\right) \Rightarrow \frac{dP_t}{d\phi}$$




$$\delta\phi = \frac{2\pi}{M}$$

$$m = 1, 2, ..., M$$

$p^j_{t,i}$ : transverse momentum of the *i*th particle in the *m*th angular bin.

$N_m$ : total number of particles in the *m*th angular bin.

> **It contains the information from both kinetic expansion and multiplicity distribution!**

- **Mean transverse momentum** **in a given azimuthal angle bin:**

$$\langle p_t(\phi_m)\rangle = \frac{1}{N_{\text{event}}}\sum_{j=1}^{N_{\text{event}}}\left(\frac{1}{N_m}\sum_{i=1}^{N_m}(p^j_{t,i}(\phi_m))\right) \Rightarrow \frac{dp_t}{d\phi}$$

> **It measures the radial expansion only!**

## ◆ Definitions of various flows:

**Azimuthal multiplicity distribution:**

$$\frac{dN}{d\phi} \propto 1 + \sum_{n=1}^{\infty} \underline{2v_n(N)} \cos\left(n(\phi - \psi_r)\right)$$

**Azimuthal total transverse momentum distribution:**

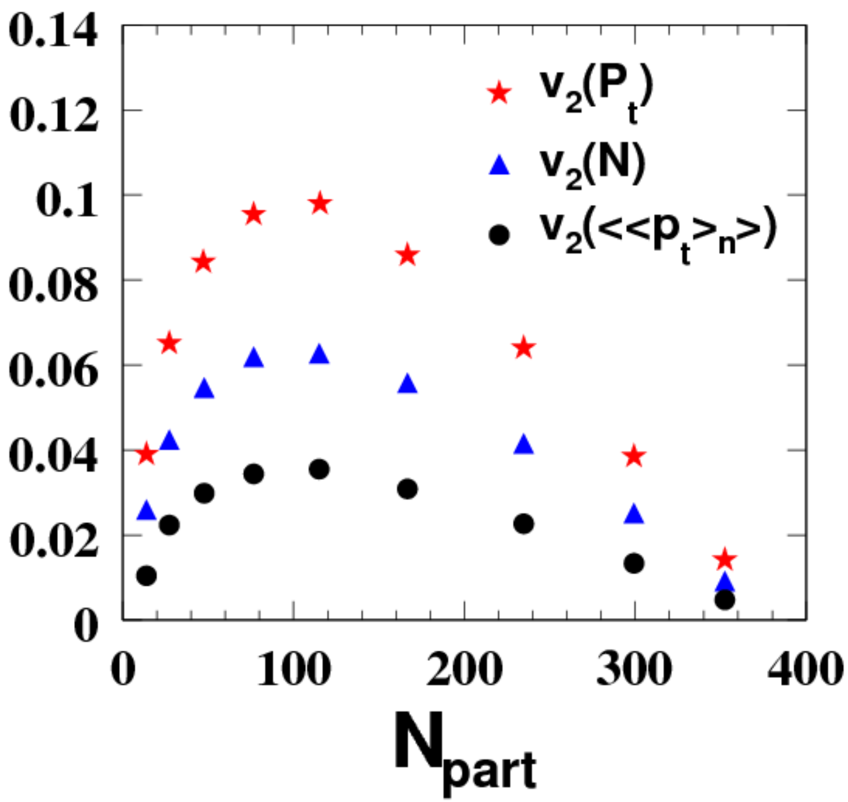
$$\frac{dP_t}{d\phi} \propto 1 + \sum_{n=1}^{\infty} \underline{2v_n(P_t)} \cos\left(n(\phi - \psi_r)\right)$$

**Azimuthal mean transverse momentum distribution:**

$$\frac{dp_t}{d\phi} \propto 1 + \sum_{n=1}^{\infty} \underline{2v_n(p_t)} \cos\left(n(\phi - \psi_r)\right)$$

# ◆ Centrality dependence of various anisotropic flows

AMPT with string melting
for Au+Au coll. at 200GeV




- They show similar centrality dependence.
- $v_2(<<p_t>>)$ is the smallest, $v_2(N)$ is in the middle, and $v_2(P_t)$ is largest, as it counts the anisotropy from both multiplicity and transverse momentum distributions.

- **Azimuthal distribution of mean transverse momentum can measure the radial expansion.**

## ◆ Suggested measurement: azimuthal dis. of mean transverse rapidity

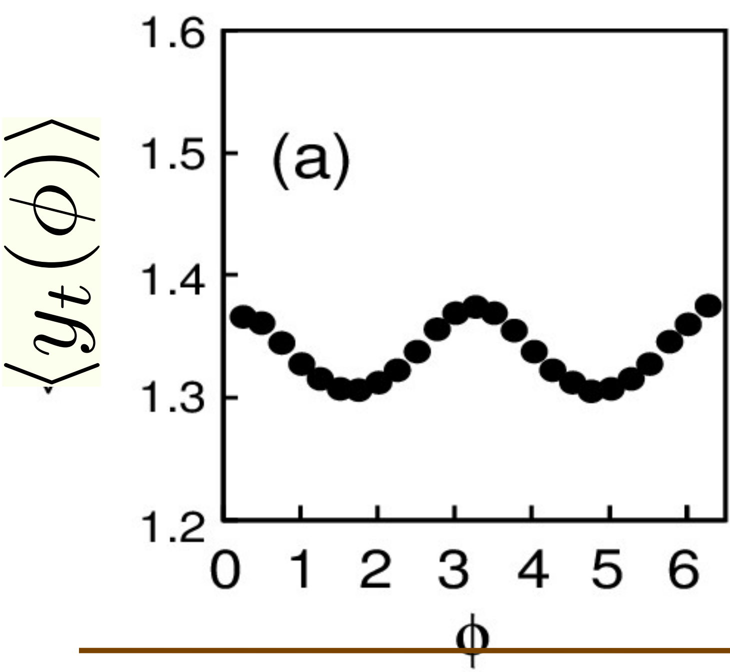
**Similarly, we can define the mean transverse rapidity:**

$$\langle y_t(\phi_m)\rangle = \frac{1}{N_{\mathrm{event}}}\sum_{j=1}^{N_{\mathrm{event}}}\left(\frac{1}{N_m}\sum_{i=1}^{N_m}(y^j_{t,i}(\phi_m)\right)$$

$$y^j_{t,i} = \ln\frac{m^j_{t,i}+p^j_{t,i}}{m_0}$$

**It relates to the total radial flow rapidity.**

AMPT with string melting for Au+Au coll. at 200GeV




**It is well fitted by:**

$$\langle y_t(\phi)\rangle = y_{t0} + y_{t2}\cos(2\phi)$$

**with** $y_{t0} = 1.3371 \pm 0.0001$

**Isotropic mean transverse rapidity:**

**isotropic expansion + thermal motion**

**and** $y_{t2} = 0.0334 \pm 0.0002$

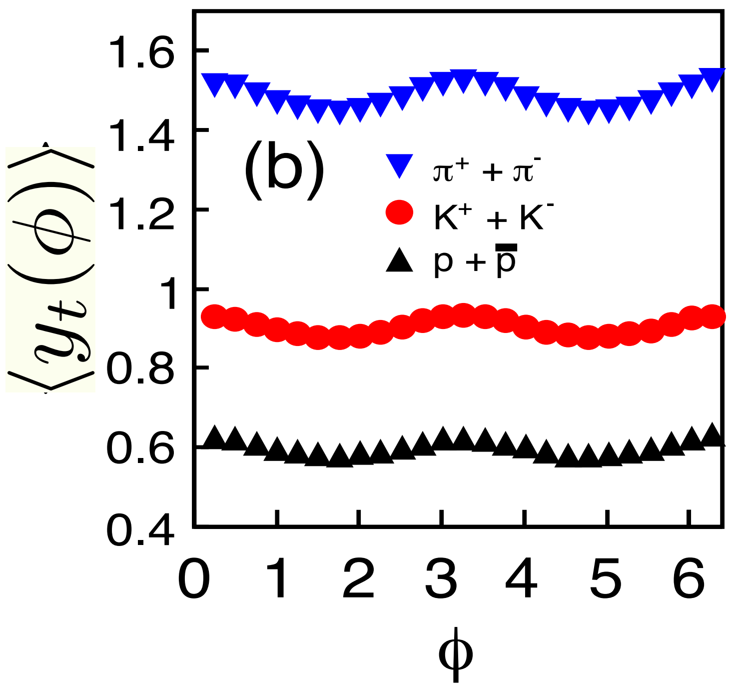
**Anisotropic mean transverse rapidity:**

➔ **anisotropic rapidity.**

# 3. The physics of measured $dy_t/d\phi$

◆ **Particle mass dependence of suggested distribution**

AMPT with string melting for Au+Au coll. at 200GeV




**Thermal motion:** temperature

particle mass

**At fixed T:** lighter particle,

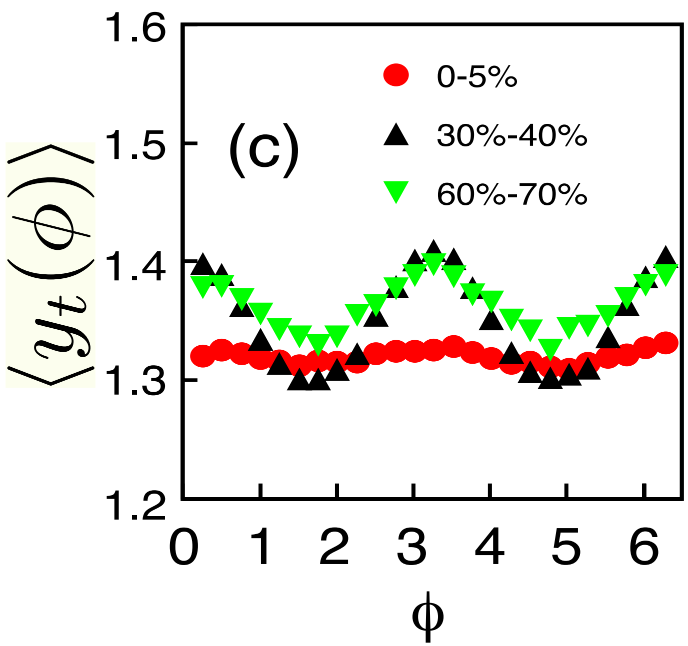
larger thermal velocity

| Mass | Particles | $y_{t0}$ |
|---|---|---|
| ↓ | Pions | ↑ |
| | Kaons | |
| | Protons | |

➢ **Their isotropic parts are ordered as expected random thermal motion!**

## ◆ Centrality dependence of suggested distribution:

AMPT with string melting
for Au+Au coll. at 200GeV



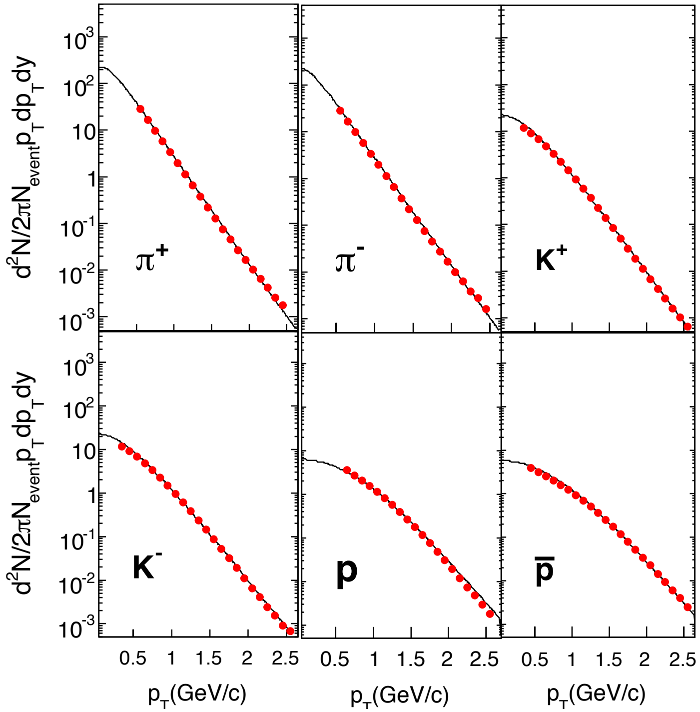

- The distributions is almost azimuthal angle independent in central collisions, but dependent in non-central coll.
- Large anisotropy in mid-central collisions, and small anisotropy in peripheral collisions.

- **Consistent with the fact that anisotropic expansion appears in non-central collisions, and is the largest in mid-central collisions !**

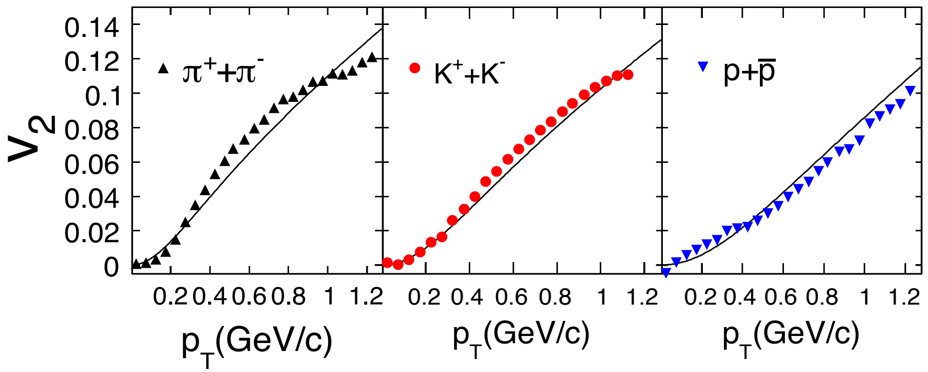
# 4. Measured $y_{t2}$ and extracted $\rho_2$

◆ **Extracted $\rho_2$ by blast-wave parameterization:**

Fitting $p_t$ spectrum of 6 particles




Fitting differential elliptic flow




Extracted parameters:

$$T = 96.1 \pm 1.0$$

$$\rho_0 = 0.73 \pm 0.01$$

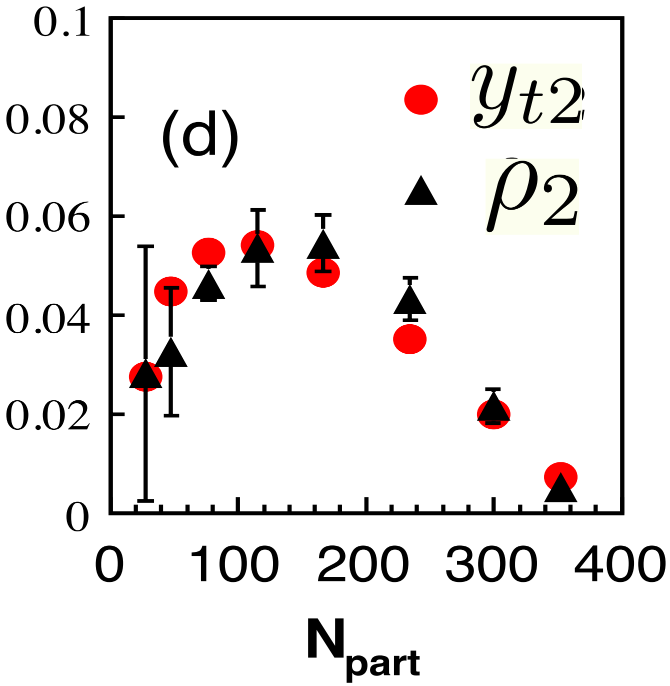
$$\rho_2 = 0.035 \pm 0.003$$

Measured: $y_{t2} = 0.0334 \pm 0.0002$

➢ **Extracted anisotropic rapidity parameter is consistent with measured anisotropic part of mean transverse rapidity!**

## ◆ Centrality dependence of extracted $\rho_2$ and measured $y_{t2}$

AMPT with string melting
for Au+Au coll. at 200GeV




- ➢ At each of centrality, the extracted anisotropic radial flow parameter is close to that from measured anisotropic part of mean transverse rapidity.
- ➢ They show consistent centrality dependence.

➢ **Provides a model independent way to get the anisotropic rapidity !**

# 5. Summary

- We suggest the measurements for the azimuthal distribution of mean transverse rapidity.
- It consists of two parts: isotropic, and anisotropic mean transverse rapidity.

  Isotropic part: isotropic radial expansion + thermal motion

  Consistent with the mass ordering

  Anisotropic part: anisotropic radial expansion

  Centrality dependence is consistent with extracted anisotropic radial rapidity
- It provides a model independent way to get anisotropic rapidity. It is helpful for hydrodynamic calculations, and a model independent determination of shear viscosity.

Thanks!