The phase diagram of QCD from lattice simulations

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Understanding the QCD phase diagram means understanding strongly interacting matter under extreme conditions (T, density, etc..)

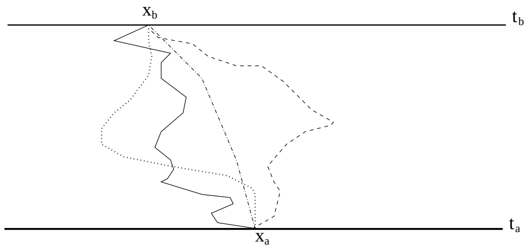
- **Are quarks and gluons confined forever?**
- **What is the fate of chiral symmetry breaking?**
- **Is there any true phase transition where thermodynamics shows a critical behavior? (Diverging correlation lengths, latent heat, etc.)**
- **What are the properties of the new possible phases?**

Such questions are fundamental for various fields, including astrophysics and cosmology, and are the primary motivation for heavy ion collision experiments.

- **Most interesting things happen at a scale ($\lesssim$ GeV) where the QCD coupling is large and perturbation theory fails.**
- **One of the best known approaches is then to compute it numerically: Lattice QCD (see talk by Jan Pawlowski for analytic approaches)**

LATTICE QCD IN BRIEF

The starting point is the path-integral approach to Quantum Mechanics and Quantum Field Theory, opened by R. Feynman in 1948.



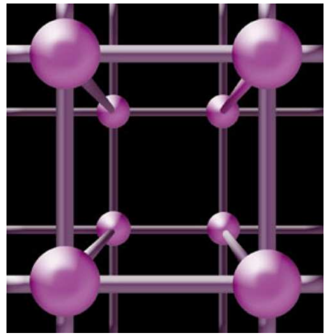
$$\longrightarrow \langle 0|O|0\rangle \Rightarrow \int \mathcal{D}\varphi e^{-S[\varphi]} O[\varphi]$$

The QCD path integral is discretized on a finite space-time lattice

$\Longrightarrow$ finite number of integration variables

For QCD, integration variables are 3×3 unitary matrices, $U_\mu(n)$, living on lattice links (elementary parallel transporters)

(K.G. Wilson, 1974)



The path-integral is then computed by Monte-Carlo algorithms which sample field configurations proportionally to $e^{-S[U]}$

$$\langle O\rangle = \frac{1}{Z}\int \mathcal{D}U e^{-S[U]} O[U] \quad \simeq \quad \bar{O} = \frac{1}{M}\sum_{i=1}^{M} O[U^{\{i\}}]$$

The thermal QCD partition function is naturally rewritten in terms of an Euclidean path integral with a compactified temporal extension

$$S_{QCD} = \int d^4x \left(\sum_f \bar{\psi}_i^f \left(D_{ij}^{\mu} \gamma_{\mu}^E + m_f \delta_{ij} \right) \psi_j^f + \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu} \right) \to \bar{\psi} M[U] \psi + S_G[U]$$

$$Z(V,T) = \mathrm{Tr} \left(e^{-\frac{H_{\mathrm{QCD}}}{T}} \right) \Rightarrow \int \mathcal{D}U \mathcal{D}\psi \mathcal{D}\bar{\psi} e^{-(S_G[U] + \bar{\psi} M[U] \psi)} = \int \mathcal{D}U e^{-S_G[U]} \det M[U]$$

As long as $\mathcal{D}U e^{-S_G} \det M[U]$ is positive, it can be interpreted as a probability distribution $\mathcal{D}U \mathcal{P}[U]$ over gauge link configurations, which can be sampled by proper algorithms

$$T = \frac{1}{\tau} = \frac{1}{N_t a(\beta, m)}$$

τ **is the extension of the compactified time**

Sample averages give access to equilibrium properties (energy density, specific heat, etc.)

To understand the nature of phase transitions, we study different growing spatial sizes and look for possible singularities in the infinite volume limit: finite size scaling

Uncertainties

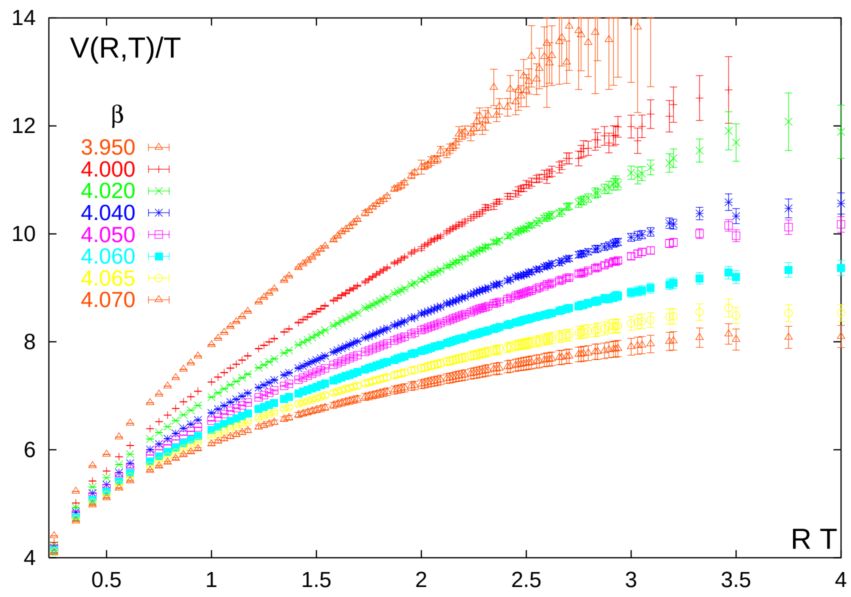
- **statistical: finite sample, error** $\sim 1/\sqrt{\text{sample size}}$.
- **systematic: finite box size** L**, finite lattice spacing** a**, unphysical quark masses.**

Given enough computer power, uncertainties can be kept under control. Different groups, adopting different discretizations, converge to consistent results.

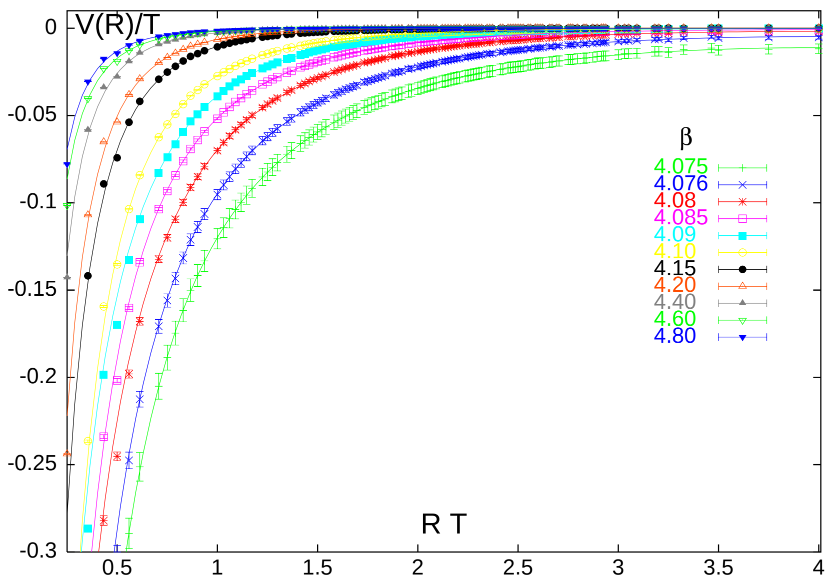
Projects employing $L \gg 1$ **fm,** a **well below 0.1 fm and physical quark masses, require more than 100 Teraflop*year (almost** 10^{22} **floating point operations)**

Finite T transition

Clear evidence for deconfinement is obtained both in the pure gauge theory (quenched approximation) and in presence of dynamical fermions



low T



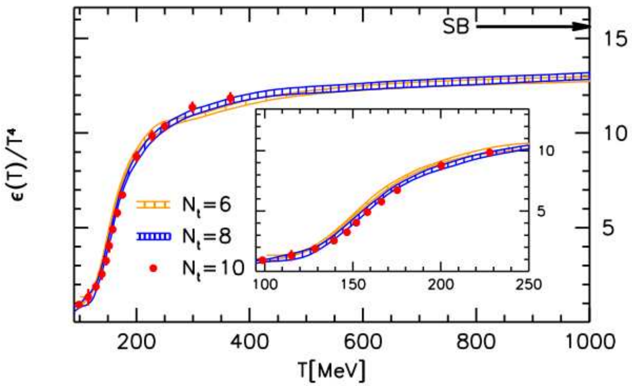
high T

Taken from O. Kaczmarek, F. Karsch, E. Laermann and M. Lutgemeier, Phys. Rev. D 62, 034021 (2000)

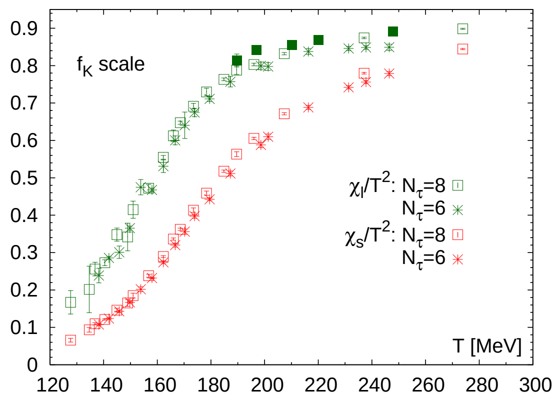
The confining potential which is present at low T, disappears at high T.

The liberation of color degrees of freedom is clearly visible in thermodynamical quantities and coincides with chiral symmetry restoration.

energy density | **u/d and s number fluctuations** | **chiral condensate**



arXiv:1007.2580



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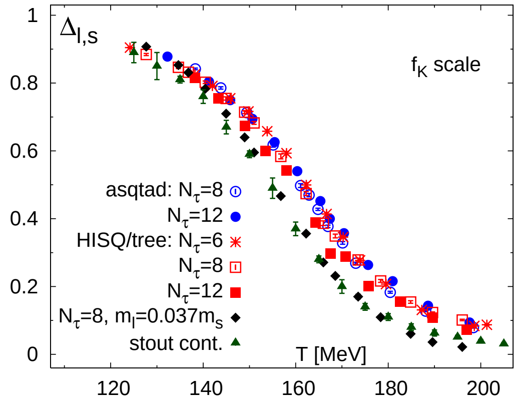


arXiv:1111.1710

Temperature and nature of the transition

S. Borsanyi *et al.* **JHEP 1009, 073 (2010)** $T_c = 155(6)$ **MeV** **(stout link stag. discretization, $a_{min} \simeq 0.08$ fm)**

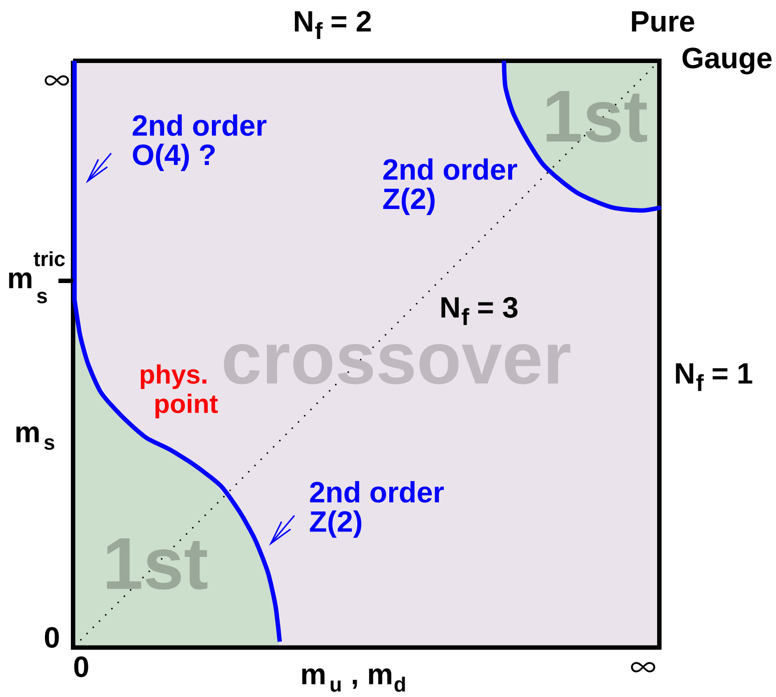
A. Bazavov *et al.*, **PRD 85, 054503 (2012)** $T_c = 154(9)$ **MeV** **(HISQ/tree stag. discretization, $a_{min} \simeq 0.1$ fm)**

No exact symmetries are known for QCD. Then, it is not granted that a true transition takes place

Indeed the physical point is consistent with a crossover (no discontinuity) (Aoki et al., Nature 443, 675 (2006)): either the transition is extremely weak (hence not phenomenologically relevant) or absent

However, we can play as God, and change the quark mass spectrum (almost) at will

One can thus study the nature of the transition as a function of u/d, s quark masses



This is the commonly accepted scenario

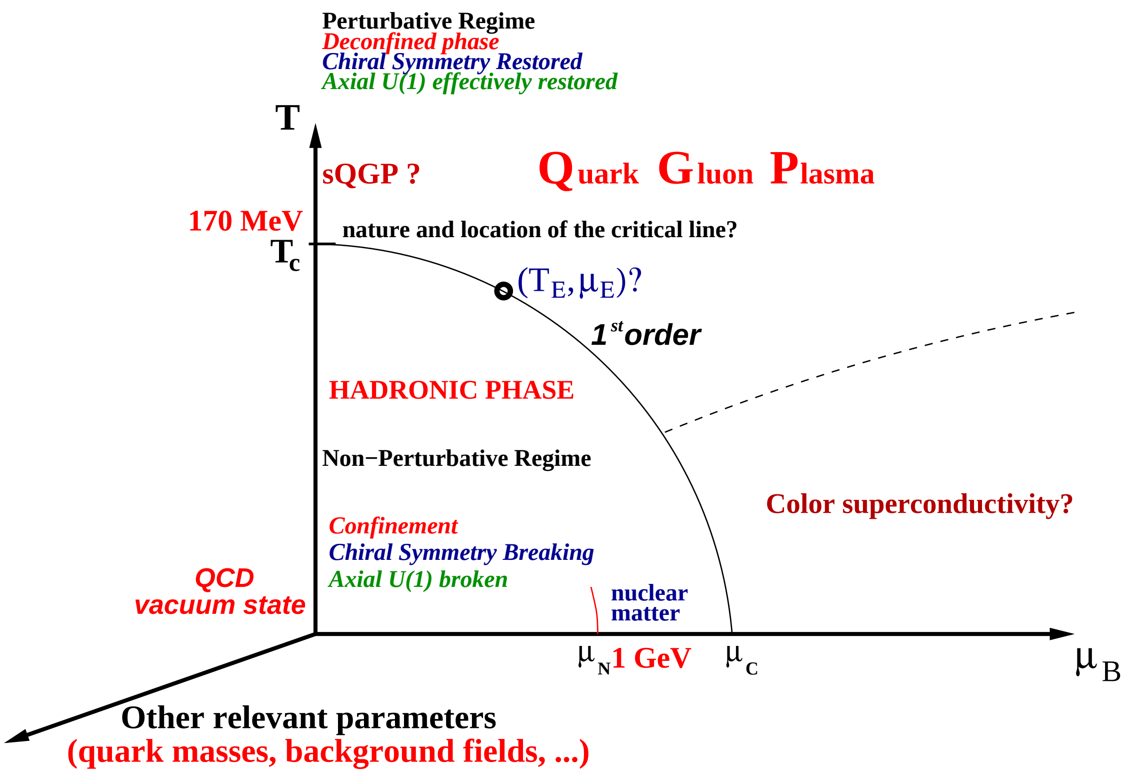
$\Longleftarrow$ COLUMBIA PLOT

A true, first order transition is present for very light or very heavy quark masses

Unsettled issues in the chiral limit of $N_f = 2$: 2nd order or first order?

(Bonati, Cossu, M.D., Di Giacomo, Pica, '05, '07; Bonati, M.D., de Forcrand, Philipsen, Sanfilippo, '11)

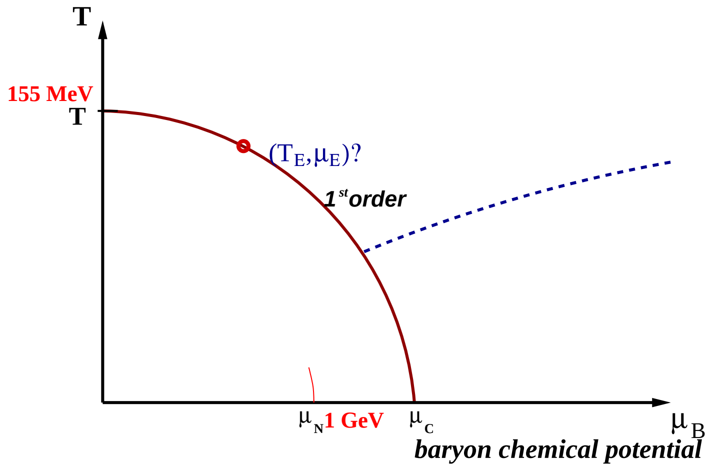
The QCD phase diagram: not just temperature ...



What we would like to know:

- **Location and nature of deconfinement/chiral symmetry restoration as a function of other external parameters (μ_B, external fields, ...)**
- **Properties of the various phases of strongly interacting matter**

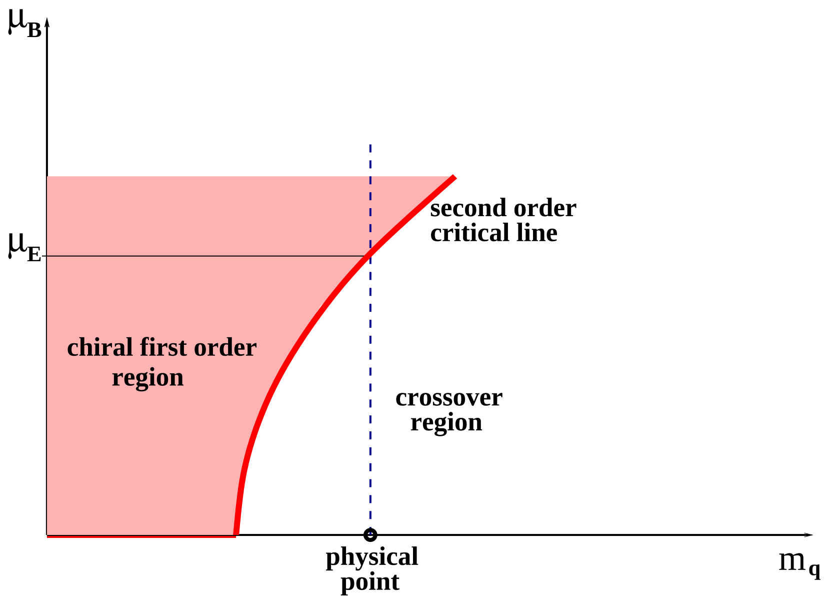
Moving to finite baryon density



If no transition at $\mu_B = 0$ at the physical point, and if the transition is first order at large μ_B, a critical (second order) endpoint is expected in the T, μ_B plane.

It would have clear experimental signatures: critical fluctuations (M. A. Stephanov, K. Rajagopal and E. V. Shuryak, PRL 81, 4816 (1998))

How is that expected to happen? The commonly accepted scenario is that:



The first order region present for $\mu_B = 0$ and small quark masses (Columbia plot) grows as μ_B grows, crossing the physical point at the critical endpoint.

The critical endpoint is a "chiral" critical endpoint in this scenario

Problems in lattice QCD at $\mu_B \neq 0$

$$Z(\mu_B, T) = \mathrm{Tr}\left(e^{-\frac{H_{\mathrm{QCD}}-\mu_B N_B}{T}}\right) = \int \mathcal{D}U e^{-S_G[U]} \det M[U, \mu_B]$$

$\det M[\mu_B]$ **complex** $\Longrightarrow$ **Monte Carlo simulations are not feasibile.**

This is usually known as the sign problem.

We can then rely on a few approximate methods, viable only for small μ_B/T, like

- **Taylor expansion of physical quantities around $\mu = 0$**

  Bielefeld-Swansea collaboration 2002; R. Gavai, S. Gupta 2003

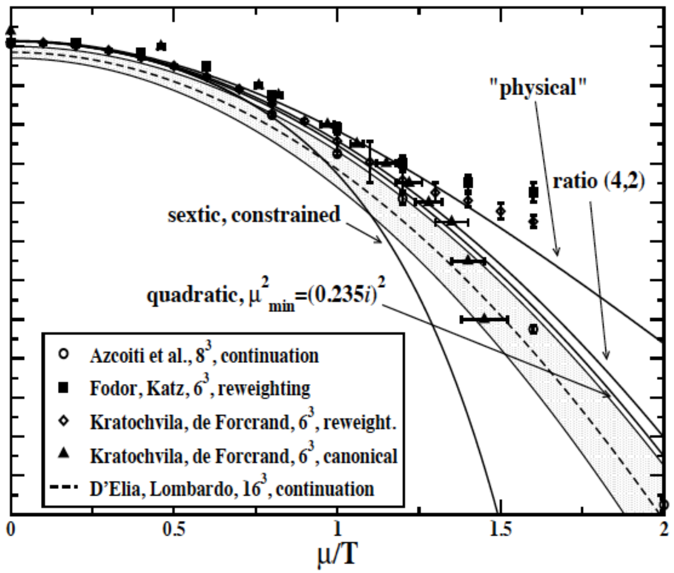
- **Reweighting (complex phase moved from the measure to observables)**

  Barbour et al. 1998; Z. Fodor and S, Katz, 2002

- **Simulations at imaginary chemical potentials (plus analytic continuation)**

  Alford, Kapustin, Wilczek, 1999; de Forcrand, Philipsen, 2002; M.D'E., Lombardo 2003.

An example: the critical line $T_c(\mu_B)$



Comparison of various methods to extract $T_c(\mu_B)/T_c(0)$ as a function of μ_B (4 stag. flavors)

S. Kratochvila and P. de Forcrand, PoS LAT2005 (2006) 167;

P. Cea, L. Cosmai, M. D'E., A. Papa, PRD 81, 094502 (2010).

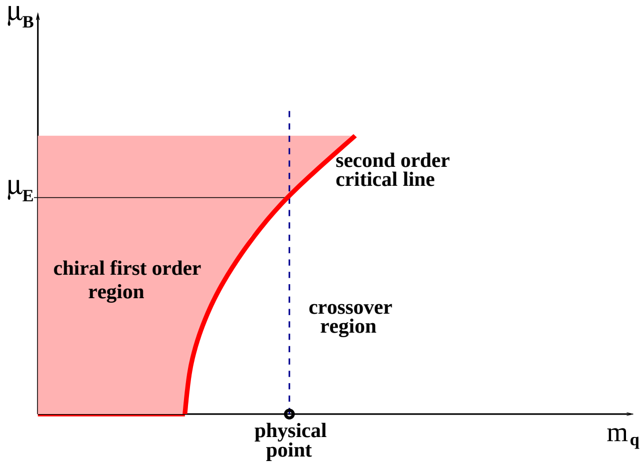
Various different methods agree for the curvature $\partial T/\partial\mu^2$ of the critical line at $\mu^2 = 0$.

In more physical cases we obtain for the curvature $T_c(\mu_q)/T_c(0) = 1 - A\left(\frac{\mu_q}{T}\right)^2$

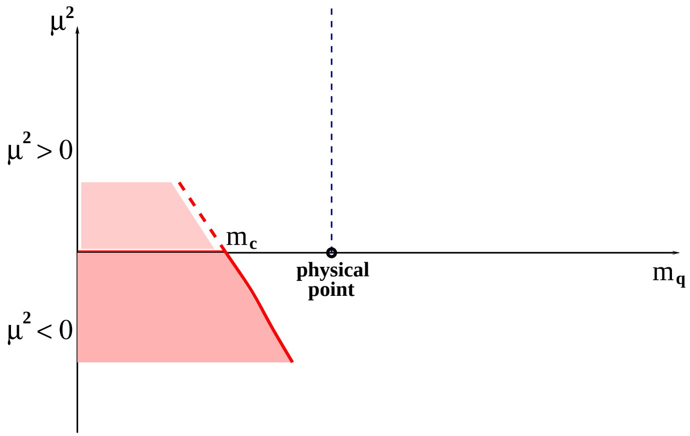
- **A = 0.051(4)** **($N_f = 2$, $m_\pi \sim 280$ MeV, analytic cont., de Forcrand, Philipsen, hep-lat/0205016)**
- **A = 0.052(2)** **(as above, $m_\pi \sim 400$ MeV Cea, Cosmai, D'E., Papa, Sanfilippo, arXiv:1202.5700)**
- **A = 0.059(2)(4)** **($N_f = 2+1$, chiral+continuum limit, Taylor, O. Kaczmarek** ***et al.*** **arXiv:1011.3130)**
- **A = 0.07-0.09** **($N_f = 2+1$, physical point, Taylor, G. Endrodi** ***et al.*** **arXiv:1102.1356)**

However, systematics get out of control at larger μ_B, with large uncertainties regarding the location and the very existence of a possible QCD critical endpoint.

A different question: if the critical endpoint exist, is it chiral?



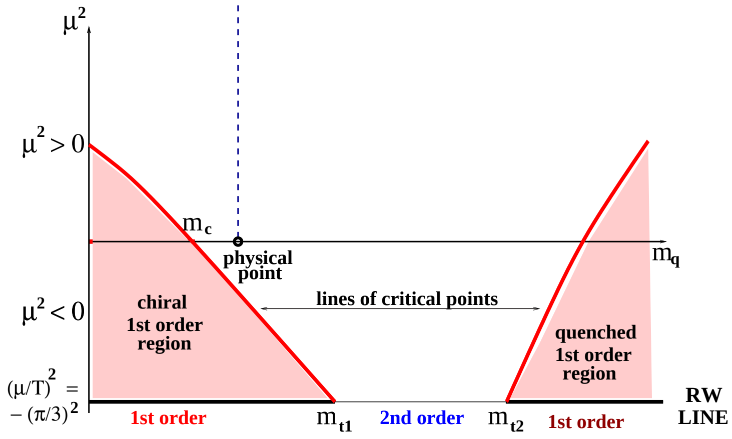
Standard scenario: the first order region at small quark masses (chiral region) grows with μ until it crosses the physical point

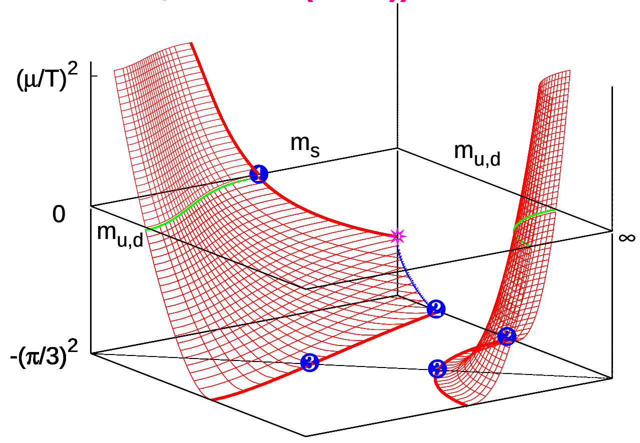


Simulations at imaginary chemical potential ($\mu^2 < 0$) predict instead that the first order region shrinks for $\mu^2 > 0$.

P. de Forcrand and O. Philipsen, JHEP 0701, 077 (2007), JHEP 0811, 012 (2008).

This has been recently reinterpreted in terms of the general structure of the phase diagram at $\mu^2 < 0$ (M. D'E., F. Sanfilippo, Phys. Rev. D80, 111501 (2009); P. de Forcrand, O. Philipsen, PRL 105 (2010) 152001; C. Bonati, G. Cossu, M. D'E., F. Sanfilippo, Phys. Rev. D83, 054505 (2011))





- **At $\mu/T = i\pi/3$ QCD has an exact Z_2 symmetry (similar to charge conjugation) which spontaneouly breaks at T_c. The transition must be first or second order, with tricritical points inbetween.**

- **The conjecture is that such phase structure propagates up to $\mu = 0$, i.e. that the "Columbia plot" originates from it. No relation would then exist between the first order regions in the Columbia plot and a possible endpoint at real μ.**

- **The conjecture can be completely verified for $\mu^2 < 0$. In this way one can also approach issues like the order of the chiral transition for $N_f = 2$ QCD (C. Bonati, P. de Forcrand, M. D'E., O. Philipsen and F. Sanfilippo, arXiv:1201.2769 and in progress)**

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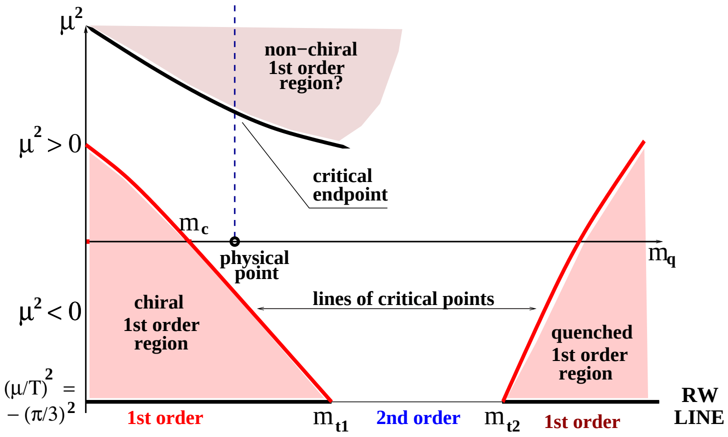

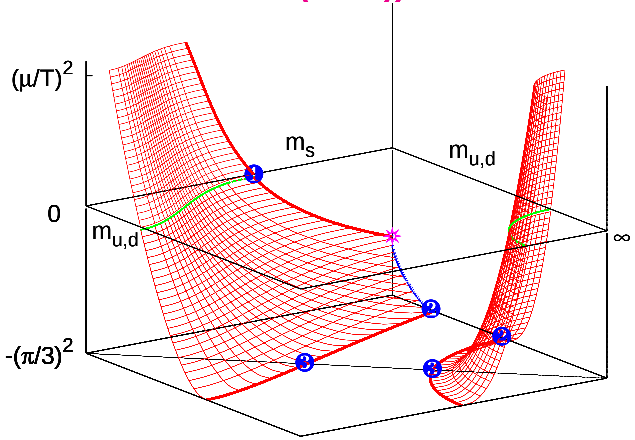

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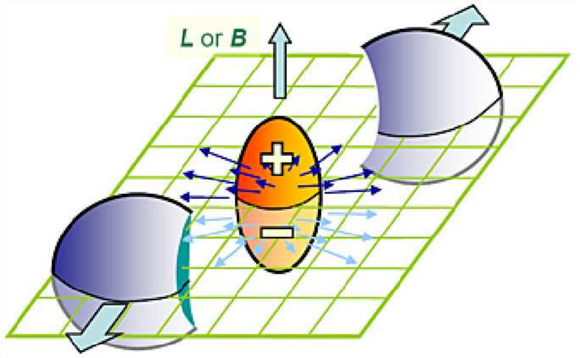
Another extension of the phase diagram:

Strong interactions in strong magnetic fields



- in non-central heavy ion collisions, largest magnetic fields ever created (B up to 10^{15} Tesla at LHC)

- possible strong fields in the early Universe, at the time of the QCD transition (B up to 10^{16} Tesla)

Many new phenomena are predicted, going from the chiral magnetic effect (CME) to a modification of the phase transition and of the thermal medium

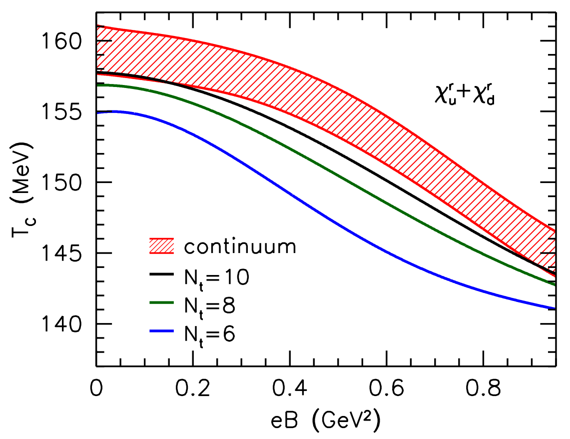
- How does T_c depend on B? Does the nature of the transition change?
- Do chiral symmetry restoration and deconfinement get disentangled?
- Is strongly interacting matter diamagnetic or paramagnetic?

All these issues are perfectly explorable. Just place an additional $U(1)$ background field in the covariant derivative, no sign problem for purely magnetic fields.

Many studies have explored the phase transition in the presence of a magnetic field

(M. D'E., S. Mukherjee, F. Sanfilippo, PRD 82, 051501 (2010); G. S. Bali *et al*, JHEP 1202, 044 (2012); E. -M. Ilgenfritz et al., PRD 85, 114504 (2012))

from G. S. Bali et al., JHEP 1202, 044 (2012)



STATUS

Deconfinement and chiral symmetry restoration stay entangled (at least for eB up to 1 GeV^2).

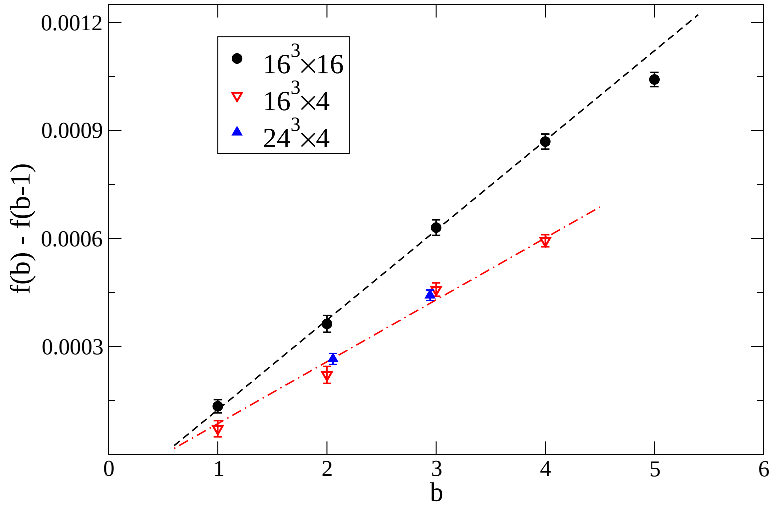
The transition moves to lower T as B increases, it becomes sharper

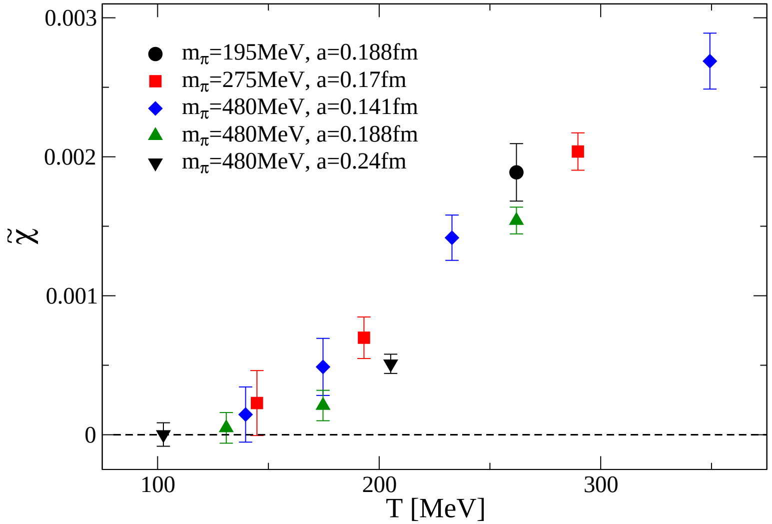
e.m. fields, even if directly coupled to quarks, can strongly affect gluodynamics.

Large effects on gluon field distribution (M. D. and F. Negro, PRD 83, 114028 (2011)); Large effects on the action density ((E. -M. Ilgenfritz et al., PRD 85, 114504 (2012)); G. S. Bali et al., PRD 86, 094512 (2012); JHEP 1304, 130 (2013)) and likely on the gluon condensate; possible generation of effective θ term in the presence of CP-odd e.m. backgrounds (M. D., M. Mariti and F. Negro, PRL 110, 082002 (2013))

Magnetic susceptibility of strongly interacting matter

C. Bonati, M. D., M. Mariti, F. Negro and F. Sanfilippo, arXiv:1307.8063





- The idea is to determine the B-dependent part of the free energy density, $\Delta f(B)$. Ultraviolet divergences cured by subtracting $T = 0$ contributions, then $\Delta f(B) = -\tilde{\chi} B^2/2\mu_0 + O(B^4)$ we can determine the magnetic susceptibility.

- **RESULT:** the QGP is strongly paramagnetic (compare, e.g., $\tilde{\chi} \simeq 2.8 \times 10^{-4}$ for Platinum) Confirmed by other studies (L. Levkova, C. DeTar, arXiv:1309.1142; G. Endrodi, talk at Lattice 2013)

- Future studies should better resolve the region around T_c: presently $\tilde{\chi}$ vanishes within errors below T_c.

CONCLUSIONS AND REFLECTIONS

- Present computational resources permit to obtain consistent and reliable predictions about the phase diagram of QCD and the properties of strongly interacting matter at finite T and in presence of external sources such as magnetic background fields

- Some cases exist where we still do not have full control over systematic errors, like QCD at large baryon density or the computation of transport properties, where major progress could be achieved by future breakthroughs.

- Of course, a full theoretical comprehension of the phase diagram, and of QCD in general, will require to understand not only when and how quarks and gluons confine/deconfine, but also what is the physical mechanism at the basis of color confinement and what its relation to chiral symmetry breaking.