Exotic Glue in the Nucleus?

Gluonic Transversity Structure Functions from Lattice QCD

Phiala Shanahan, Will Detmold

Massachusetts Institute of Technology

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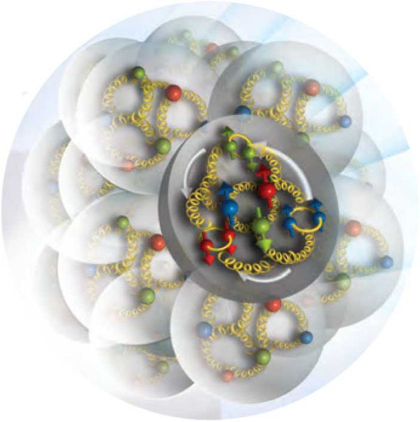
Outline

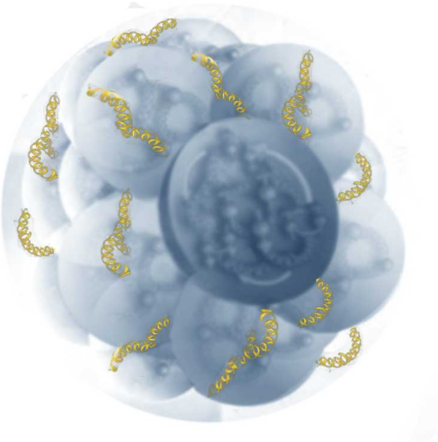
Motivation

Electron Ion Collider:
The Next QCD Frontier
Understanding the glue
that binds us all

'Exotic' Glue in the Nucleus



'Exotic' Glue in the Nucleus



'Exotic' Glue

Contributions to gluon observables that are not from nucleon degrees of freedom.

Exotic glue operator:

operator in nucleon $= 0$

operator in nucleus $\neq 0$

Outline

Double Helicity Flip Gluon Structure Function: $\Delta(x, Q^2)$

Jaffe and Manohar (1989)

Leading-twist, double-helicity-flipping structure function $\Delta(x, Q^2)$ sensitive to exotic glue in the nucleus

- Clear signature for exotic glue in nuclei with spin ≥ 1:
  NO analogous twist-2 quark PDF $\rightarrow$ unambiguous
- Experimentally measurable (JLab LOI 2016, James Maxwell's talk)
- Moments are calclable on the lattice

First Lattice Study: arXiv:1606.04505

- First moment of $\Delta(x, Q^2)$ in spin-1 ϕ (or ρ) meson

Double Helicity Flip Gluon Structure Function: $\Delta(x, Q^2)$

Hadronic tensor for inelastic lepton scattering from a polarized spin-one target:

$$\frac{1}{4\pi}\int d^4x\, e^{iq\cdot x}\langle p, E'|[j_\mu(x), j_\nu(0)]|p, E\rangle$$

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Target Polarization Vector

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Helicity projection:

$$W_{\mu\nu,\alpha\beta} = \sum_{hH,h'H'} P(hH, h'H')_{\mu\nu,\alpha\beta}A_{hH,h'H'}$$

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Double helicity flip amplitude:

$$\Delta(x, Q^2) = A_{+-,-+} = A_{-+,+-}$$

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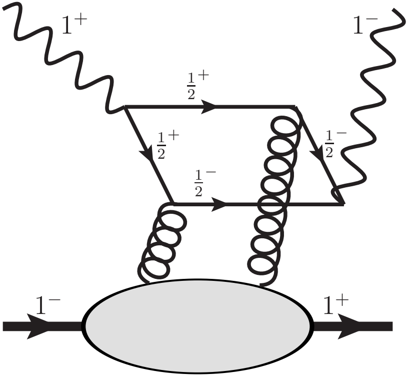

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Double Helicity Flip Gluon Structure Function: $\Delta(x, Q^2)$

Parton model interpretation

For a target in the infinite momentum frame polarized in the $\hat{x}$ direction perpendicular to its momentum,

$$\Delta(x, Q^2) \propto \int_x^1 \frac{dy}{y^3} \left(g_{\hat{x}}(y, Q^2) - g_{\hat{y}}(y, Q^2) \right)$$

$g_{\hat{x},\hat{y}}(y, Q^2)$: probability of finding a gluon with momentum fraction y linearly polarized in the $\hat{x}$, $\hat{y}$ direction

"How much more momentum of transversely polarized particle carried by gluons aligned rather than perpendicular to it in the transverse plane"

Double Helicity Flip Gluon Structure Function: $\Delta(x, Q^2)$

Operator Product Expansion to relate to matrix elements of operator

Symmetrize and trace subtract in $\mu_1, \ldots, \mu_n$

$$\langle pE' | \mathcal{O}_{\mu\nu\{\mu_1 \ldots \mu_n\}} - \mathrm{Tr} | pE \rangle$$
$$= (-2i)^{n-2} \underline{S} \left[(p_\mu E'^*_{\mu_1} - p_{\mu_1} E'^*_\mu)(p_\nu E_{\mu_2} - p_{\mu_2} E_\nu) + (\mu \leftrightarrow \nu) \right] p_{\mu_3} \ldots p_{\mu_n} \boxed{A_n(Q^2)} \ldots,$$

Reduced Matrix Element

where

Symmetrize and trace subtract in $\mu_1, \ldots, \mu_n$

$$\boxed{\mathcal{O}_{\mu\nu\mu_1 \ldots \mu_n}} = \underline{S} \left[G_{\mu\mu_1} \overleftrightarrow{D}_{\mu_3} \ldots \overleftrightarrow{D}_{\mu_n} G_{\nu\mu_2} \right]$$

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Optical theorem, dispersion relation for hadronic forward scatt. amplitude, analytic continuation give **moments**:

$$\int_0^1 dx x^{n-1}\Delta(x, Q^2) = \frac{\alpha_s(Q^2)}{3\pi}\frac{A_n(Q^2)}{n+2}, \quad n = 2, 4, 6\ldots,$$

Unpolarized scattering: symmetric piece of hadronic tensor $W_{\mu\nu}$, $\rightarrow$ even n

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Moment of Structure Function

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Outline

Lattice Operators

First moment of $\Delta(x, Q^2)$

$\Updownarrow$

Matrix elt. of $\mathcal{O}_{\mu\nu\mu_1\mu_2} = S[G_{\mu\mu_1} G_{\nu\mu_2}]$

- Relate $\mathcal{O}_{\mu\nu\mu_1\mu_2}$ to Euclidean operator
- Find linear combs. of Euclidean operator (with different indices) that
  1. Transform irriducibly under appropriate representations of $H(4)$
  2. Don't mix with same or lower-dimensional quark or gluon operators

     - 3 irreps. of dimension 2, 6, 2, i.e., 10 basis vectors
- Lattice simulation of matrix element in ϕ meson (spin-1)

Lattice Details

Luscher-Weisz gauge action with a clover-improved quark action

L/a	T/a	β	am_l	am_s
24	64	6.1	-0.2800	-0.2450
a (fm)	L (fm)	T (fm)	m_π (MeV)	m_K (MeV)
0.1167(16)	2.801(29)	7.469(77)	450(5)	596(6)
m_ϕ (MeV)	$m_\pi L$	$m_\pi T$	N_{cfg}	N_{src}
1040(3)	6.390	17.04	1042	10^5

- All ϕ polarization states ($\{1,2,3\}$ or $\{+,-,0\}$)
    - on-diagonal
    - off-diagonal
- Momenta up to (1,1,1) in lattice units (1 unit $\sim$ 0.4GeV)
- HYP smearing of gauge fields in operator (2-5 steps)

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SYSTEMATICS IGNORED

- Quark mass effects
- Volume effects
- Discretization effects
- Renormalization (for now)

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Extraction of A_2

Moment of Structure Function

Reduced Matrix Element

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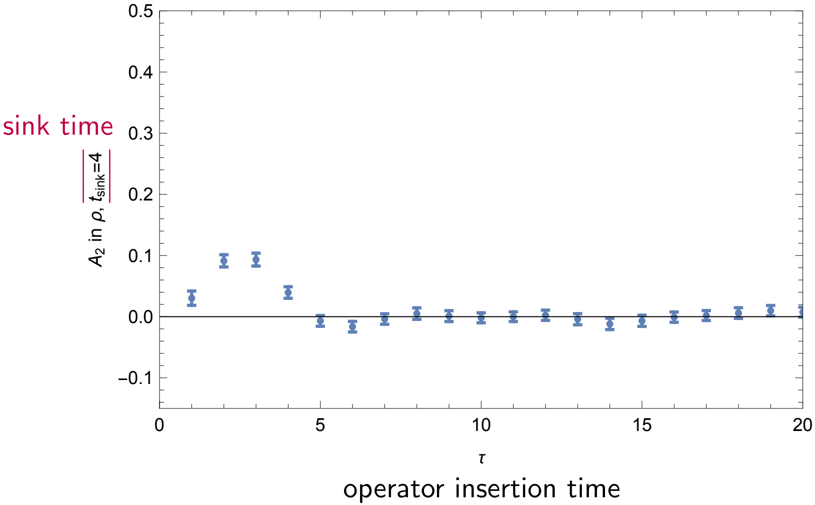
We calculate on the lattice:

factors of m and p

$$\left[\frac{C^{EE'}_{\text{3pt}}}{C^{EE'}_{\text{2pt}}}\right](t_{\text{sink}},\tau) \propto \boxed{A_2}, \qquad 0 \ll \tau \ll t_{\text{sink}}$$

Extraction of A_2: 3pt/2pt ratio



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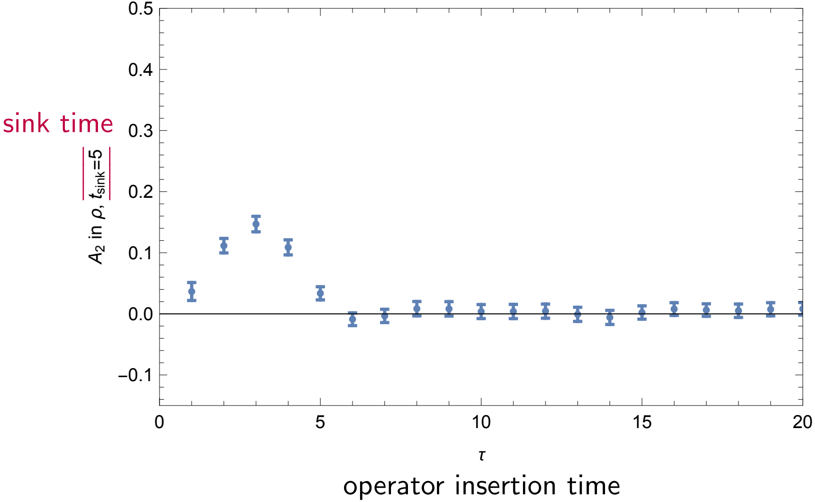
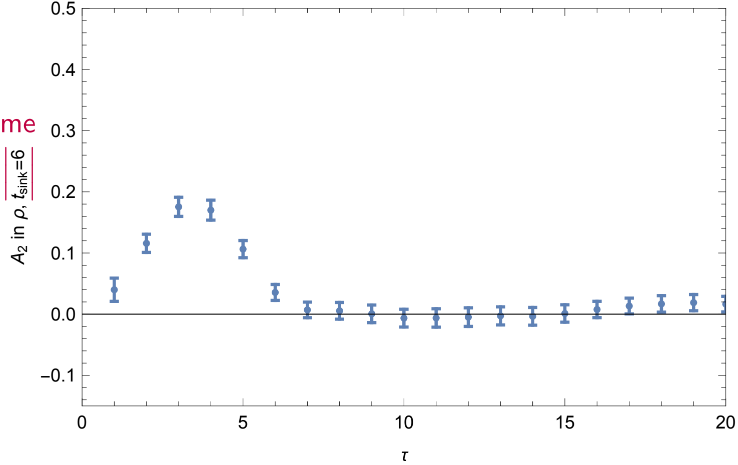


Extraction of A_2: 3pt/2pt ratio

sink time



operator insertion time

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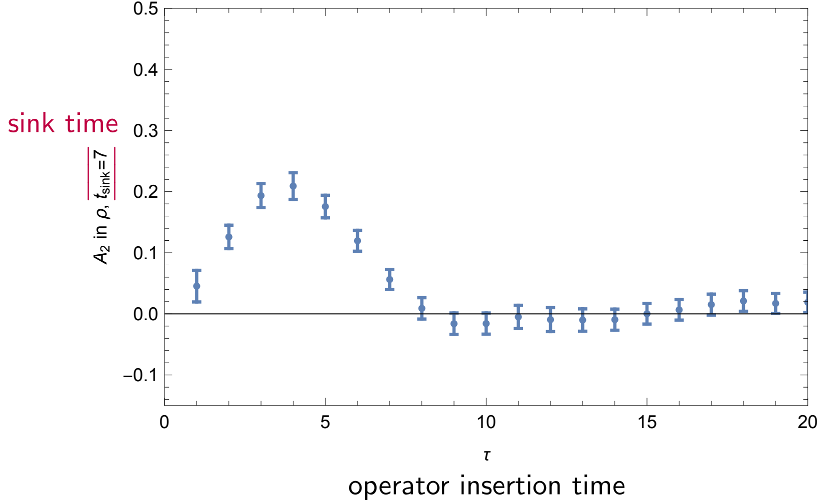

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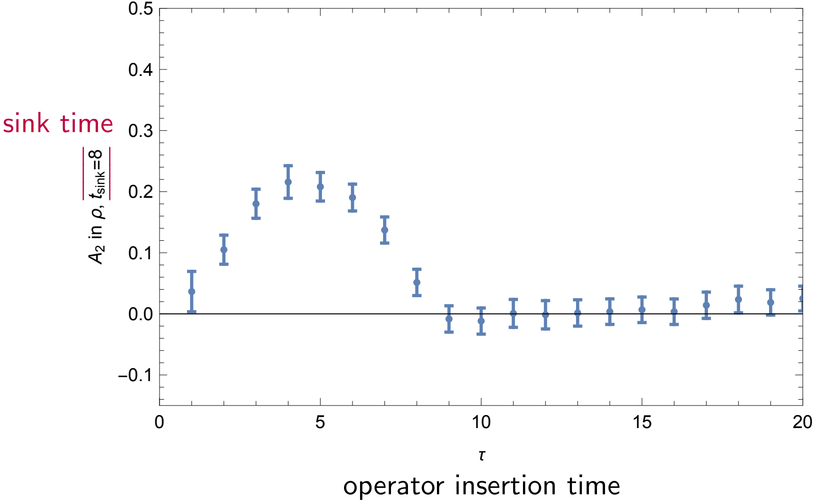

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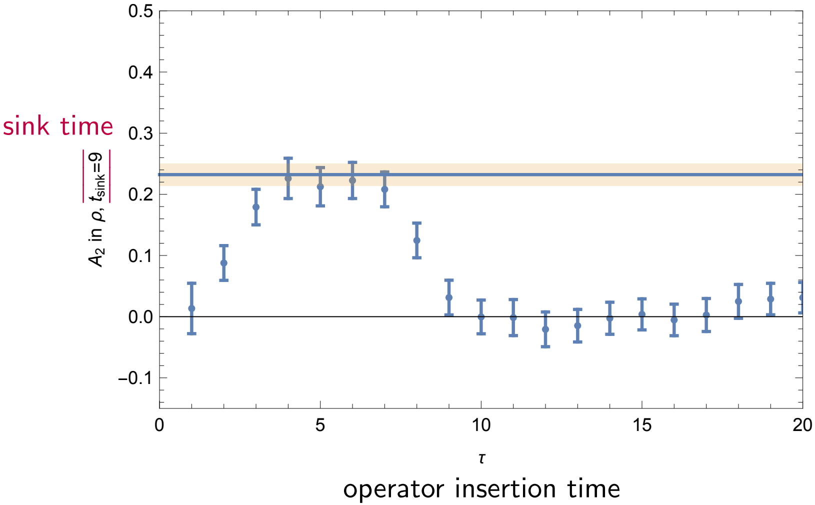

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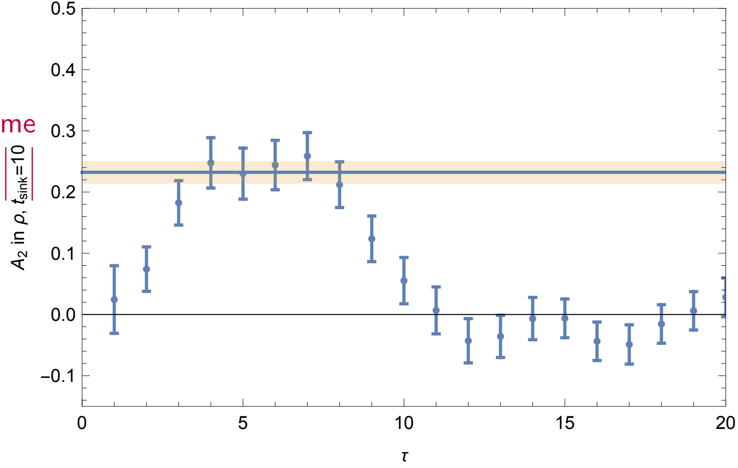

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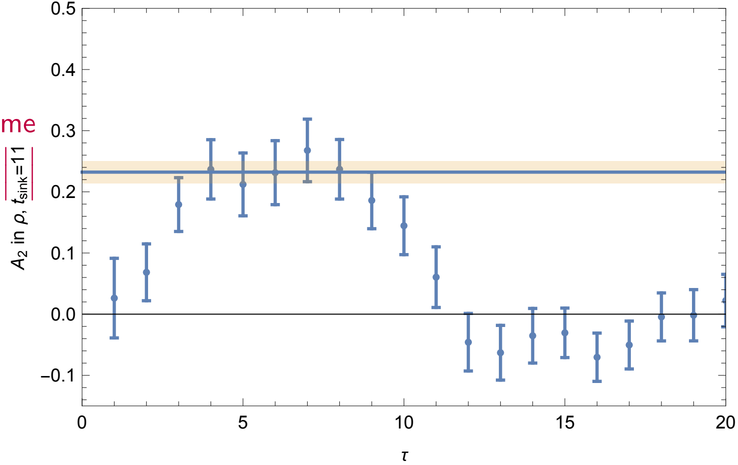

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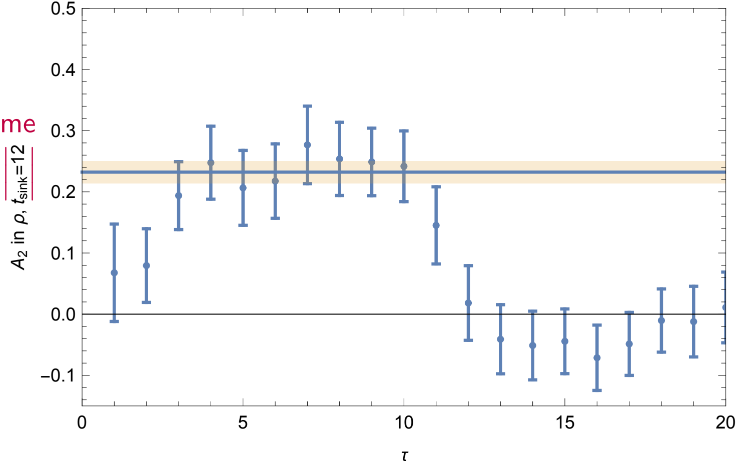

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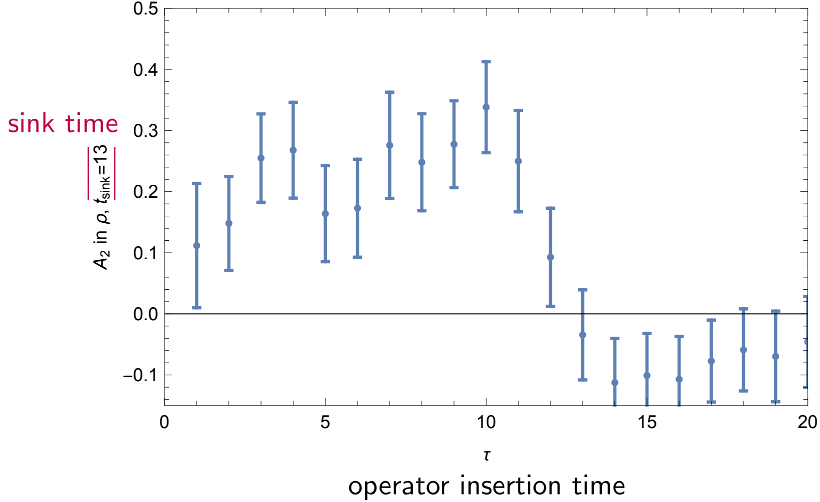

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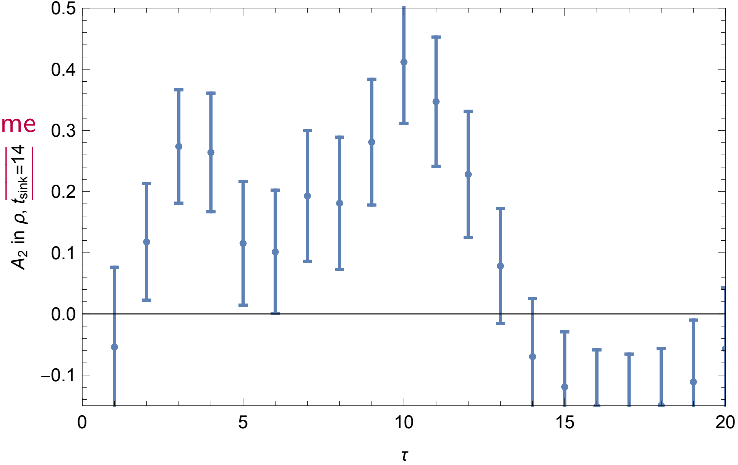
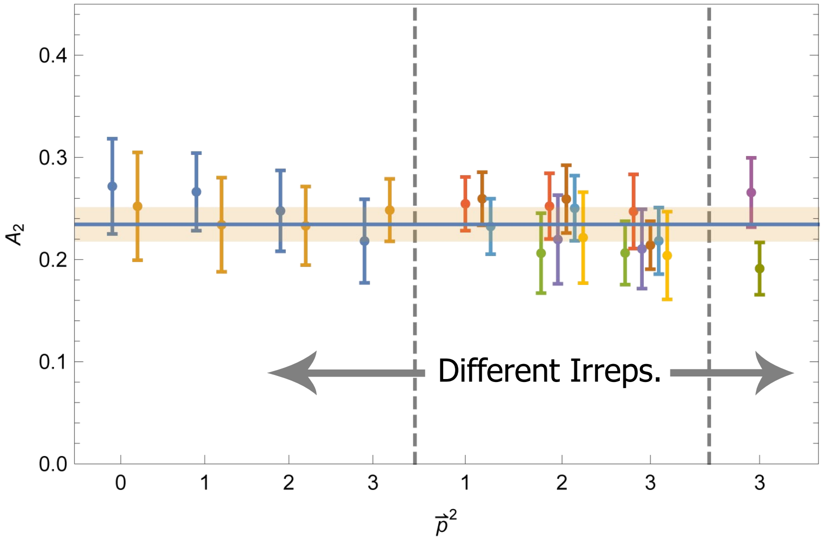


operator insertion time

Outline

UNRENORMALISED reduced matrix element: ϕ meson



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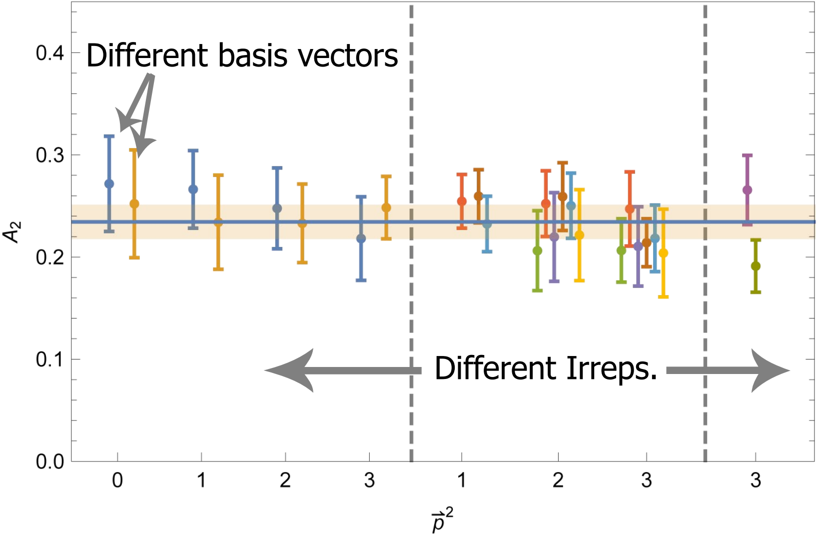

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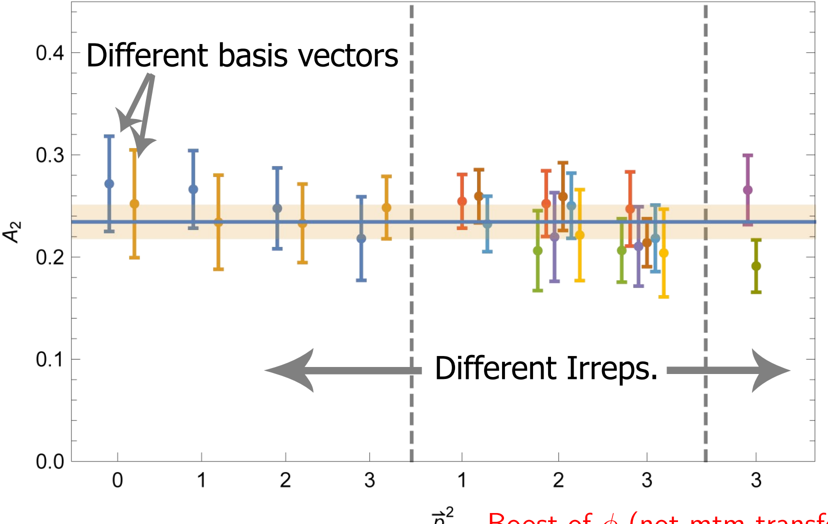

Boost of ϕ (not mtm transfer)

Results: ϕ meson

ROBUST NON-ZERO SIGNAL

Proof of principle: similar signal in a nucleus $\Leftrightarrow$ exotic glue

SYSTEMATICS IGNORED

- Quark mass effects
- Volume effects
- Discretization effects
- Renormalization

Soffer bound analogue

Explore gluon structure of ϕ meson more generally

Soffer bound for transversity quark distributions:

$$|\delta q(x)| \leq \frac{1}{2}\left(q(x) + \Delta q(x)\right)$$

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Soffer bound for transversity quark distributions:

Transversity

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Direct analogue for leading moments of gluon distributions:

$$|A_2| \leq \frac{1}{2} B_2$$

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$G_{\mu\mu_1} G_{\nu\mu_2}$

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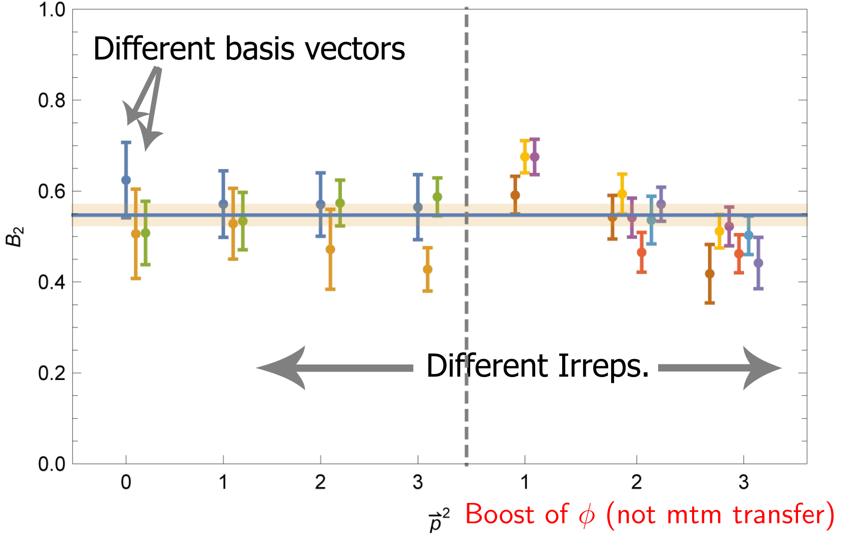
$$|\boxed{A_2}| \leq \frac{1}{2}\boxed{B_2}$$

$\widetilde{G}_{\mu_1\alpha}G_{\mu_2}{}^{\alpha} \to 0$

UNRENORMALISED reduced matrix element: ϕ meson



Soffer bound analogue

If we assume approx. the same renormalisation for A_2 and B_2:

$$\underbrace{|\boxed{A_2}|}_{G_{\mu\mu_1}G_{\nu\mu_2}} \leq \frac{1}{2}\underbrace{\boxed{B_2}}_{G_{\mu_1\alpha}G_{\mu_2}{}^{\alpha}}$$

$$0.25 \leq \frac{1}{2}\, 0.6$$

First two moments of quark distributions: Soffer bound saturated to 80% (lattice QCD, Diehl *et al.* 2005)

Outline

Summary

NON-ZERO signal for 'exotic glue' operator
in the ϕ (or ρ) meson

Proof of principle: similar signal in a nucleus $\Leftrightarrow$ exotic glue

Explore gluon structure of hadrons more generally
e.g., Soffer bound analogue

BUT: SYSTEMATICS IGNORED
$\Rightarrow$ no physically meaningful number (yet)

Summary - What else can we do?

Future prospects for Gluonic Lattice QCD Relevant to the EIC

- Exotic glue in nuclei
- Nucleon off-forward gluon transversity
- 'Gluonic radii'
- Gluon spin in the nucleon
- . . .