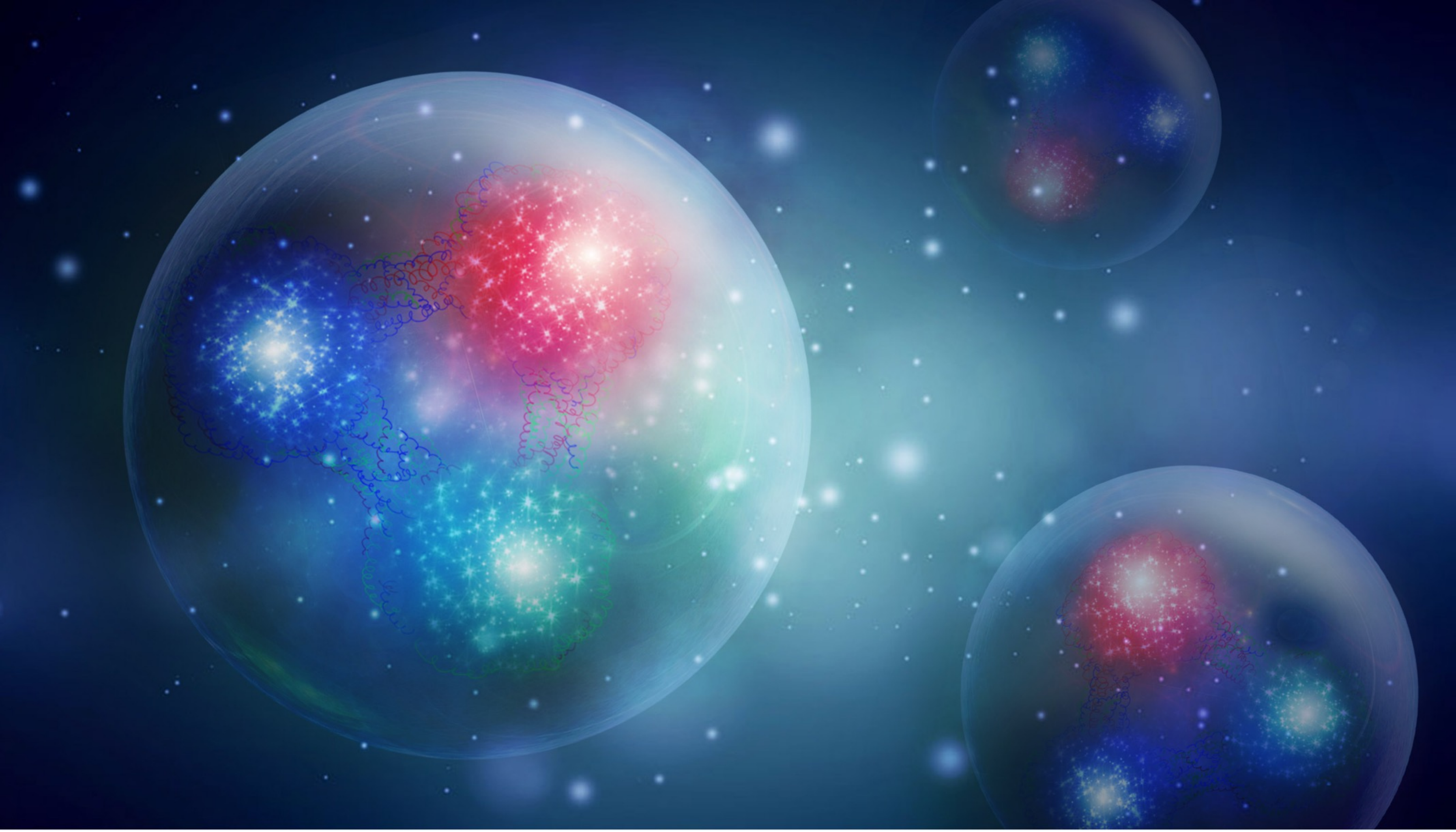


Rapidity factorization approach and EIC

Andrey Tarasov

Electron Ion Collider User Group Meeting 2016, July 8, 2016

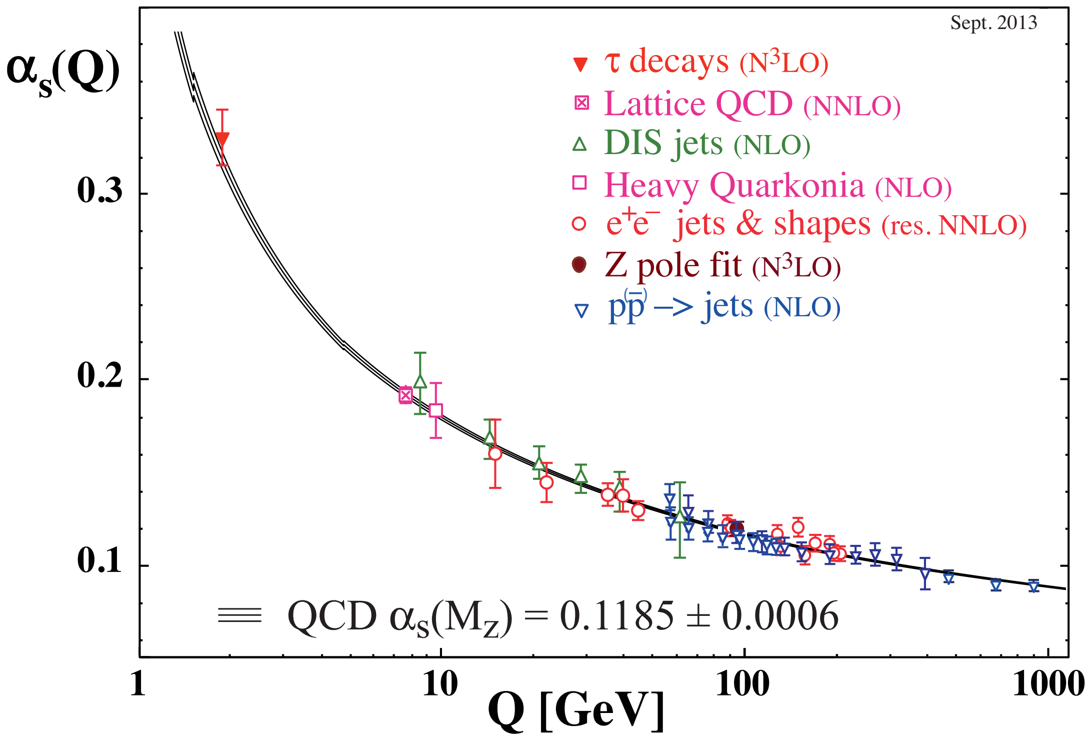
U.S. DEPARTMENT OF ENERGY

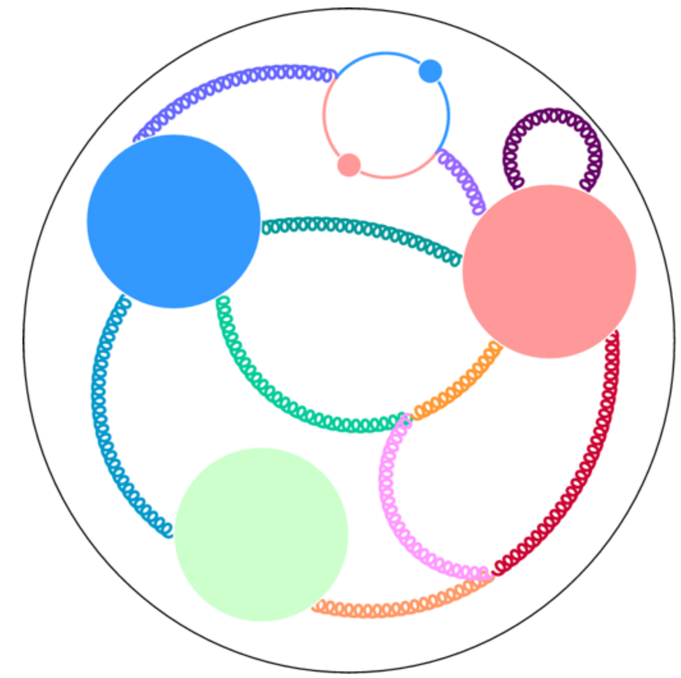
JSA

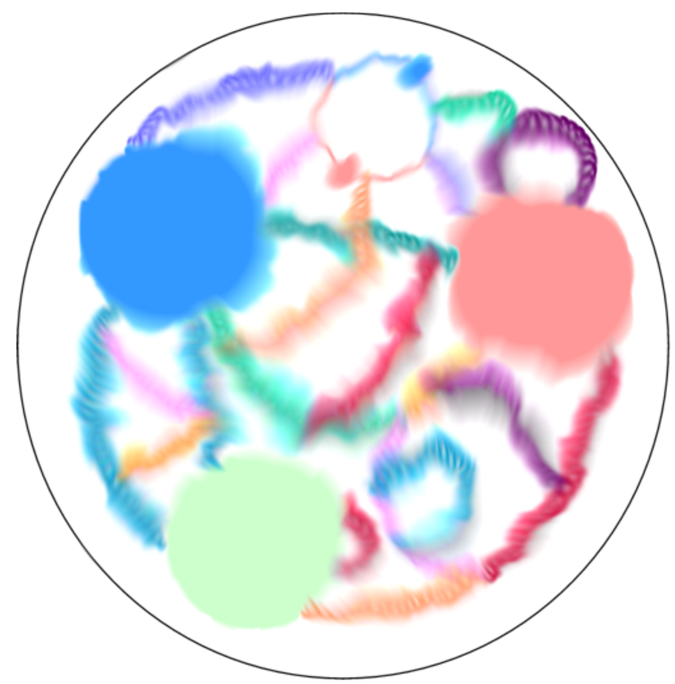
Jefferson Lab

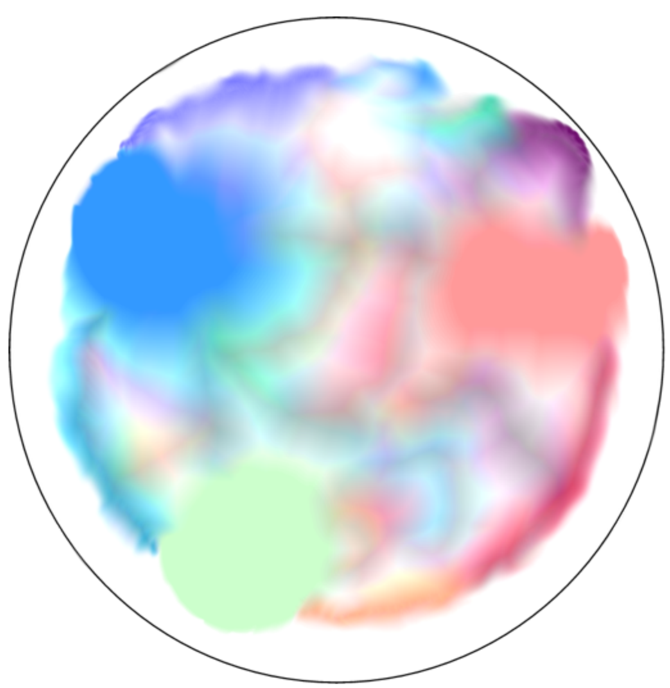
HADRON AT DIFFERENT SCALES

$Q^2\ (GeV^2)$



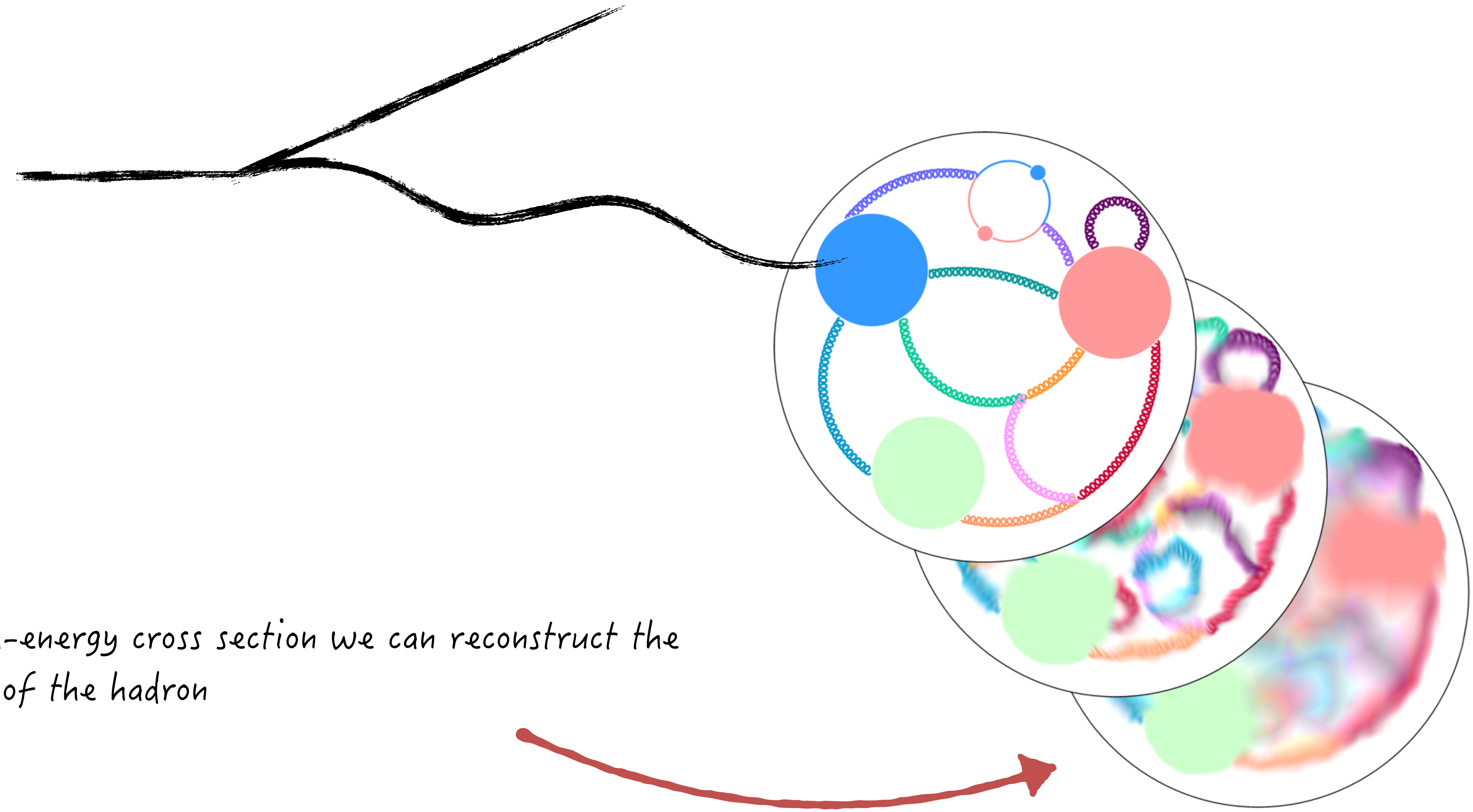






This "layers" are not independent

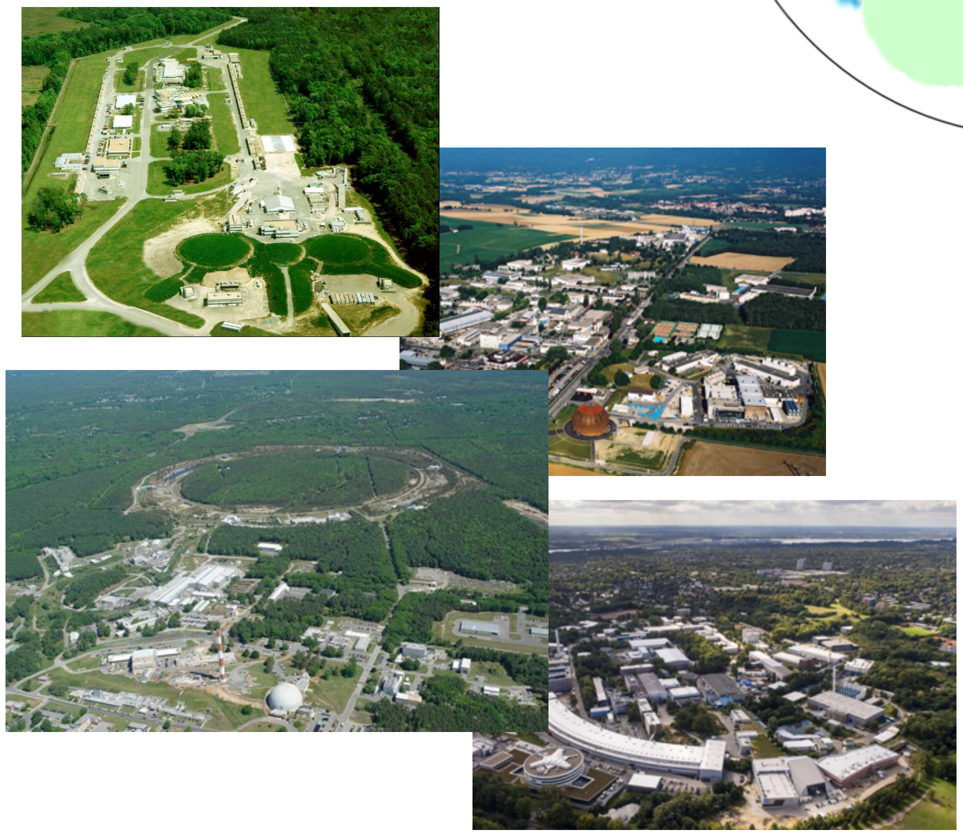
Factorization

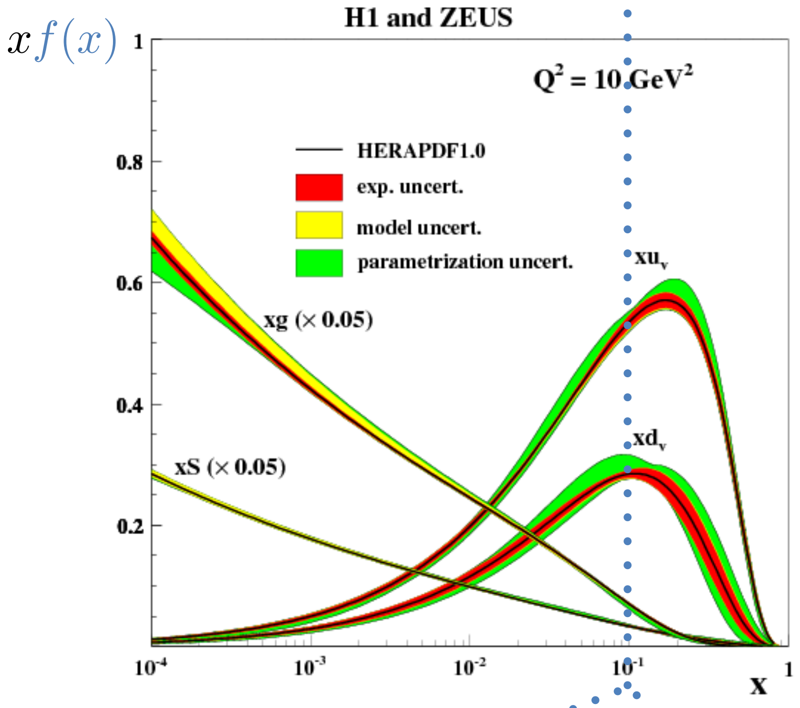


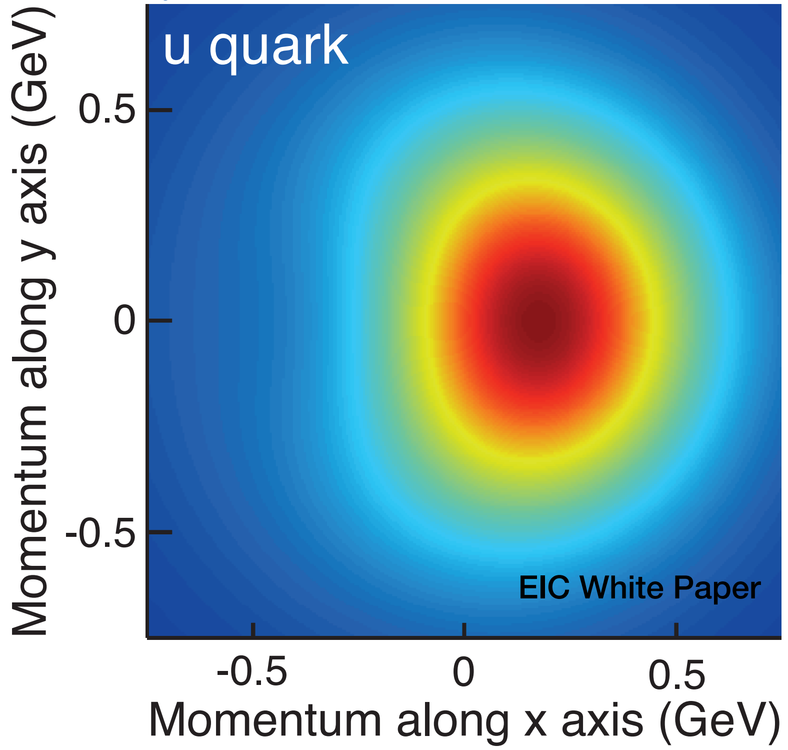
From high-energy cross section we can reconstruct the structure of the hadron

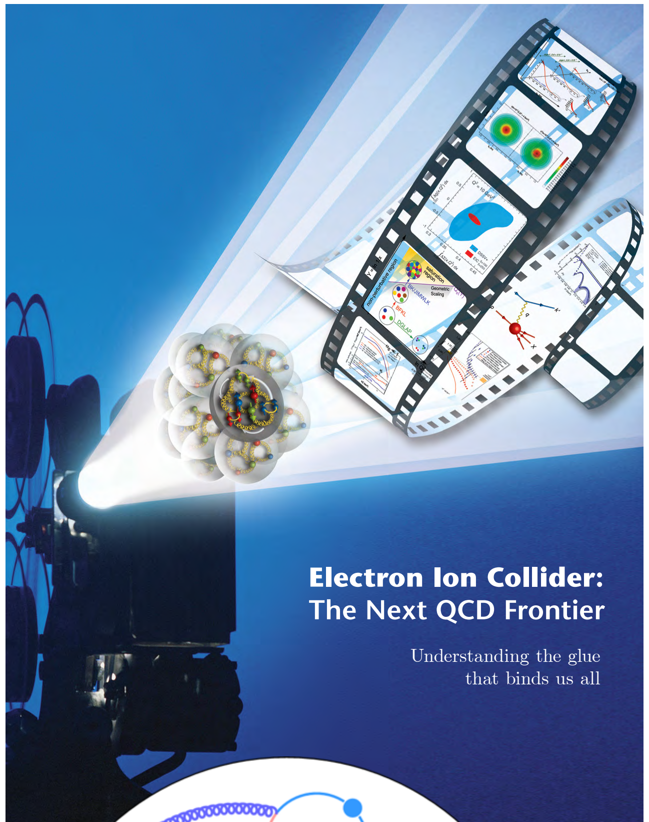
Factorization: "layers" are connected

Collinear and TMD factorization







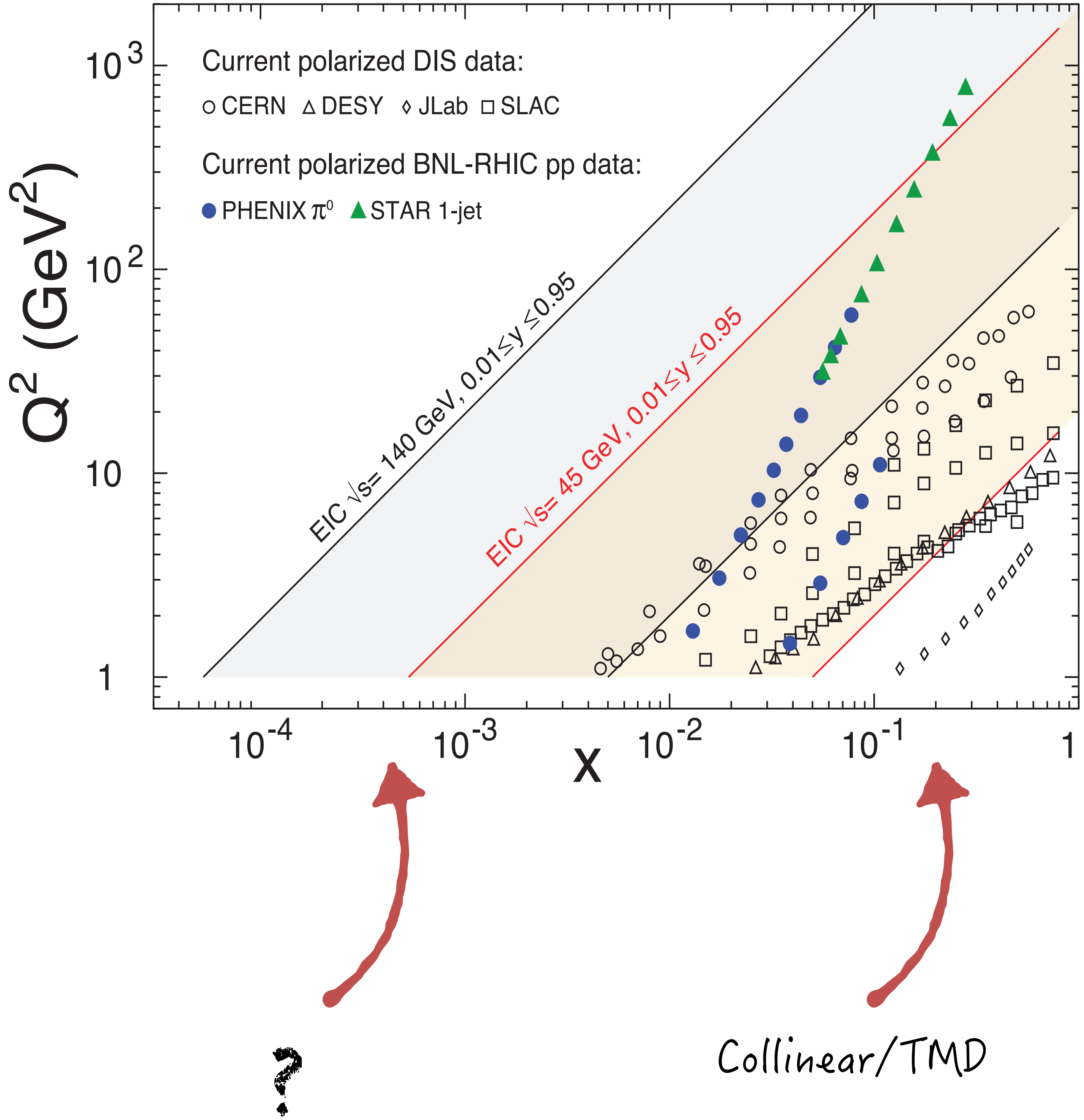


coherent contributions from many nucleons effectively amplify the gluon density being probed.

The EIC was designated in the 2007 Nuclear Physics Long Range Plan as "embodying the vision for reaching the next QCD frontier" [1]. It would extend the QCD science programs in the U.S. established at both the CEBAF accelerator at JLab and RHIC at BNL in dramatic and fundamentally important ways. The most intellectually pressing questions that an EIC will address that relate to our detailed and fundamental understanding of QCD in this *frontier* environment are:

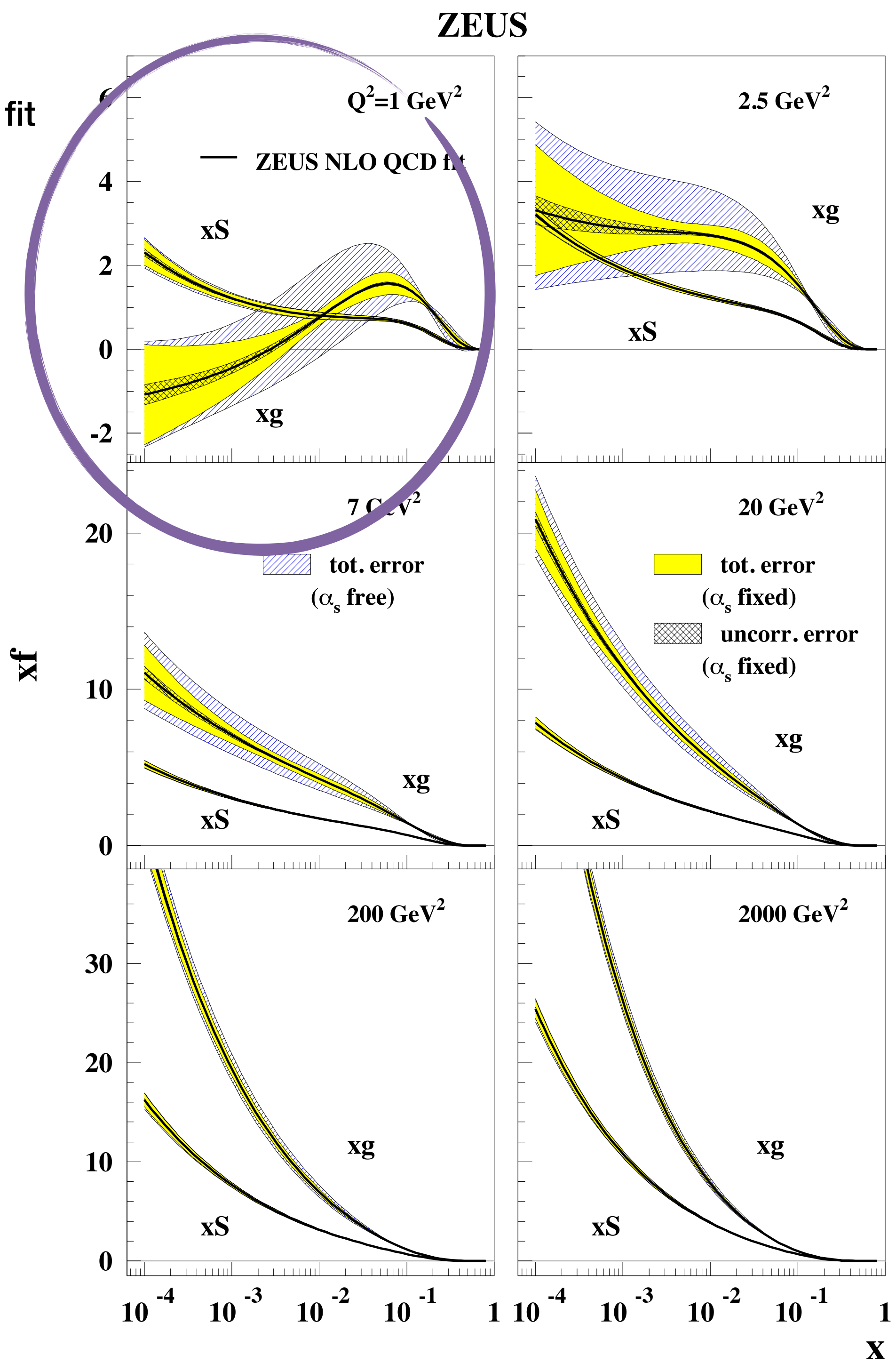
- **How are the sea quarks and gluons, and their spins, distributed in space and momentum inside the nucleon?** How are these quark and gluon distributions correlated with overall nucleon properties, such as spin direction? What is the role of the orbital motion of sea quarks and gluons in building the nucleon spin?
- **Where does the saturation of gluon densities set in?** Is there a simple boundary that separates this region from that of more dilute quark-gluon matter? If so, how do the distributions of quarks and gluons change as one crosses the boundary? Does this saturation produce matter of universal properties in the nucleon and all nuclei viewed at nearly the speed of light?
- **How does the nuclear environment affect the distribution of quarks and gluons and their interactions in nuclei?** How does the transverse spatial distribution of gluons compare to that in the nucleon? How does nuclear matter respond to a fast moving color charge passing through it? Is this response different for light and heavy quarks?

EIC White Paper

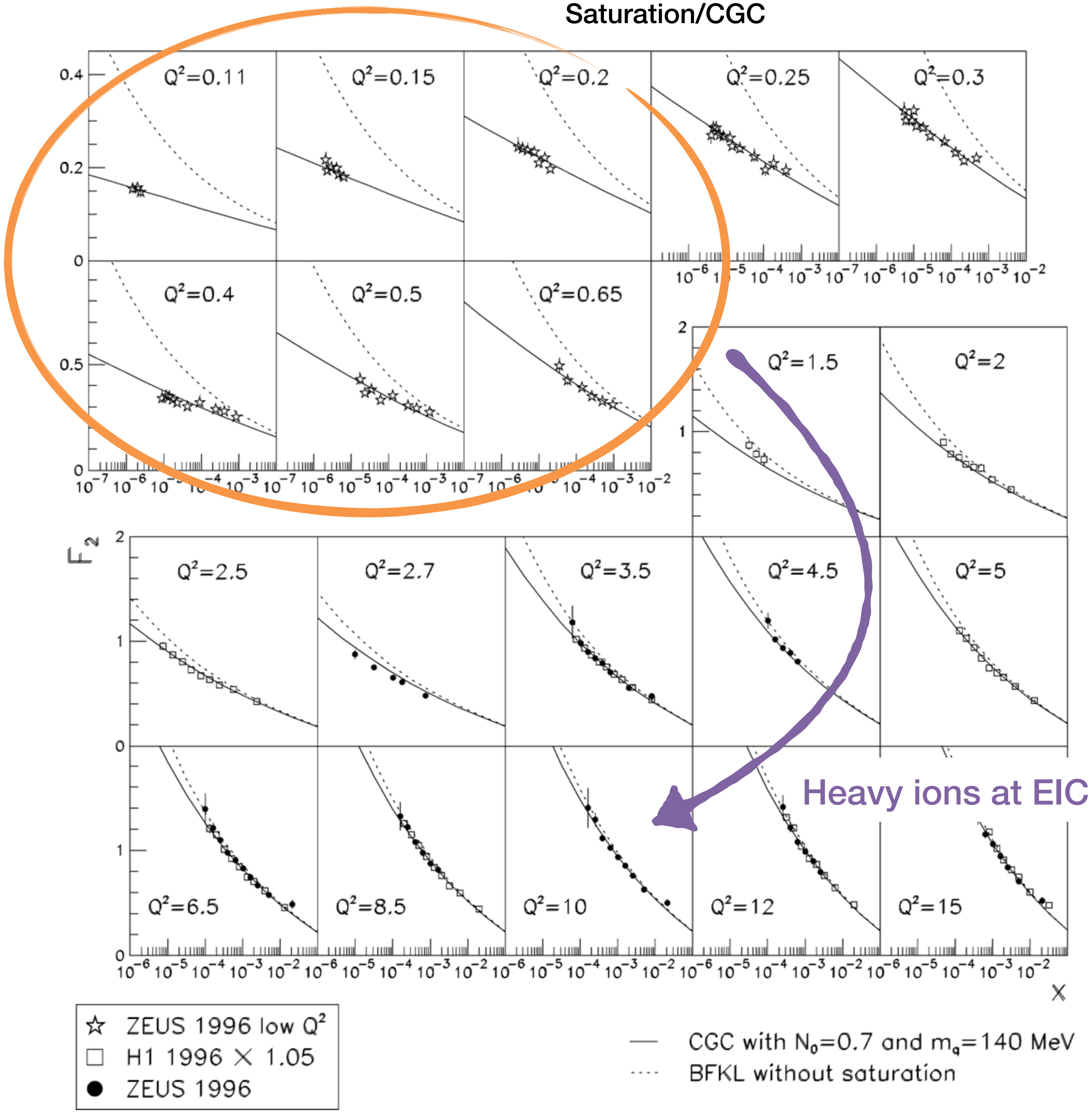


DGLAP vs. BFKL/BK

DGLAP fit



Saturation/CGC

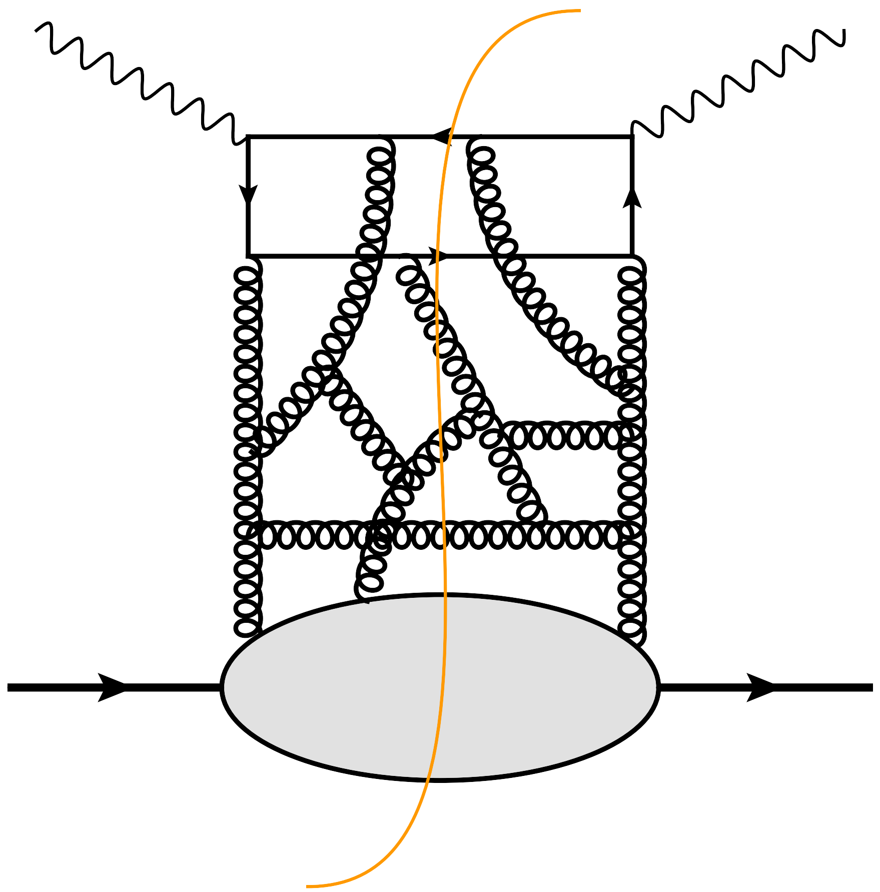


E. Iancu, K. Itakura, S. Munier
Phys. Lett. B 590 (2004) 199

Particle production

Total cross section (collinear distributions):

$$W_{\mu\nu} = \frac{1}{4\pi} \sum_X \int d^4 z e^{iqz} \langle P|j_\mu(z)|X\rangle\langle X|j_\nu(0)|P\rangle$$

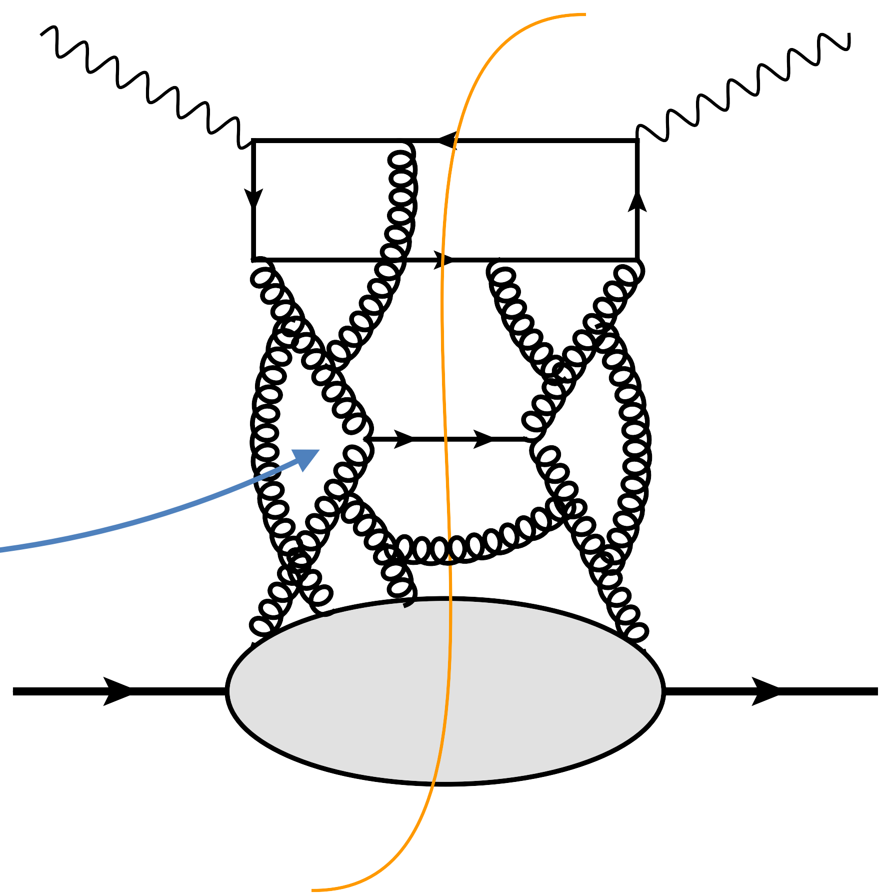


Particle production (TMD distributions):

$$\sigma_{\mu\nu} = \frac{1}{2\pi} \sum_X \int d^4 w e^{iqw} \langle p|j_\mu(w)|\Phi + X\rangle\langle \Phi + X|j_\nu(0)|p\rangle$$

Scalar particle production:

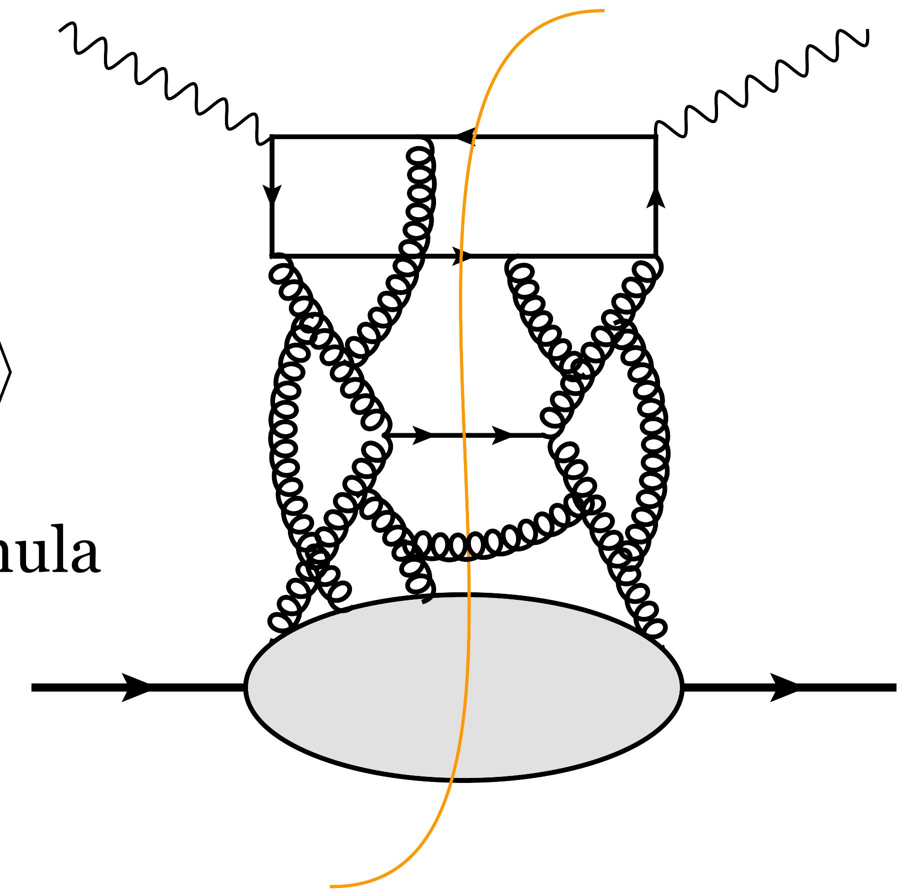
$$S_\Phi = \lambda \int d^4 z \; F^a_{\mu\nu}(z) F^{a\mu\nu}(z)\Phi(z)$$



Particle production

Particle production (TMD distributions):

$$\sigma_{\mu\nu} = \frac{1}{2\pi}\sum_X \int d^4w e^{iqw} \langle p | j_\mu(w) | \Phi + X\rangle \langle \Phi + X | j_\nu(0) | p\rangle$$



reduction formula

$$\sigma_{\mu\nu} = \frac{\lambda^2}{2\pi}\int d^4w d^4x d^4y e^{iqw-ikx+iky}$$
$$\times \sum_X \langle p | \tilde{T}\{j_\mu(w) F^2(x)\} | X\rangle \langle X | T\{F^2(y) j_\nu(0)\} | p\rangle$$

different time-ordering

functional integral

$$\sum_X |X\rangle\langle X|$$

$$\sigma_{\mu\nu} = \frac{\lambda^2}{2\pi}\int d^4w d^4x d^4y e^{iqw-ikx+iky} \int^{\tilde{A}(t_f)=A(t_f)} D\tilde{A} D\bar{\tilde{\psi}} D\tilde{\psi} DA D\bar{\psi} D\psi$$
$$\times \Psi_p^*(\vec{\tilde{A}}(t_i), \tilde{\psi}(t_i)) e^{-iS_{\mathrm{QCD}}(\tilde{A},\tilde{\psi})} \tilde{j}_\mu(w) \tilde{F}^2(x) F^2(y) j_\nu(0) e^{iS_{\mathrm{QCD}}(A,\psi)} \Psi_p(\vec{A}(t_i), \psi(t_i))$$

Kinematic variables

Light-like vectors:

$$p_1^\mu \simeq q^\mu + \frac{Q^2}{s} P^\mu \qquad p_2^\mu \simeq P^\mu - \frac{M^2}{s} q^\mu$$

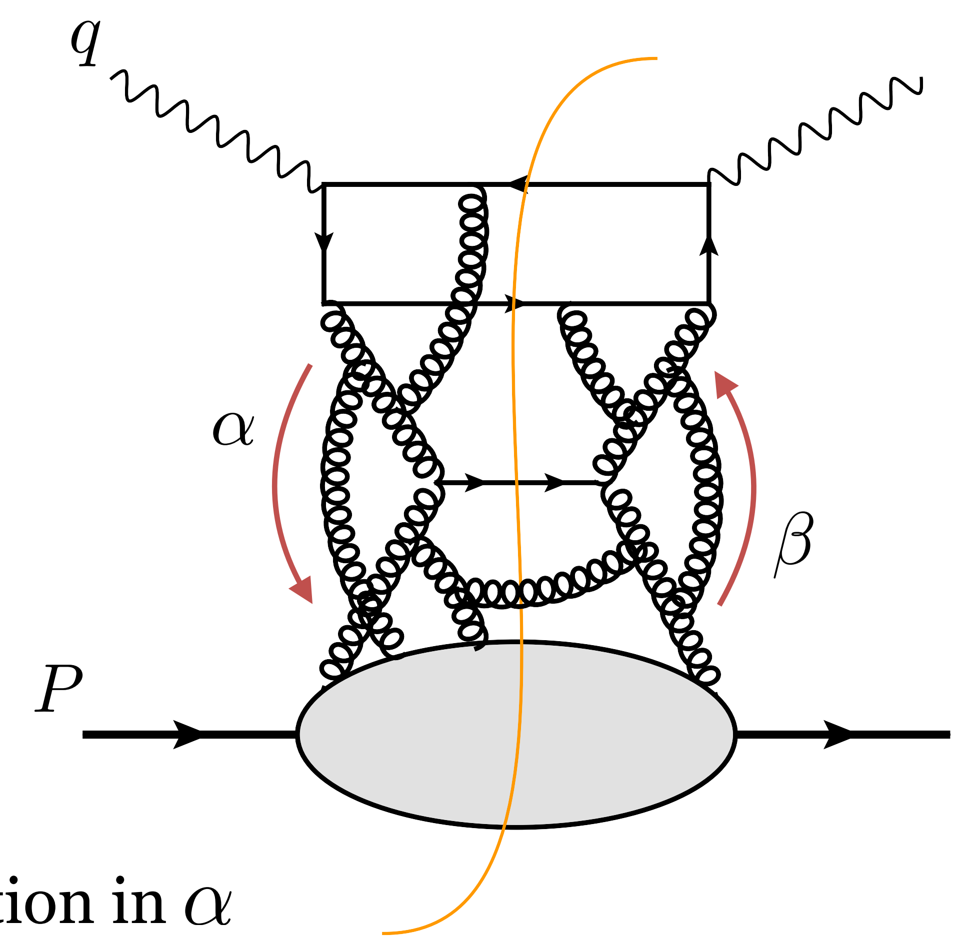
Sudakov momentum decomposition:

$$p^\mu = \alpha p_1^\mu + \beta p_2^\mu + p_\perp^\mu$$

Coordinate vector decomposition:

$$z^\mu = \frac{2}{s} z_* p_1^\mu + \frac{2}{s} z_\bullet p_2^\mu + z_\perp^\mu$$

$$z_* = \sqrt{\tfrac{s}{2}} z_+ \qquad z_\bullet = \sqrt{\tfrac{s}{2}} z_-$$



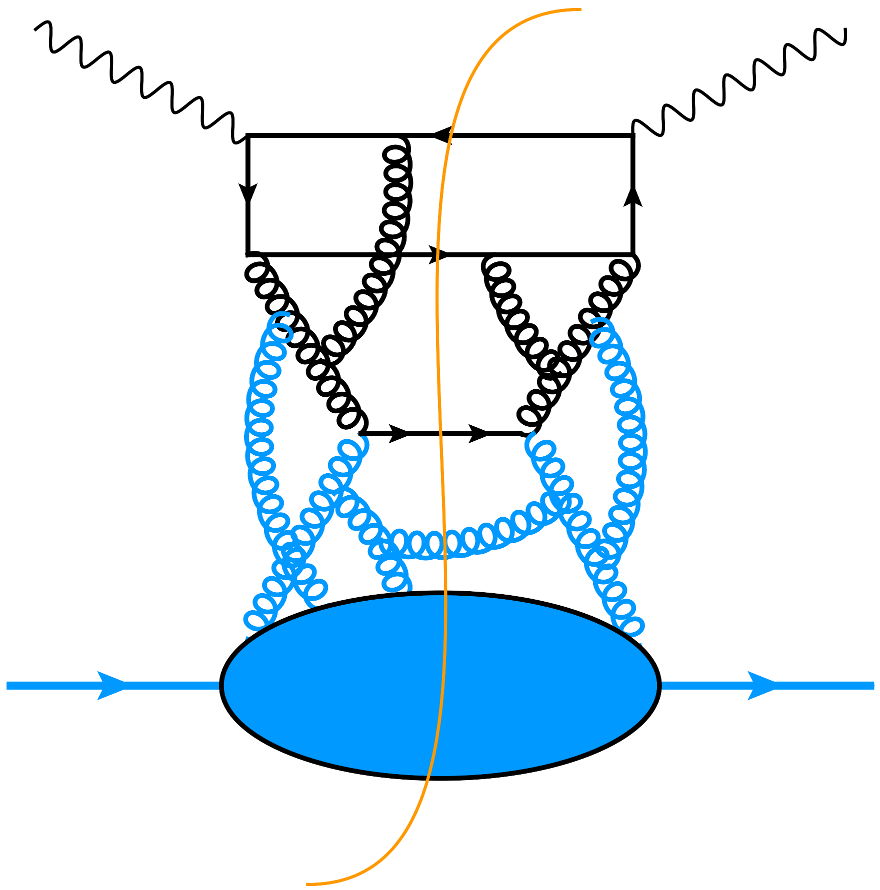
separation in α

$$\sigma_{\mu\nu} = \frac{\lambda^2}{2\pi} \int d^4w d^4x d^4y e^{iqw - ikx + iky} \int^{\tilde{A}(t_f) = A(t_f)} D\tilde{A} D\tilde{\bar{\psi}} D\tilde{\psi} DA D\bar{\psi} D\psi$$

$$\times \Psi_p^*(\vec{\tilde{A}}(t_i), \tilde{\psi}(t_i)) e^{-iS_{\rm QCD}(\tilde{A}, \tilde{\psi})} \tilde{j}_\mu(w) \tilde{F}^2(x) F^2(y) j_\nu(0) e^{iS_{\rm QCD}(A,\psi)} \Psi_p(\vec{A}(t_i), \psi(t_i))$$

RAPIDITY FACTORIZATION

$$\sigma_{\mu\nu} = \frac{\lambda^2}{2\pi}\int d^4w d^4x d^4y e^{iqw-ikx+iky}\int^{\tilde{A}(t_f)=A(t_f)} D\tilde{A}D\bar{\tilde{\psi}}D\tilde{\psi}DAD\bar{\psi}D\psi$$
$$\times\ \Psi_p^*(\vec{\tilde{A}}(t_i),\tilde{\psi}(t_i))e^{-iS_{\mathrm{QCD}}(\tilde{A},\tilde{\psi})}\tilde{j}_\mu(w)\tilde{F}^2(x)F^2(y)j_\nu(0)e^{iS_{\mathrm{QCD}}(A,\psi)}\Psi_p(\vec{A}(t_i),\psi(t_i))$$



$$A \to A|_{\alpha>\sigma} + A|_{\alpha<\sigma}$$

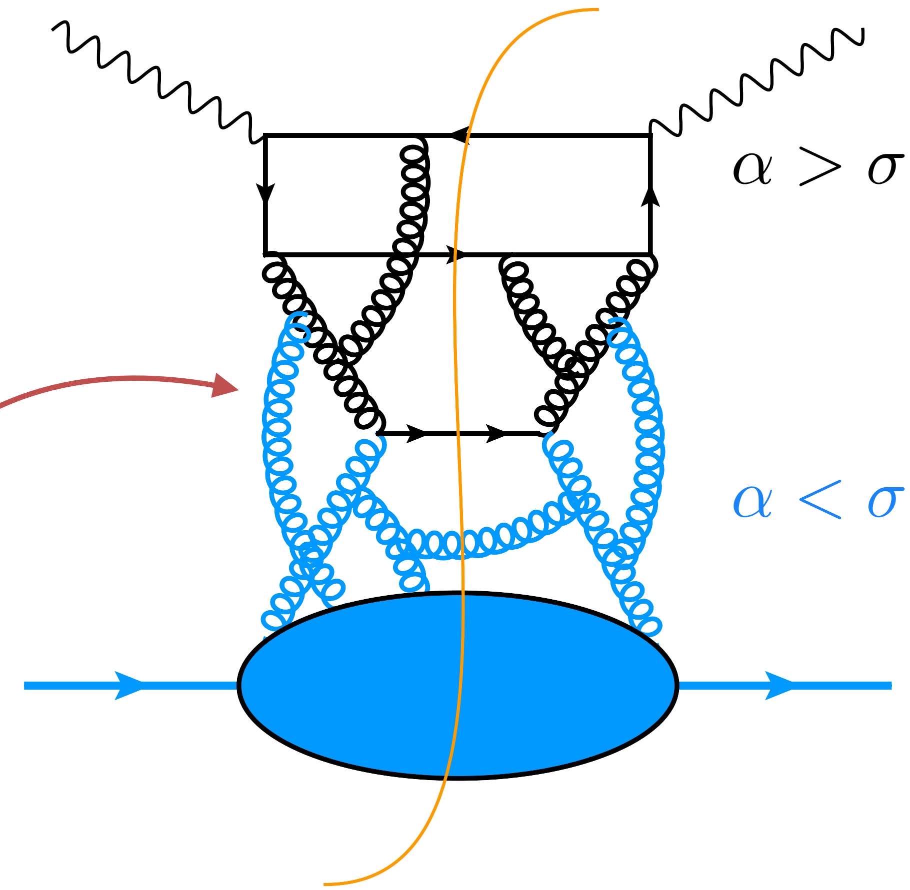
$$\sigma_{\mu\nu} = \frac{\lambda^2}{2\pi}\int d^4w d^4x d^4y e^{iqw-ikx+iky}\int^{\tilde{A}(t_f)=A(t_f)} D\tilde{A}D\bar{\tilde{\psi}}D\tilde{\psi}DAD\bar{\psi}D\psi$$
$$\times\ \Psi_p^*(\vec{\tilde{A}}(t_i),\tilde{\psi}(t_i))e^{-iS_{\mathrm{QCD}}(\tilde{A},\tilde{\psi})}e^{iS_{\mathrm{QCD}}(A,\psi)}\Psi_p(\vec{A}(t_i),\psi(t_i))$$
$$\times \sum_X \langle p|\tilde{T}\{j_\mu(w)F^2(x)\}|X\rangle_{\tilde{A}}\langle X|T\{F^2(y)j_\nu(0)\}|p\rangle_A$$

separate dependence on A fields

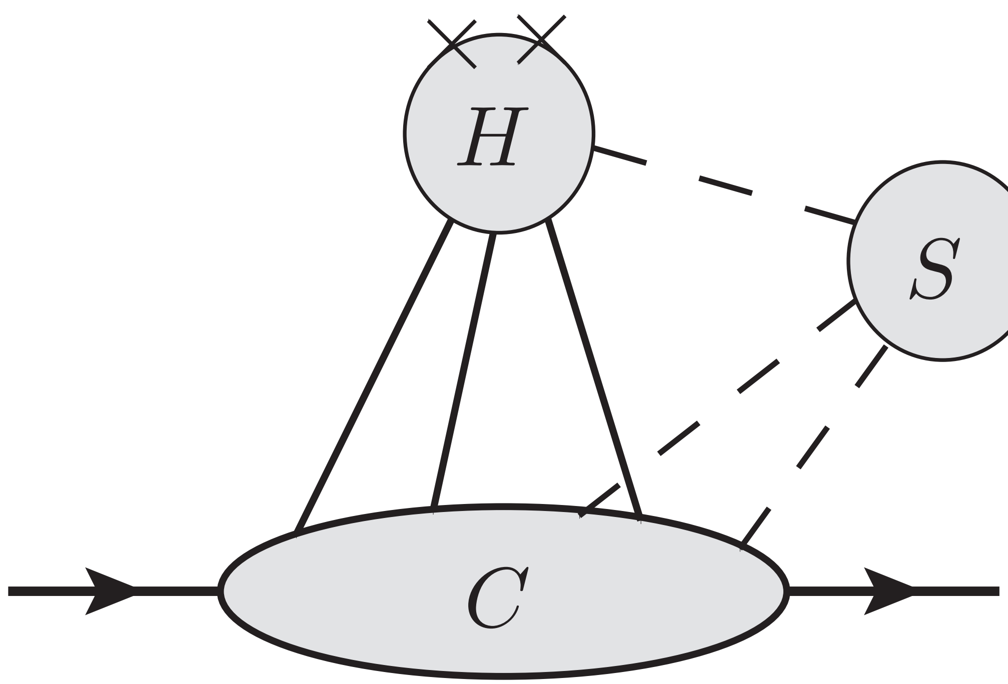
COLLINEAR/TMD FACTORIZATION

$$\sigma_{\mu\nu} = \frac{\lambda^2}{2\pi}\int d^4w\, d^4x\, d^4y\, e^{iqw-ikx+iky}\int^{\tilde{A}(t_f)=A(t_f)} D\tilde{A}\, D\tilde{\bar{\psi}}\, D\tilde{\psi}\, DA\, D\bar{\psi}\, D\psi$$
$$\times\ \Psi_p^*(\vec{\tilde{A}}(t_i), \tilde{\psi}(t_i))\, e^{-iS_{\mathrm{QCD}}(\tilde{A},\tilde{\psi})}\, e^{iS_{\mathrm{QCD}}(A,\psi)}\, \Psi_p(\vec{A}(t_i), \psi(t_i))$$
$$\times \sum_X \langle p|\tilde{T}\{j_\mu(w)F^2(x)\}|X\rangle_{\tilde{A}} \langle X|T\{F^2(y)j_\nu(0)\}|p\rangle_A$$

Don't specify kinematics



Region analysis in collinear/TMD factorization:

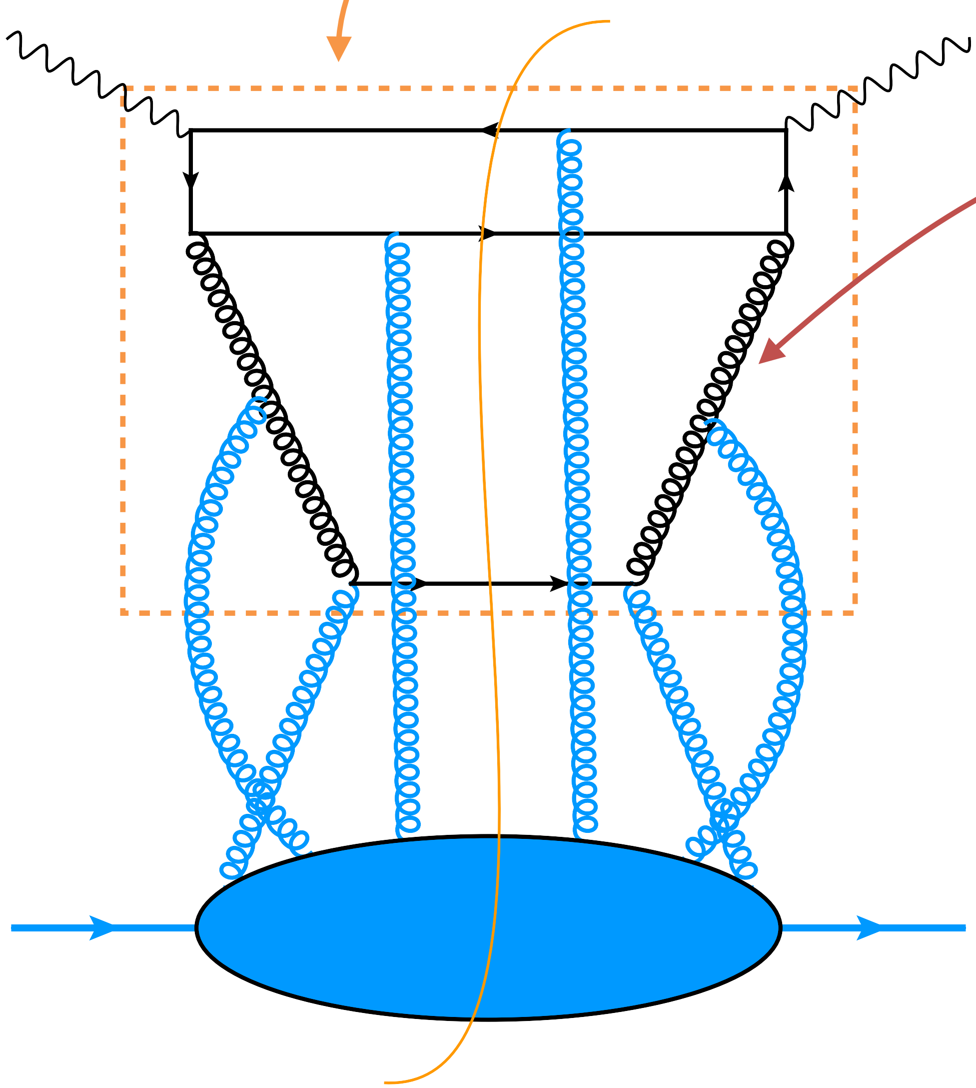


- Hard
- Soft
- Collinear
- Glauber

Rapidity factorization

$$\sigma_{\mu\nu} = \frac{\lambda^2}{2\pi}\int d^4w d^4x d^4y e^{iqw-ikx+iky}\int^{\tilde{A}(t_f)=A(t_f)} D\tilde{A}D\tilde{\bar{\psi}}D\tilde{\psi}DAD\bar{\psi}D\psi$$
$$\times\ \Psi_p^*(\vec{\tilde{A}}(t_i),\tilde{\psi}(t_i))e^{-iS_{\rm QCD}(\tilde{A},\tilde{\psi})}e^{iS_{\rm QCD}(A,\psi)}\Psi_p(\vec{A}(t_i),\psi(t_i))$$
$$\times \sum_X \langle p|\tilde{T}\{j_\mu(w)F^2(x)\}|X\rangle_{\tilde{A}}\langle X|T\{F^2(y)j_\nu(0)\}|p\rangle_A$$

Integrate over fast fields



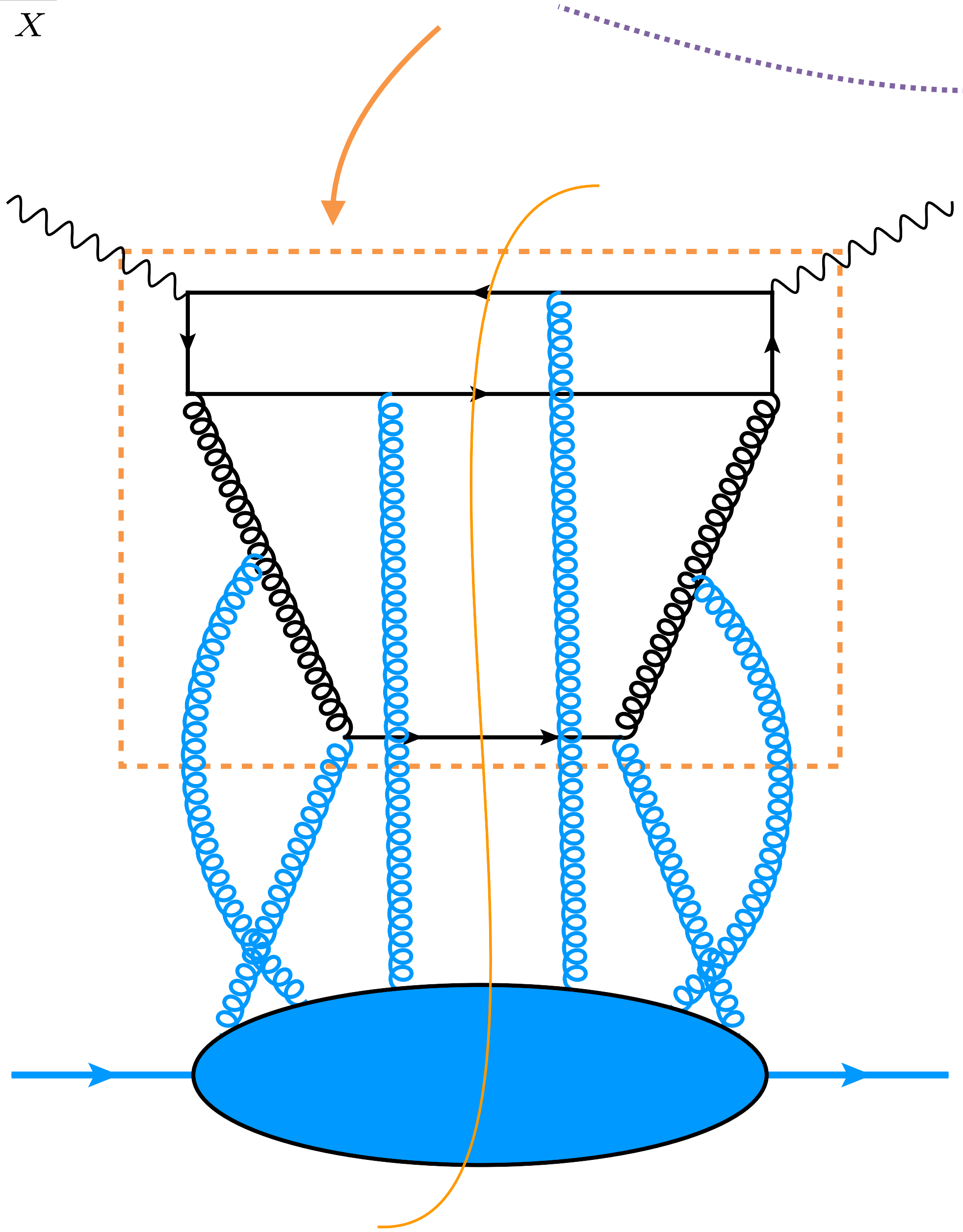
Gluon propagator in external field

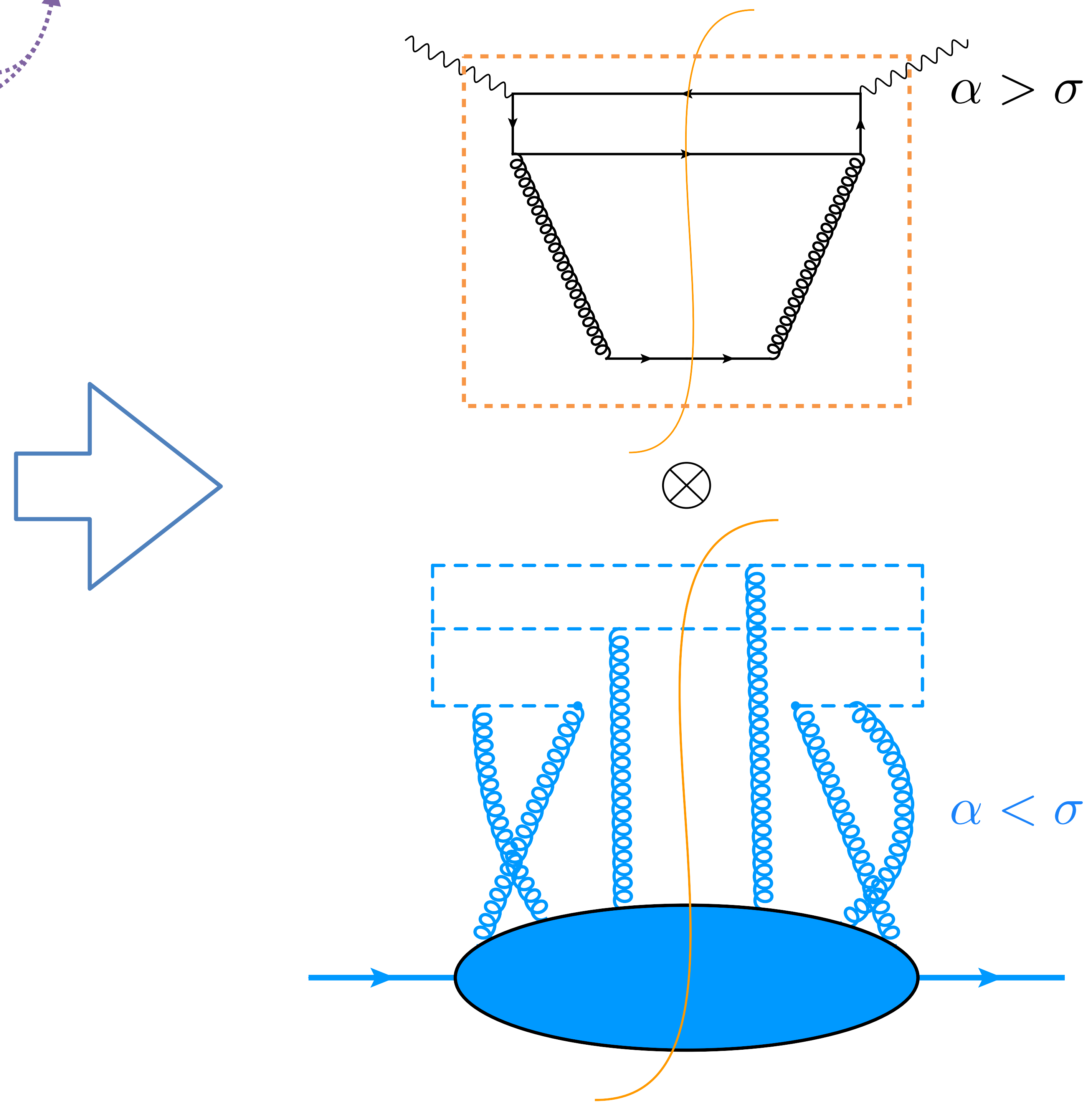
$$\langle A_\mu(z)F_{*j}(y)\rangle = \frac{i}{2}\int_\sigma^\infty \frac{đ\alpha}{2\alpha}e^{-i\alpha(y-z)_\bullet}$$
$$\times\ (z_\perp|e^{-i\frac{p_\perp^2}{\alpha s}(y-z)_*}(\alpha s g_{\mu j}^\perp + 2p_{2\mu}p_j)|y_\perp)[-\infty, y_*]_y$$

Separate interaction (factorization)

RAPIDITY FACTORIZATION

$$\sigma_{\mu\nu} = \frac{\lambda^2}{2\pi}\int d^4w d^4x d^4y e^{iqw-ikx+iky}\int^{\tilde{A}(t_f)=A(t_f)} D\tilde{A}D\tilde{\bar{\psi}}D\tilde{\psi}DAD\bar{\psi}D\psi$$
$$\times\ \Psi_p^*(\vec{\tilde{A}}(t_i),\tilde{\psi}(t_i))e^{-iS_{\rm QCD}(\tilde{A},\tilde{\psi})}e^{iS_{\rm QCD}(A,\psi)}\Psi_p(\vec{A}(t_i),\psi(t_i))$$
$$\times\sum_X \langle p|\tilde{T}\{j_\mu(w)F^2(x)\}|X\rangle_{\tilde{A}}\langle X|T\{F^2(y)j_\nu(0)\}|p\rangle_A$$



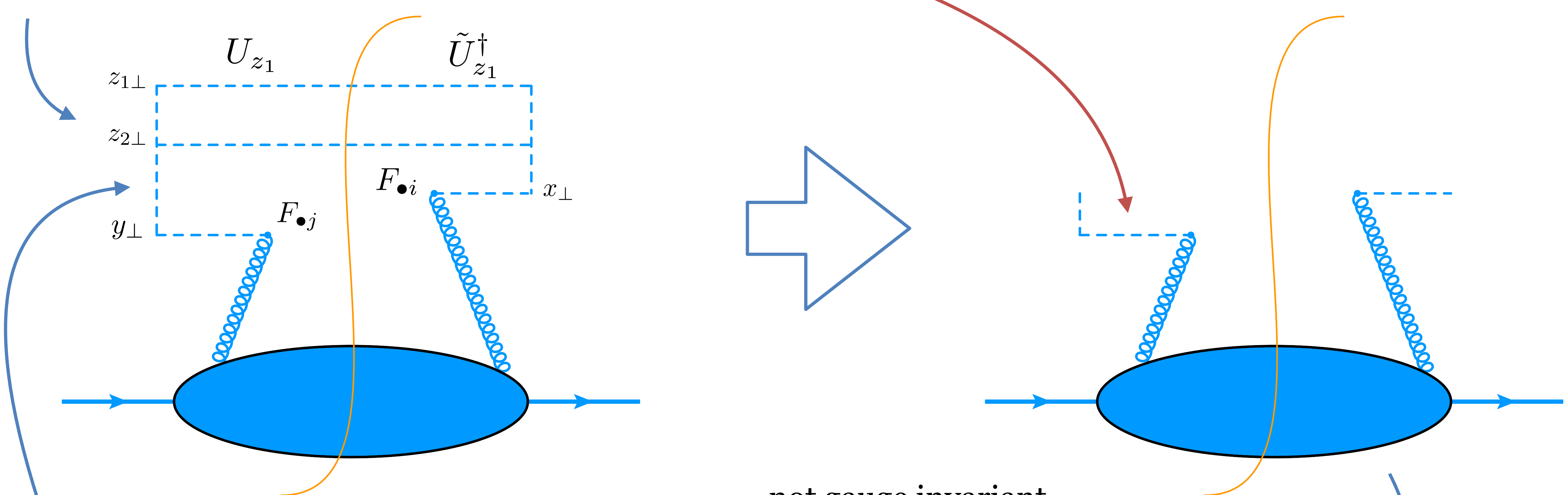


WILSON LINES

$$tr\langle P|\tilde{T}\{U_{z_2}[z_{2\perp},x_\perp]_{-\infty}[-\infty,x_*]_x F_{\bullet i}(x_*,x_\perp)[x_*,-\infty]_x[x_\perp,z_{1\perp}]_{-\infty}U^\dagger_{z_1}\}$$
$$\times T\{U_{z_1}[z_{1\perp},y_\perp]_{-\infty}[-\infty,y_*]_y F_{\bullet j}(y_*,y_\perp)[y_*,-\infty]_y[y_\perp,z_{2\perp}]_{-\infty}U^\dagger_{z_2}\}|P\rangle$$

two different T-products

gauge invariant



space(light)-like distance

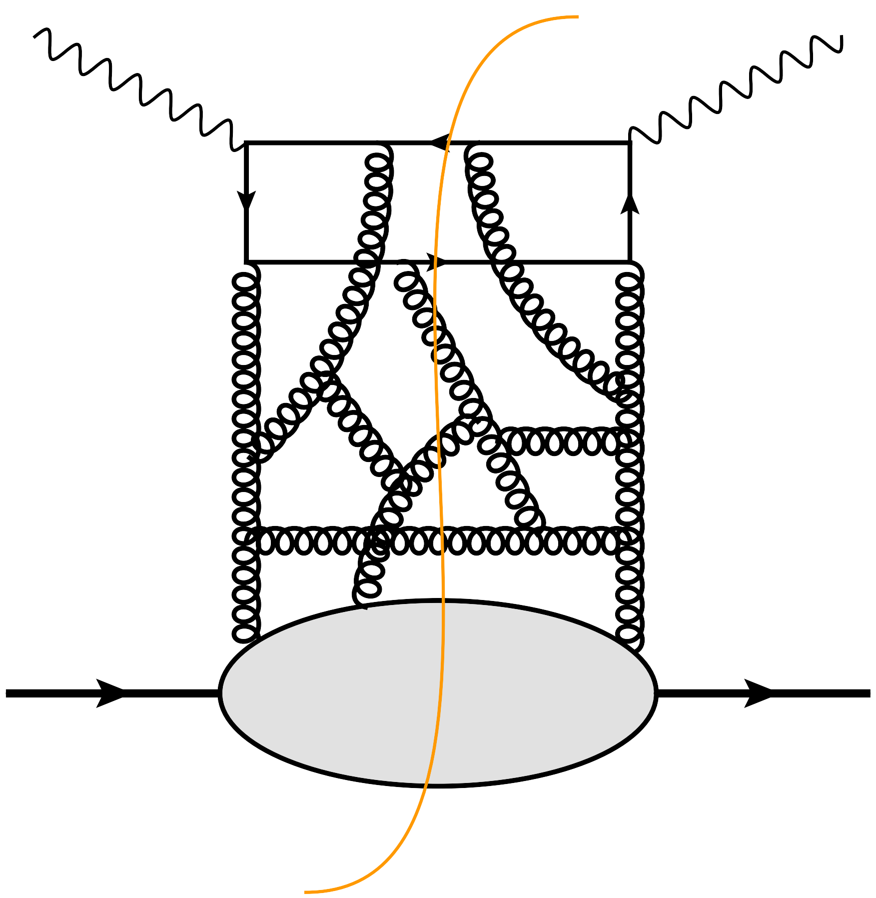
not gauge invariant

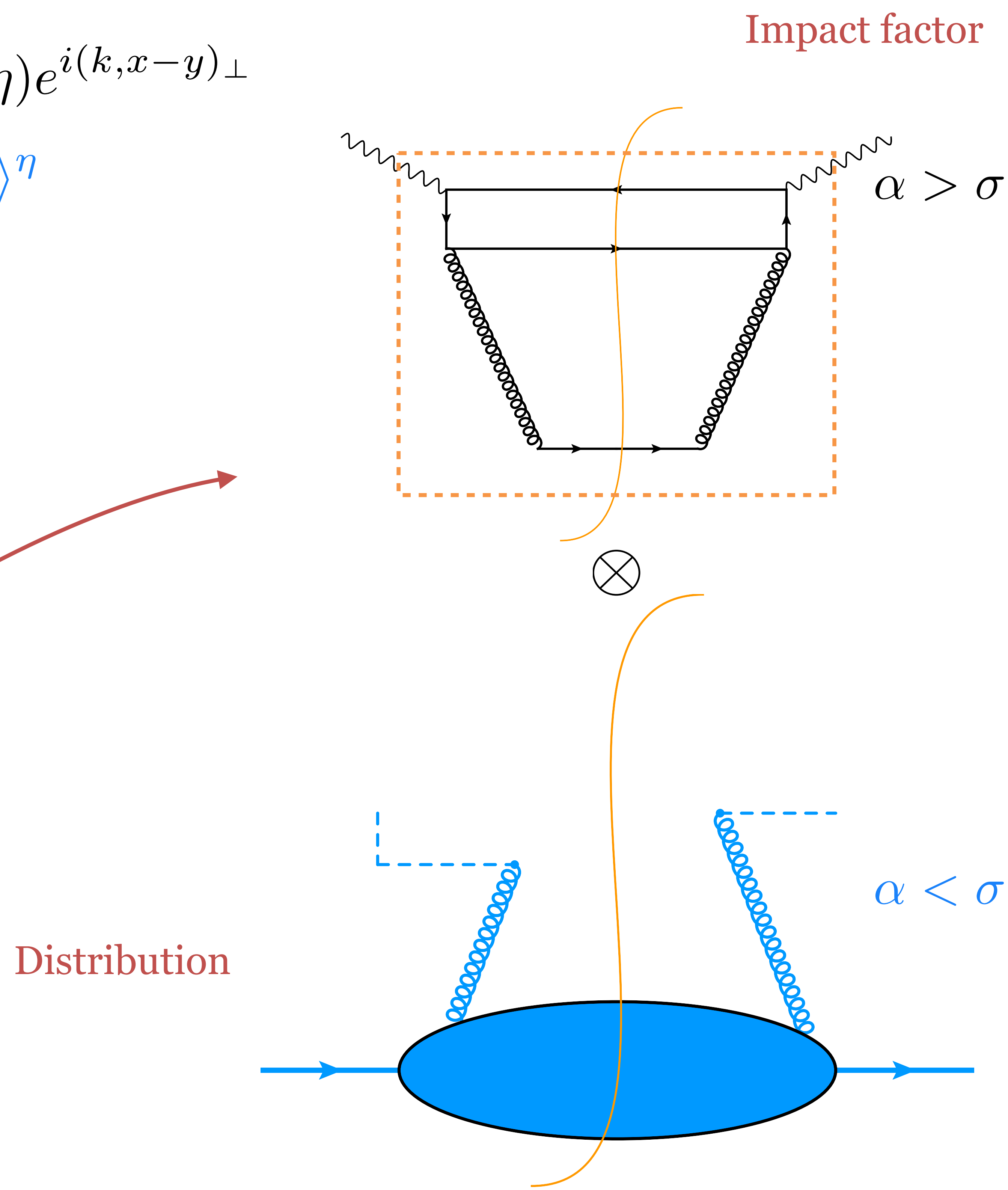
$$\mathrm{tr}\langle P|[y_\perp,x_\perp]_{-\infty}[-\infty_*,x_*]_x F_{\bullet i}(x_*,x_\perp)[x_*,-\infty_*]_x\times[x_\perp,y_\perp]_{-\infty}[-\infty_*,y_*]_y F_{\bullet j}(y_*,y_\perp)[y_*,-\infty_*]_y|P\rangle$$

Cross section

$$\lambda^{-2}\sigma_{\mu\nu} \;=\; \int d^2x_\perp d^2y_\perp \; \mathcal{I}^{ij}_{\mu\nu}(x_\perp, y_\perp;\eta)e^{i(k,x-y)_\perp}$$

$$\times\langle\!\langle p|\tilde{\mathcal{F}}^a_i(\beta_B, x_\perp)[x_\perp, y_\perp]^{ab}_{-\infty}\mathcal{F}^b_j(\beta_B, y_\perp)|p\rangle\!\rangle^\eta$$



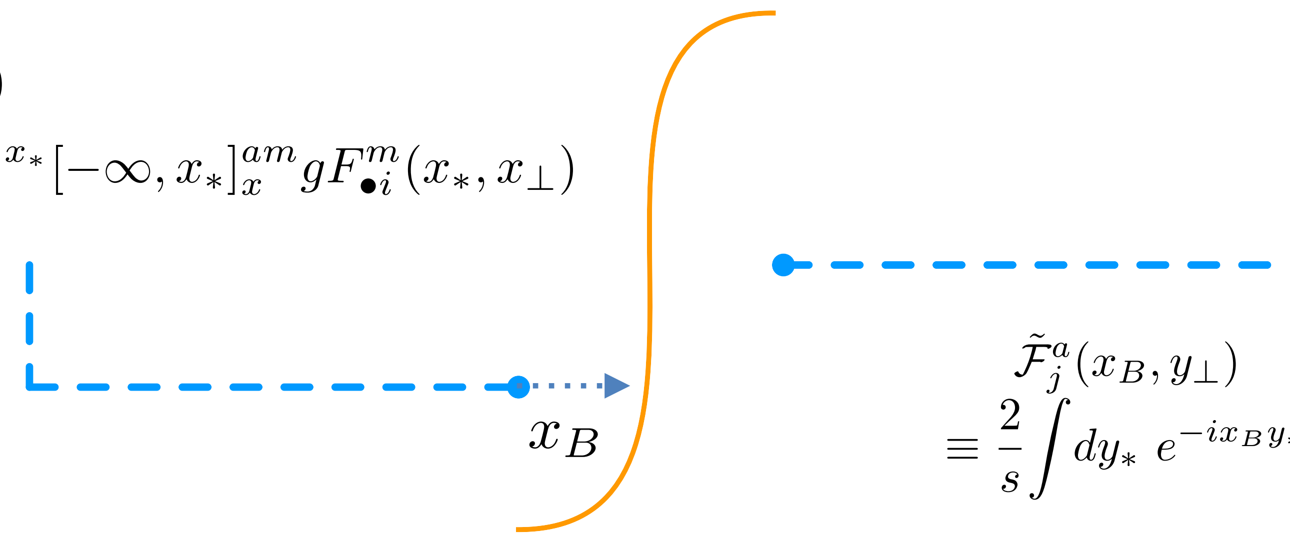


TMD DISTRIBUTION

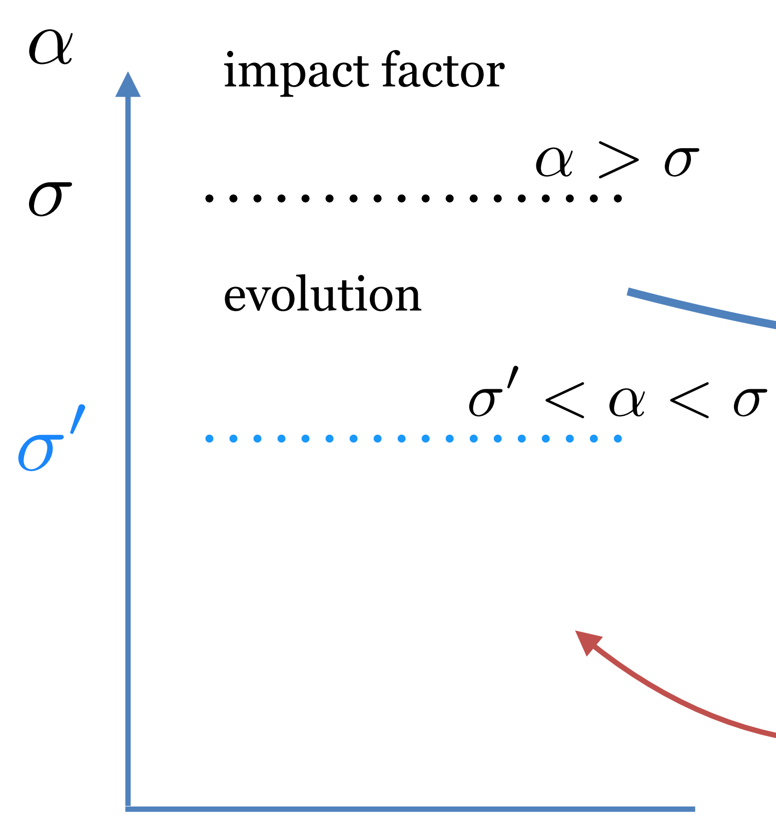
$$\langle P|\mathcal{F}_i^a(x_*,x_\perp)[x_\perp,y_\perp]^{ab}_{-\infty}\mathcal{F}_j^b(y_*,y_\perp)|P\rangle^\sigma$$

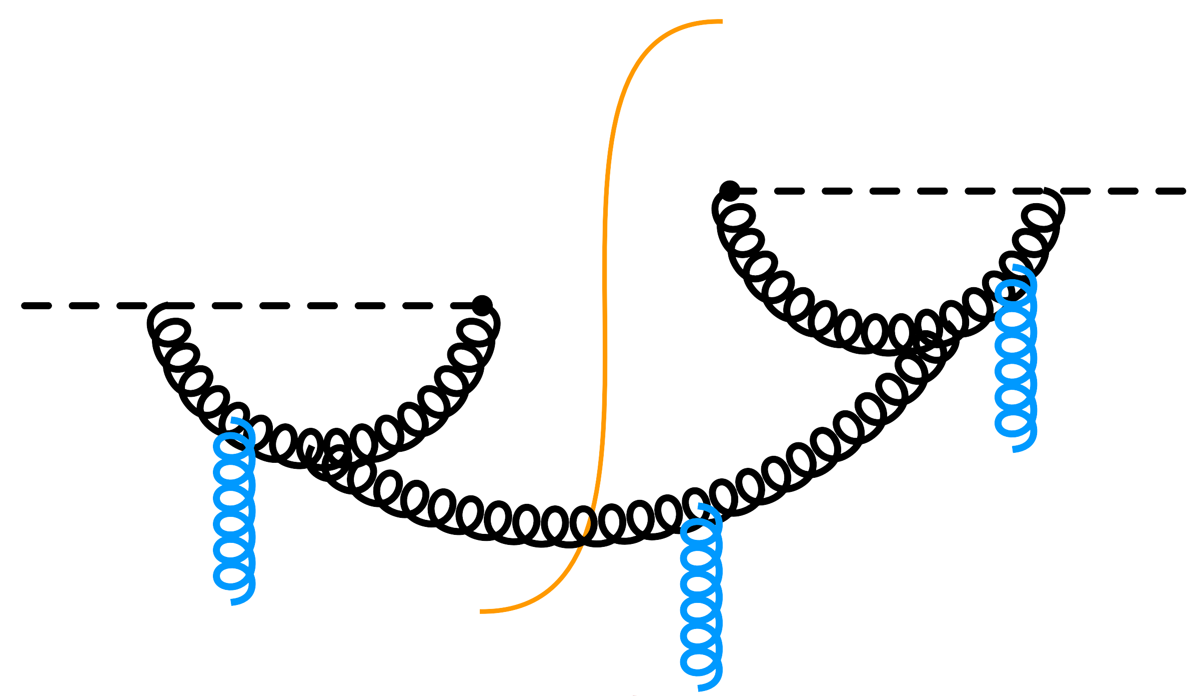
regulator

$$\mathcal{F}_i^a(x_B,x_\perp) \equiv \frac{2}{s}\int dx_*\; e^{ix_Bx_*}[-\infty,x_*]_x^{am}\, gF^m_{\bullet i}(x_*,x_\perp)$$



$$\tilde{\mathcal{F}}_j^a(x_B,y_\perp) \equiv \frac{2}{s}\int dy_*\; e^{-ix_By_*}\, g\tilde{F}^m_{\bullet j}(y_*,y_\perp)[y_*,-\infty]_y^{ma}$$



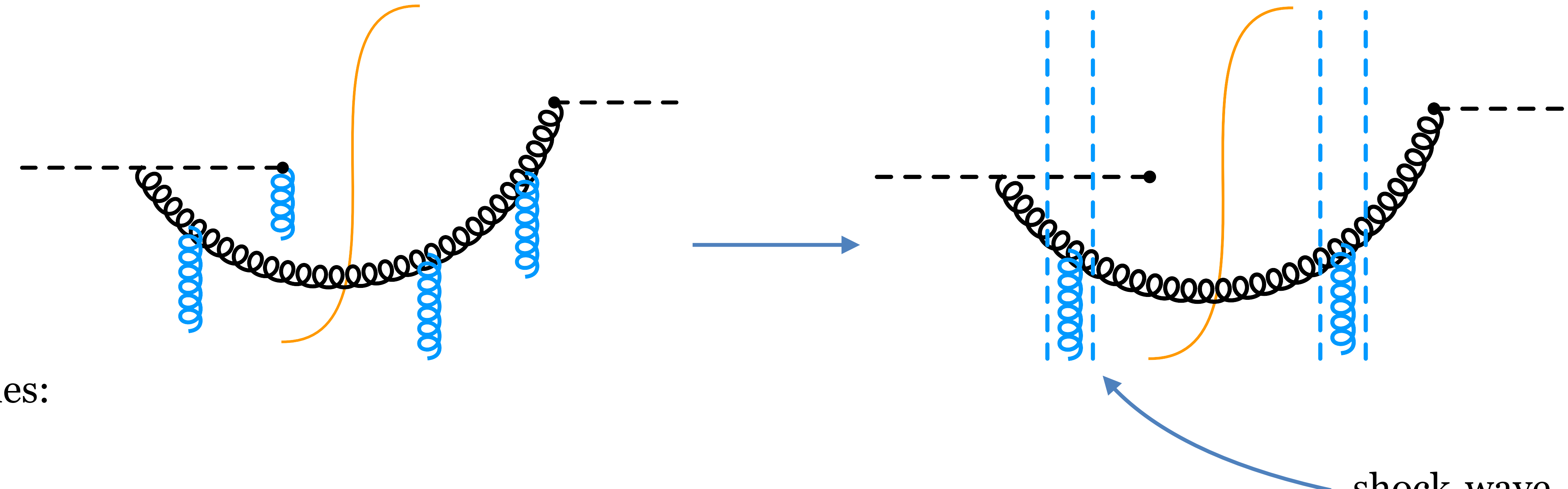


Can apply different approximations

Small-x limit

$x_B \to 0$

$k_\perp^2 \ll s$



Past-point Wilson lines:

shock-wave

$$\frac{d}{d\eta} U_i^a(z_1) U_j^a(z_2)$$
$$= -\frac{g^2}{8\pi^3} \mathrm{Tr}\Big\{ (i\partial_i^{z_1} + U_i^{z_1}) \Big[\int d^2 z_3 (U_{z_1}^\dagger U_{z_3} - 1) \frac{z_{12}^2}{z_{13}^2 z_{23}^2} (U_{z_3}^\dagger U_{z_2} - 1) \Big] (-i \overleftarrow{\partial}_j^{z_2} + U_j^{z_2}) \Big\}$$

Ian Balitsky, A.T. 2016

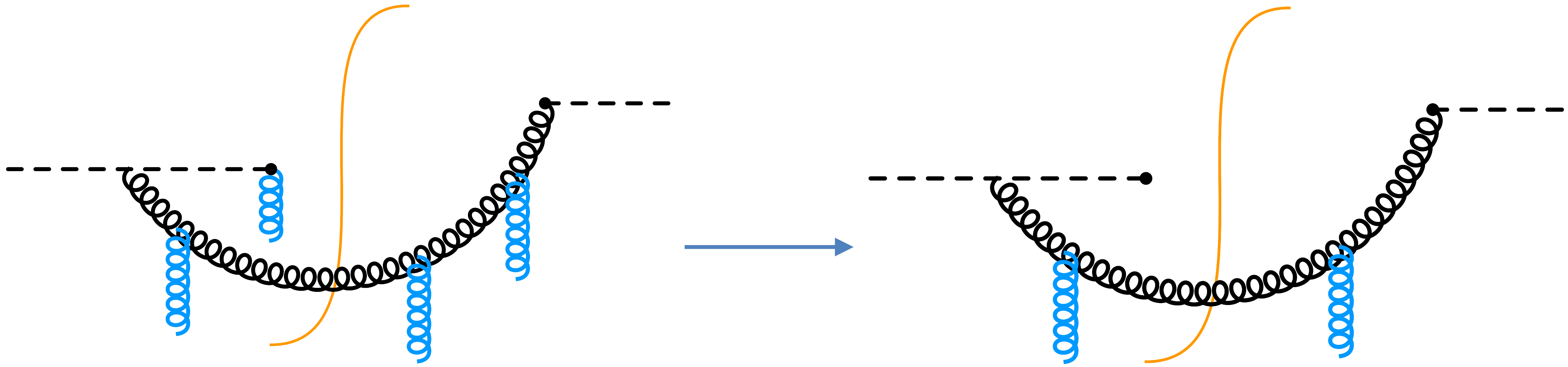
BFKL/BK dynamics

non-linear evolution
equation

MODERATE-X LIMIT

$x_B \to 1$

$k_\perp^2 \sim s$



$$\frac{d}{d\ln\sigma}\alpha_s\mathcal{D}(\beta_B,z_\perp,\sigma)$$
$$= \frac{\alpha_s N_c}{\pi}\int_{\beta_B}^{1}\frac{dz'}{z'}\Big\{J_0\Big(|z_\perp|\sqrt{\sigma s\beta_B\frac{1-z'}{z'}}\Big)\Big[\Big(\frac{1}{1-z'}\Big)_+ + \frac{1}{z'} - 2 + z'(1-z')\Big]\alpha_s\mathcal{D}\Big(\frac{\beta_B}{z'},z_\perp,\sigma\Big)$$
$$+ \frac{4}{m^2}(1-z')z'z_\perp^2 J_2\Big(|z_\perp|\sqrt{\sigma s\beta_B\frac{1-z'}{z'}}\Big)\alpha_s\mathcal{H}''\Big(\frac{\beta_B}{z'},z_\perp,\sigma\Big)\Big\}$$

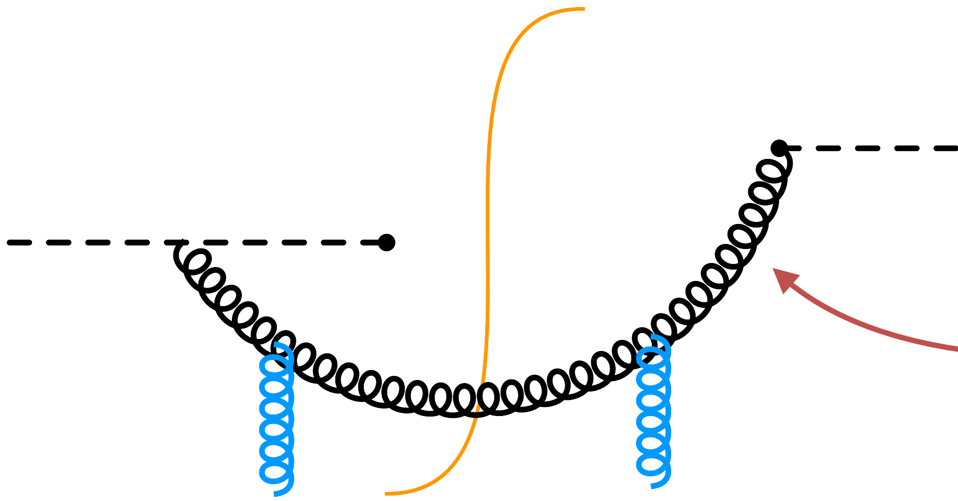
$z_\perp = 0$

Rapidity evolution for collinear distribution

$$\frac{d}{d\ln\sigma}\alpha_s\mathcal{D}(\beta_B,0_\perp,\sigma) = \frac{\alpha_s}{\pi}N_c\int_{\beta_B}^{1}\frac{dz'}{z'}\Big[\Big(\frac{1}{1-z'}\Big)_+ + \frac{1}{z'} - 2 + z'(1-z')\Big]\alpha_s\mathcal{D}\Big(\frac{\beta_B}{z'},0_\perp,\sigma\Big)$$

DGLAP dynamics

INTERMEDIATE REGIME



Neglect any non-linear contribution

Linear evolution equation (we can solve it)

$$
\begin{aligned}
&\frac{d}{d\ln\sigma}\mathcal{R}_{ij}(\beta_B,p_\perp;\sigma)\\
&= -\alpha_s N_c\int \bar{d}^2k_\perp\Big\{\Big[\Big(\frac{(2p-k)_k}{\sigma\beta_B s+p_\perp^2}\,\frac{\sigma\beta_B s g_{\mu i}-2(p-k)^\perp_\mu(p-k)_i}{\sigma\beta_B s+(p-k)_\perp^2}-2\frac{(p-k)^\perp_\mu g_{ik}+p_i g_{\mu k}}{\sigma\beta_B s+p_\perp^2}\Big)\\
&\times\Big(\frac{\sigma\beta_B s\delta^\mu_j-2(p-k)^\mu_\perp(p-k)_j}{\sigma\beta_B s+(p-k)_\perp^2}\,\frac{(2p-k)_l}{\sigma\beta_B s+p_\perp^2}-2\frac{(p-k)^\mu_\perp g_{jl}+\delta^\mu_l p_j}{\sigma\beta_B s+p_\perp^2}\Big)\\
&+2g_{ik}\Big(\frac{(p-k)_j(2p-k)_l-2p_j(p-k)_l}{(p-k)_\perp^2(\sigma\beta_B s+p_\perp^2)}+\frac{(p-k)_j(2p-k)_l}{(\sigma\beta_B s+(p-k)_\perp^2)(\sigma\beta_B s+p_\perp^2)}+\frac{2g_{jl}}{\sigma\beta_B s+p_\perp^2}\Big)\\
&+2g_{lj}\Big(\frac{(p-k)_i(2p-k)_k-2p_i(p-k)_k}{(p-k)_\perp^2(\sigma\beta_B s+p_\perp^2)}+\frac{(p-k)_i(2p-k)_k}{(\sigma\beta_B s+(p-k)_\perp^2)(\sigma\beta_B s+p_\perp^2)}+\frac{2g_{ik}}{\sigma\beta_B s+p_\perp^2}\Big)\Big]\\
&\times\ \theta\Big(1-\beta_B-\frac{(p-k)_\perp^2}{\sigma s}\Big)\mathcal{R}^{kl}\Big(\beta_B+\frac{(p-k)_\perp^2}{\sigma s},k_\perp\Big)\\
&-\ \frac{2}{k_\perp^2}[\delta^k_i(k_jp^l-2k^lp_j)+\delta^l_j(k_ip^k-2p_ik^k)]\mathrm{V.p.}\frac{1}{\sigma\beta_B s-(p-k)_\perp^2}\mathcal{R}_{kl}(\beta_B,p_\perp;\eta)\\
&-4\Big[\frac{\theta\big(1-\beta_B-\frac{(p-k)_\perp^2}{\sigma s}\big)}{(p-k)_\perp^2}\mathcal{R}_{ij}\Big(\beta_B+\frac{(p-k)_\perp^2}{\sigma s},k_\perp;\eta\Big)-\mathrm{V.p.}\frac{\sigma\beta_B s}{k_\perp^2(\sigma\beta_B s-k_\perp^2)}\mathcal{R}_{ij}(\beta_B,p_\perp;\eta)\Big]\Big\}
\end{aligned}
$$

Moderate-x (DGLAP).
$\beta_B \sim 1$ and $p_\perp^2 \sim (x-y)_\perp^{-2} \sim s$

Sudakov evolution
$\beta_B \sim 1$ and $p_\perp^2 \sim (x-y)_\perp^{-2} \ll s$

Small-x (BFKL).
$\beta_B \ll 1$ and $p_\perp^2 \sim (x-y)_\perp^{-2} \ll s$

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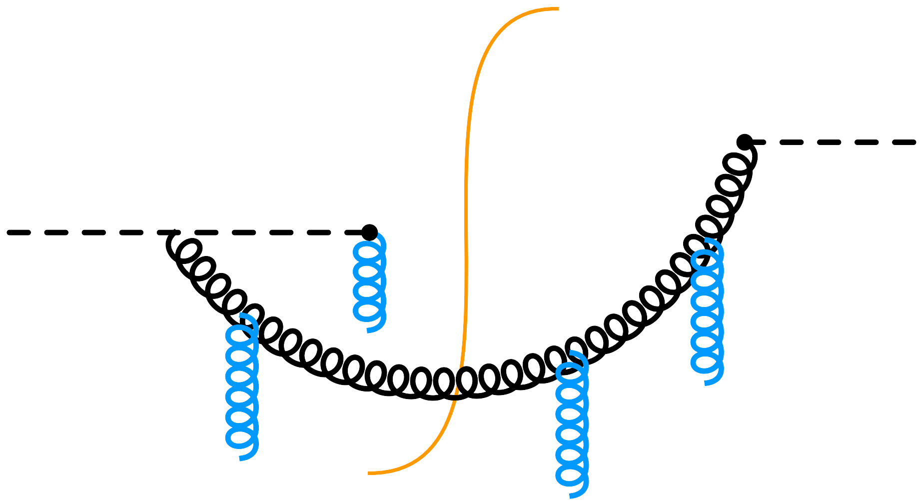
INTERMEDIATE REGIME



Non-linear effects

$$
\begin{aligned}
&\frac{d}{d\ln\sigma}\tilde{\mathcal{F}}_i^a(\beta_B,x_\perp)\mathcal{F}_j^a(\beta_B,y_\perp)\\
=\;& -\alpha_s \mathrm{Tr}\Big\{\int đ^2k_\perp (x_\perp|\Big\{U^\dagger\frac{1}{\sigma\beta_B s+p_\perp^2}(Uk_k+p_kU)\frac{\sigma\beta_B s g_{\mu i}-2k_\mu^\perp k_i}{\sigma\beta_B s+k_\perp^2}\\
-\;& 2k_\mu^\perp g_{ik}U^\dagger\frac{1}{\sigma\beta_B s+p_\perp^2}U-2g_{\mu k}U^\dagger\frac{p_i}{\sigma\beta_B s+p_\perp^2}U+\frac{2k_\mu^\perp}{k_\perp^2}g_{ik}\Big\}\tilde{\mathcal{F}}^k\big(\beta_B+\frac{k_\perp^2}{\sigma s}\big)|k_\perp)\\
\times\;& (k_\perp|\mathcal{F}^l\big(\beta_B+\frac{k_\perp^2}{\sigma s}\big)\Big\{\frac{\sigma\beta_B s\delta_j^\mu-2k_\perp^\mu k_j}{\sigma\beta_B s+k_\perp^2}(k_lU^\dagger+U^\dagger p_l)\frac{1}{\sigma\beta_B s+p_\perp^2}U\\
-\;& 2k_\perp^\mu g_{jl}U^\dagger\frac{1}{\sigma\beta_B s+p_\perp^2}U-2\delta_l^\mu U^\dagger\frac{p_j}{\sigma\beta_B s+p_\perp^2}U+2g_{jl}\frac{k_\perp^\mu}{k_\perp^2}\Big\}|y_\perp)\\
+\;& 2\tilde{\mathcal{F}}_i(\beta_B,x_\perp)(y_\perp|\frac{p^m}{p_\perp^2}\mathcal{F}_k(\beta_B)(i\overleftarrow{\partial}_l+U_l)(2\delta_m^k\delta_j^l-g_{jm}g^{kl})U^\dagger\frac{1}{\sigma\beta_B s-p_\perp^2+i\epsilon}U\\
+\;& \mathcal{F}_j(\beta_B)\frac{\sigma\beta_B s}{p_\perp^2(\sigma\beta_B s-p_\perp^2+i\epsilon)}|y_\perp)\\
+\;& 2(x_\perp|-U^\dagger\frac{1}{\sigma\beta_B s-p_\perp^2-i\epsilon}U(2\delta_i^k\delta_m^l-g_{im}g^{kl})(i\partial_k-U_k)\tilde{\mathcal{F}}_l(\beta_B)\frac{p^m}{p_\perp^2}\\
+\;& \tilde{\mathcal{F}}_i(\beta_B)\frac{\sigma\beta_B s}{p_\perp^2(\sigma\beta_B s-p_\perp^2-i\epsilon)}|x_\perp)\mathcal{F}_j(\beta_B,y_\perp)\Big\}\;+\;O(\alpha_s^2)
\end{aligned}
$$

Moderate-x (DGLAP).
$\beta_B \sim 1$ and $p_\perp^2 \sim (x-y)_\perp^{-2} \sim s$

Sudakov evolution
$\beta_B \sim 1$ and $p_\perp^2 \sim (x-y)_\perp^{-2} \ll s$

Small-x (BK). Non-linear evolution
$\beta_B \ll 1$ and $p_\perp^2 \sim (x-y)_\perp^{-2} \ll s$

Ian Balitsky, A.T. 2016

Conclussions