Quark Helicity at Small x

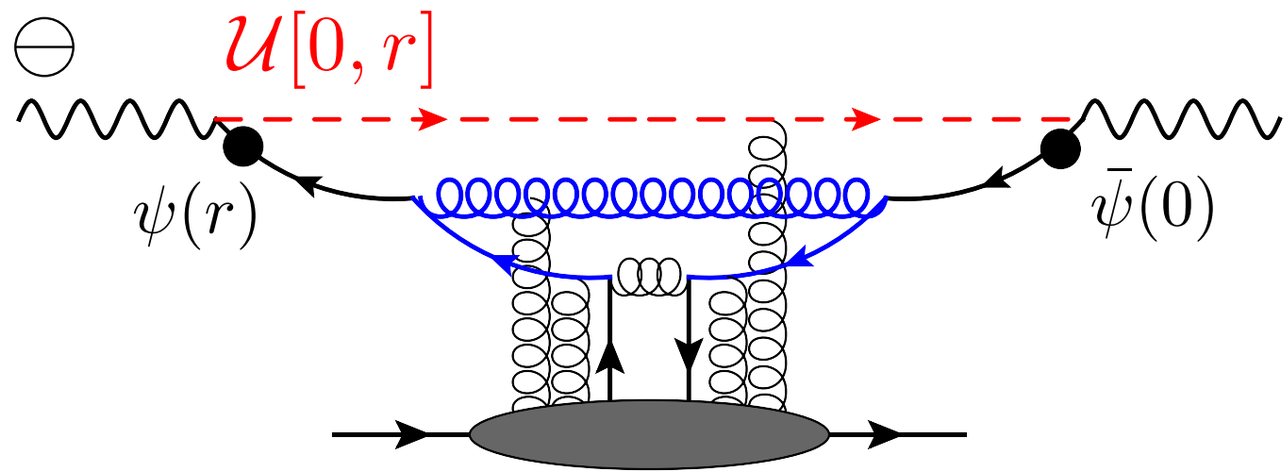
Matthew D. Sievert
with Daniel Pitonyak
and Yuri Kovchegov

Friday July 8, 2016

EIC User Group Meeting 2016

Overview

Small-x Helicity Evolution



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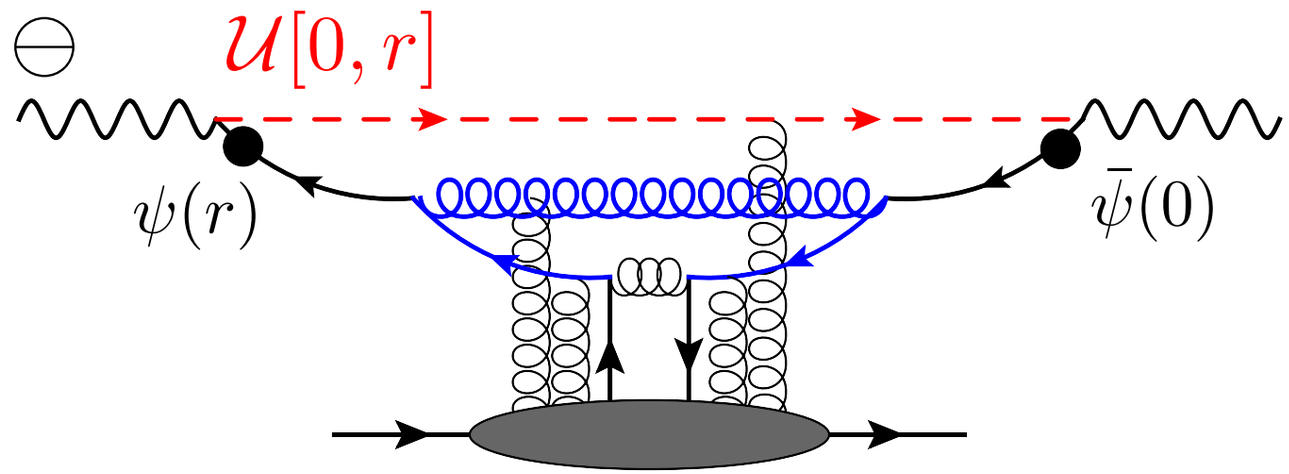

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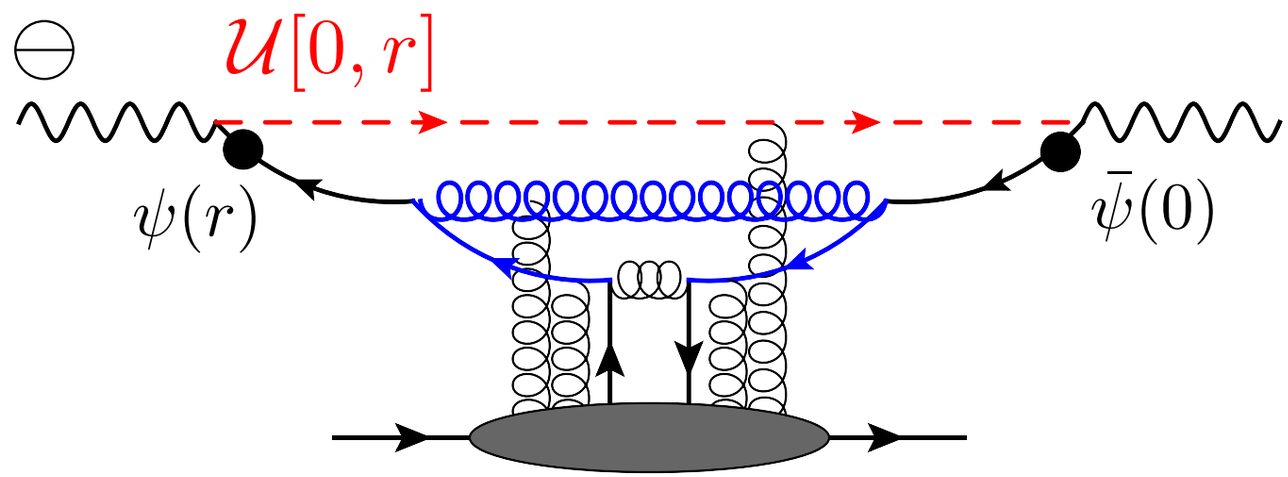

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- Early work on helicity evolution suggested a rapid growth of quark polarization at small x.
- Our modern s-channel formalism generalizes these equations, suggesting weaker growth at small x.

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Kovchegov, Pitonyak, & MS JHEP 1601 **072** (2016), + (in preparation)

Motivation: Proton Spin Puzzle

- The "Proton Spin Budget" is described by the Jaffe-Manohar Sum Rule.

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$

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$$0.001 < x < 1$$

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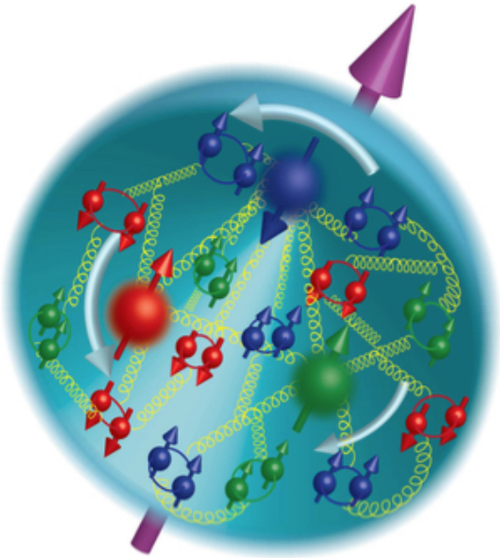
- Proton structure is much more complex than previously believed!
  - ➡ Orbital angular momentum?
  - ➡ Polarization at very small x?



Definition: TMD Parton Distributions

$$\phi_{\alpha\beta}(x, \vec{k}_\perp) = \int \frac{d^{2-}r}{(2\pi)^3}\, e^{ik\cdot r} \langle h(p,S)|\, \bar{\psi}_\beta(0)\, \mathcal{U}[0,r]\, \psi_\alpha(r)\, |h(p,S)\rangle$$

Transverse Momentum Dependent
Parton Distribution Functions

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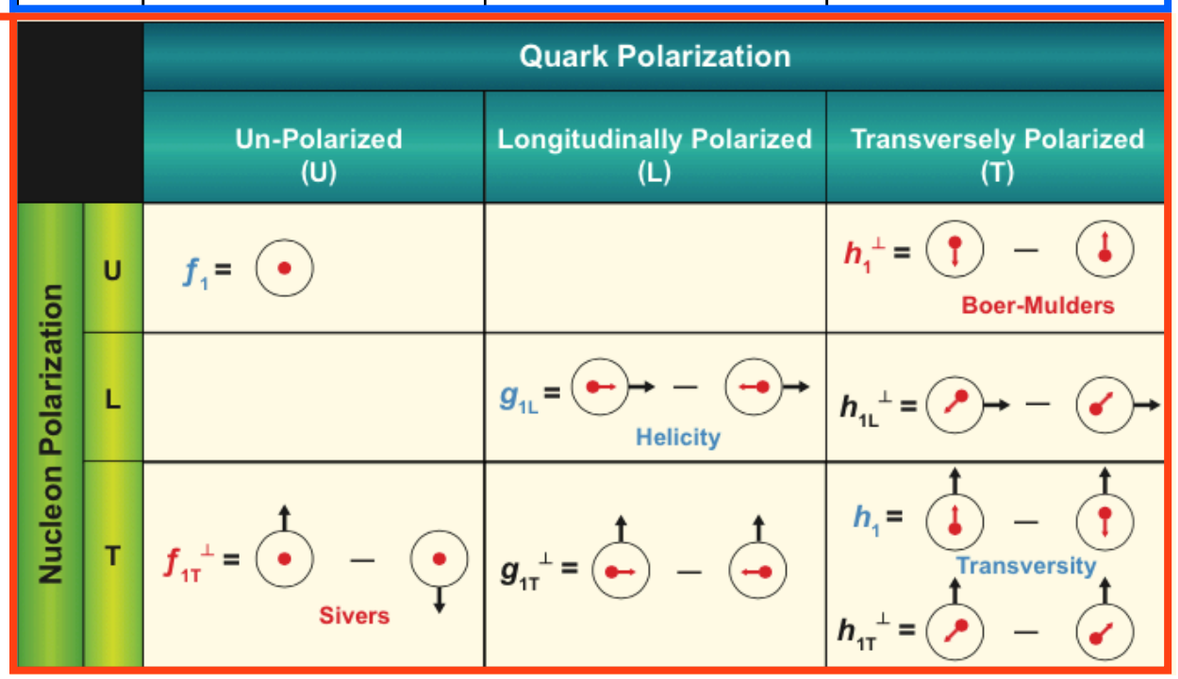

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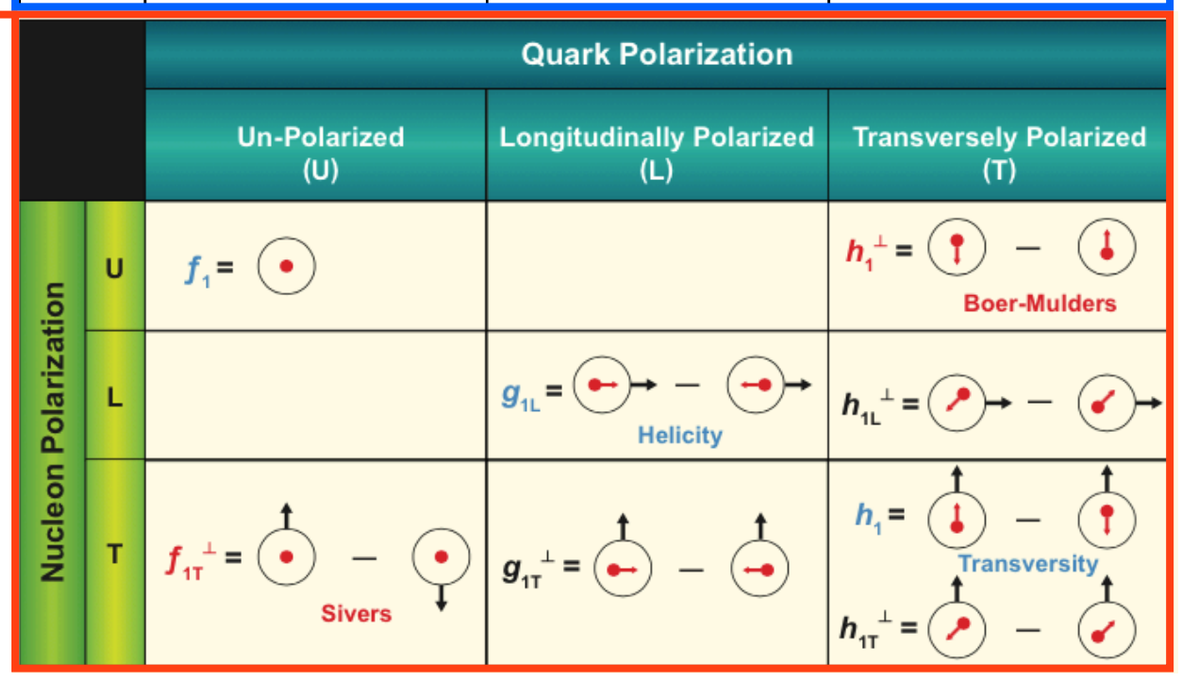
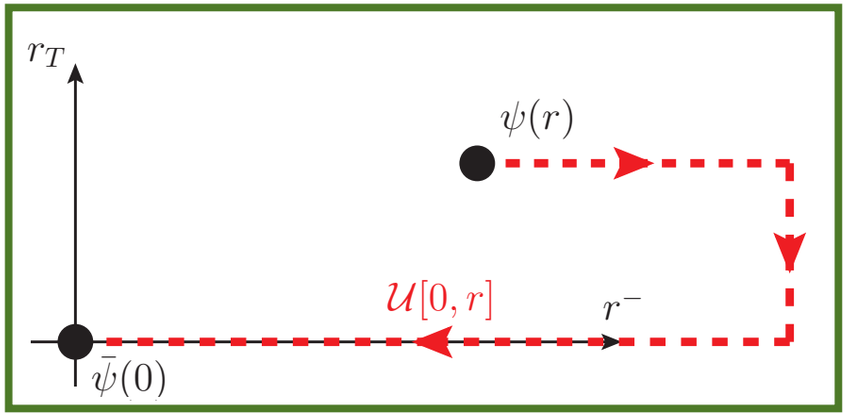


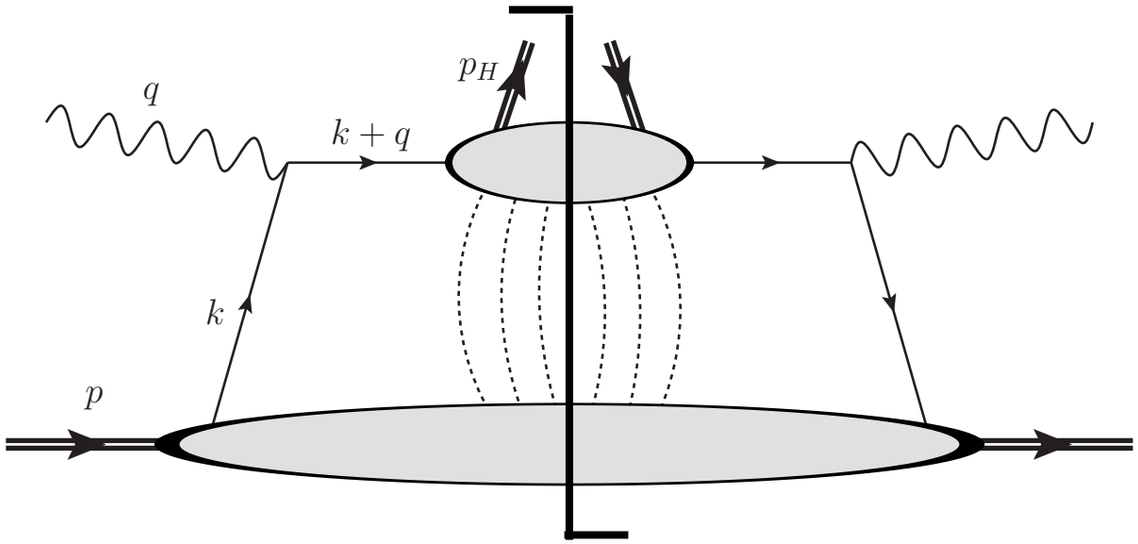


Staple-shaped Gauge Link
encodes final-state interactions

TMD's at Large x

Semi-Inclusive
Deep Inelastic Scattering (SIDIS)

$$e + p \to e' + h + X$$



Large-x Kinematics:

$$\hat{s} \sim Q^2 \gg k_T^2$$

$$x = \frac{Q^2}{\hat{s}+Q^2} \sim \mathcal{O}(1)$$

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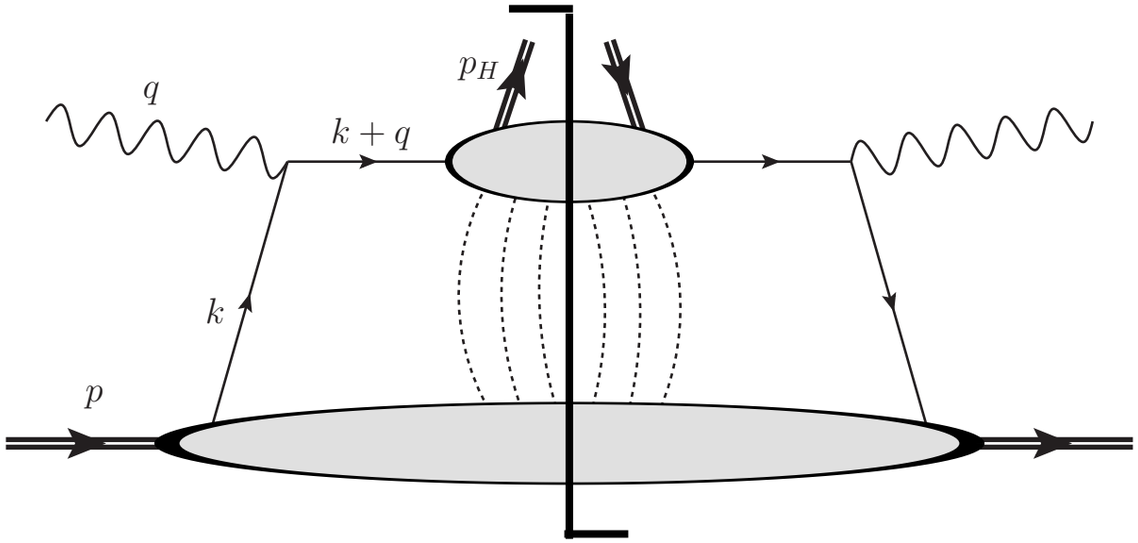

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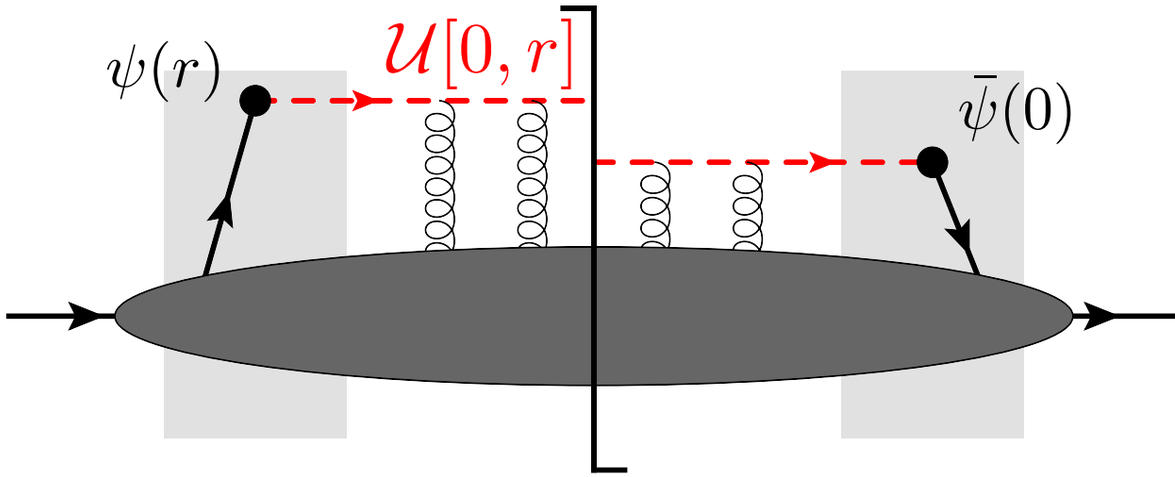

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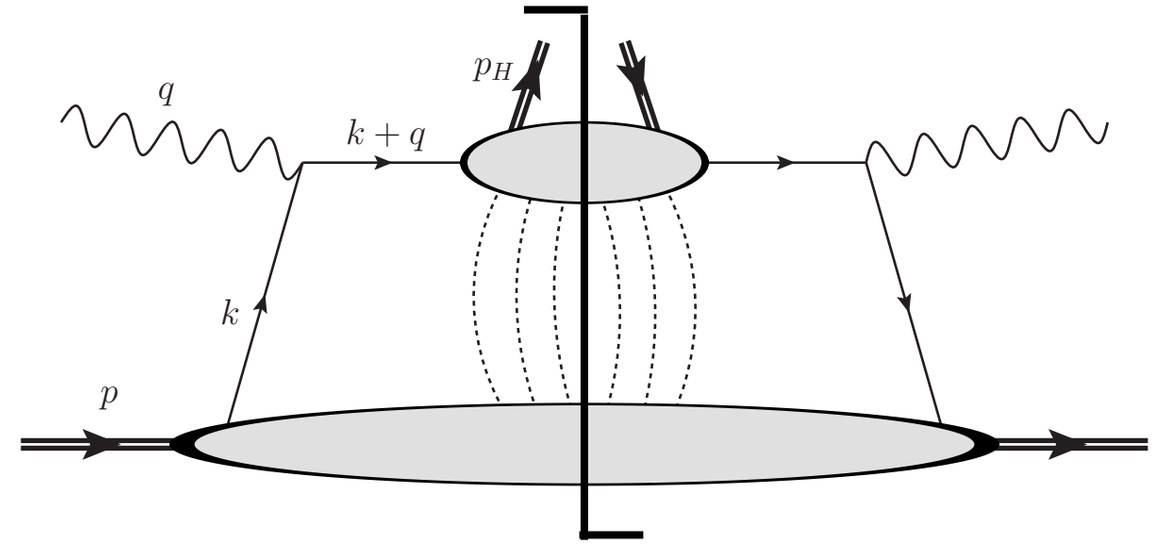

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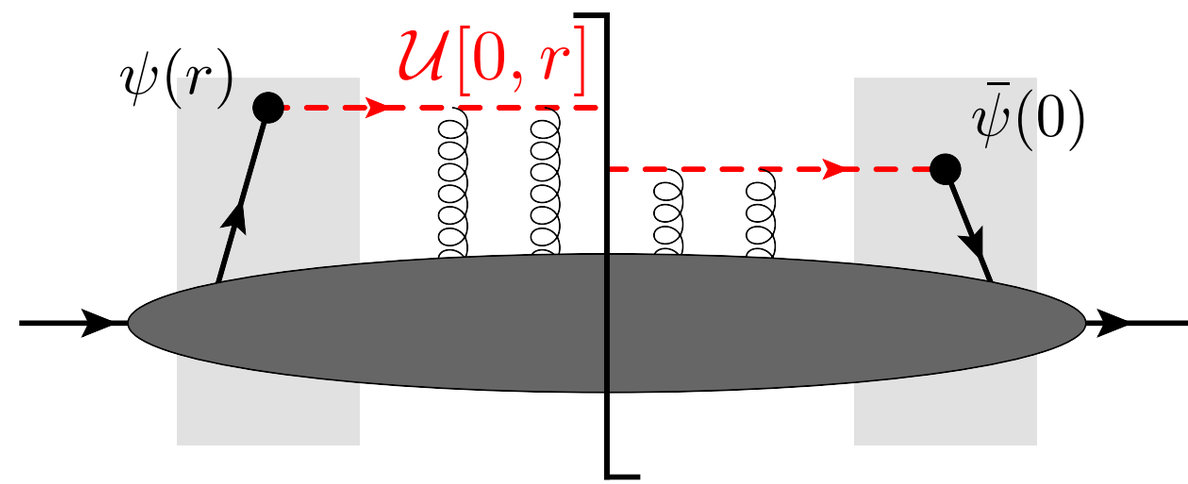

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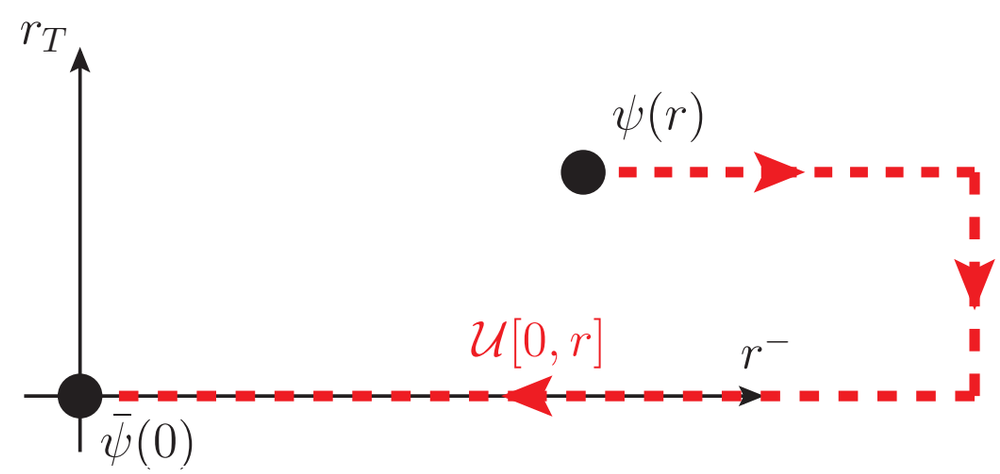

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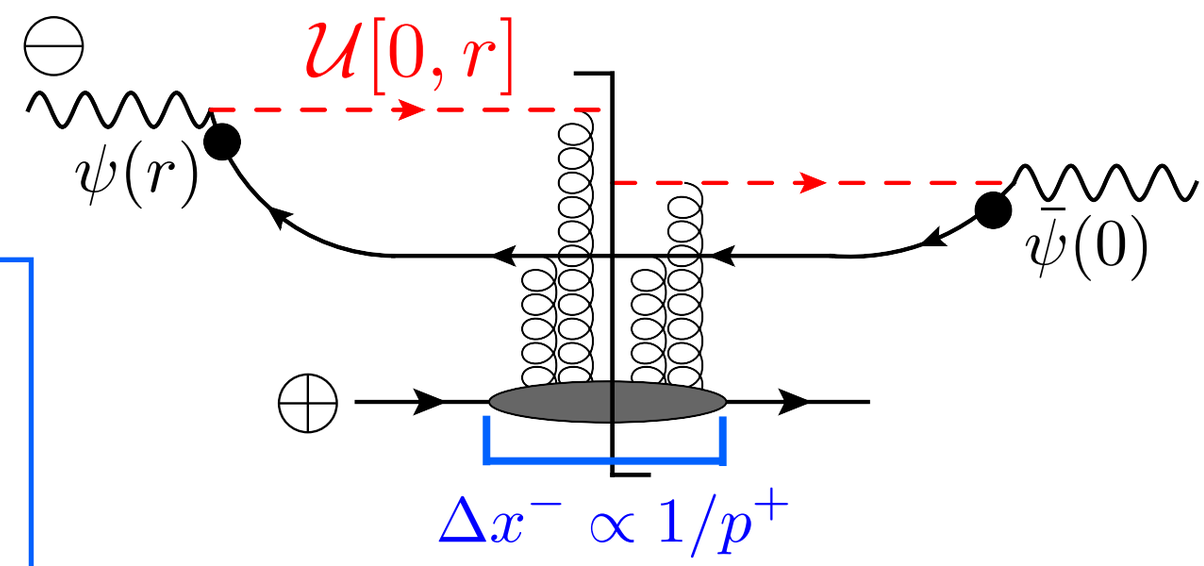

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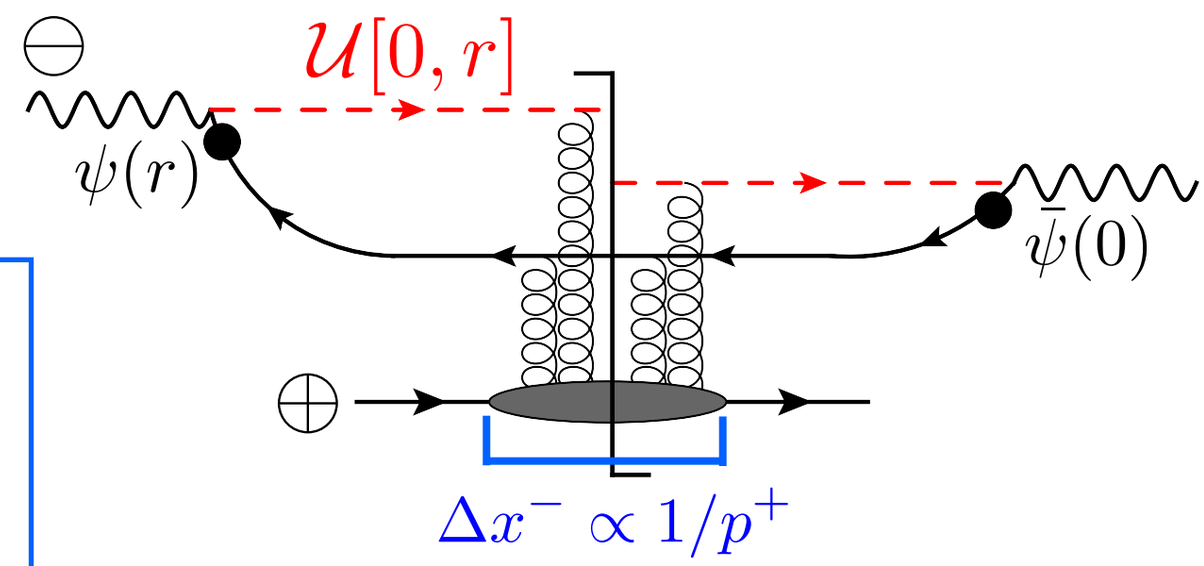

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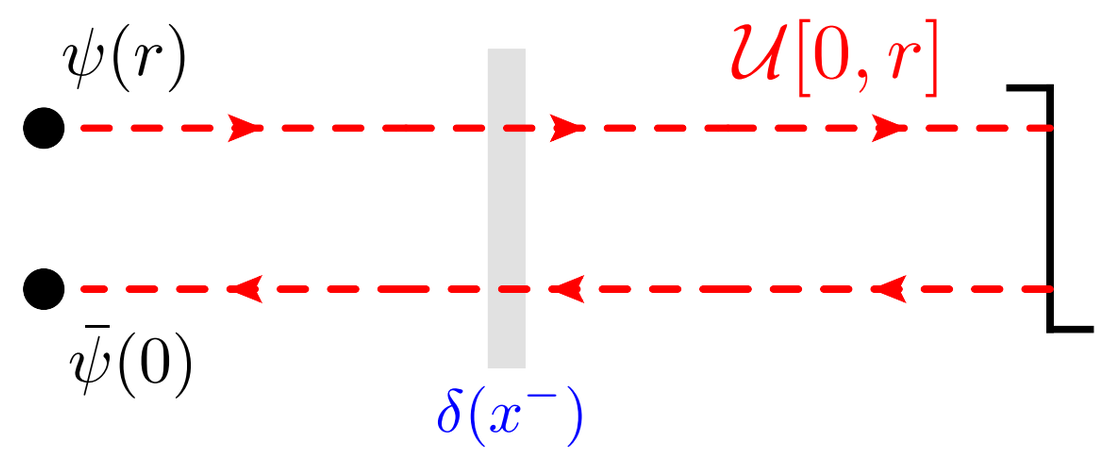
- Proton is Lorentz-contracted to a "shockwave".

➡ Gauge link covers the entire proton.

➡ Infinite dipole degrees of freedom at small x



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Small-x Initial Conditions: Classical Gluon Fields

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➡ High gluon density at small x enhances multiple scattering

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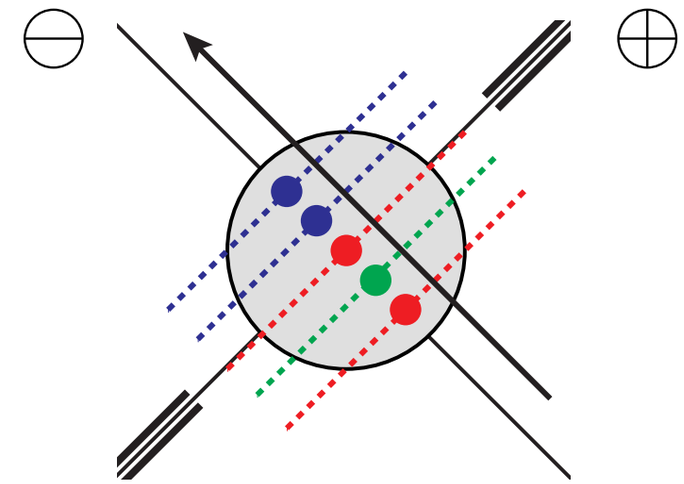

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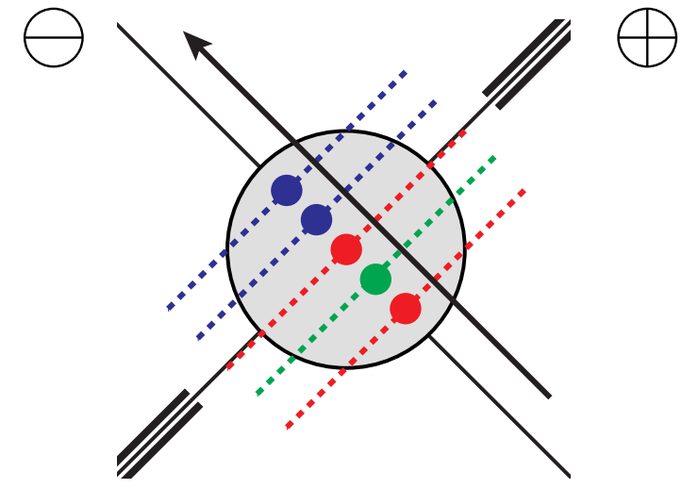

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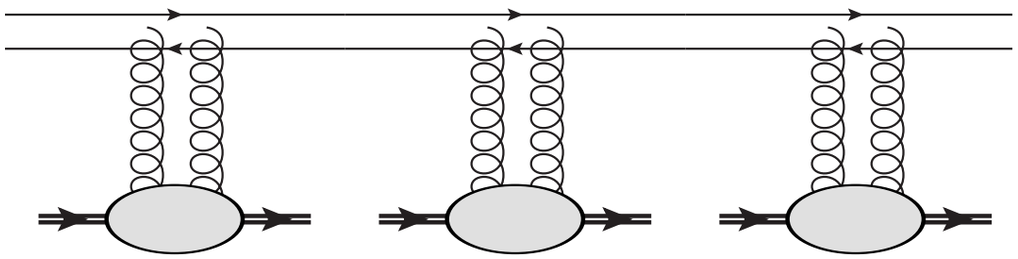
Nucleus: $\alpha_s^2 A^{1/3} \sim 1$ Proton: $\alpha_s \rho \sim 1$



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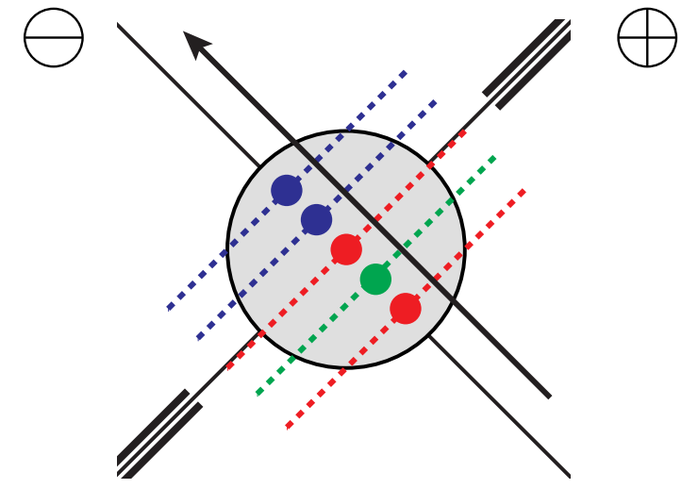

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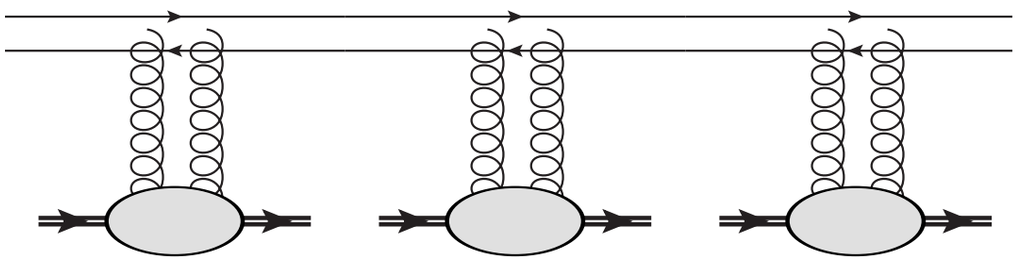
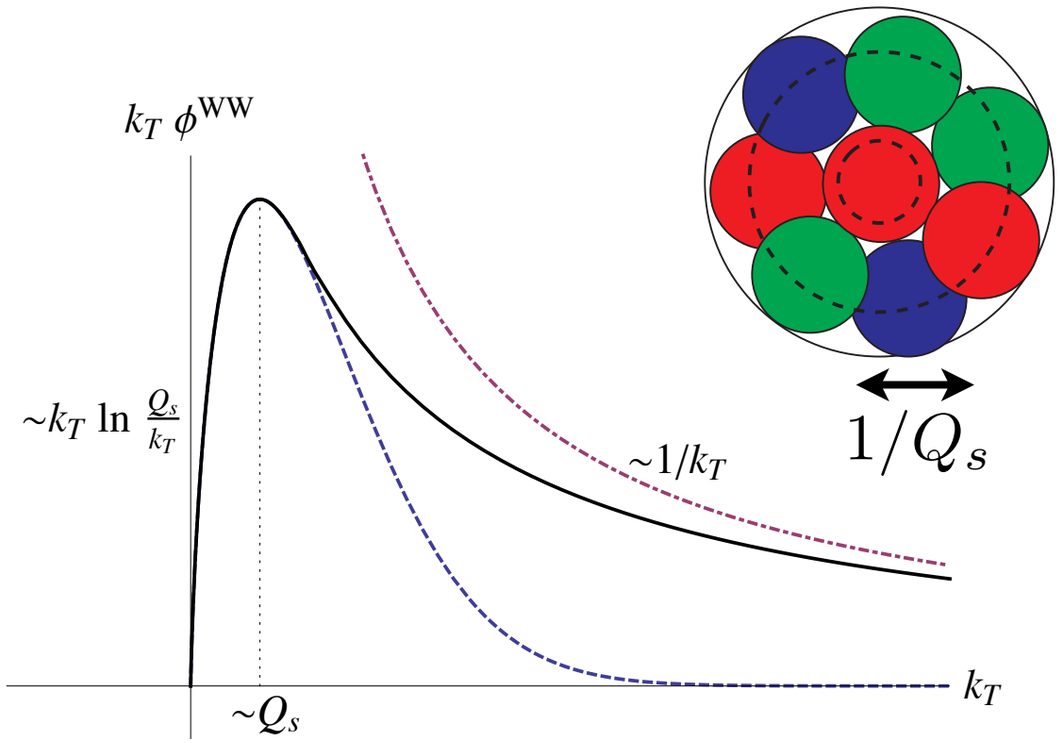
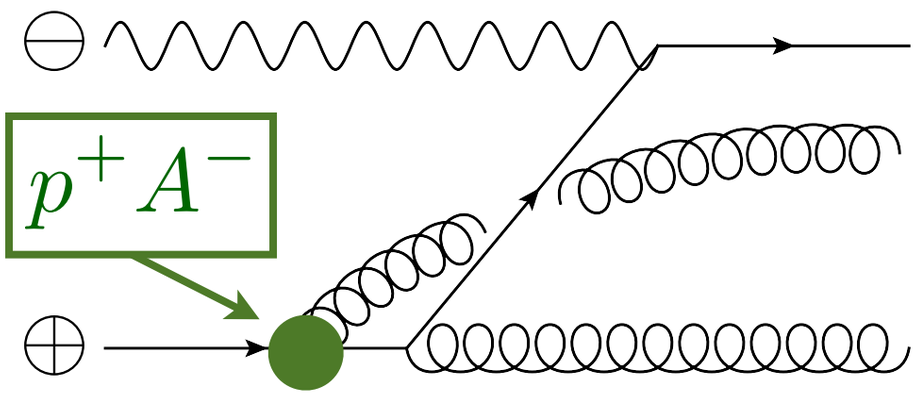


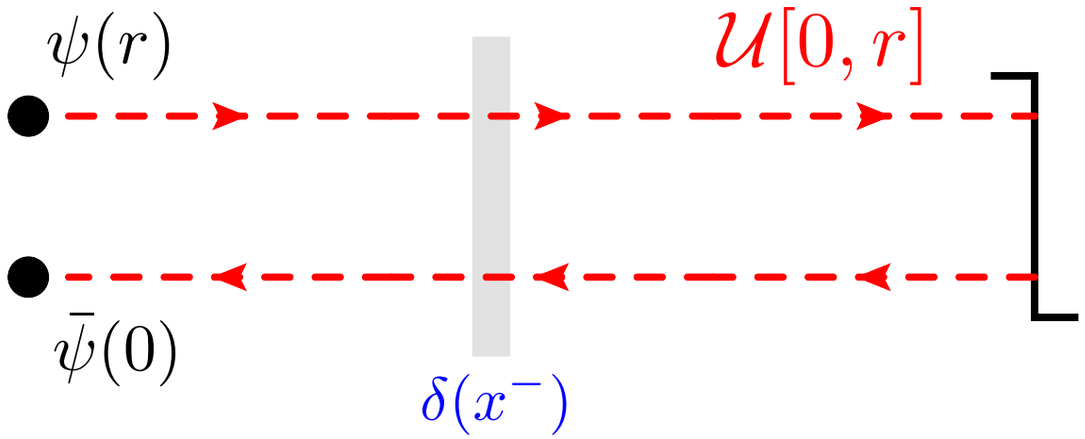
- Charge density defines a hard momentum scale which screens the IR gluon field.

Both:
$$Q_s^2 \propto \alpha_s^2 A^{1/3} \propto \alpha_s \rho$$
$$Q_s^2 \gg \Lambda^2$$



Quantum Evolution in the Light-Cone Gauge





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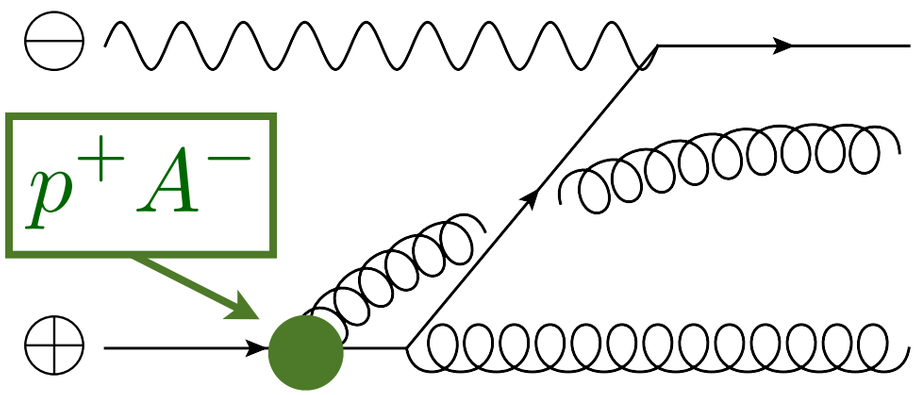

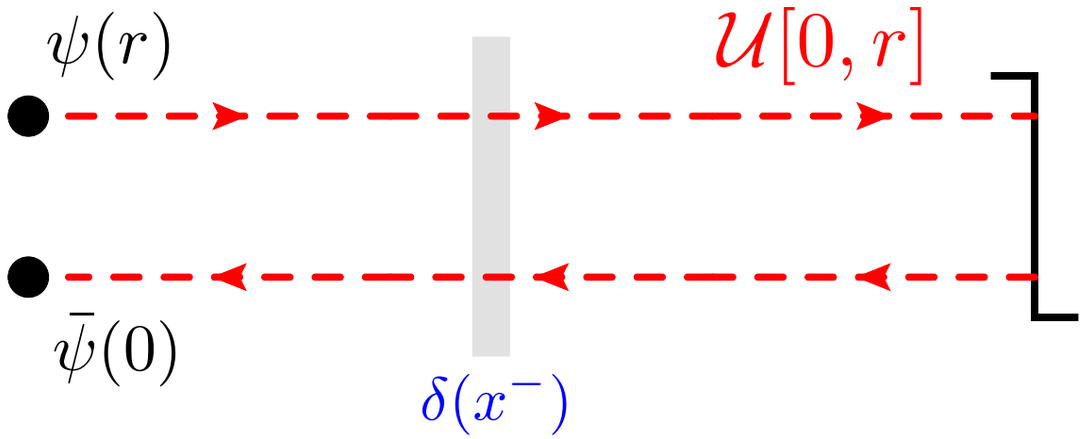

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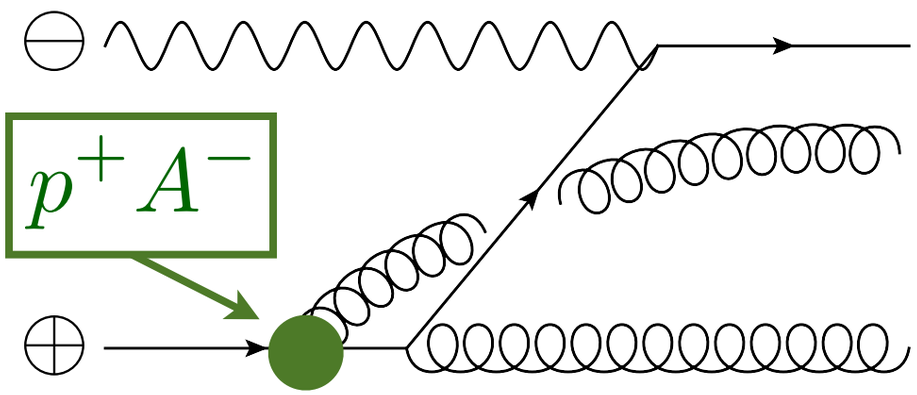

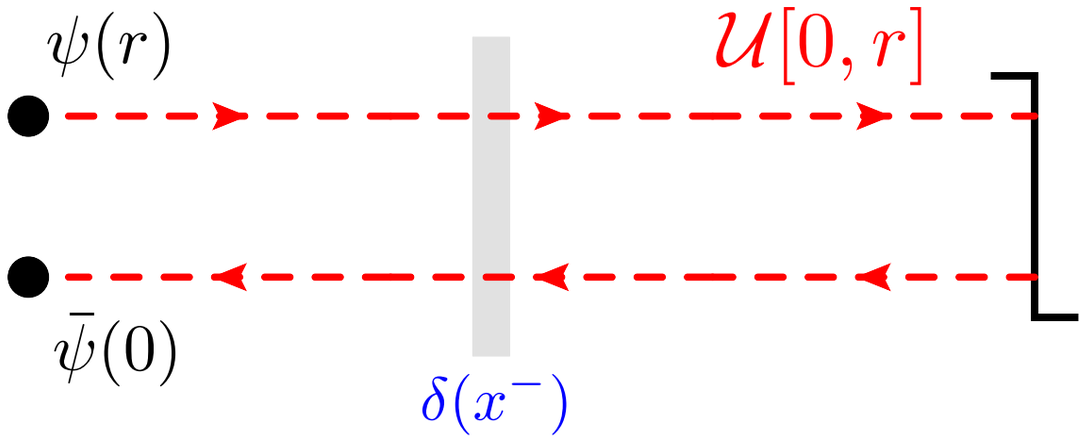

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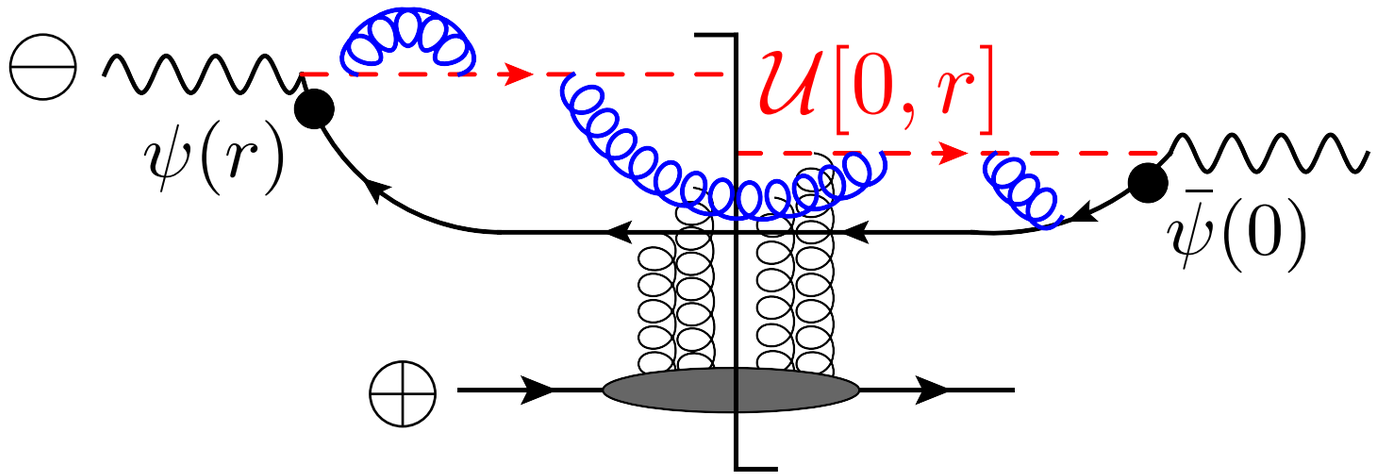
- For classical fields and leading-log evolution, $A_\perp = 0$ as well.

➡ The transverse part of the gauge link does not contribute.

Unpolarized Small-x Evolution



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- The quark dipole radiates soft gluons before and after scattering.

➡ Evolution of the dipole scattering amplitude

➡ Re-sums single logarithms of x

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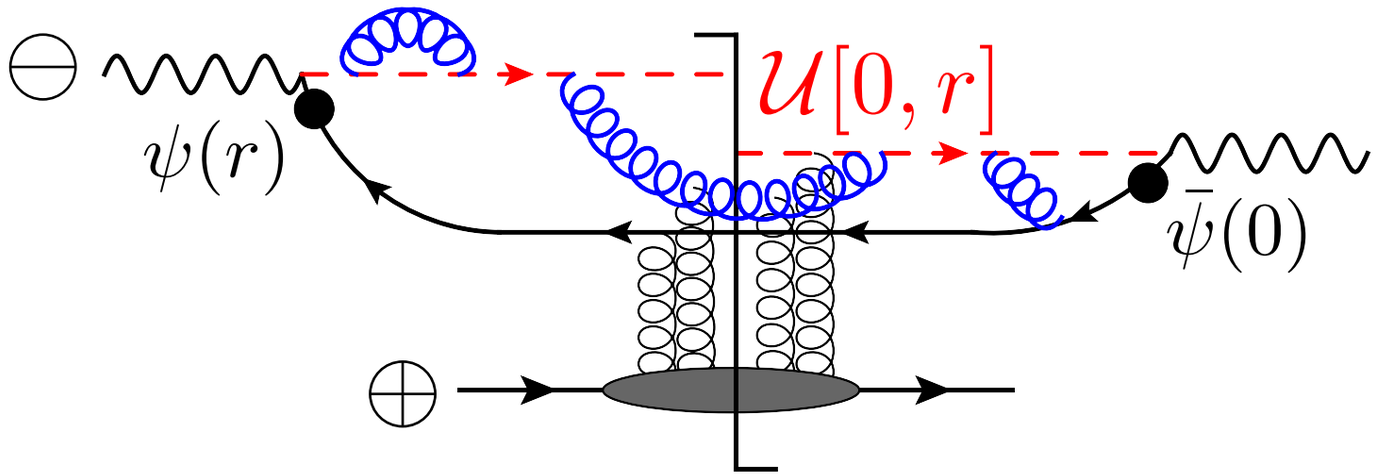

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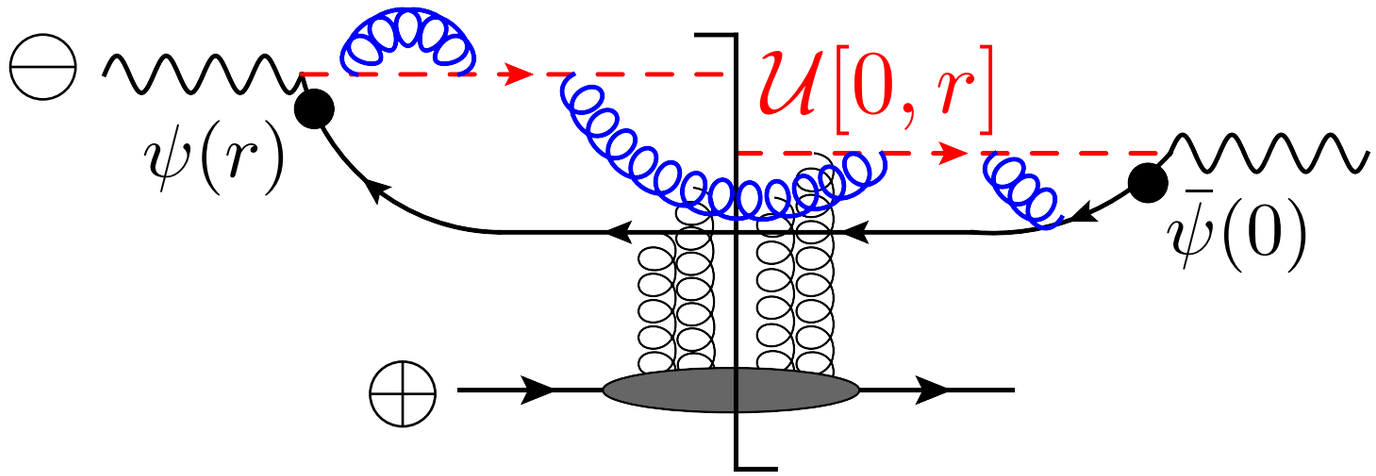

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- Evolution closes in the large N_c limit (BK eqn.) $\quad Q_s^2(x) \sim \left(\frac{1}{x}\right)^{0.3}$

Leading-Order Spin Dependence

- High energy (small x) scattering is predominantly spin independent.

➡ BK evolution: total cross section, unpolarized quark distribution.

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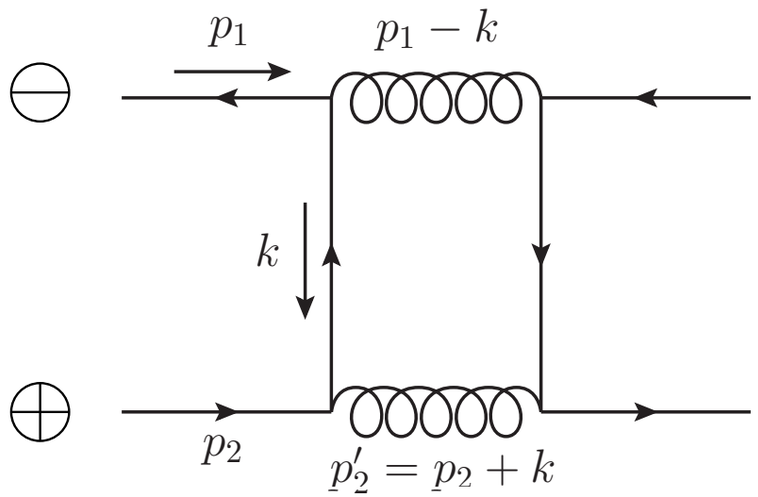

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- Transporting quark polarization to small x is suppressed!

➡ Spin asymmetries, polarized quarks are suppressed at small x.

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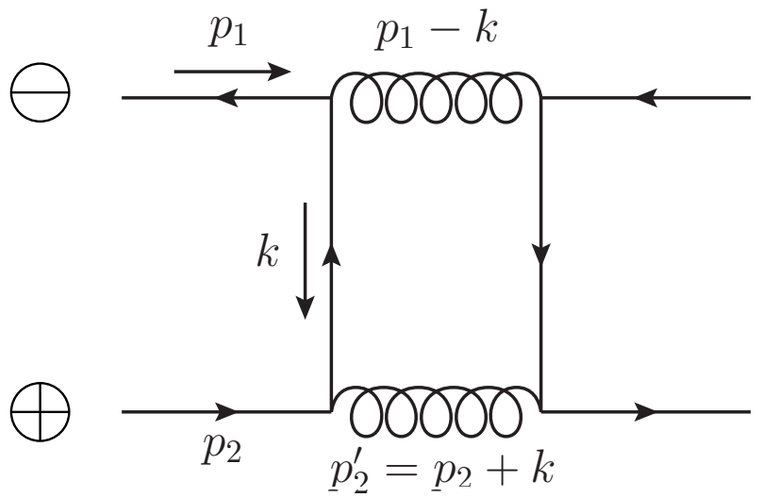

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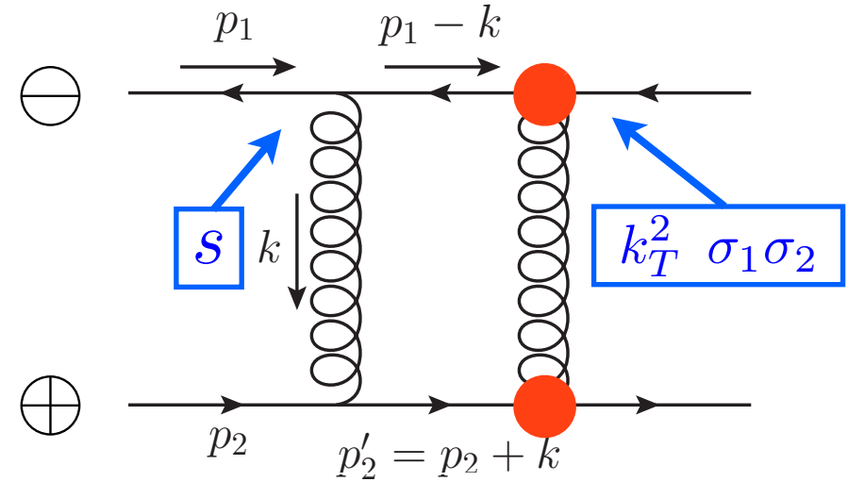

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- High energy (small x) scattering is predominantly spin independent.

➡ BK evolution: total cross section, unpolarized quark distribution.

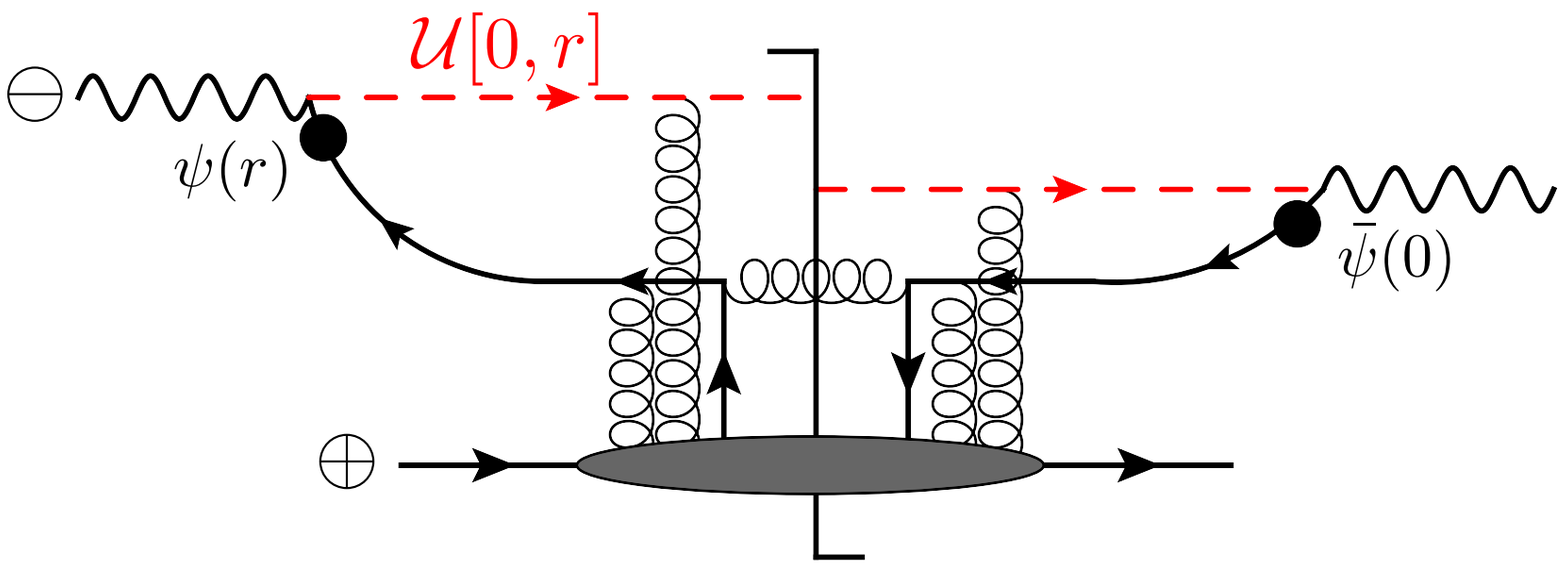
- Transporting quark polarization to small x is suppressed!

➡ Spin asymmetries, polarized quarks are suppressed at small x.

- Sub-leading gluon exchange can also transfer spin dependence.

➡ Gluon exchange can mix with quark exchange.

Spin-Dependent Initial Conditions



- "Polarized Wilson Line" - Coherent, spin-dependent scattering.

➡ One spin-dependent exchange (more are suppressed)

➡ Dressed by multiple unpolarized scattering

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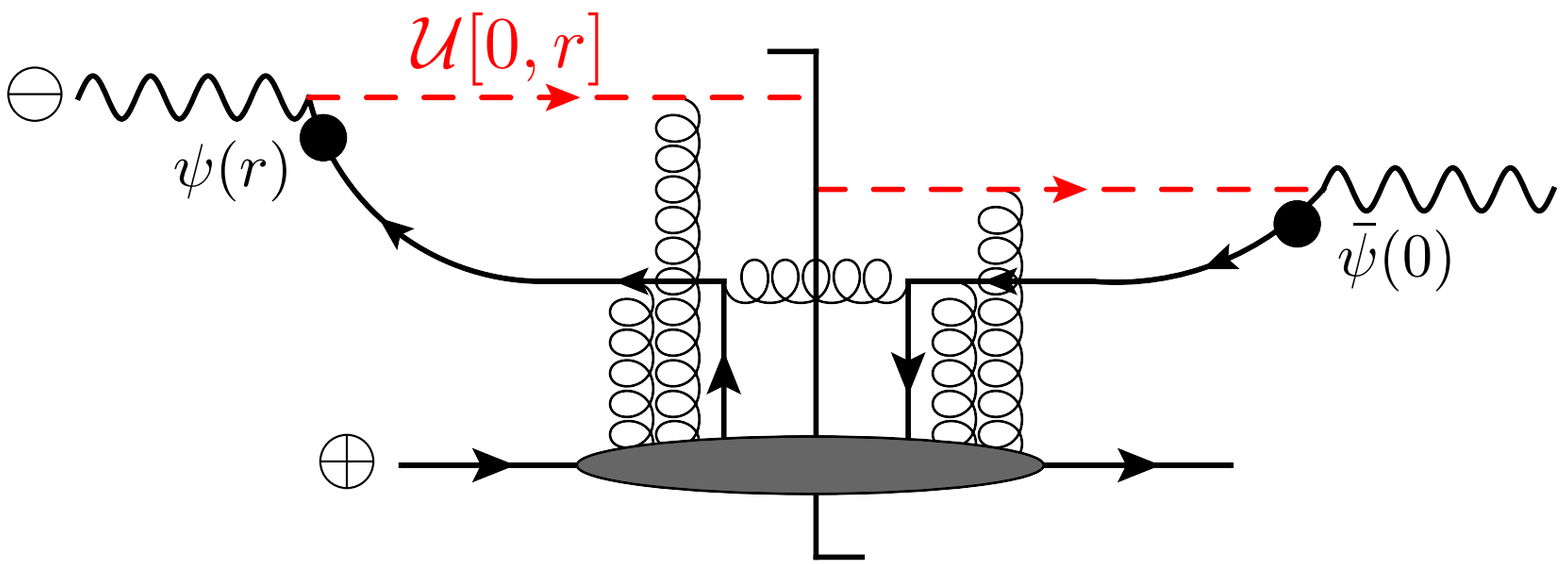

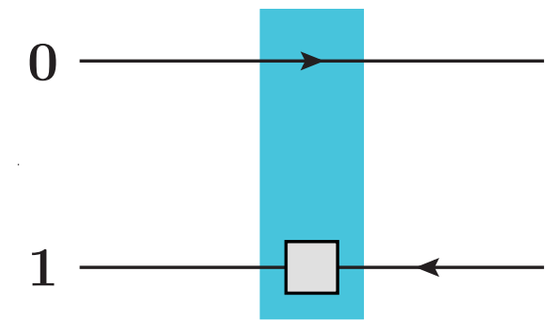

$$\frac{1}{N_c}\mathrm{Tr}[V_0 V_1^{pol\,\dagger}]$$

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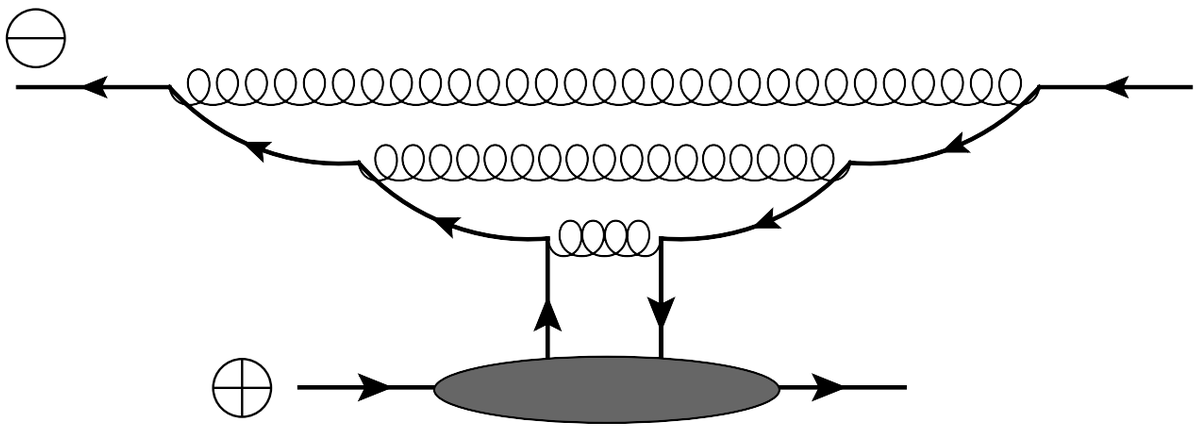
- "Polarized Dipole Amplitude":

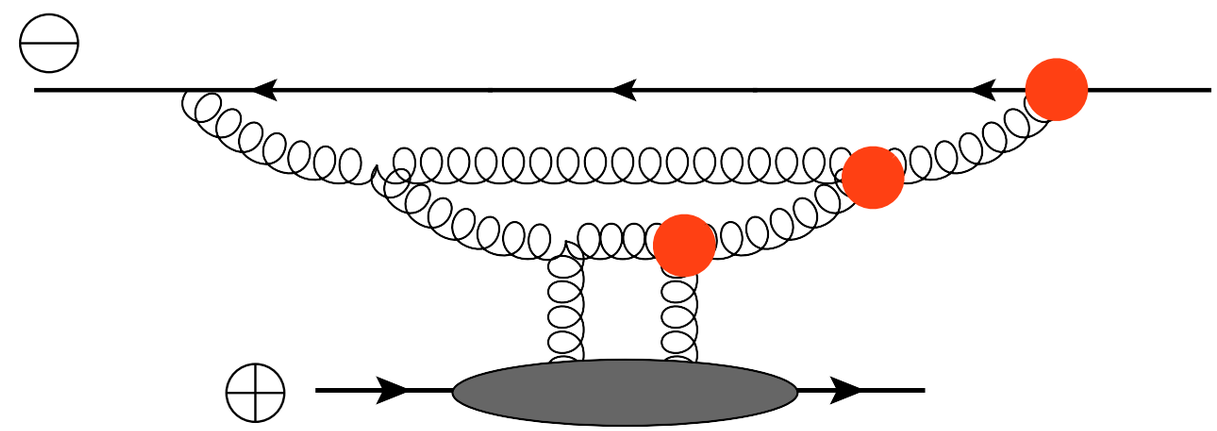
➡ Quark (gauge link) scatters by an unpolarized Wilson line.

➡ Fermion (antiquark) scatters by a polarized Wilson line.

$$G_{xy} \equiv \frac{1}{2N_c}\mathrm{Tr}\left[V_x V_y^\dagger(\sigma) + V_y(\sigma) V_x^\dagger\right]$$

Constructing Polarized Splitting Kernels





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➡ Soft quarks and soft gluons can mix (same order)

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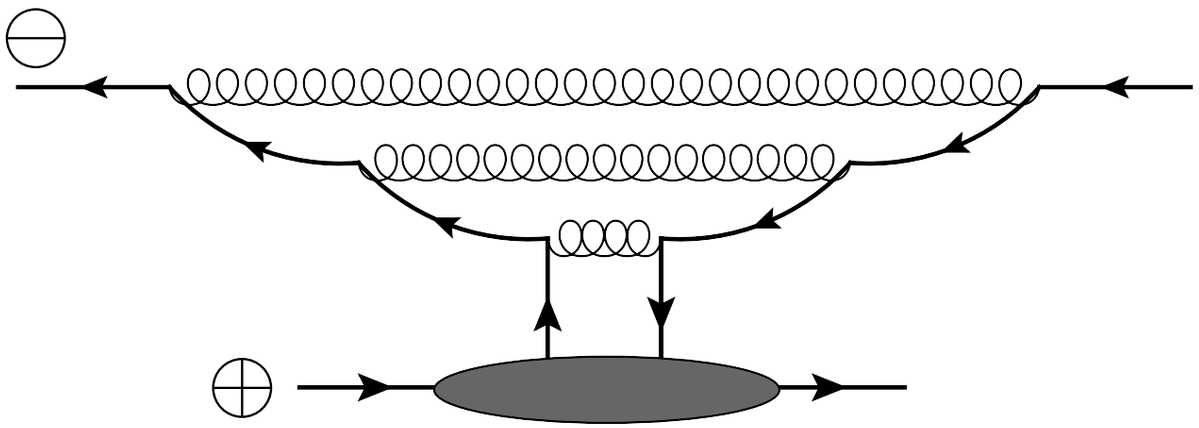

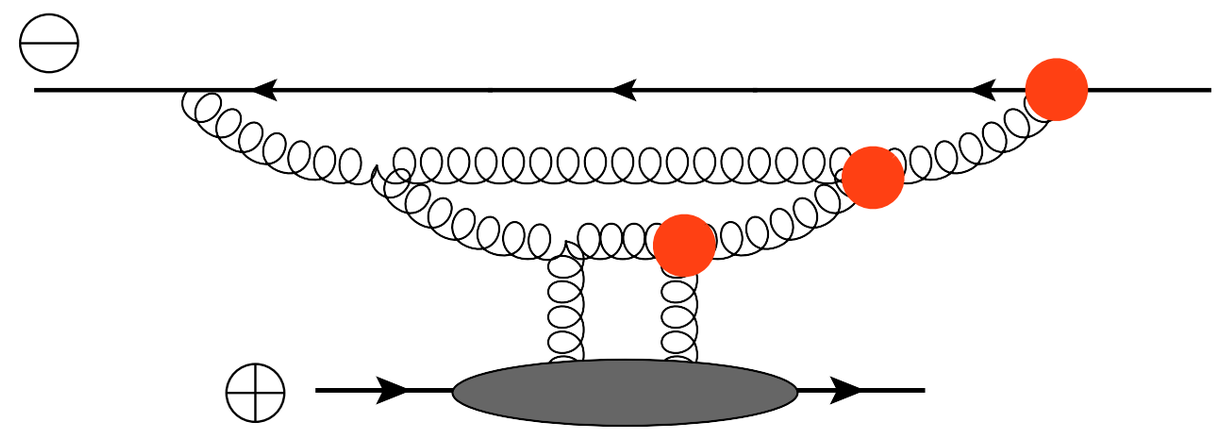

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- Requires longitudinal ordering and lifetime ordering

$$1 \gg z_1 \gg z_2 \gg \cdots \gg x \qquad \frac{k_{1T}^2}{z_1} \ll \frac{k_{2T}^2}{z_2} \ll \cdots$$

➡ Includes "infrared" phase space: $k_{1T}^2 \gg k_{2T}^2 \gg k_{1T}^2 \frac{z_2}{z_1}$

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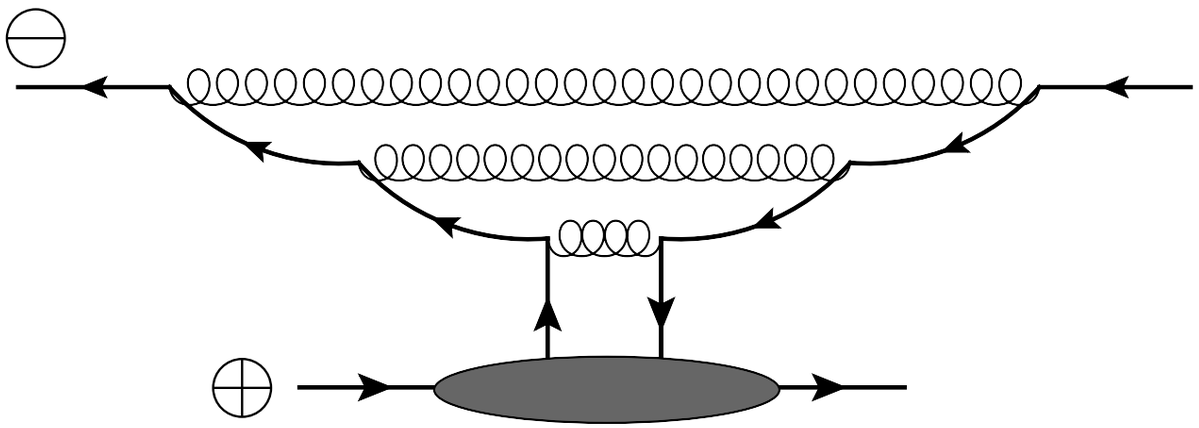

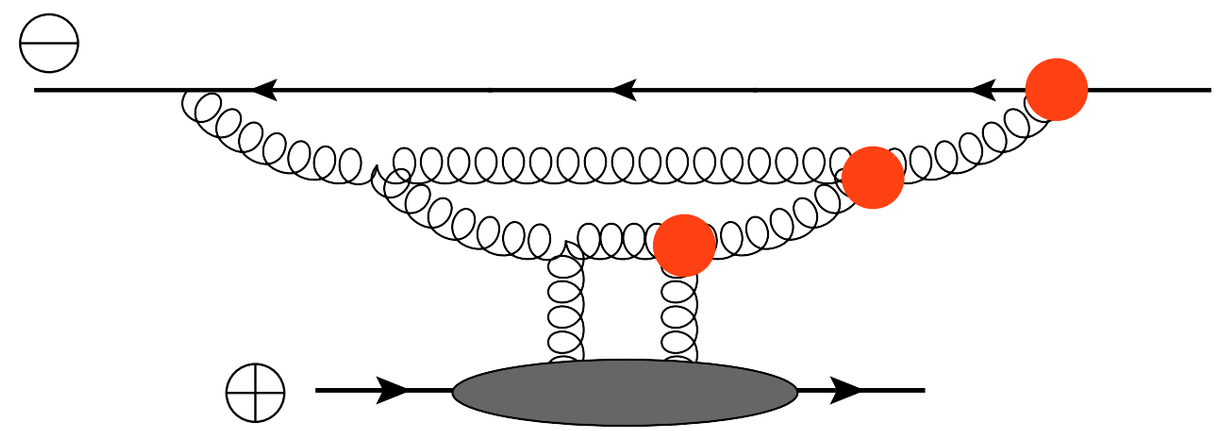

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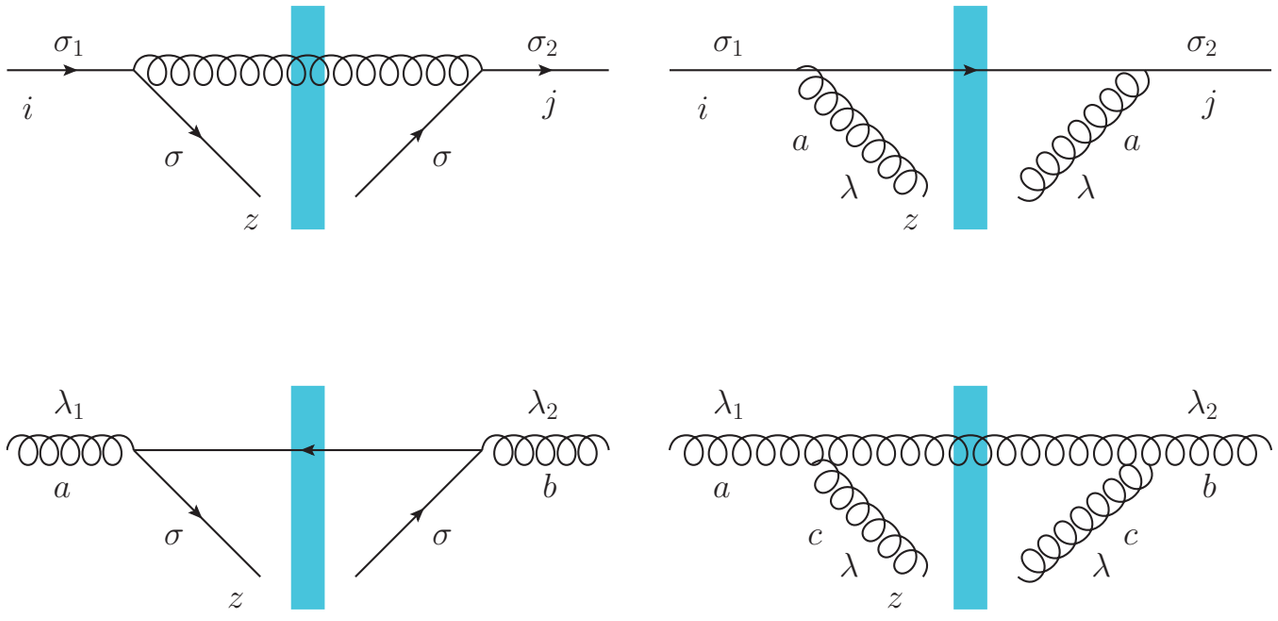
➡ Includes "infrared" phase space: $k_{1T}^2 \gg k_{2T}^2 \gg k_{1T}^2 \frac{z_2}{z_1}$

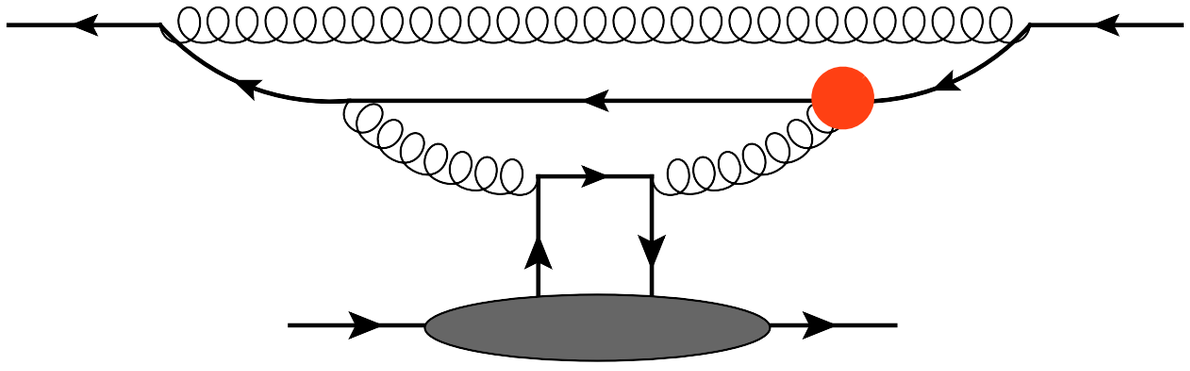
- Leads to double-log evolution.

➡ Faster evolution than unpolarized BK!

$$\alpha_s \ln^2 \frac{1}{x} \sim 1$$

Solution: Ladder Evolution





Agrees with Bartels et al.

- To solve, first keep only the kernels without unpolarized rescattering.

$$\frac{\alpha_s}{2\pi}\int\frac{dz}{z}\int\frac{dk_T^2}{k_T^2}\begin{pmatrix} C_F & 2C_F \\ -N_f & 4N_c \end{pmatrix}$$

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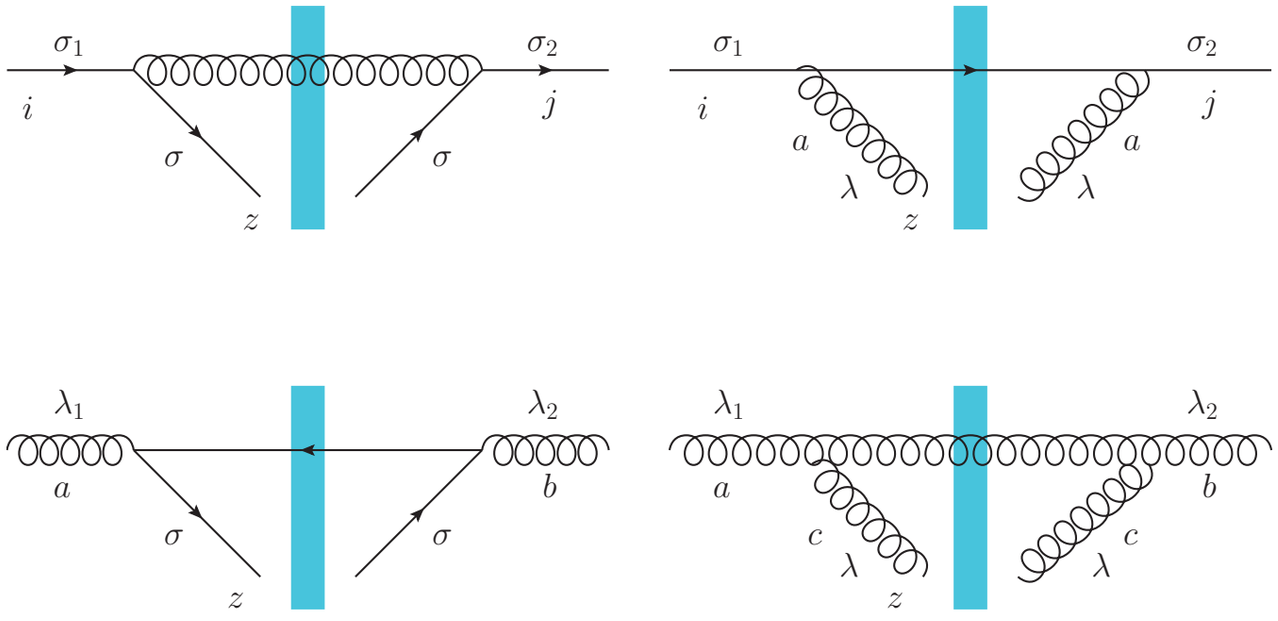

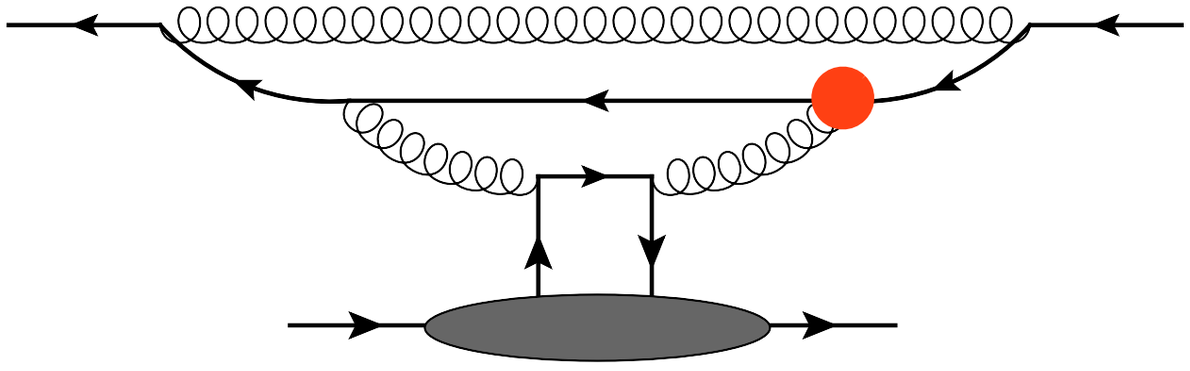

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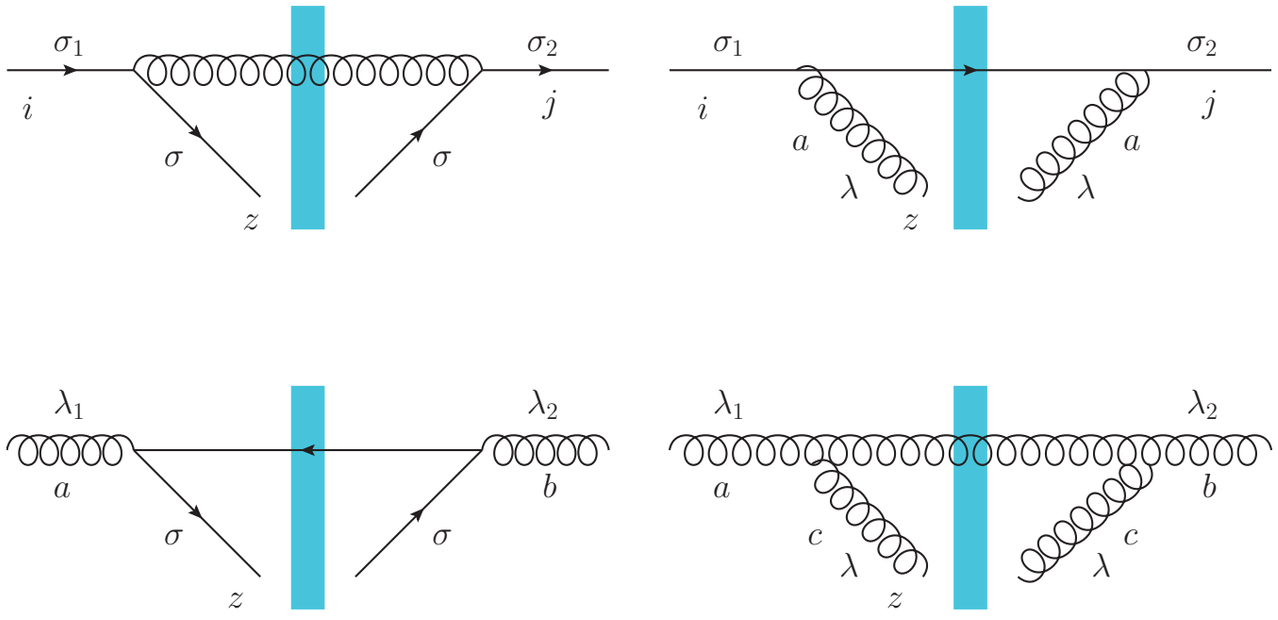

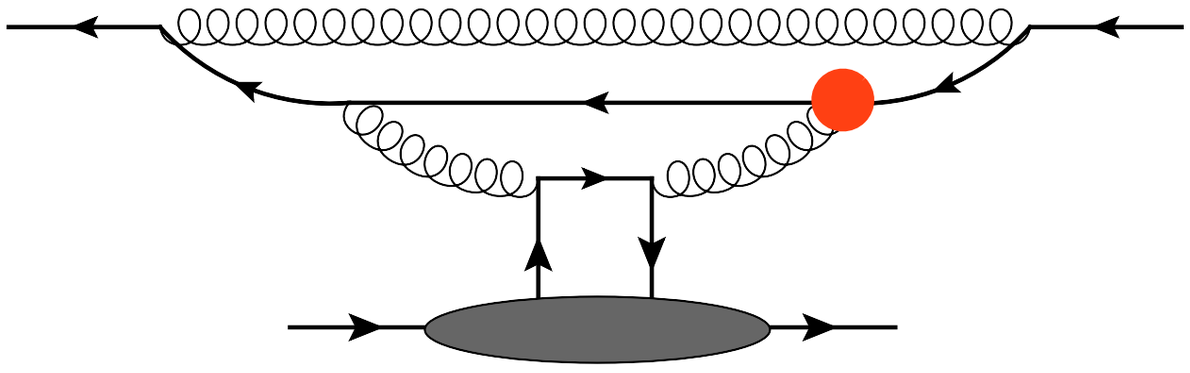

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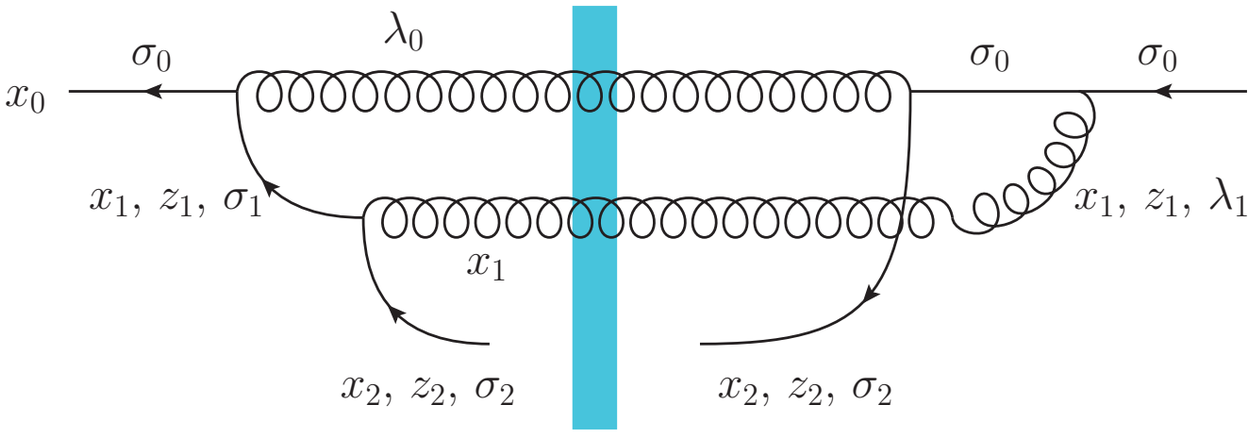
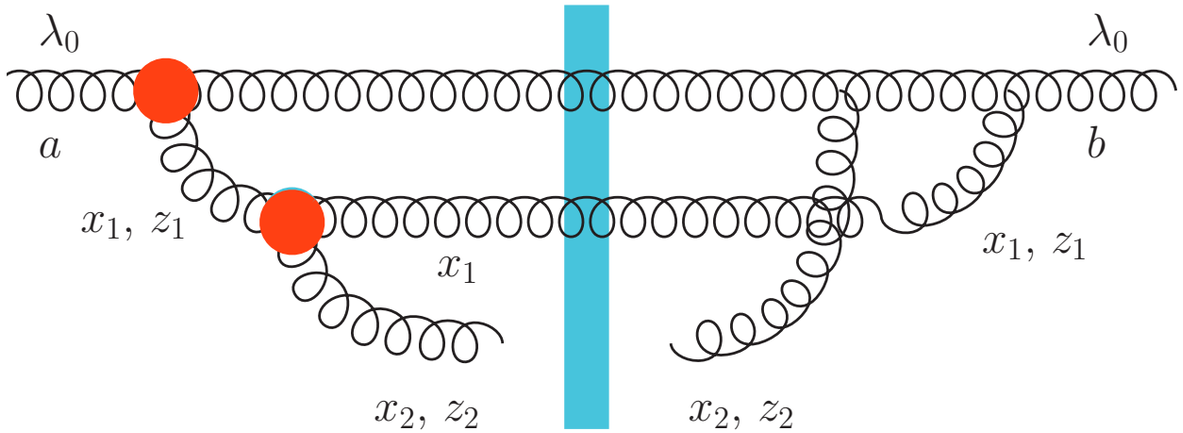
- Fast growth of quark polarization at small x!

➡ Large contribution to the proton spin?

$$S_{01} \sim \left(\frac{1}{x}\right)^{0.3}$$

The Complication: Non-Ladder Graphs





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➡ Starting at 2-loop order, polarization transfer can "jump" rungs of ladder evolution

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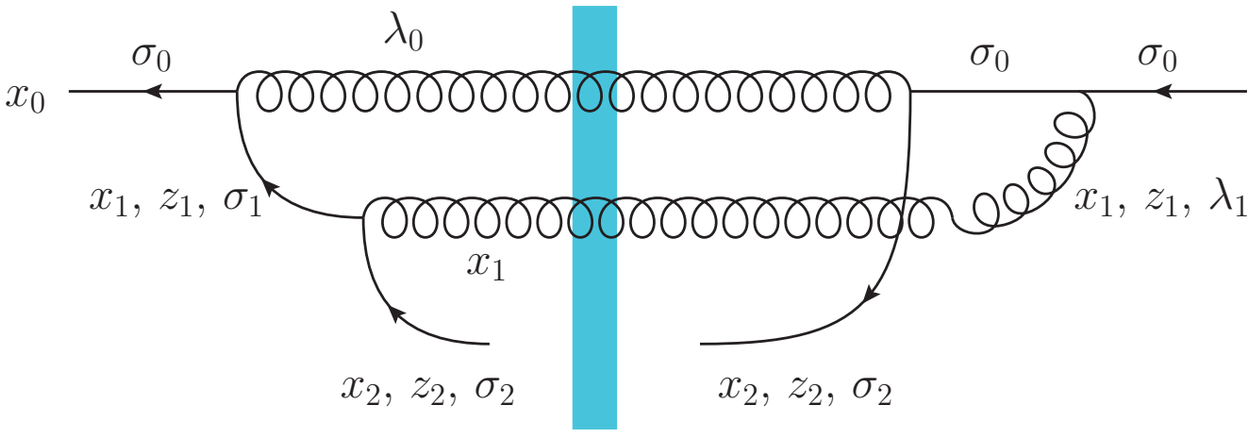

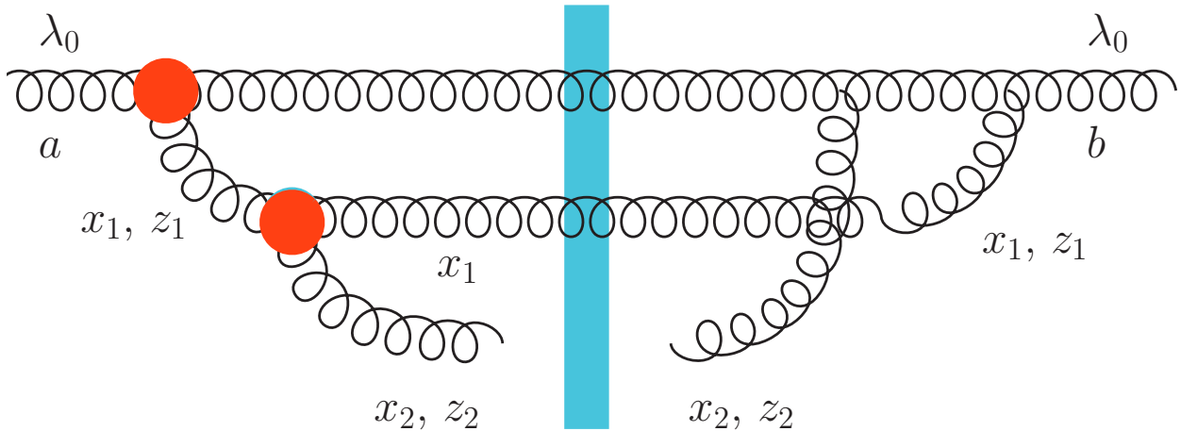

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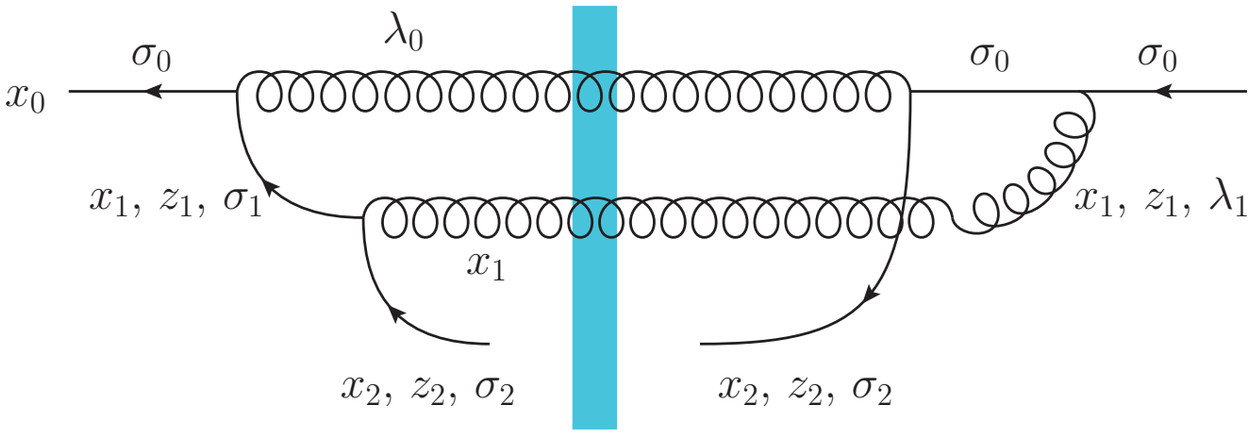

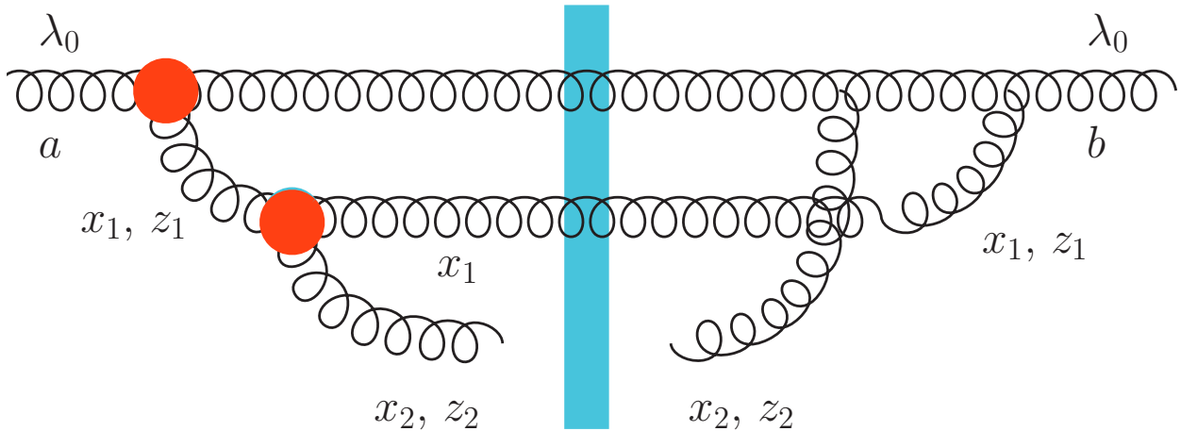

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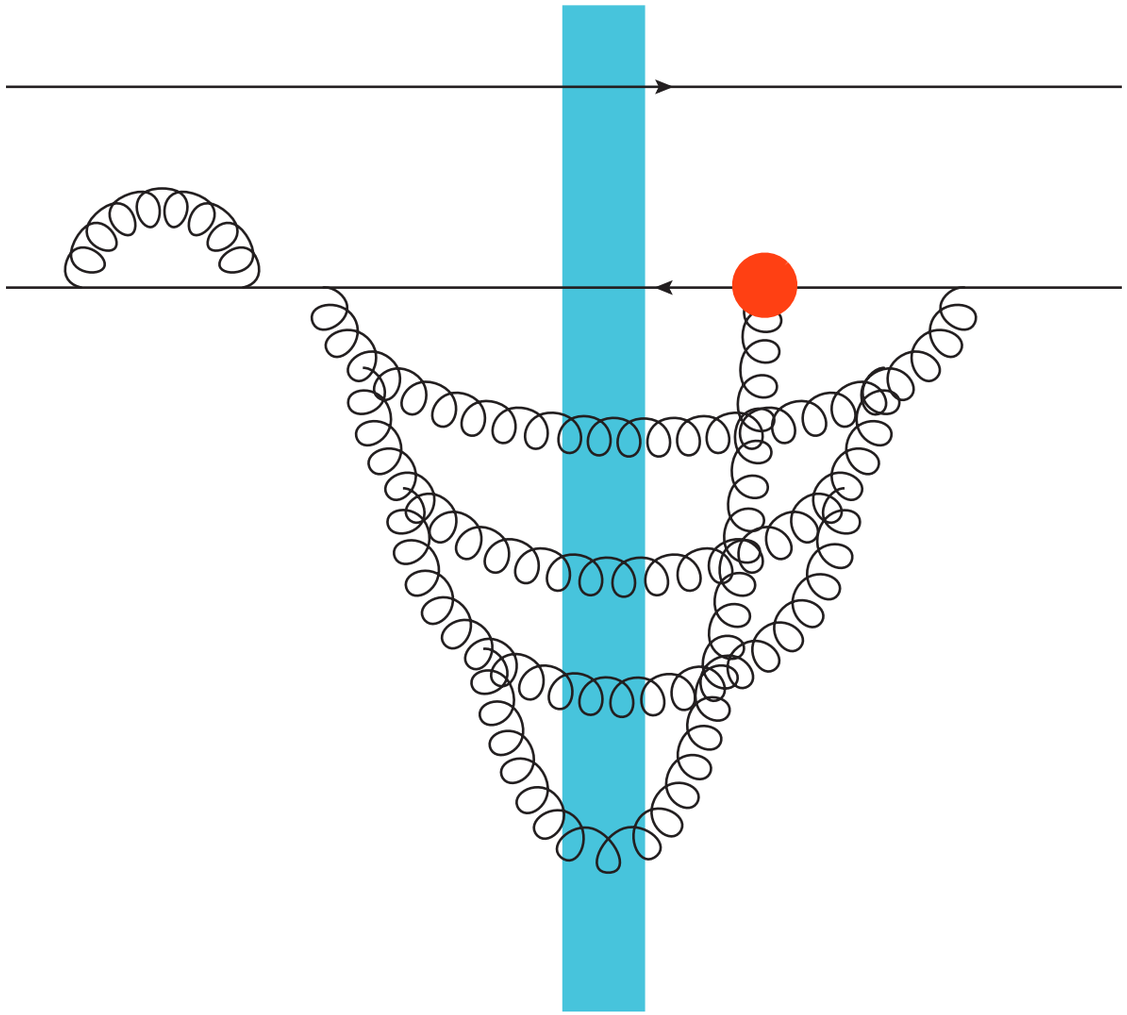
- But gluon non-ladder graphs do not cancel.

➡ Ladder evolution is an unjustified truncation

A Mess of Non-Ladder Gluons

- Non-ladder gluons can stack in complex ways which still generate double logarithms.

➡ Polarized gluons can "jump multiple rungs" of evolution



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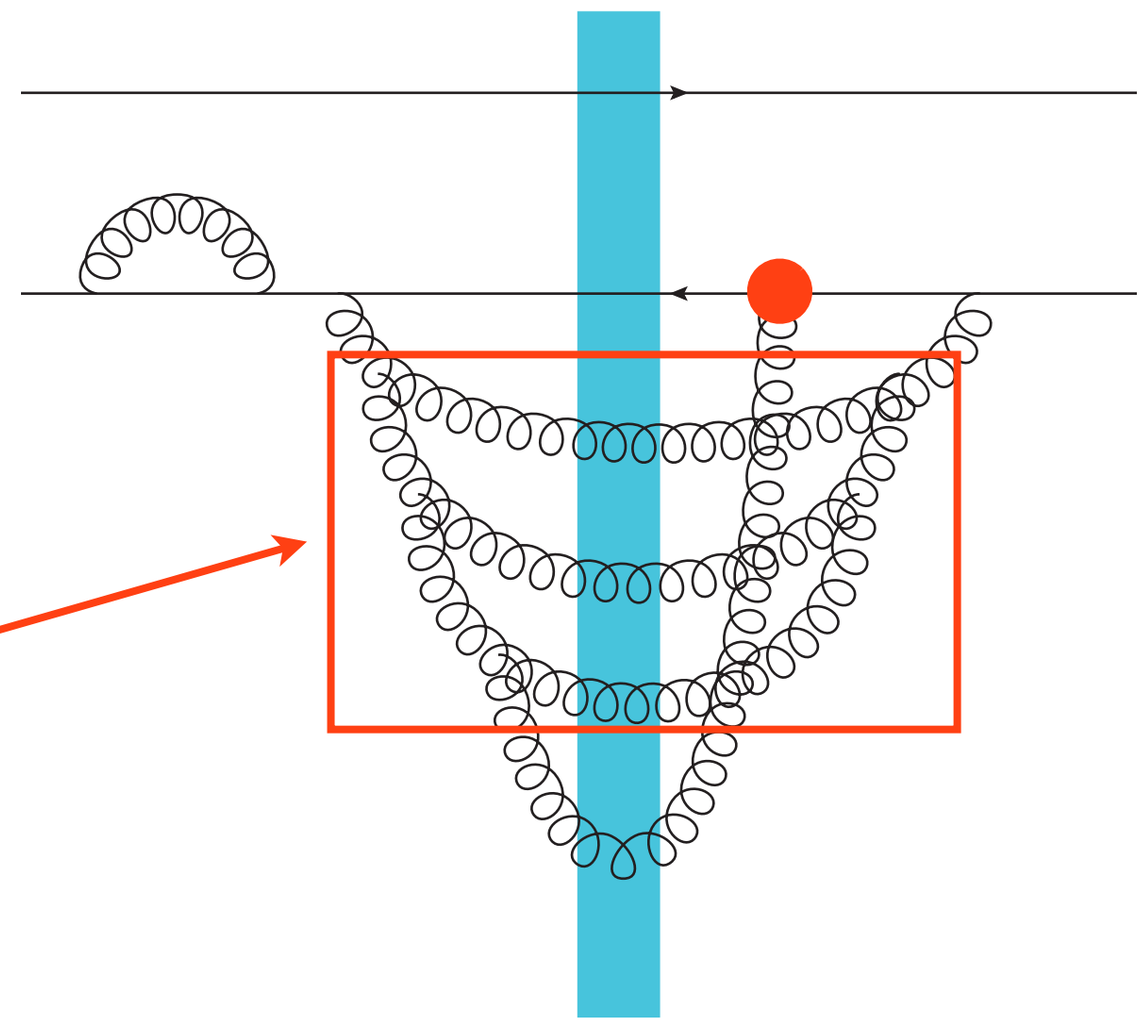

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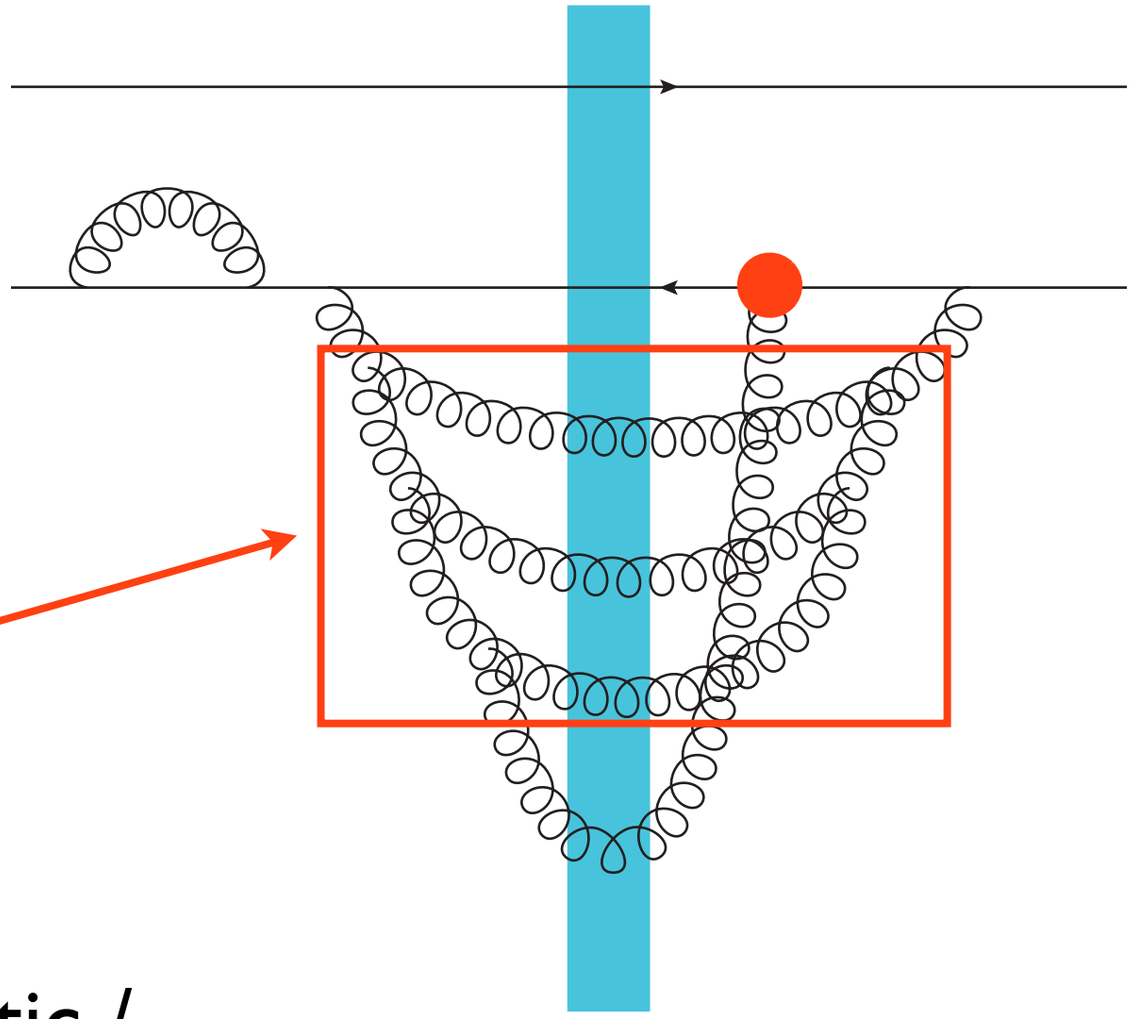
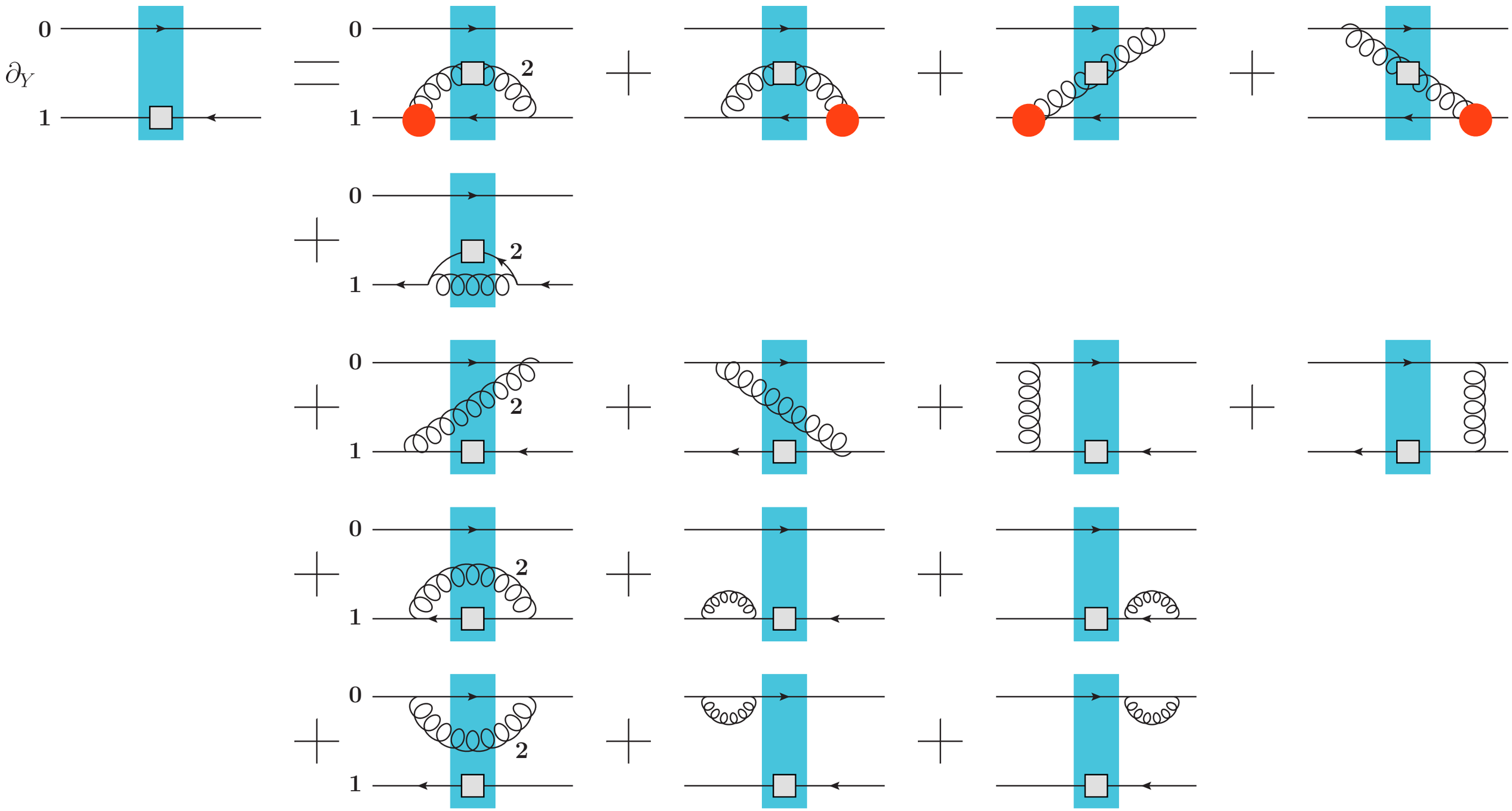
➡ Intermediate gluon emission can even be unpolarized!

➡ Makes it difficult to use standard small-x methods like the stochastic / functional description of unpolarized JIMWLK evolution



Contributions to Small-x Helicity Evolution



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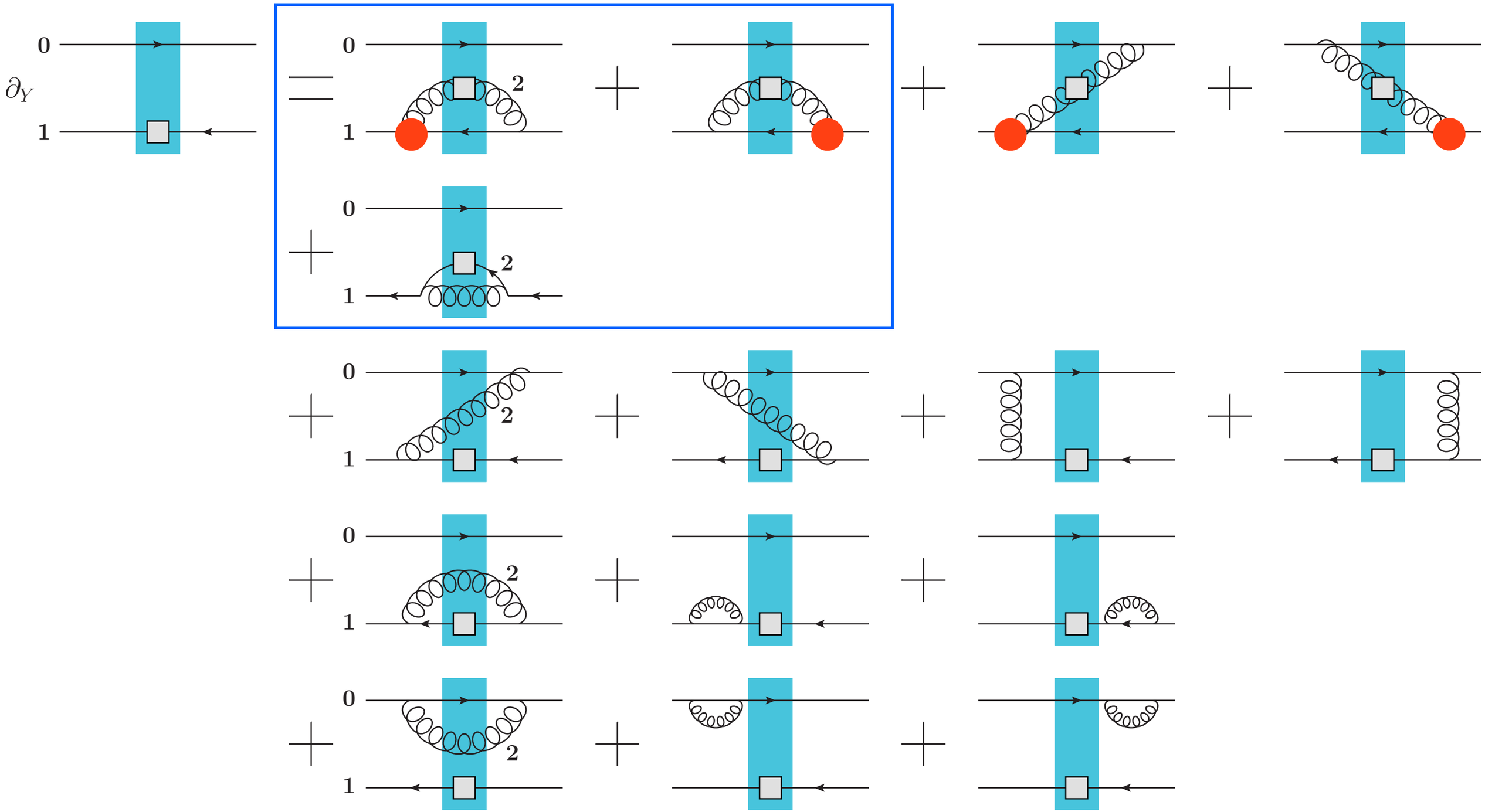
Ladder:



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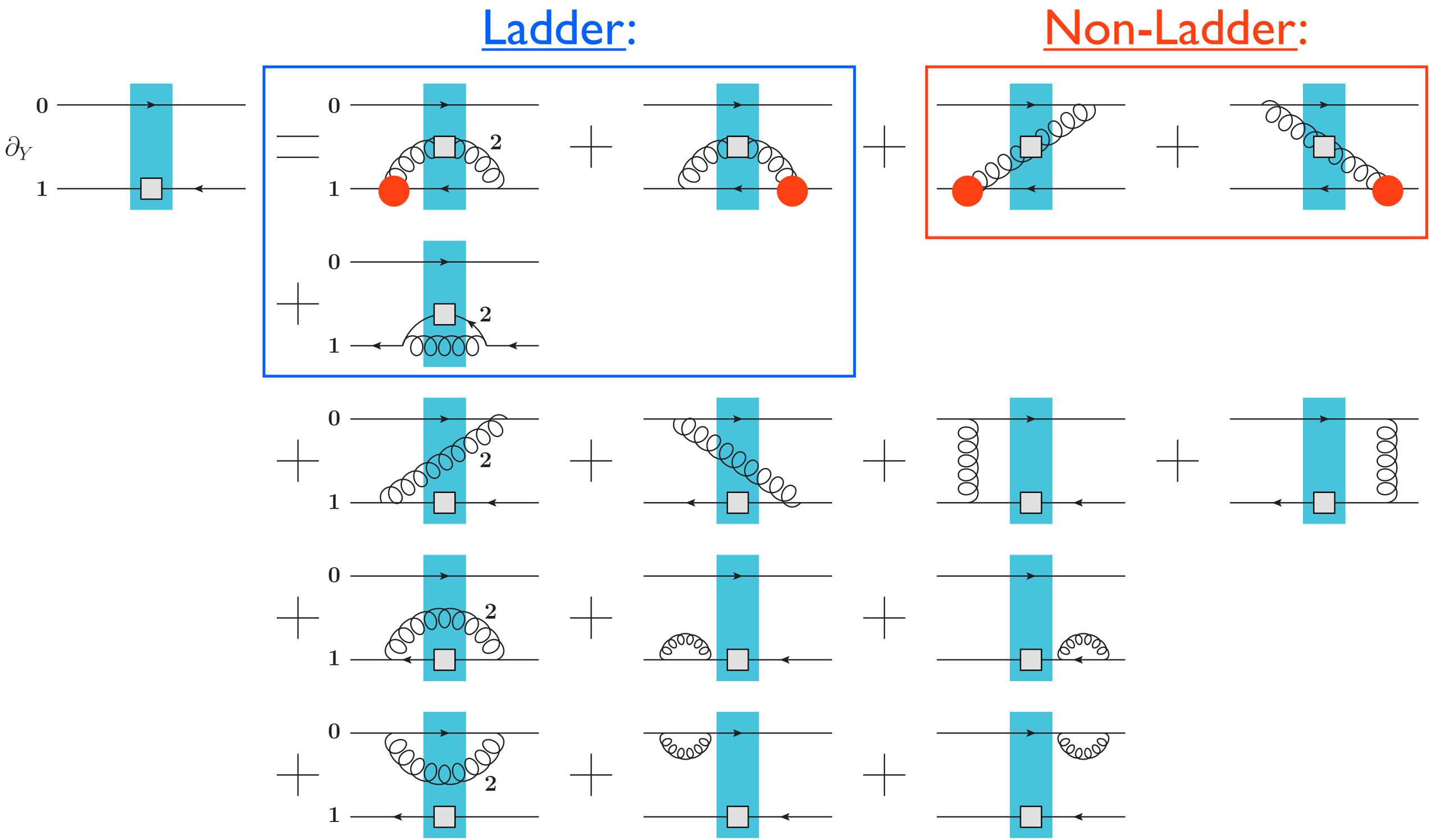
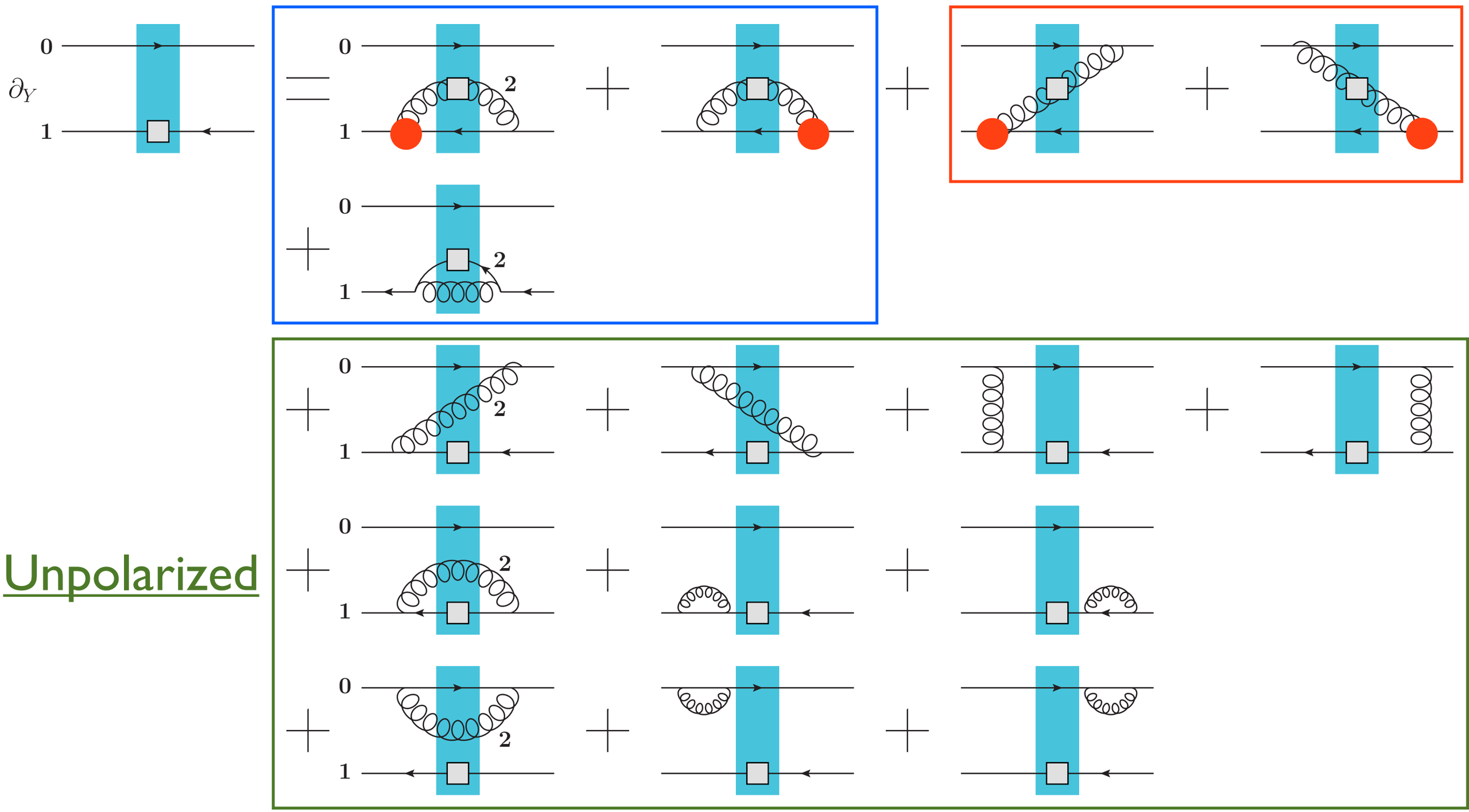


Contributions to Small-x Helicity Evolution

Ladder:

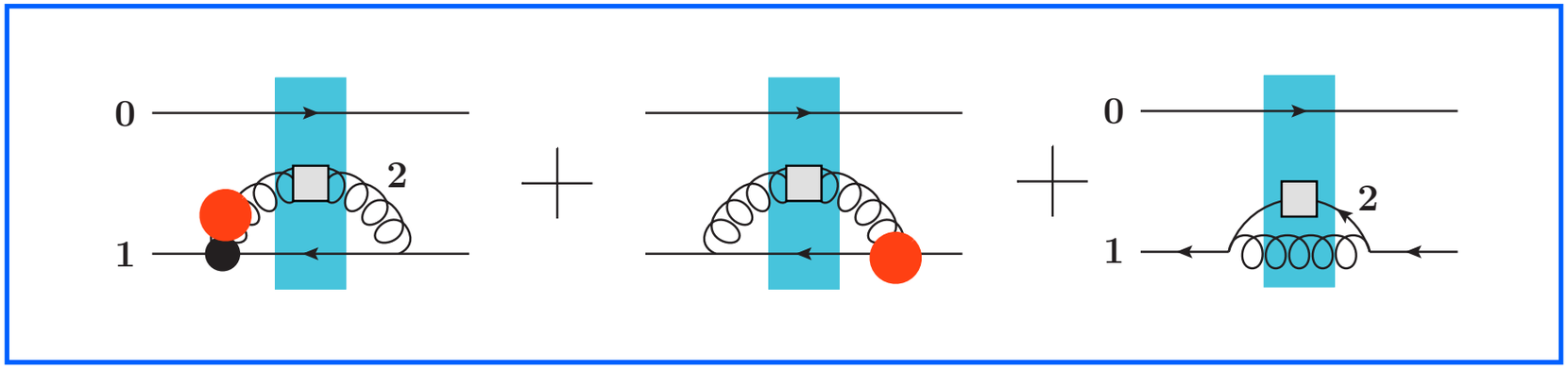
Non-Ladder:



Unpolarized

Contributions to Small-x Helicity Evolution

Ladder Graphs:



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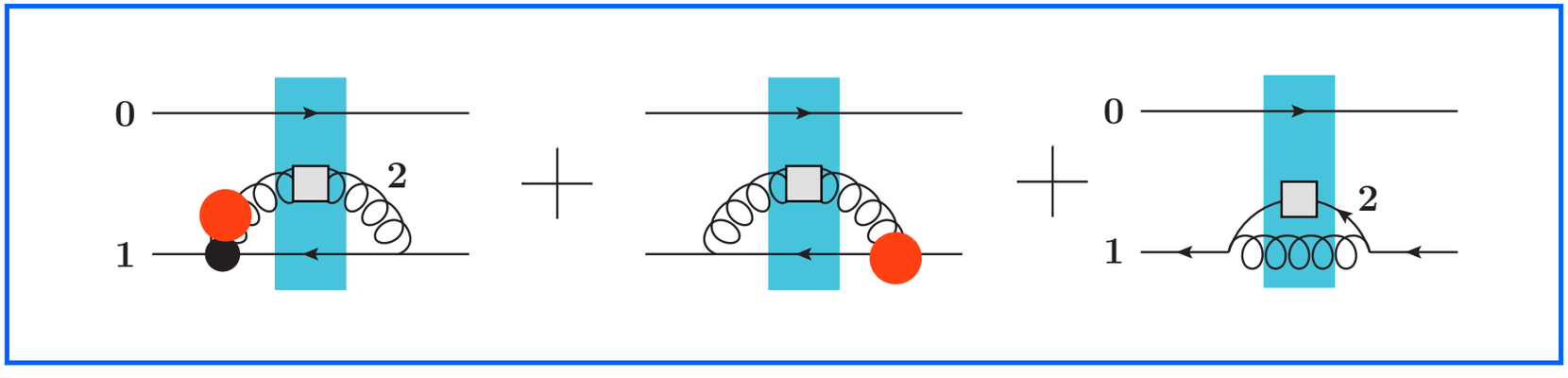

Double logarithms

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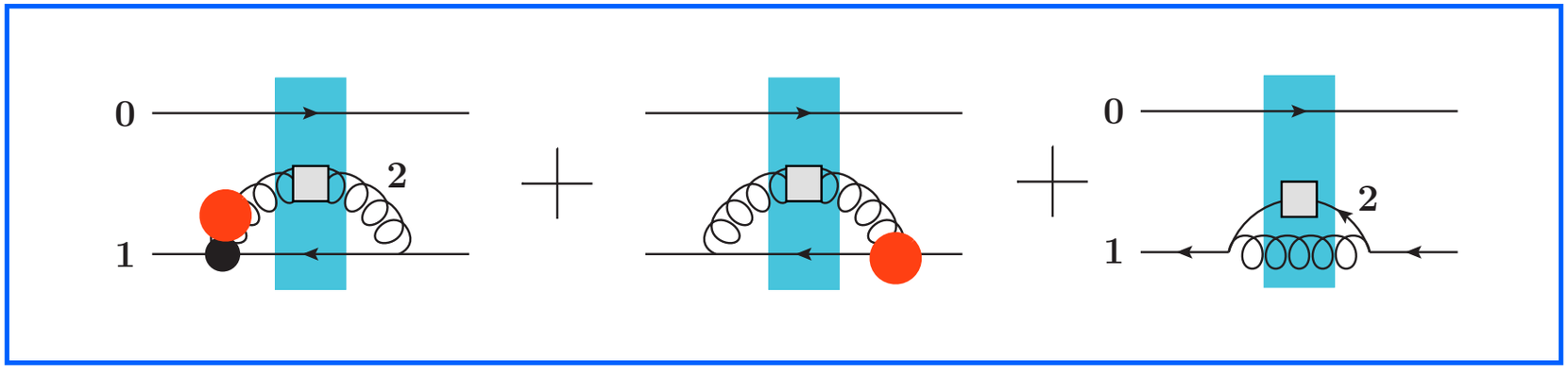

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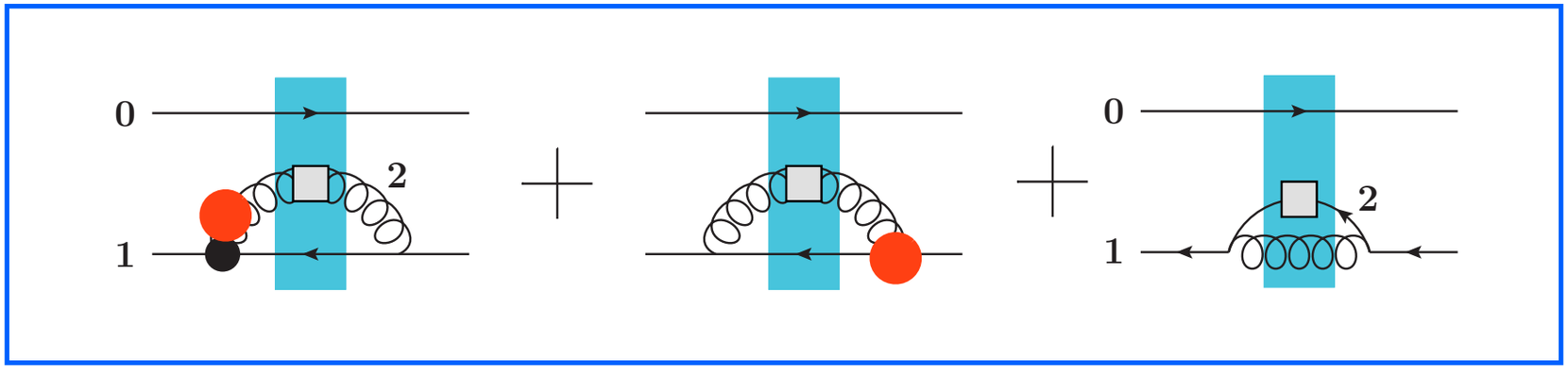

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Polarized gluon splitting

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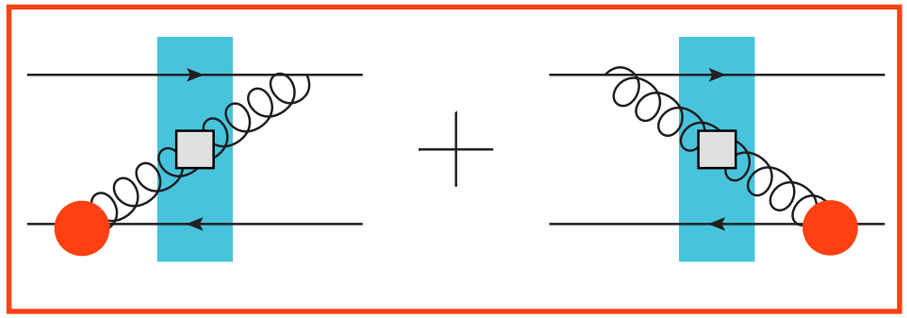
Polarized quark splitting

Contributions to Small-x Helicity Evolution

Polarized Non-Ladder Gluons:



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opposite sign

Contributions to Small-x Helicity Evolution

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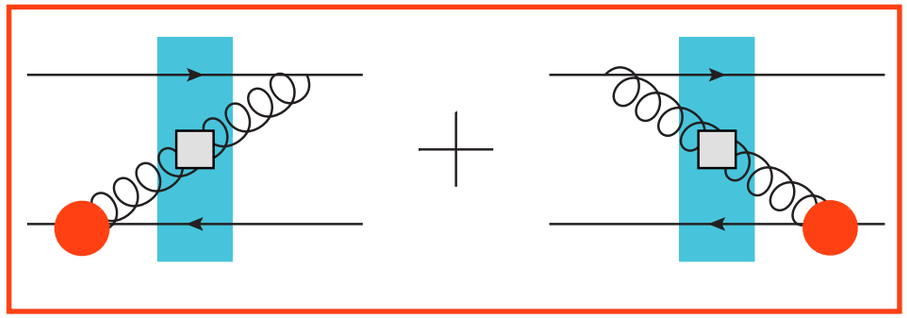

Double logarithms

only in IR phase space

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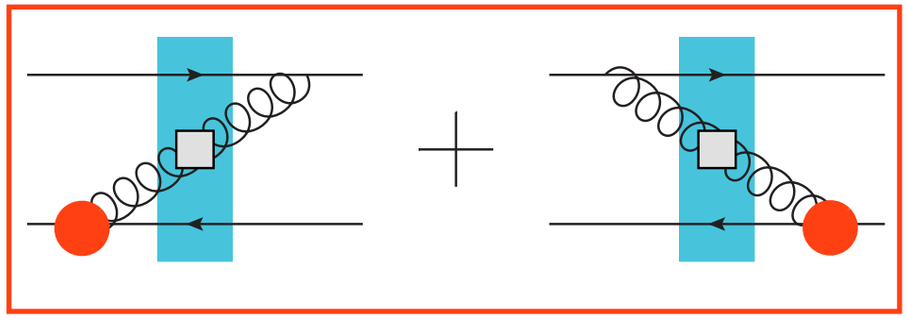

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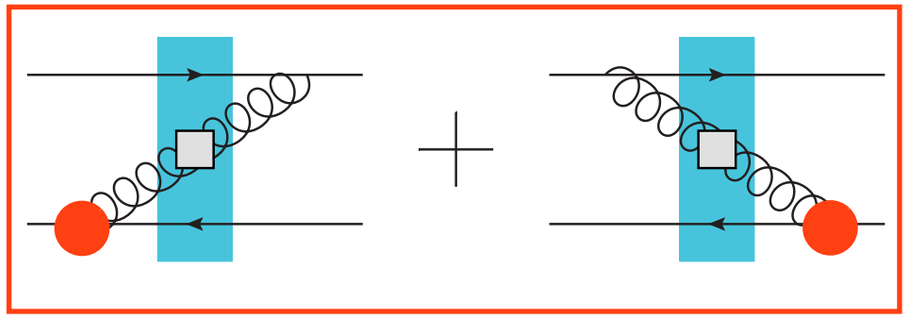
Contributions to Small-x Helicity Evolution

Polarized Non-Ladder Gluons:



Double logarithms

only in IR phase space

$$\frac{1}{N_c}\langle\langle \mathrm{Tr}[V_0 V_1^{pol\,\dagger}]\rangle\rangle(z) = -\frac{\alpha_s}{2\pi}\int_{\frac{\Lambda^2}{s}}^{z}\frac{dz'}{z'}\int_{x_{01}^2}^{x_{01}^2\frac{z}{z'}}\frac{dx_{21}^2}{x_{21}^2}\,2\,\frac{1}{N_c}\langle\langle \mathrm{Tr}[T^b V_0 T^a V_1^\dagger]\underline{(U_2^{pol})^{ba}}\rangle\rangle(z')$$

opposite sign

Polarized gluon splitting

- Non-ladder gluons cancel the IR phase space of the ladder gluons!
  - ➡ Leaves strict DGLAP-like transverse ordering for gluons
  - ➡ But not for quarks....

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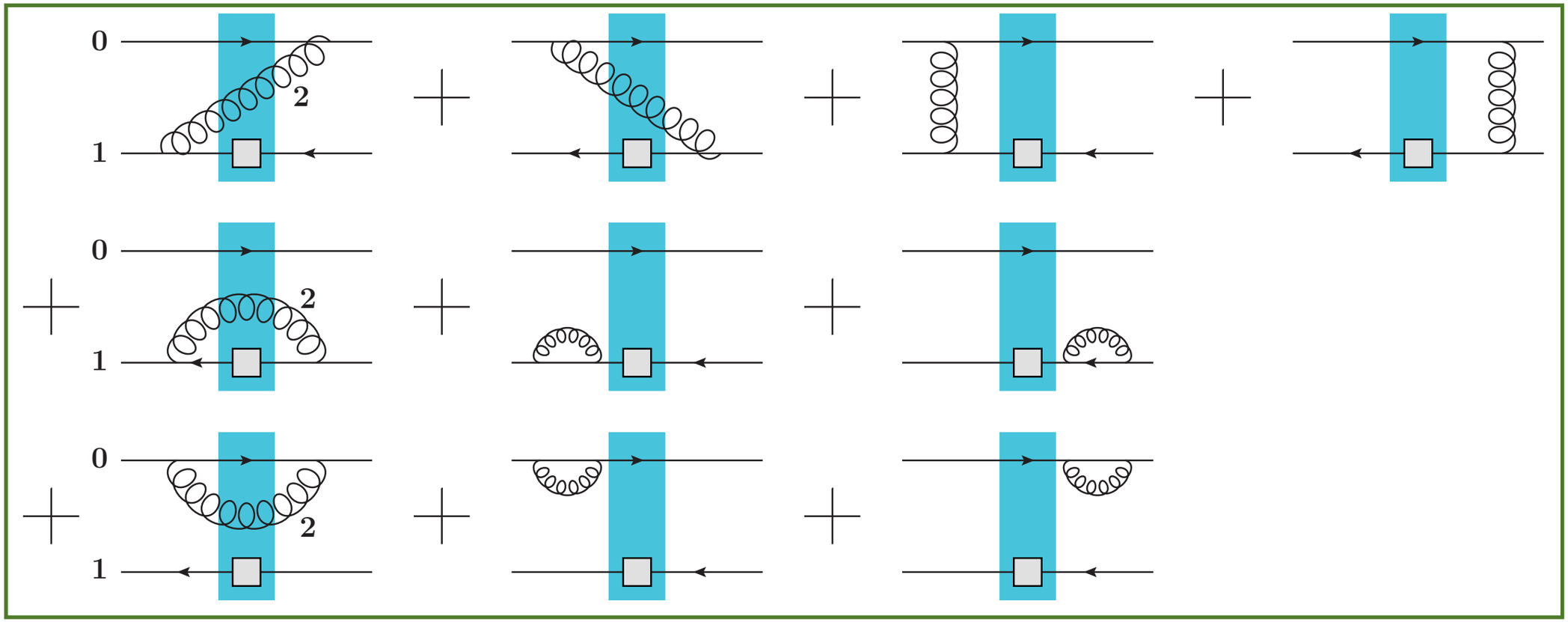
Unpolarized Gluons



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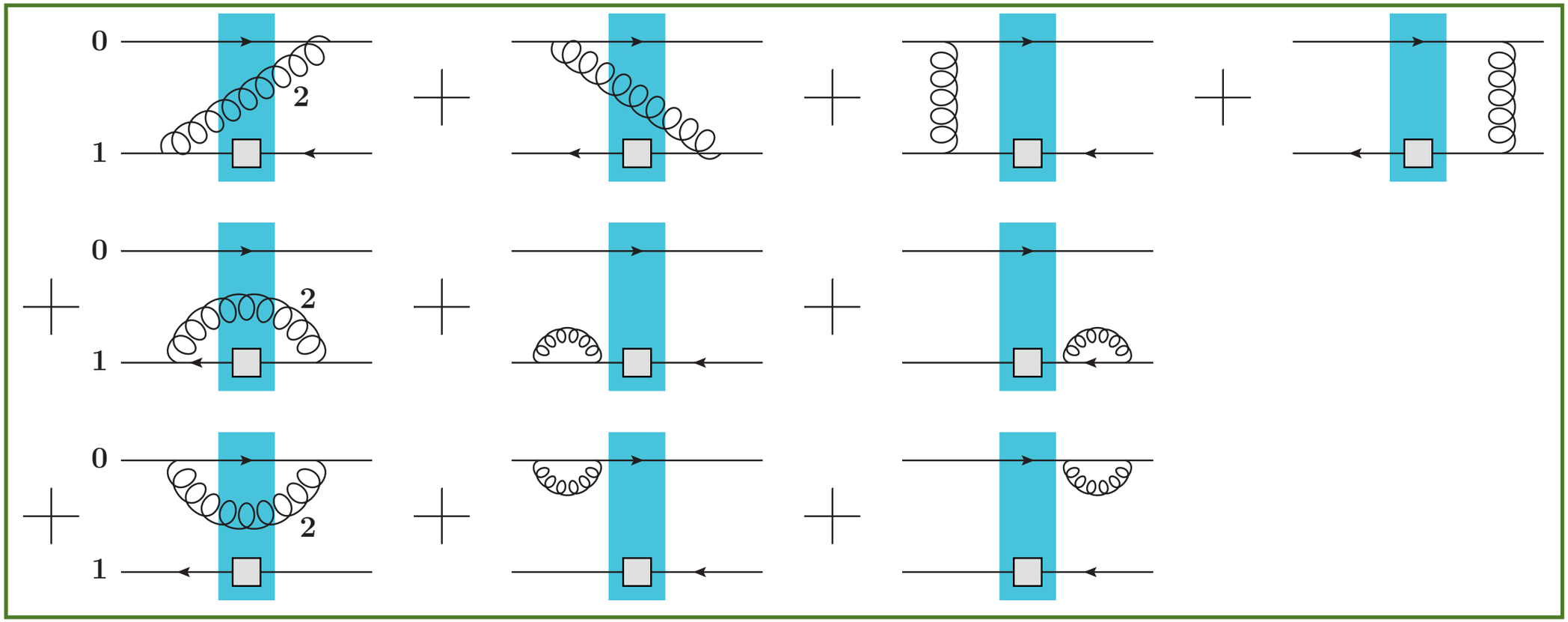

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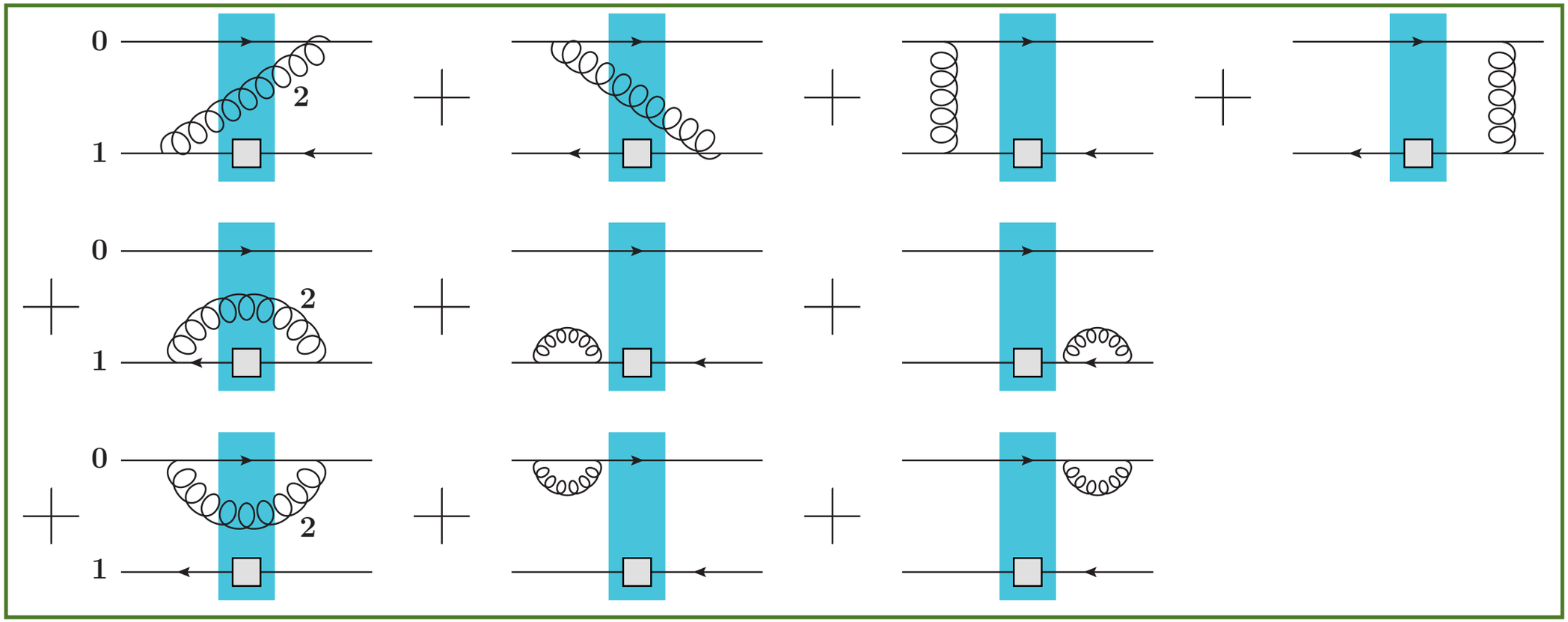

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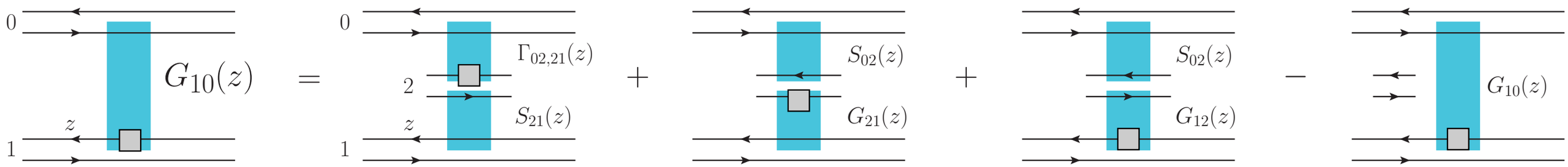
BK Kernel

Trying to Solve It: The Large N_c Approximation



- The evolution yields another infinite operator hierarchy

➡ Closes in the large N_c limit, like BK evolution.

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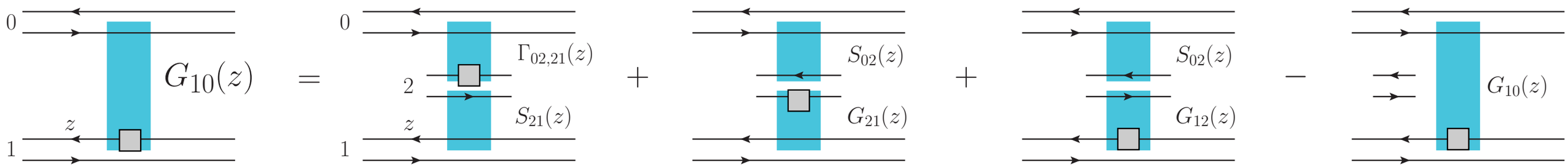

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- Lifetime ordering and IR cancellations compete to restrict the phase space

$$x_{32}^2 \ll \min\left[x_{21}^2 \frac{z'}{z''}, x_{02}^2\right]$$

➡ Polarized dipoles can depend on their "neighbors"

$$\Gamma_{02,21}(z')$$

➡ More complex than the large N_c BK equation.

Solving The Large N_c Approximation

$$G_{01}(z) = G_{01}^{(0)}(z) + \frac{\alpha_s N_c}{2\pi} \int_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int_{\frac{1}{z's}}^{x_{01}^2} \frac{dx_{21}^2}{x_{21}^2} \left[2\Gamma_{02,21}(z')S_{21}(z') + 2G_{21}(z')S_{02}(z') + G_{12}(z')S_{02}(z') - \Gamma_{01,21}(z')\right]$$

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$$\alpha_s = 0.3 \qquad xg_1(x, k_T^2) \sim x^{0.11}$$

$$\alpha_s = 0.4 \qquad xg_1(x, k_T^2) \sim \left(\frac{1}{x}\right)^{0.02}$$

Kovchegov, Pitonyak, & MS
in preparation

- Estimates are borderline between growth / suppression at small x

Outlook: The Truth Is Out There!

- How do we reconcile our much lower intercept with the previous work which generates a strong growth at small x?

$$\alpha_s = 0.3 \div 0.4 \qquad xg_1 \sim \left(\frac{1}{x}\right)^{0.39 \div 0.60}$$

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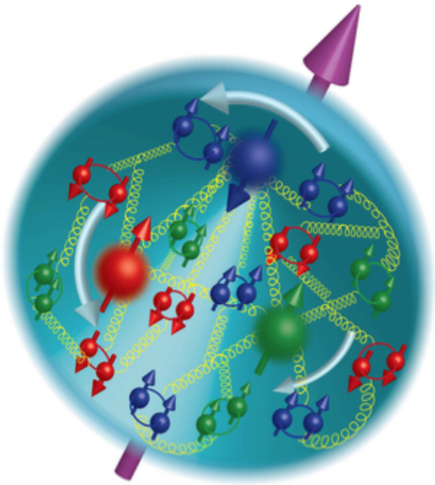
- What about other polarization observables?

Summary

- Up to 35% of the proton angular momentum is unaccounted for.
  - ➡ Is there significant polarization at small x?

$$0.001 < x < 1$$

$$\Delta\Sigma \approx 0.25 \ \ (25\%)$$
$$\Delta G \approx 0.2 \ \ (40\%)$$



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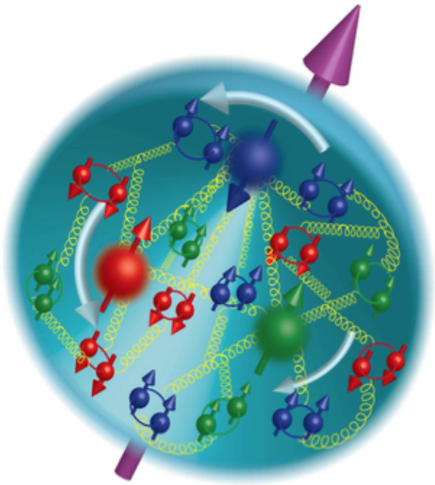

- Quark / gluon splitting leads to rich double-logarithmic evolution equations
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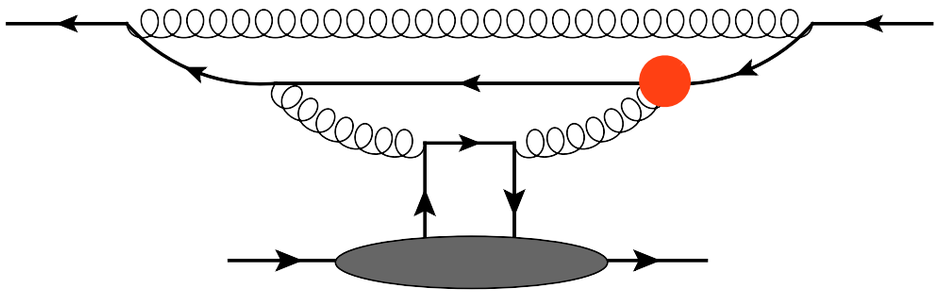

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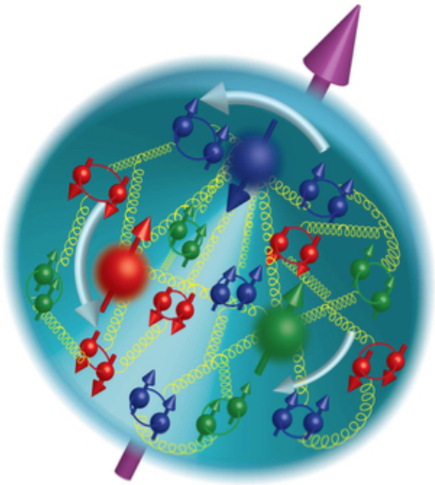

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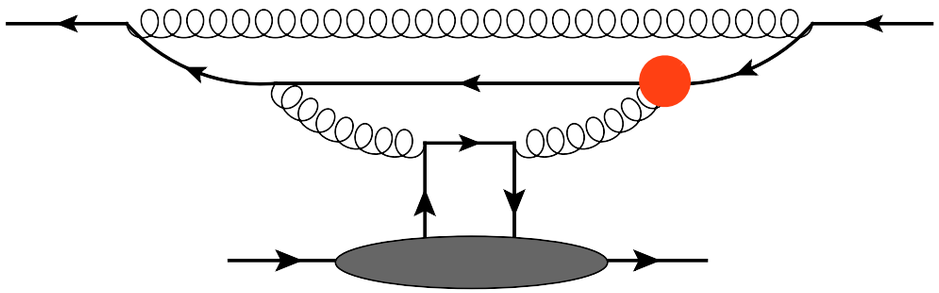

- We estimate behavior on the borderline between weak suppression and weak growth at small x.

$$\alpha_s = 0.3 \div 0.4$$

$$xg_1 \sim \left(\frac{1}{x}\right)^{0.02} \div x^{0.11}$$