Azimuthal Anisotropy in Deep Inelastic Scattering Dijet Production at High Energy

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A. Dumitru, T. Lappi and V. S. Phys.Rev.Lett. 115 (2015) 25, 252301

A. Dumitru and V. S., arXiv:1605.02739

A. Dumitru, V. S. and T. Ullrich, work in progress

Introduction

There are two different unintegrated gluon distributions (UGD):

- Dipole gluon distribution ($G^{(2)}$) + linear polarized partner ($h^{(2)}$).
  Appears in many processes. Small x evolution is well understood (BFKL/BF evolution equations)
- Weizsäcker-Williams gluon distribution ($G^{(1)}$) + linear polarized partner ($h^{(1)}$).

WW UGD appears exclusively only in dijet DIS $\rightsquigarrow$ unique probe of WW UGD in saturation regime

As in p-A or d-A, saturation in e-A $\rightsquigarrow$ decrease of back-to-back dihadron correlation

L. Zheng, E. C. Aschenauer, J. H. Lee and B. W. Xiao

Phys. Rev. D **89**, *7, 074037 (2014)*

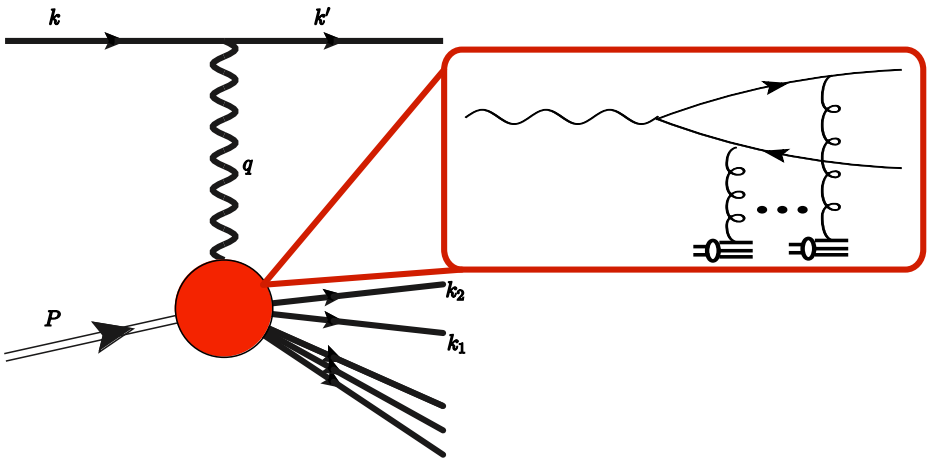
In this talk: structure of back-to-back peak

Weizsäcker-Williams gluon distribution: linearly polarized gluons in unpolarized target

P. Mulders and J. Ridrigues Phys.Rev. D63 (2001) 094021
D. Boer, P. Mulders, C. Pisano Phys.Rev. D80 (2009) 094017
A. Metz and J. Zhou Phys.Rev. D84 (2011) 051503
F. Dominguez, C. Marquet, B.-W. Xiao, F. Yuan Phys.Rev. D83 (2011) 105005
F. Dominguez, J.-W. Qiu , B.-W. Xiao, F. Yuan Phys.Rev. D85 (2012) 045003

- WW Linearly polarized gluons (partner of conventional WW) are present even in unpolarized hadrons; it contributes with $\cos(2\phi)$ azimuthal angular dependence
- Origin: averaged quantum interference of different helicity states between scattering amplitude and its complex conjugate
- It is present only at non-zero transverse momentum: transverse momentum-dependent distribution
- Small x behaviour of polarization WW Linearly polarized gluon distribution is largely unknown

Dijet production in DIS at small x



- DIS dijet production: $\gamma^* A \quad \rightarrow \quad q\,\bar{q}\;X$
- Multiple scatterings of (anti) quark are accounted for by ressumation:

$$U(\mathbf{x}) = \mathbb{P}\exp\left\{ig\int dx^- A^+(x^-,\mathbf{x}_\perp)\right\}$$

- In color dipole model this process corresponds to

$$\frac{d\sigma^{\gamma^* A\to q\bar{q}X}}{d^3k_1 d^3k_2} =$$

$$N_c\alpha_{em}e_q^2\delta(p^+ - k_1^+ - k_2^+)\int \frac{d^2x_1}{(2\pi)^2}\frac{d^2x_2}{(2\pi)^2}\frac{d^2y_1}{(2\pi)^2}\frac{d^2y_2}{(2\pi)^2}\exp\left(-i\mathbf{k}_1(\mathbf{x}_1-\mathbf{y}_1)-i\mathbf{k}_2(\mathbf{x}_2-\mathbf{y}_2)\right)$$

$$\sum_{\gamma\alpha\beta}\psi^{\mathrm{T,L}\gamma}_{\alpha\beta}(\mathbf{x}_1-\mathbf{x}_2)\psi^{\mathrm{T,L}\gamma*}_{\alpha\beta}(\mathbf{y}_1-\mathbf{y}_2)\left[1+\frac{1}{N_c}\Big(\langle\mathrm{Tr}\;U(\mathbf{x}_1)U^\dagger(\mathbf{y}_1)U(\mathbf{y}_2)U^\dagger(\mathbf{x}_2)\rangle\right.$$

$$\left.-\langle\mathrm{Tr}\;U(\mathbf{x}_1)U^\dagger(\mathbf{x}_2)\rangle-\langle\mathrm{Tr}\;U(\mathbf{y}_1)U^\dagger(\mathbf{y}_2)\rangle\Big)\right] \quad \uparrow \text{Quadrupole contribution}$$

- Splitting wave function of $\gamma^\star$ with longitudinal momentum p^+ and virtuality Q^2
- This expression can be computed without any further simplifications with quadrupole, but no direct relation to TMD result

Dijet production in DIS

- In correlation limit (almost back-to-back jets), $\mathbf{P} = (\mathbf{k}_1 - \mathbf{k}_2)/2$ is much larger than $\mathbf{q} = \mathbf{k}_1 + \mathbf{k}_2$;
  for conjugate variables, $u \ll v$, where $\mathbf{u} = \mathbf{x}_1 - \mathbf{x}_2$ and $\mathbf{v} = (\mathbf{x}_1 + \mathbf{x}_2)/2$.
  Expand in u.
- Expansion of quadrupole brings gradients of Wilson lines.
- Allows to reduce to 2 point functions

$$xG^{ij}_{WW}(\mathbf{k}) = \frac{8\pi}{S_\perp}\int \frac{d^2x}{(2\pi)^2}\frac{d^2y}{(2\pi)^2} e^{-\mathbf{k}\,(\mathbf{x}-\mathbf{y})}\langle A^i_a(\mathbf{x})A^j_a(\mathbf{y})\rangle, \quad A^i(\mathbf{x}) = \frac{1}{ig}U^\dagger(\mathbf{x})\partial_i U(\mathbf{x})$$

WW Color Electric field ↑

- Decomposition to conventional and traceless contribution

$$xG^{ij}_{WW} = \frac{1}{2}\delta^{ij} x\, G^{(1)} - \frac{1}{2}\left(\delta^{ij} - 2\frac{q^i q^j}{q^2}\right) x\, h^{(1)}_\perp$$

- Beyond leading order in expansion: 2-nd part of talk

MV model results

- $G^{(1)}$ and $h_\perp^{(1)}$ can be analytically computed in Gaussian approximation.
- In particular, using McLerran-Venugopalan (MV) model

$$g^2 \langle A^{-a}(z_1^+, z_1) A^{-b}(z_2^+, z_2) \rangle = \delta^{ab}\, \delta(z_1^+ - z_2^+)\, \mu^2(z^+)\, L_{z_1 z_2} \ ,$$

It was obtained

$$xh^{(1)}(x, q^2) = \frac{N_c S_\perp}{2\pi^3 \alpha_s} \int d|r|\, |r| J_2(|r|\,|q|) \left[1 - \exp\left(-\frac{Q_s^2 r^2}{4} \log \frac{1}{r^2 \Lambda_{\rm IR}^2} \right) \right] \frac{1}{r^2 \log \frac{1}{r^2 \Lambda_{\rm IR}^2}} \ ,$$

$$xG^{(1)}(x, q^2) = \frac{N_c S_\perp}{2\pi^3 \alpha_s} \int d|r|\, |r| J_0(|r|\,|q|) \left[1 - \exp\left(-\frac{Q_s^2 r^2}{4} \log \frac{1}{r^2 \Lambda_{\rm IR}^2} \right) \right] \frac{1}{r^2}$$

Limiting cases:

- $\Lambda_{IR} \ll q \ll Q_s, \quad xh^{(1)} \propto q^0$ and $xG^{(1)} \propto \ln \frac{Q_s^2}{q^2}$
- $q \gg Q_s, \quad xh^{(1)} \approx xG^{(1)} \propto 1/q^2$

A. Metz and J. Zhou Phys.Rev. D84 (2011) 051503

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Correlations limit results for $\gamma^*_{\|,\perp}$

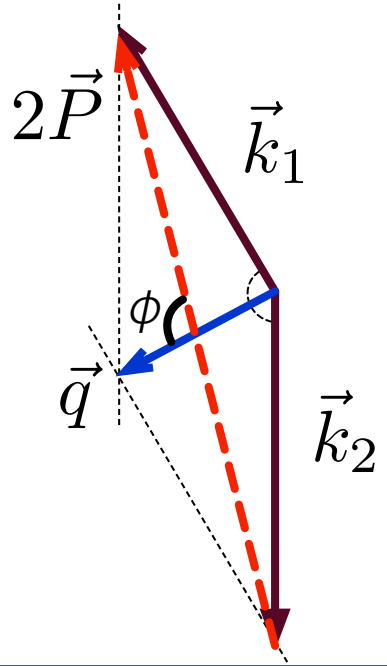
$$E_1E_2\frac{d\sigma^{\gamma^*_L A\to q\bar{q}X}}{d^3k_1d^3k_2d^2b} = \alpha_{em}e_q^2\alpha_s\delta\left(x_{\gamma^*}-1\right)z^2(1-z)^2\frac{8\epsilon_f^2P_\perp^2}{(P_\perp^2+\epsilon_f^2)^4}\times\left[xG^{(1)}(x,q_\perp)+\underline{\cos(2\phi)}\ xh_\perp^{(1)}(x,q_\perp)\right]$$

$$E_1E_2\frac{d\sigma^{\gamma^*_T A\to q\bar{q}X}}{d^3k_1d^3k_2d^2b} = \alpha_{em}e_q^2\alpha_s\delta\left(x_{\gamma^*}-1\right)z(1-z)\left(z^2+(1-z)^2\right)\frac{\epsilon_f^4+P_\perp^4}{(P_\perp^2+\epsilon_f^2)^4}$$

$$\times\left[xG^{(1)}(x,q_\perp)-\frac{2\epsilon_f^2P_\perp^2}{\epsilon_f^4+P_\perp^4}\ \underline{\cos(2\phi)}\ xh_\perp^{(1)}(x,q_\perp)\right]$$

z is long. momentum fraction of photon carried by quark $\qquad \epsilon_f^2 = z(1-z)Q^2$

- Jets are almost back-to-back. Note: this is not about suppression of back-to-back peak, but rather about the structure of back to back correlation.
- **Azimuthal anisotropy is in angle between P and q, denoted by ϕ.**
- Is $h_\perp^{(1)}$ important at small x?



$$2\vec{P} = \vec{k}_1 - \vec{k}_2$$
$$\vec{q} = \vec{k}_1 + \vec{k}_2$$

Numerics: small x evolution

- McLerran-Venugopalan initial conditions at $Y = \ln x_0/x = 0$

$$S_{\rm eff}[\rho^a] = \int dx^- d^2x_\perp \, \frac{\rho^a(x^-, x_\perp)\,\rho^a(x^-, x_\perp)}{2\mu^2}$$

for

$$U(x_\perp) = \mathbb{P}\exp\left\{ ig^2 \int dx^- \frac{1}{\nabla_\perp^2} t^a \rho^a(x^-, x_\perp) \right\}$$

- Quantum evolution at $Y > 0$ is accounted for by solving JIMWLK-B using Langevin method

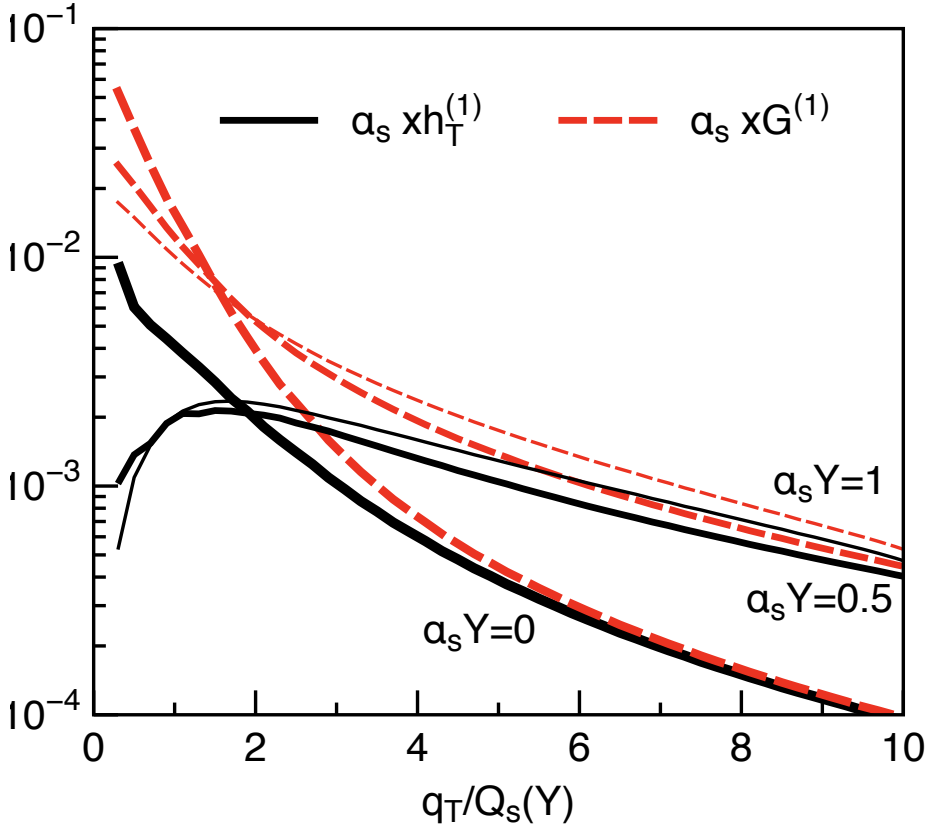
$$\partial_Y U(z) = U(z)\frac{i}{\pi}\int d^2u \frac{(z-u)^i \eta^i(u)}{(z-u)^2} - \frac{i}{\pi}\int d^2v\, U(v)\frac{(z-v)^i\eta^i(v)}{(z-v)^2} U^\dagger(v) U(z)\,.$$

The Gaussian white noise $\eta^i = \eta^i_a t^a$ satisfies $\langle \eta^a_i(z)\rangle = 0$ and

$$\langle \eta^a_i(z)\, \eta^b_j(y)\rangle = \alpha_s\, \delta^{ab}\delta_{ij}\delta^{(2)}(z-y).$$

L. D. McLerran and R. Venugopalan, Phys. Rev. D49, 2233 (1994)
J.-P. Blaizot, E. Iancu and H. Weigert, Nucl. Phys. A713, 441 (2003)
T. Lappi and H. Mäntysaari, Eur. Phys. J. C73, 2307 (2013)

Small x evolution



Reminder of McLerran-Venugopalan model results

$$xh_\perp^{(1)} = \frac{S_\perp}{2\pi^3\alpha_s}\frac{N_c^2-1}{N_c}\int_0^\infty drr\,\frac{J_2(q_\perp r)}{r^2\ln\frac{1}{r^2\Lambda^2}}\left(1-\exp\left(-\frac{1}{4}r^2Q_s^2\right)\right)$$

$$xG^{(1)} = \frac{S_\perp}{2\pi^3\alpha_s}\frac{N_c^2-1}{N_c}\int_0^\infty drr\,\frac{J_2(q_\perp r)}{r^2}\left(1-\exp\left(-\frac{1}{4}r^2Q_s^2\right)\right)$$

Large $q_\perp \gg Q_s$: $xh_\perp^{(1)} = xG^{(1)} \propto 1/q_\perp^2$

Small $q_\perp \ll Q_s$: $xh_\perp^{(1)} \propto q_\perp^0 \quad xG^{(1)} \propto \ln\frac{Q_s^2}{q_\perp^2}$

- at large $q_\perp$, saturation of positivity bound $h_\perp^{(1)} \to G^{(1)}$, as also was found in pert. twist 2 calculations of small x field of fast quark
- at small $q_\perp$, $h_\perp^{(1)}/G^{(1)} \to 0$
- both functions decrease fast as functions of $q_\perp$: best measured when $q_\perp \approx Q_s$. Nuclear target!

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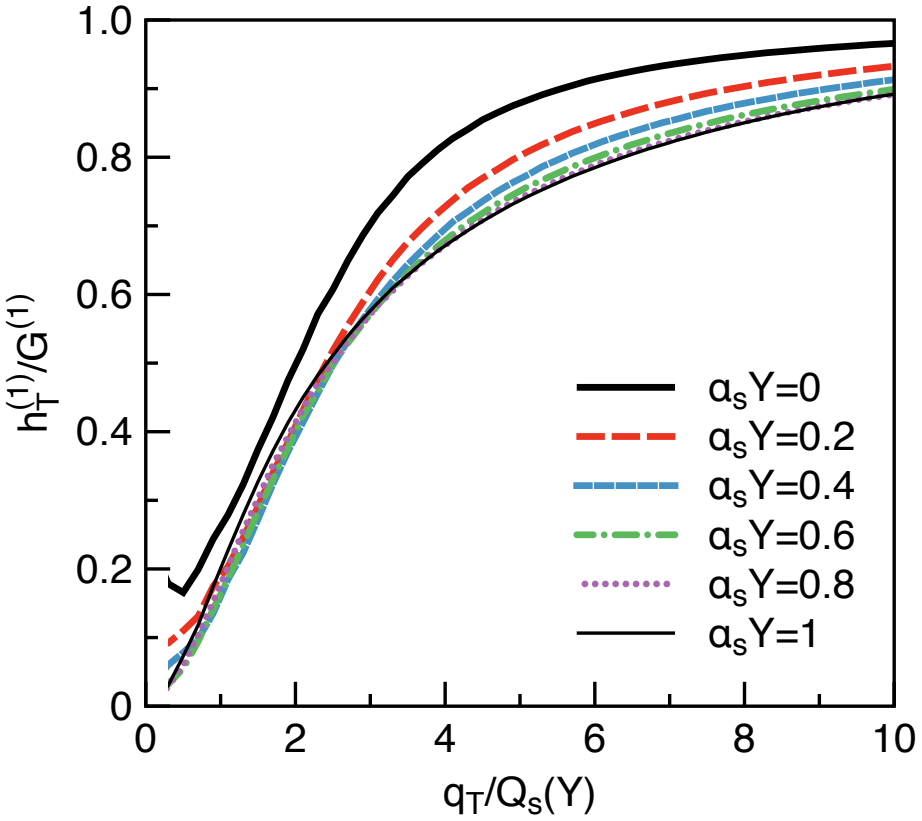
Small x evolution II



- Fast departure from MV ($\alpha_s Y = 0$)
- Slow evolution towards smaller x
- $h_\perp^{(1)}$ is large at small x
- Note: $q_\perp$ is scaled by exponentially growing $Q_s(Y)$: ratio at fixed $q_\perp$ decreases with rapidity.
  Emission of small x gluons reduces degree of polarization.
- Approximate scaling at small x.

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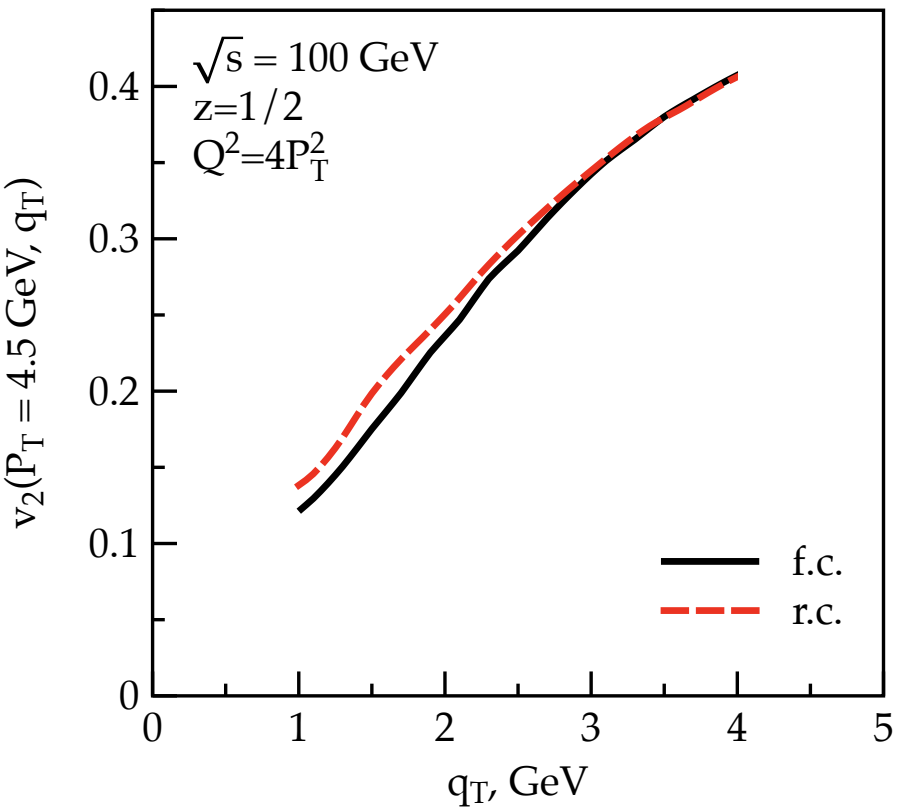
Second harmonics of azimuthal anisotropy: $q_\perp$-dependence

- By analogy to HIC

$$v_2(P_\perp, q_\perp) = \langle \cos 2\phi \rangle$$

- Fixed coupling results ("f.c.") are for $\alpha_s = 0.15$
- At this fixed $P_\perp$ not very significant dependence on prescription for α_s
- Increase of v_2 is due to increasing $h_\perp^{(1)}(q_\perp)/G^{(1)}(q_\perp)$



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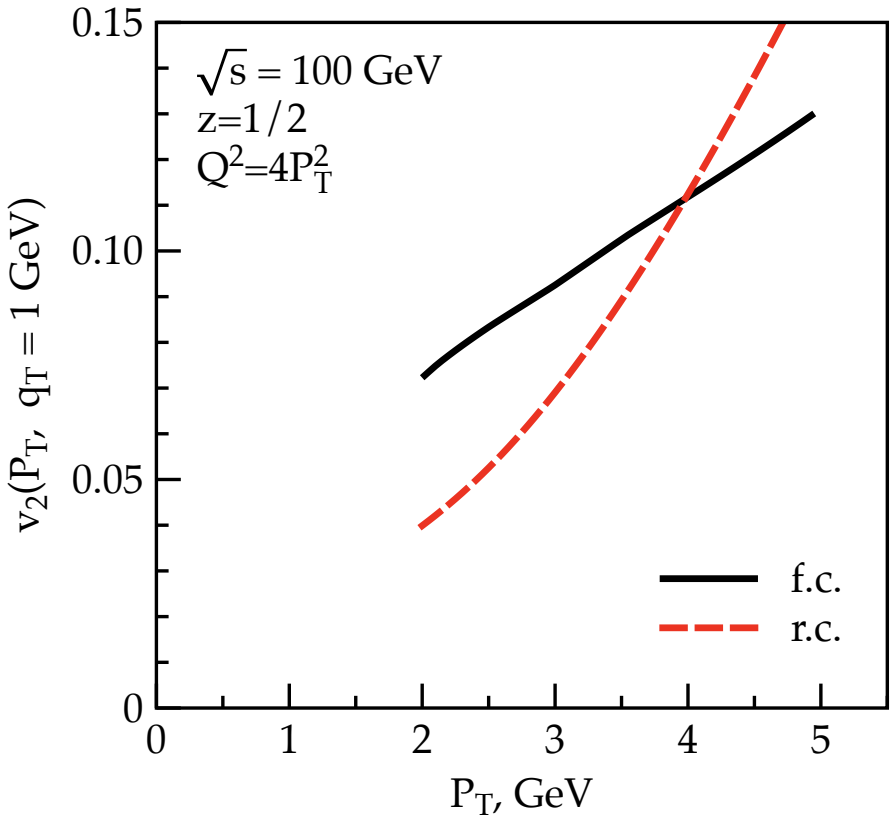
Second harmonics of azimuthal anisotropy: $P_\perp$-dependence



- Fixed coupling results significantly different from running coupling
- Large azimuthal anisotropy in both cases
- Increasing $P_\perp$ increases x and suppresses evolution effects driving v_2 towards its MV value

$$x = \frac{1}{s}\left(q_\perp^2 + \frac{1}{z(1-z)}P_\perp^2\right)$$

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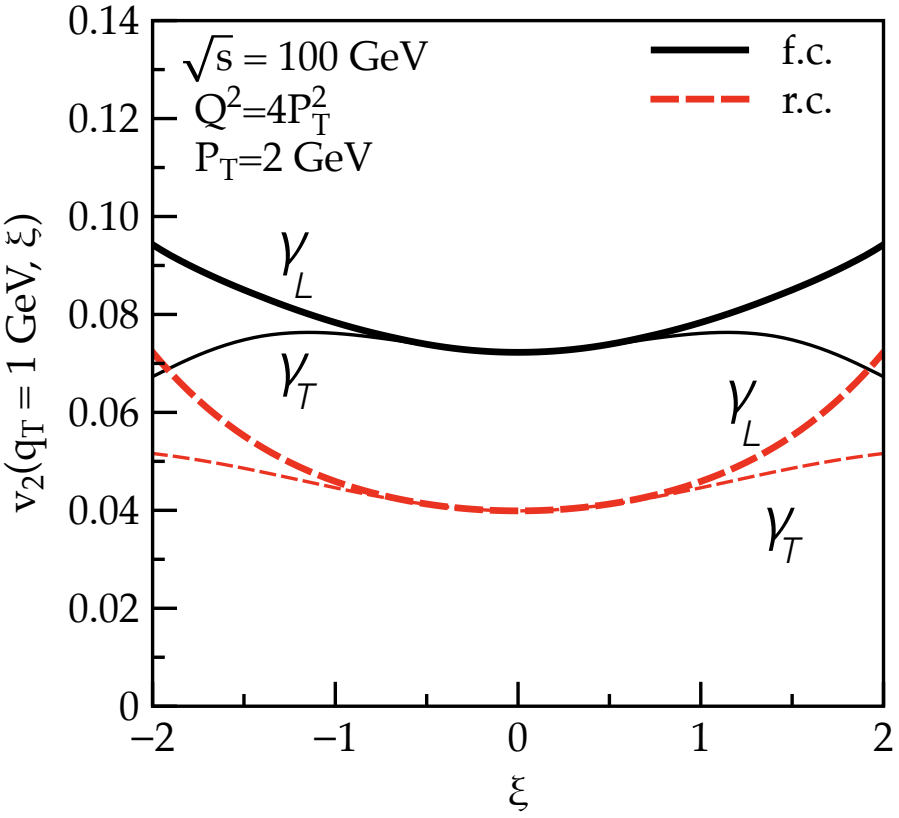
Dependence on longitudinal momentum

- To probe longitudinal structure

$$\xi = \ln \frac{1-z}{z}$$

- Long-range "rapidity" correlation
- Mild increase for large ξ because asymmetric dijets probe target at larger values of x



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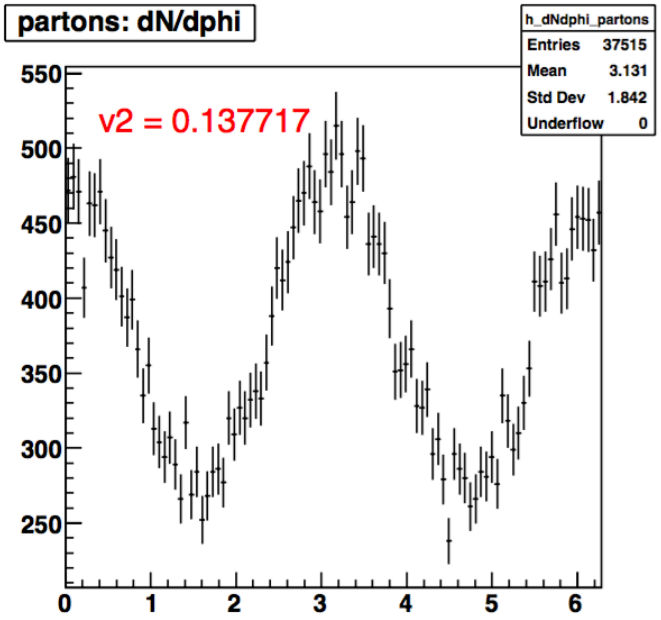
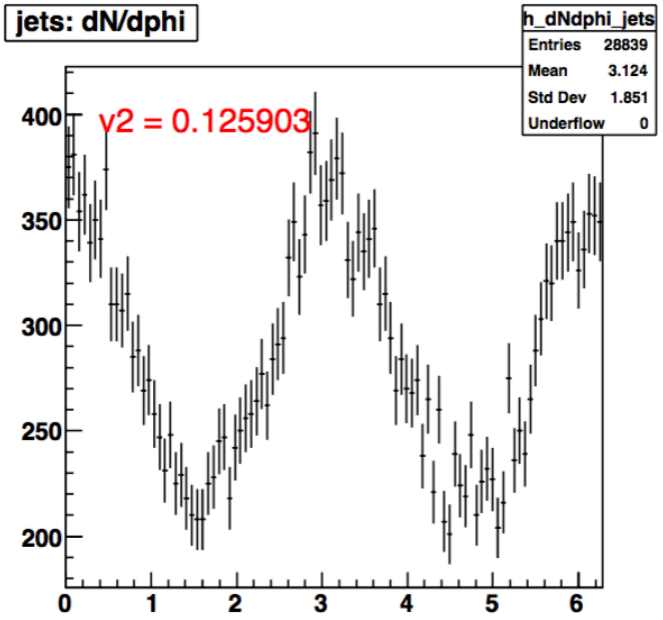
Monte Carlo Event generator

- DIS event with random Q^2, W^2, photon polarization, as well as $P_\perp$ and $q_\perp$
- Input: $\sqrt{s}$ and A
- Q_s and target area are adjusted according to A
- Output: Parton 4-momentum etc
- Pythia afterburner → particles
- Jet reconstruction





- 1 < qt < 1.5
- 2 < Pt < 2.5
- pol=1 (L)

A. Dumitru, V. S. and T. Ullrich work in progress

First correction to correlation limit at small x I

- Lets return to general small x expression for dijet cross section

$$\frac{d\sigma^{\gamma^* A\to q\bar{q}X}}{d^3k_1 d^3k_2} =$$

$$N_c\alpha_{em}e_q^2\delta(p^+ - k_1^+ - k_2^+)\int \frac{d^2x_1}{(2\pi)^2}\frac{d^2x_2}{(2\pi)^2}\frac{d^2y_1}{(2\pi)^2}\frac{d^2y_2}{(2\pi)^2}\exp\left(-i\mathbf{k}_1(\mathbf{x}_1-\mathbf{y}_1)-i\mathbf{k}_2(\mathbf{x}_2-\mathbf{y}_2)\right)$$

$$\sum_{\gamma\alpha\beta}\psi_{\alpha\beta}^{\mathrm{T,L}\gamma}(\mathbf{x}_1-\mathbf{x}_2)\psi_{\alpha\beta}^{\mathrm{T,L}\gamma *}(\mathbf{y}_1-\mathbf{y}_2)\left[1+\frac{1}{N_c}\Big(\langle \mathrm{Tr}\, U(\mathbf{x}_1)U^\dagger(\mathbf{y}_1)U(\mathbf{y}_2)U^\dagger(\mathbf{x}_2)\rangle\right.$$

$$\left.-\langle \mathrm{Tr}\, U(\mathbf{x}_1)U^\dagger(\mathbf{x}_2)\rangle-\langle \mathrm{Tr}\, U(\mathbf{y}_1)U^\dagger(\mathbf{y}_2)\rangle\Big)\right] \quad \uparrow \text{Quadrupole contribution}$$

- For arbitrary $\mathbf{k}_1$ and $\mathbf{k}_2$, one expects presence of non-trivial $\langle\cos 2n\phi\rangle, n\in Z$
- First correction to correlation limit (suppressed by $1/P^2$) includes terms $\propto(\mathbf{q}\cdot\mathbf{P})^4$ and thus results in $\langle\cos 4\phi\rangle\neq 0$

First correction to correlation limit at small x II

- Derivation is tedious but straight forward (see details in 1605.02739)
- Expectation of Wilson lines

$$Q(\mathbf{x}_1, \mathbf{x}_2; \mathbf{x}_2', \mathbf{x}_1') = 1 + \frac{\langle \mathrm{Tr}\, U(\mathbf{x}_1) U^\dagger(\mathbf{x}_1') U(\mathbf{x}_2') U^\dagger(\mathbf{x}_2)\rangle - \langle \mathrm{Tr}\, U(\mathbf{x}_1) U^\dagger(\mathbf{x}_2)\rangle - \langle \mathrm{Tr}\, U(\mathbf{x}_1') U^\dagger(\mathbf{x}_2')\rangle}{N_c}$$

is expanded in series wrt $\mathbf{u} = \mathbf{x}_1 - \mathbf{x}_2$ and $u' = \mathbf{x}_1' - \mathbf{x}_2'$:

$$Q = u_i u_j' \mathcal{G}^{i,j}(v, v') + u_i u_j' u_k' u_l' \mathcal{G}^{i,jkl}(v, v') + u_i u_j u_k u_l' \mathcal{G}^{ijk,l}(v, v') + u_i u_j u_k' u_l' \mathcal{G}^{ij,kl}(v, v') + \cdots$$

- Only following combination is relevant (momentum space)

$$\mathcal{G}^{ijmn}(x, q^2) = \mathcal{G}^{i,jmn}(x, q^2) + \mathcal{G}^{ijm,n}(x, q^2) - \frac{2}{3}\mathcal{G}^{ij,mn}(x, q^2)$$

- $\mathcal{G}^{ijmn}(x, q^2)$ brings corrections to isotropic and $\langle cos 2\phi \rangle$, as well as non-trivial $\langle cos 4\phi \rangle$. I will concentrate only on $\langle cos 4\phi \rangle$.

First correction to correlation limit at small x III

- The amplitude $\propto \cos 4\phi$ is determined by

$$\Phi_2(x, q^2) = -\frac{2N_c}{\alpha_s} \mathfrak{P}_3^{ijmn} \mathcal{G}^{ijmn}(x, q^2) \, .$$

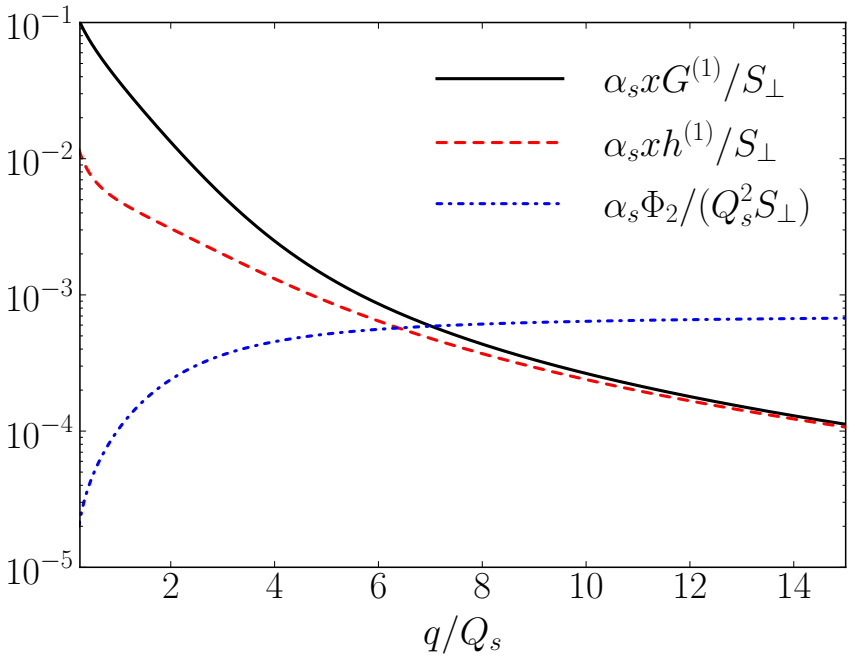
where $\mathfrak{P}_3^{ijmn}$ is projector extracting $\propto \cos 4\phi$

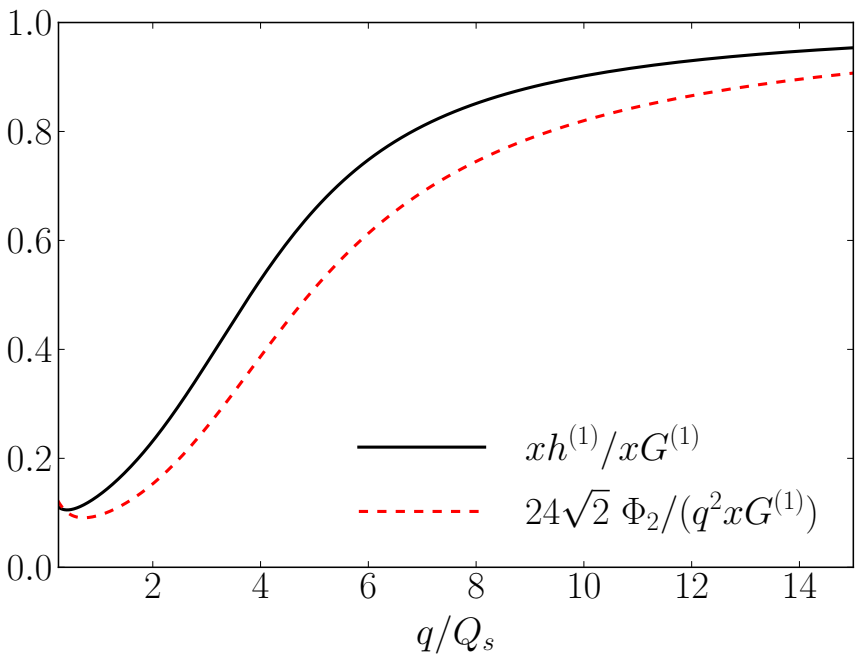
- For MV model

$$\Phi_2(q^2) = \frac{N_c}{\sqrt{2}\, 3\pi\alpha_s} \frac{S_\perp}{(2\pi)^2} \int \frac{d|r|}{|r|^3} J_4(|r|\,|q|) \left[\frac{2}{\ln \frac{1}{r^2\Lambda_{\rm IR}^2}} \left\{ 1 - \exp\left(-\frac{Q_s^2 r^2}{4} \log \frac{1}{r^2\Lambda_{\rm IR}^2} \right) \right\} \right.$$
$$\left. + \frac{5}{\ln^2 \frac{1}{r^2\Lambda_{\rm IR}^2}} \left\{ 1 - \exp\left(-\frac{Q_s^2 r^2}{4} \log \frac{1}{r^2\Lambda_{\rm IR}^2} \right) \left[1 + \frac{Q_s^2 r^2}{4} \log \frac{1}{r^2\Lambda_{\rm IR}^2} \right] \right\} \right]$$

- $\Phi_2(q^2)$ is positive-definite function
- Limiting cases:
  $\Lambda_{\rm IR} \ll q \ll Q_s \quad \Phi_2(q^2) \sim (N_c/\alpha_s \log Q_s^2/\Lambda_{\rm IR}^2)\, S_\perp q^2$
  $q \gg Q_s, \Phi_2(q^2) \to (N_c/\sqrt{2}\, 24\pi\alpha_s)\, (S_\perp/4\pi^2)\, Q_s^2$

MV results





These functions determine the amplitudes of the $\cos 2n\phi$ contributions to the dijet angular distributions for $n = 0, 1, 2$, respectively.

A. Dumitru and V. S., arXiv:1605.02739

DiJet cross section

DiJet cross section to this order

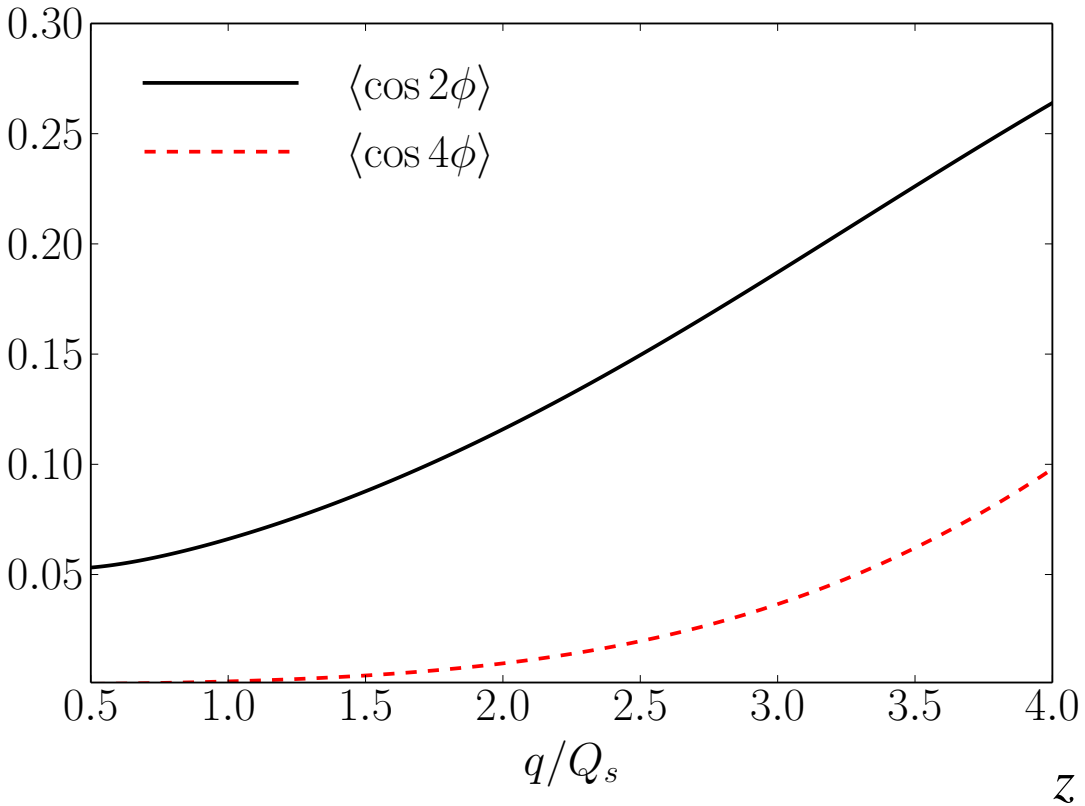
$$\frac{d\sigma^{\gamma_T^* A\to q\bar{q}X}}{d^2k_1 dz_1 d^2k_2 dz_2} = \alpha_s\alpha_{em}e_q^2\left(z_1^2+z_2^2\right)\left[\frac{P^4+\epsilon_f^4}{(P^2+\epsilon_f^2)^4}\left(xG^{(1)}(x,q^2)-\frac{2\epsilon_f^2P^2}{P^4+\epsilon_f^4}xh^{(1)}(x,q^2)\cos 2\phi+O\left(\frac{1}{P^2}\right)\right) - \frac{48\epsilon_f^2P^4}{\sqrt{2}\,(P^2+\epsilon_f^2)^6}\Phi_2(x,q^2)\cos 4\phi\right]$$

$$\frac{d\sigma^{\gamma_L^* A\to q\bar{q}X}}{d^2k_1 dz_1 d^2k_2 dz_2} = 8\alpha_s\alpha_{em}e_q^2 z_1 z_2\epsilon_f^2\left[\frac{P^2}{(P^2+\epsilon_f^2)^4}\left(xG^{(1)}(x,q^2)+xh^{(1)}(x,q^2)\cos 2\phi+O\left(\frac{1}{P^2}\right)\right) + \frac{48P^4}{\sqrt{2}\,(P^2+\epsilon_f^2)^6}\Phi_2(x,q^2)\cos 4\phi\right].$$

A. Dumitru and V. S., arXiv:1605.02739

MV results

$\langle \cos 2\phi \rangle$ and $\langle \cos 4\phi \rangle$ in $\gamma_L^* + A \to q + \bar{q}$ dijet production from MV model:



$z = 1/2$, $P = 4.5 Q_s$

A. Dumitru and V. S., arXiv:1605.02739

Conclusions

In correlation limit:

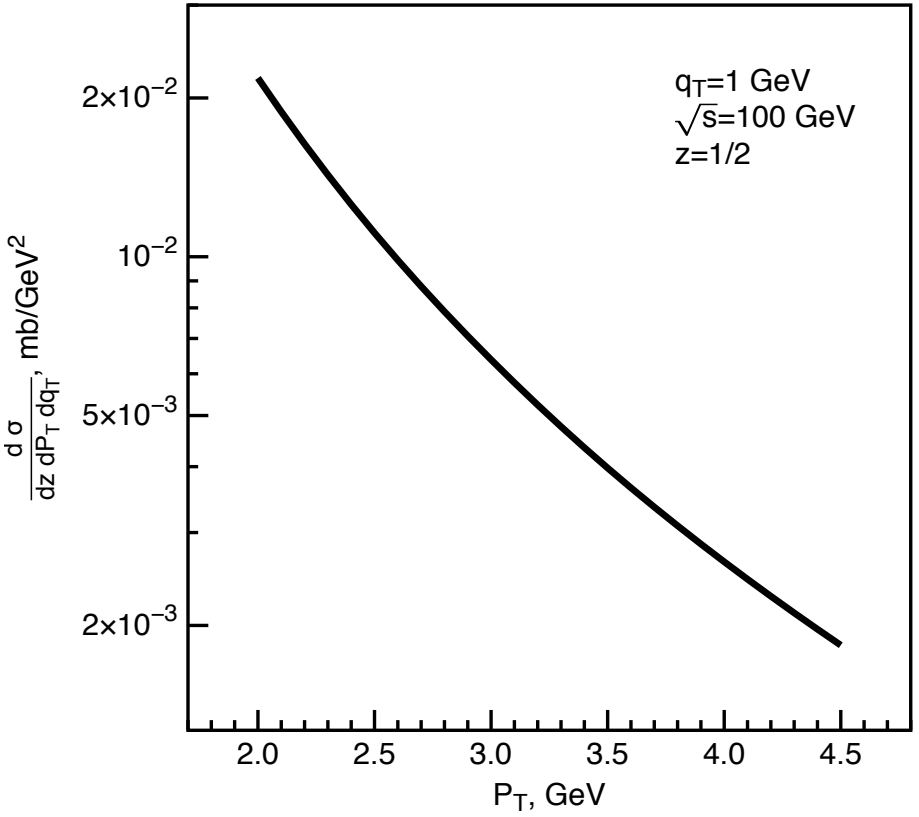
- DIS dijets to probe WW gluon distribution
- Classical McLerran-Venugopalan model gives large relative anisotropy at large momentum, both $G^{(1)}$ and $h_\perp^{(1)}$ are proportional to $1/q_\perp^2$
- Small x evolution in JIMWLK-B: $h^{(1)}$ grows as fast as $G^{(1)}$
- Not significant dependence on prescription for α_s
- Simulations and backgrounds: anisotropy is present in MC events summed over polarization and different distributions of q, z, $P_\perp$, $q_\perp$ etc. It is also present after including Pythia shower

First correction to correlations limit:

- Nontrivial contribution to $\langle \cos 4\phi \rangle$
- Corresponding amplitude has distinct dependence on q: constant at large q, proportional to q^2 at small q.
- As expected this contribution is suppressed by $1/P^2$ at nearly correlation limit.

Cross-section for signal

- Cross-section summed with respect to γ^* polarizations and integrated over angles
- $\sqrt{s}$ is given for $\gamma^* A$ CM



A. Dumitru, and V. S., 2015

Physical interpretation

- Conventional WW: probability distribution

$$\delta_{ij} = \varepsilon_+^{*i}\varepsilon_+^j + \varepsilon_-^{*i}\varepsilon_-^j$$

- Gluon helicity: difference of probability distributions

$$i\epsilon_{ij} = \varepsilon_+^{*i}\varepsilon_+^j - \varepsilon_-^{*i}\varepsilon_-^j$$

- $h^{(1)}$: transverse spin correlation function of gluons in two orthogonal polarization states

$$2\frac{q^i q^j}{q^2} - \delta^{ij} = i(\varepsilon_+^{*i}\varepsilon_-^j - \varepsilon_-^{*i}\varepsilon_+^j)$$

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Weizsäcker-Williams gluon distribution

- Contribution to azimuthal anisotropy of dijet production

$$E_1 E_2 \frac{d\sigma^{\gamma_L^* A \to q\bar{q}X}}{d^3k_1 d^3k_2 d^2b} = \alpha_{em} e_q^2 \alpha_s \delta\left(x_{\gamma^*} - 1\right) z^2 (1-z)^2 \frac{8\epsilon_f^2 P_\perp^2}{(P_\perp^2 + \epsilon_f^2)^4}$$

$$\times \left[xG^{(1)}(x, q_\perp) \; + \; \underline{\cos(2\phi)} \; xh_\perp^{(1)}(x, q_\perp) \right] .$$

$$E_1 E_2 \frac{d\sigma^{\gamma_T^* A \to q\bar{q}X}}{d^3k_1 d^3k_2 d^2b} = \alpha_{em} e_q^2 \alpha_s \delta\left(x_{\gamma^*} - 1\right) z(1-z)(z^2 + (1-z)^2) \frac{\epsilon_f^4 + P_\perp^4}{(P_\perp^2 + \epsilon_f^2)^4}$$

$$\times \left[xG^{(1)}(x, q_\perp) \; - \; \frac{2\epsilon_f^2 P_\perp^2}{P_\perp^4 + \epsilon_f^4} \; \underline{\cos(2\phi)} \; xh_\perp^{(1)}(x, q_\perp) \right] .$$

z is long. momentum fraction of photon carried by quark $\qquad \epsilon_f^2 = z(1-z)Q^2$