RESUMMATION IN PDFs

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My personal (fairly impressive) record:

Argonne 3 - Fermilab 0

INTRODUCTION

A WORD ON PDFs

$$\sigma(x,Q) = \sigma_0 C\left(\frac{x}{x_1 x_2}, \alpha_s(\mu)\right) \otimes f_1(x_1,\mu) \otimes f_2(x_2,\mu)$$

Measure

Compute

Extract

- coefficient functions (NLO,NNLO, N^3LO)
- parton evolution (NLO,NNLO)
- EW corrections
- quark mass effects
- target-mass corrections
- …

LARGE LOGS IN PERTURBATIVE QCD

- Real emissions diagrams are singular for soft/collinear emissions
- These singularities are cancelled by virtual counterparts
- Finite logarithmic pieces are left over, e.g.

$$V = \kappa$$

$$\alpha_s \int \frac{d\theta}{\theta}\frac{dz}{z}\Theta\left(\kappa - z\theta\right) - \alpha_s \int \frac{d\theta}{\theta}\frac{dz}{z} = -\alpha_s \int \frac{d\theta}{\theta}\frac{dz}{z}\left[\Theta\left(z\theta - \kappa\right)\right]$$

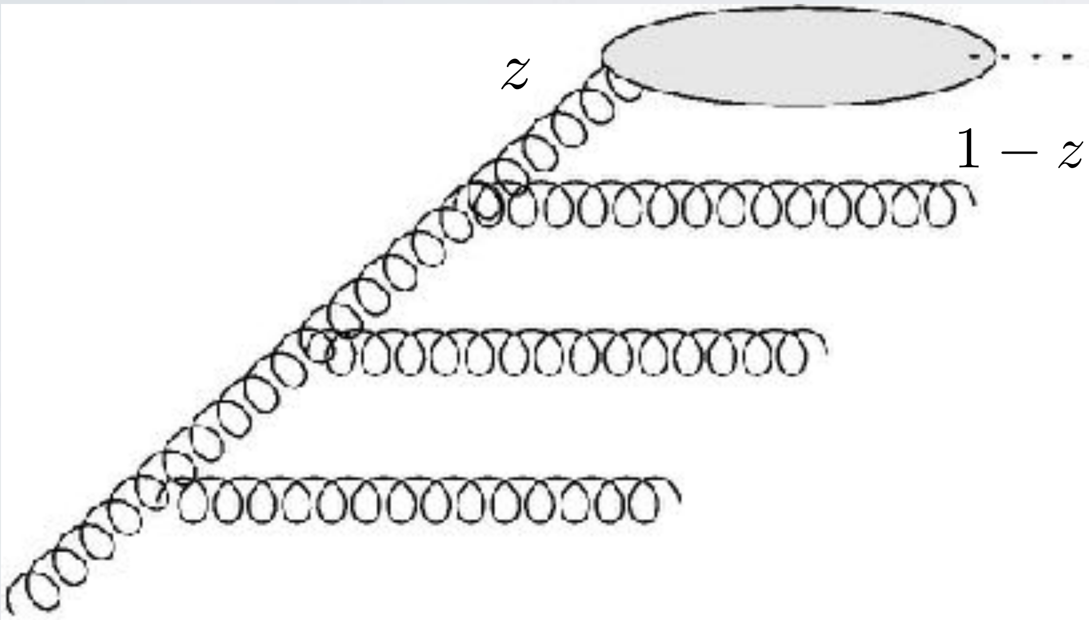
real emission **virtual correction**

$$= -\frac{1}{2}\alpha_s \ln^2 \kappa$$

- These corrections are important for observables *V* that insist on only small deviations from lowest order kinematics (V~0)
- Real radiation is constrained to a small corner of phase space and the logarithms are large
    - event (jet) shapes, e.g. thrust (jet mass): $V=1-T$ ($V= m_{jet}/p_T$)
    - production at threshold: $V=1-M^2/s$
    - transverse momentum: $V= p_T/M$ ...

THRESHOLD & HIGH ENERGY

- higher-order corrections correspond to the emission of extra quarks/gluons



$$\sim \quad \alpha_s^k \left[\frac{\log^{2k-1}(1-z)}{1-z} \right]_+ , \; z \to 1$$

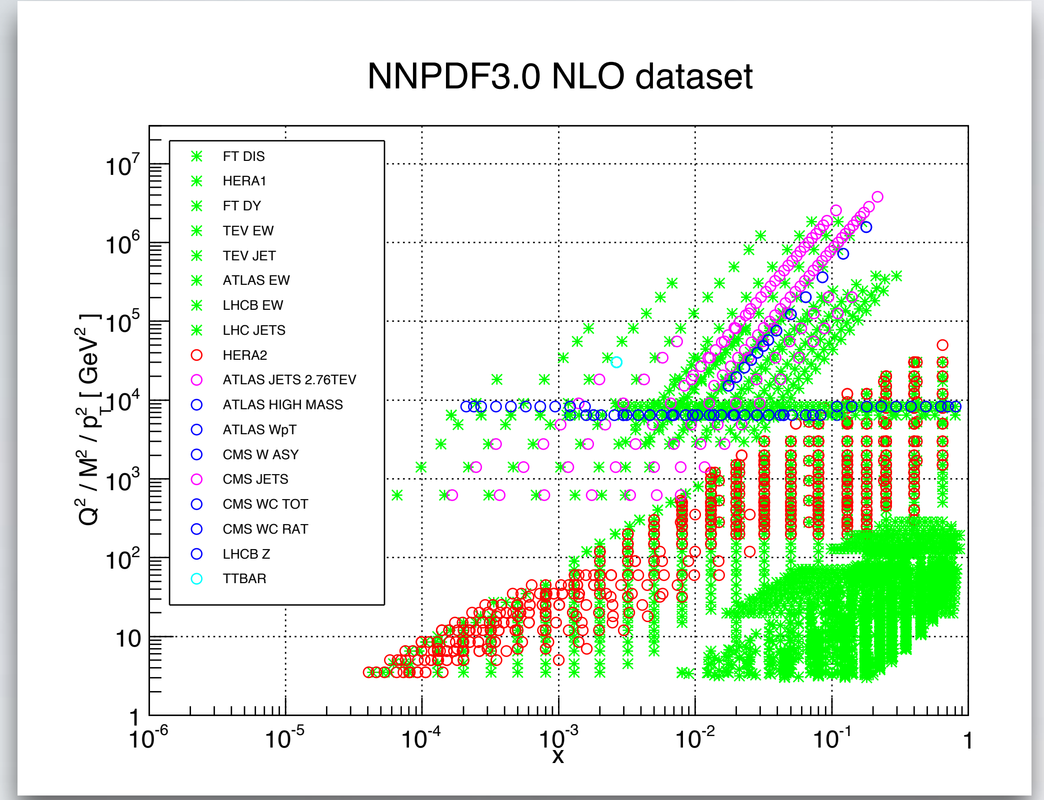
$$\alpha_s^k \frac{\log^{k-1} z}{z} , \; z \to 0$$

We will most conveniently work in Mellin space

Soft-gluon resummation: $z \to 1$ ➪ logs of N

BFKL resummation: $z \to 0$ ➪ poles in N (typically at $N=0$)

RESUMMATION AND PDFs



- Current data sets span several order of magnitude in Q^2 and x
- Do we trust FO everywhere?
- Is it ok to use standard PDFs with resummed calculation?

Resum what?

Observable: $$\sigma = \sigma_0\, C(\alpha_s(\mu)) \otimes f(\mu) \left[\otimes f(\mu)\right]$$

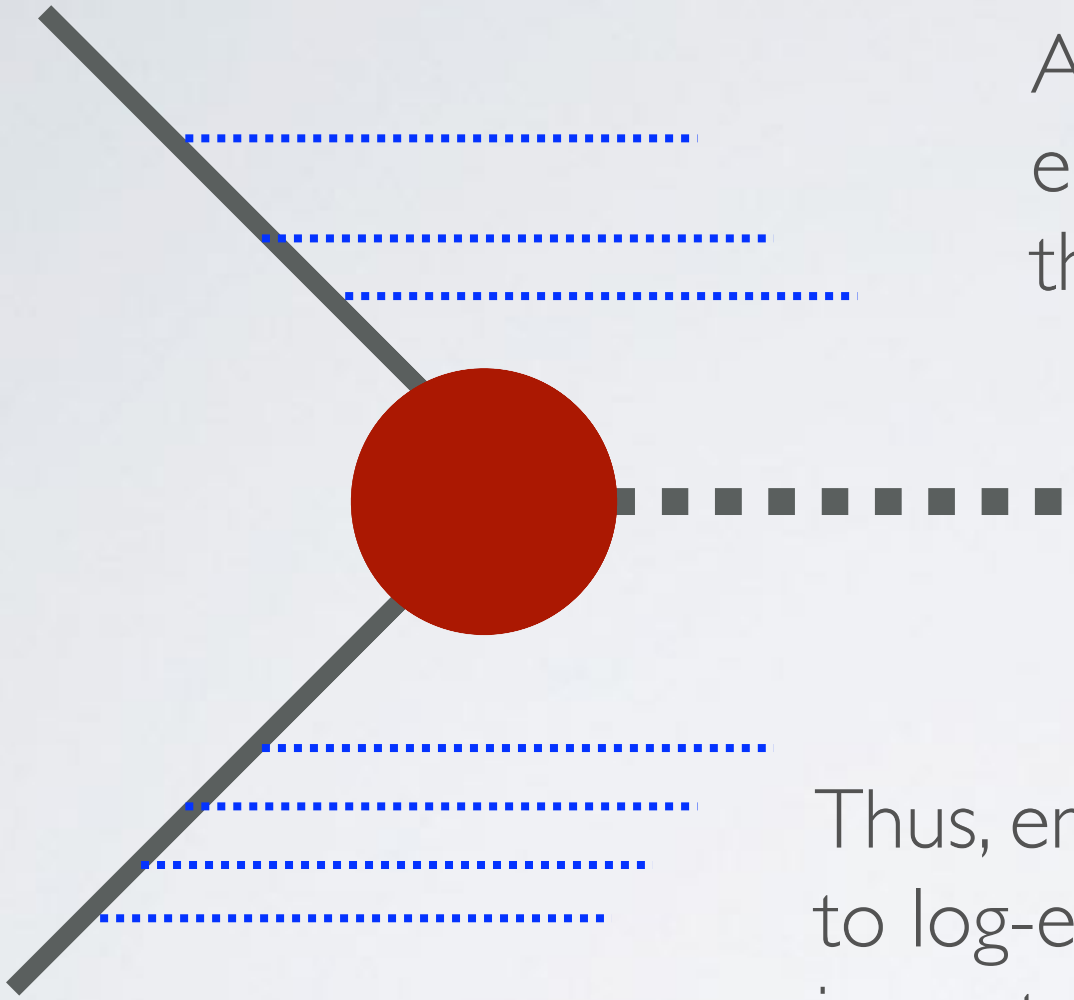
Evolution: $$\mu^2 \frac{d}{d\mu^2} f(\mu) = P(\alpha_s(\mu)) \otimes f(\mu)$$

	observable coefficient functions $C(\alpha_s(\mu))$	evolution splitting functions $P(\alpha_s(\mu))$
large-x	(N)NNLL	—
small-x	LLx	NLLx

- We work in collinear factorization
- We will distinguish between coefficient functions and splitting functions

THRESHOLD (LARGE-X) RESUMMATION

PRODUCTION AT THRESHOLD



Absolute threshold: the initial-state energy is just enough to produce the final state with invariant mass Q

$$x = \frac{Q^2}{s} \to 1$$

Thus, emissions are forced to be soft, leading to log-enhanced contributions order-by-order in perturbation theory

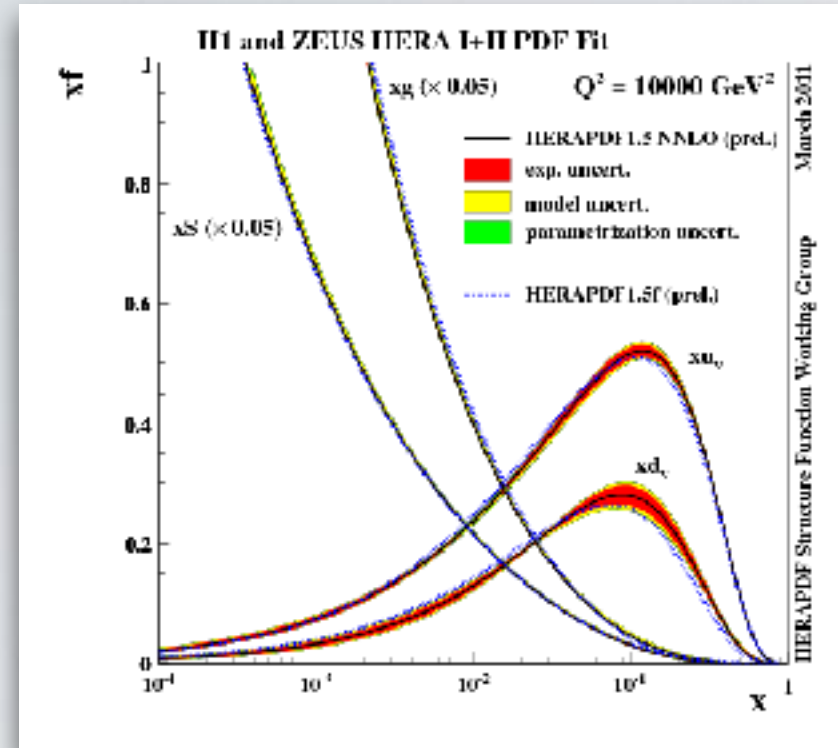
$$\text{LO} : Q^2 = \hat{s}$$

$$\text{beyond LO} : Q^2 = z\hat{s}$$

$$C(z, \alpha_s) \sim \sigma_0 \sum_{n=1} \sum_{k=-1}^{2n-1} \alpha_s^n \left[\frac{\ln^k(1-z)}{1-z} \right]_+$$

$\delta(1-z)$

WHY THRESHOLD AT THE LHC ?



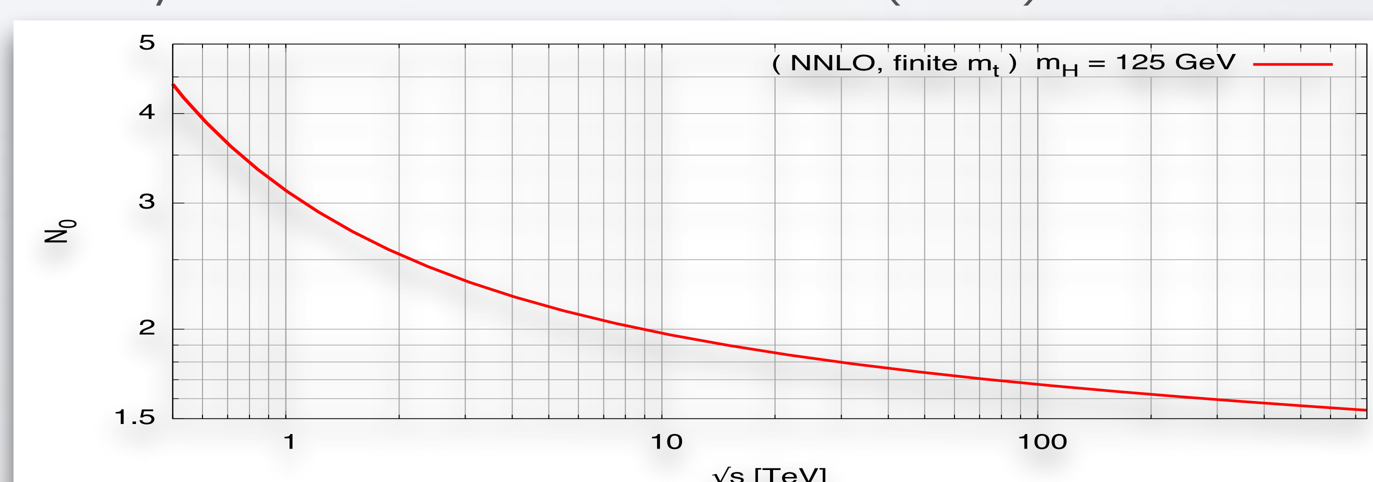
Gluon PDF shows a steep increase at low *x*

$$\hat{s} = x_1 x_2 s$$

Region of partonic threshold is enhanced in the convolution

- More precise argument in Mellin space: a saddle-point approximation indicates the region that gives the bulk of the contribution to the inverse Mellin integral.
- This region turns out to be fairly narrow around the (real) saddle-point.

Bonvini, Forte Ridolfi (2012)



THRESHOLD RESUMMATION

Momentum space: singular and distributional terms for $z \to 1$

Mellin space: terms that do not vanish at large N

$$C_{\rm res}(N,\alpha_s) = \bar{g}_0\big(\alpha_s,\mu_{\rm F}^2\big)\exp\bar{\mathcal{S}}(\alpha_s,N),$$

$$\bar{\mathcal{S}}(\alpha_s,N) = \int_0^1 dz\,\frac{z^{N-1}-1}{1-z}\left(\int_{\mu_{\rm F}^2}^{m_{\rm H}^2\frac{(1-z)^2}{z}}\frac{d\mu^2}{\mu^2}2A\big(\alpha_s(\mu^2)\big)+D\big(\alpha_s([1-z]^2m_{\rm H}^2)\big)\right),$$

$$\bar{g}_0(\alpha_s,\mu_{\rm F}^2) = 1+\sum_{k=1}^{\infty}\bar{g}_{0,k}(\mu_{\rm F}^2)\alpha_s^k,$$

Anastasiou *et al.* (2014)

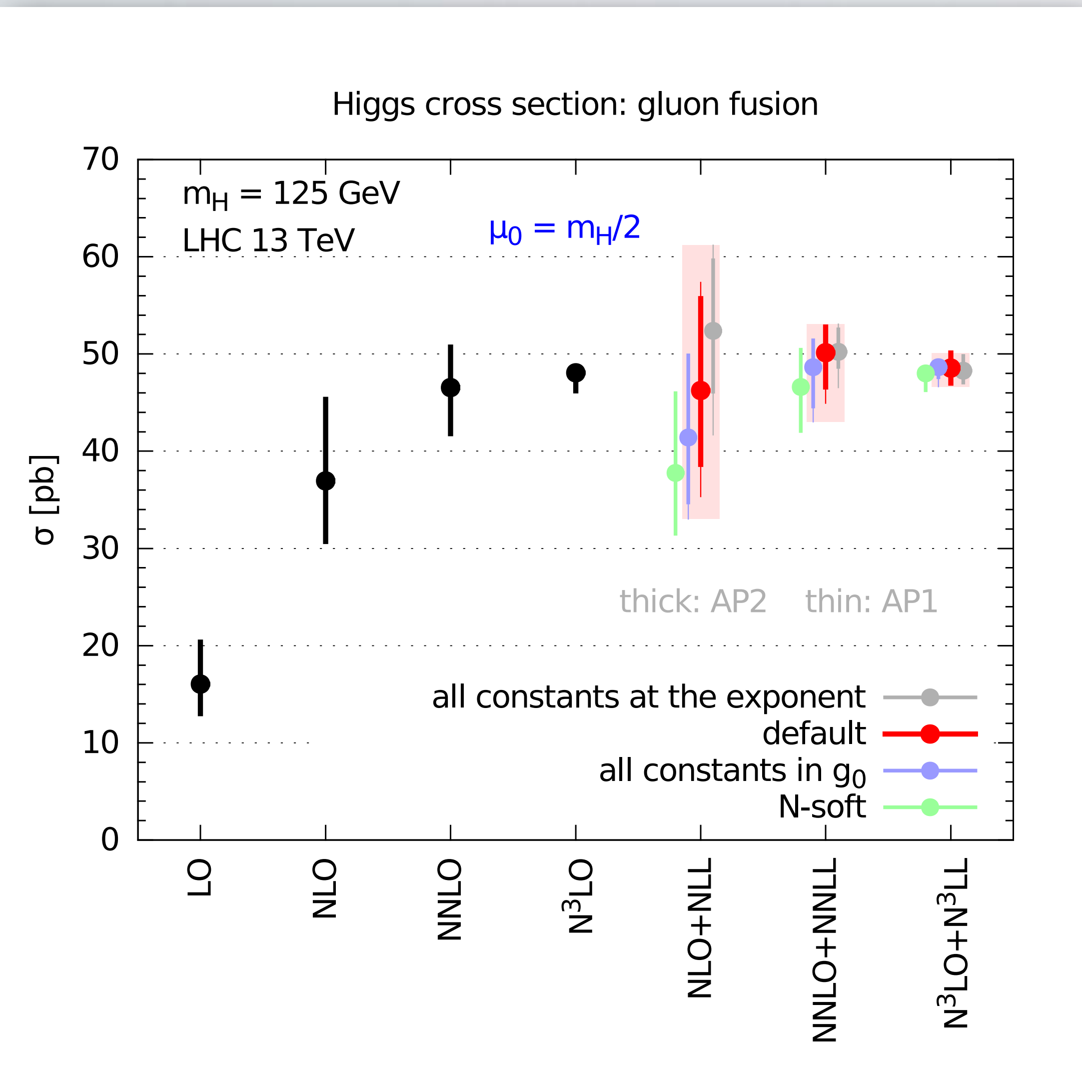
$$A(\alpha_s) = \sum_{k=1}^{\infty}A_k\alpha_s^k, \qquad D(\alpha_s) = \sum_{k=1}^{\infty}D_k\alpha_s^k,$$

Catani *et al.* (2002)
Moch, Vogt (2005)
Laenen, Magnea (2005) [...]

- constants can go in the exponent of in front of it
- state of the art N^{3}LL (but the 4-loop cusp)
- next-to-eikonal can be important (e.g. $(1\text{-}z)^2/z$)
- systematic studies underway

Laenen *et al.* (2015, 2016),
Larkoski, Neill, Stewart (2015)

AN EXAMPLE: HIGGS IN GLUON FUSION



- resummed (and matched) converges faster than pure FO
- resummation is perturbative, i.e. captures the effect of the first few orders, so that N^{3}LO+N^{3}LL ~ N^{3}LO
- they provide further handles to estimate uncertainty from missing higher orders (e.g. subleading logs)

order	σ [pb]	
N^{3}LO	$48.1^{+0.1}_{-1.8}$	scale variation
N^{3}LO	48.1 ± 2.0	$\overline{\text{CH}}$ at 95% DoB
N^{3}LO+N^{3}LL	48.5 ± 1.9	scale+resummation variations
all-order estimate	48.7	from accelerated fixed-order series
all-order estimate	48.9	from accelerated resummed series

N^{3}LO: Anastasiou *et al.* (2015)

Bonvini, SM (2014),
Bonvini, SM, Muselli, Rottoli (2016)

see also Catani *et al.* (2014), Ahmed *et al.* (2014/2015), Schmidt and Spira (2015), …
also Becher *et al.* in SCET

PDFs AT LARGE X

$$\sigma(x,Q) = \sigma_0 C\left(\frac{x}{x_1 x_2}, \alpha_s(\mu)\right) \otimes f_1(x_1,\mu) \otimes f_2(x_2,\mu)$$

Bonvini, SM +
subset of NNPDF (2015)

- coefficient functions contain large-x logs
- PDF evolution doesn't (in MSbar)

$$P_{gg}(x) \sim \frac{A(\alpha_s)}{(1-x)_+}$$

Process	observable	resummation available
DIS	$d\sigma/dx/dQ^2$ (NC, CC, charm, ...)	YES
DY Z/γ	$d\sigma/dM^2/dY$	YES
DY W	differential in the lepton kinematics	NO
$t\bar{t}$	total σ	YES
jets	inclusive $d\sigma/dp_t/dY$	YES/NO

it should be easy to compute

two different calculations exist at NLL$^{(*)}$ but no public implementation

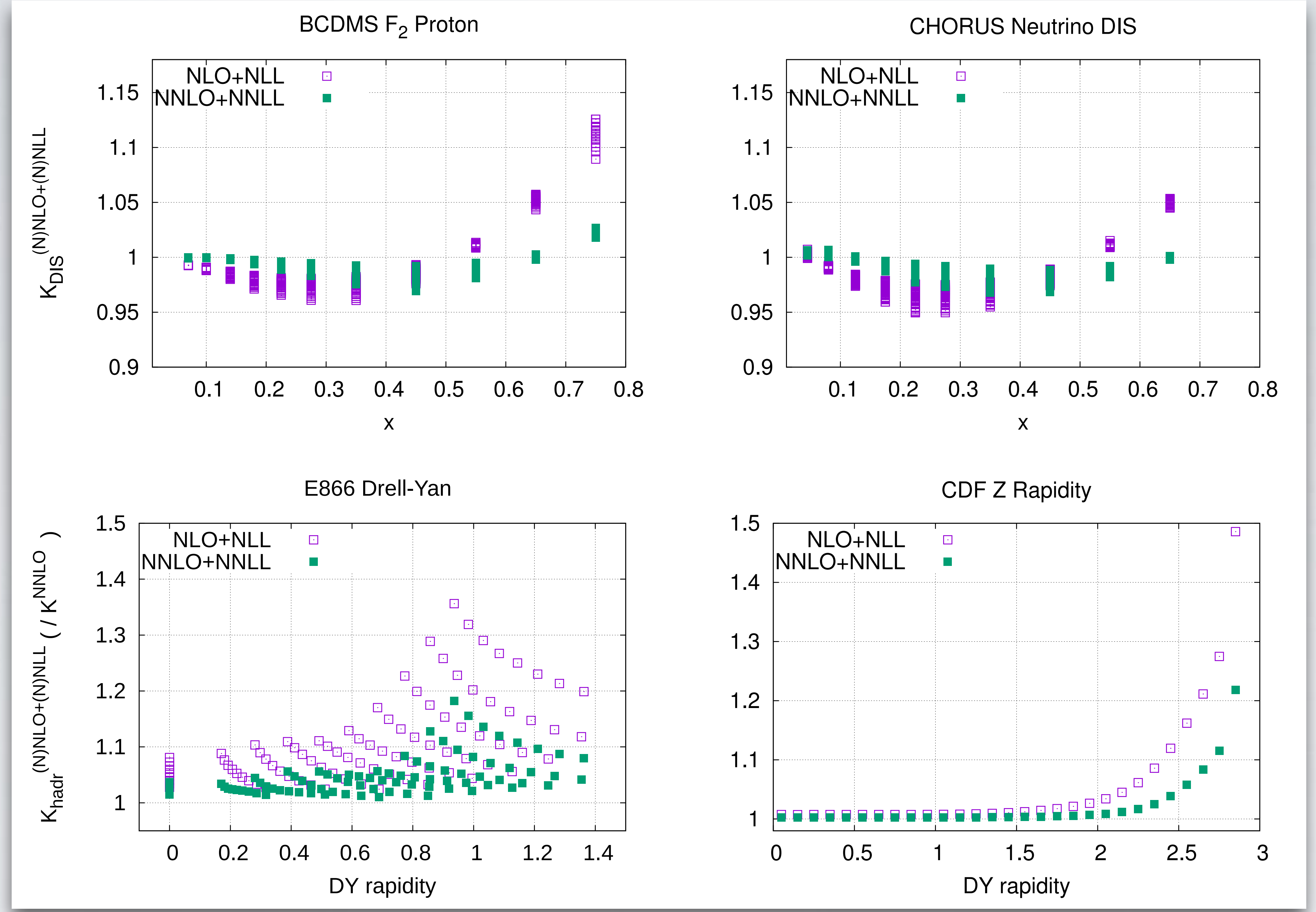
- performing a resummed fit is relatively straightforward
- data set is restricted: no jets
- $^{(*)}$global vs non-global

de Florian, Vogelsang (2007, 2013)
Kidonakis, Owens (2000)

DIS, DY available from TROLL (TROLL Resums Only Large-x Logarithms)
www.ge.infn.it/~bonvini/troll

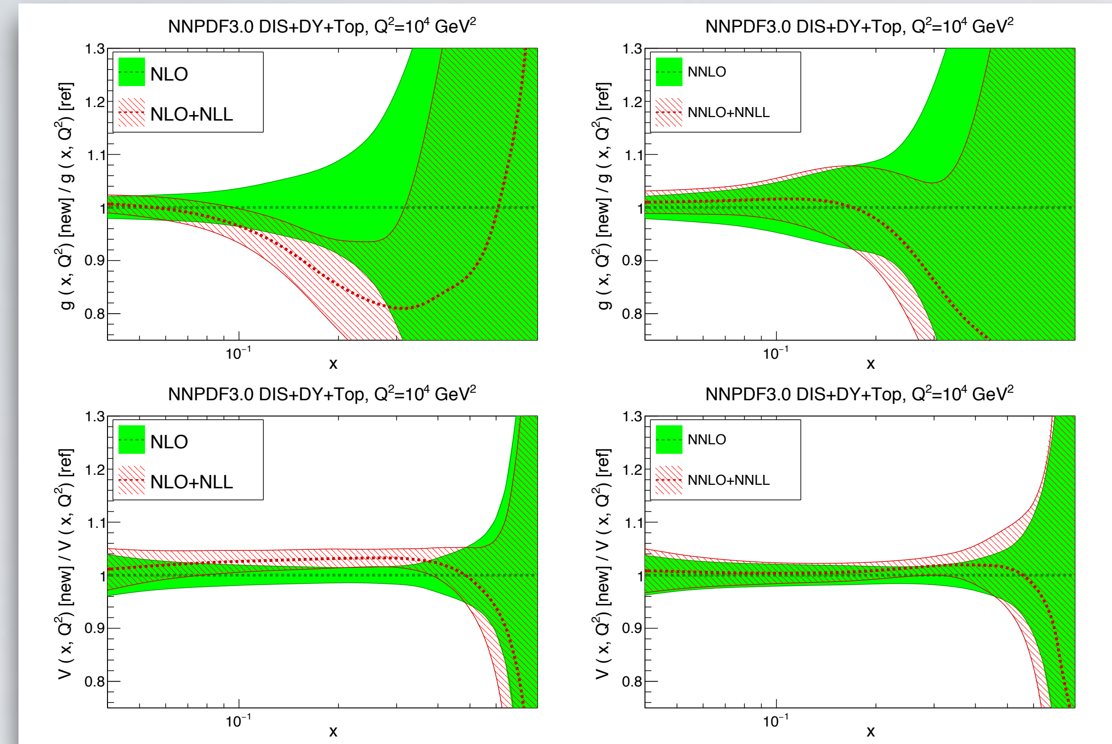
$t\bar{t}$ available from top++ www.alexandermitov.com/software

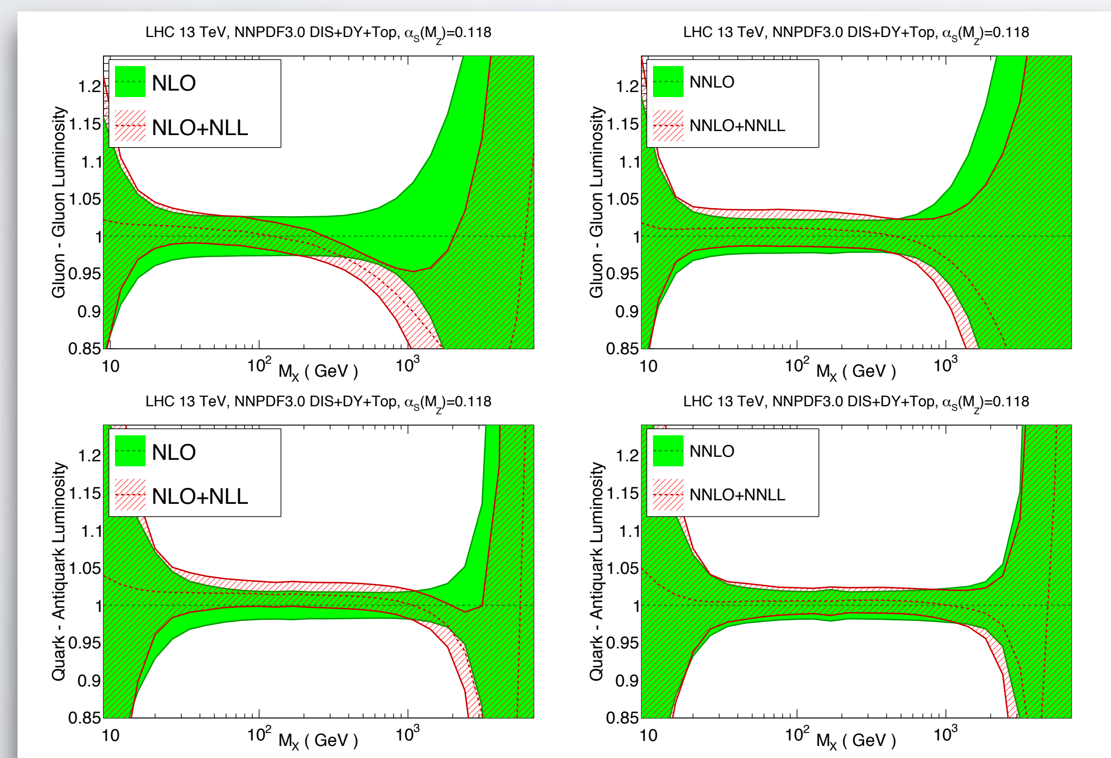
EFFECTS ON THEORY PREDICTIONS



K-factors reduced when NNLO is included (resummation is perturbative)

PDFs WITH THRESHOLD RESUMMATION



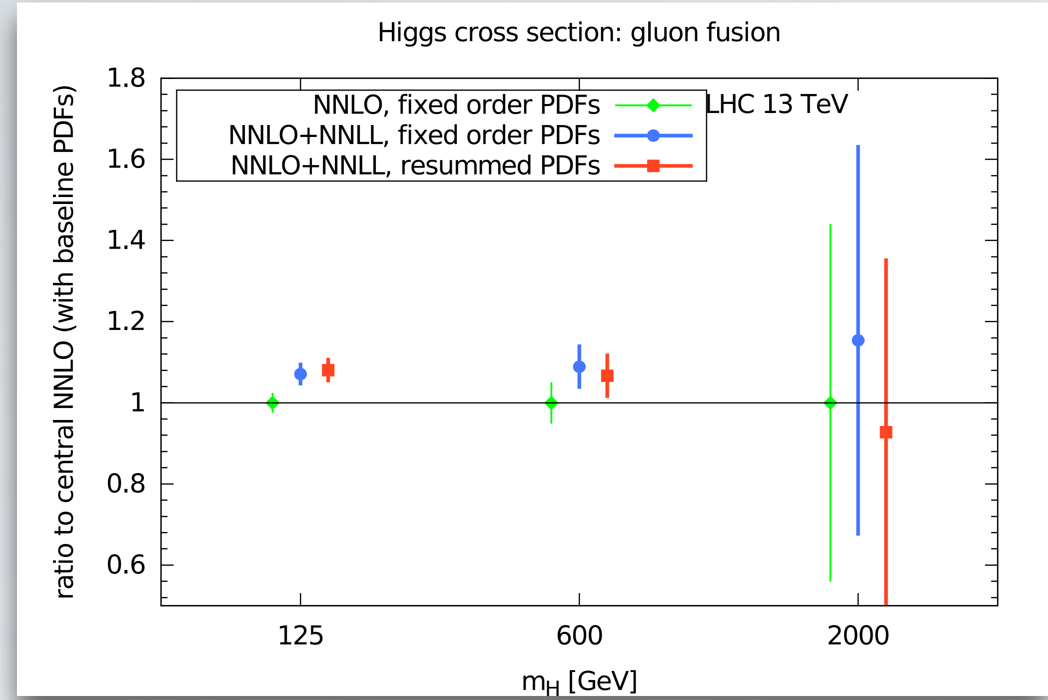


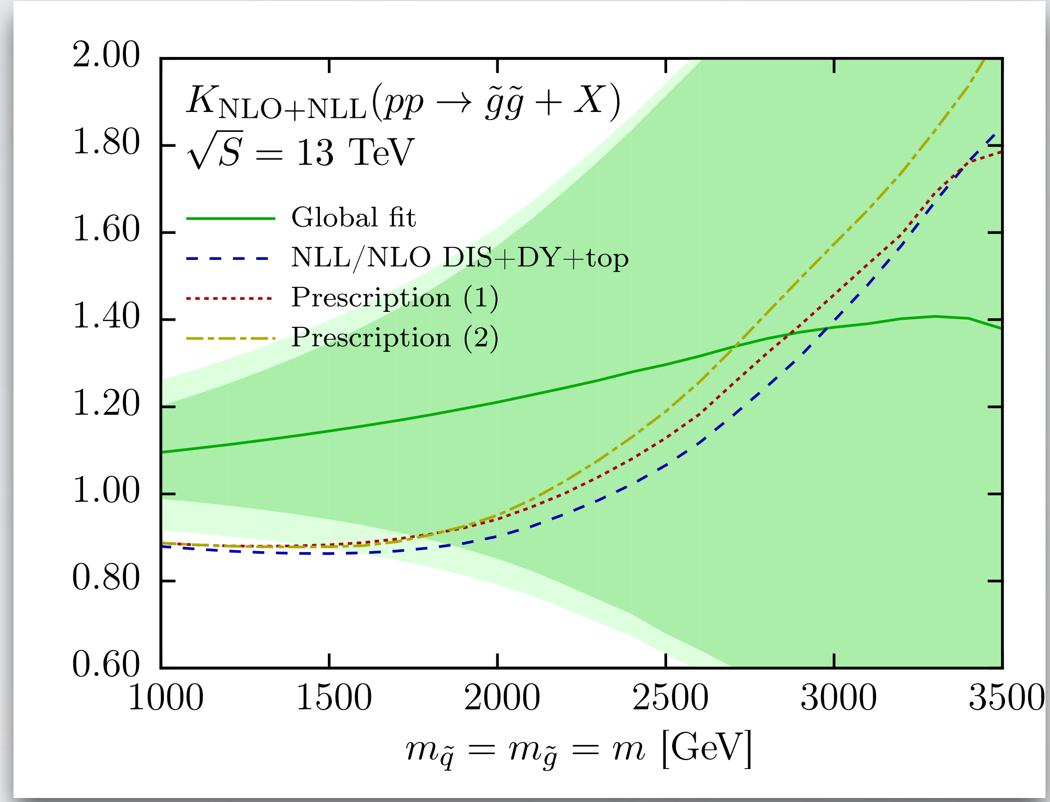
Experiment	NNPDF3.0 DIS+DY+top			
	NLO	NNLO	NLO+NLL	NNLO+NNLL
NMC	1.39	1.34	1.36	1.30
SLAC	1.17	0.91	1.02	0.92
BCDMS	1.20	1.25	1.23	1.28
CHORUS	1.13	1.11	1.10	1.09
NuTeV	0.52	0.52	0.54	0.44
HERA-I	1.05	1.06	1.06	1.06
ZEUS HERA-II	1.42	1.46	1.45	1.48
H1 HERA-II	1.70	1.79	1.70	1.78
HERA charm	1.26	1.28	1.30	1.28
DY E866	1.08	1.39	1.68	1.68
DY E605	0.92	1.14	1.12	1.21
CDF Z rap	1.21	1.38	1.10	1.33
D0 Z rap	0.57	0.62	0.67	0.66
ATLAS Z 2010	0.98	1.21	1.02	1.28
ATLAS high-mass DY	1.85	1.27	1.59	1.21
CMS 2D DY 2011	1.22	1.39	1.22	1.41
LHCb Z rapidity	0.83	1.30	0.51	1.25
ATLAS CMS top prod	1.23	0.55	0.61	0.40
Total	1.233	1.264	1.246	1.269

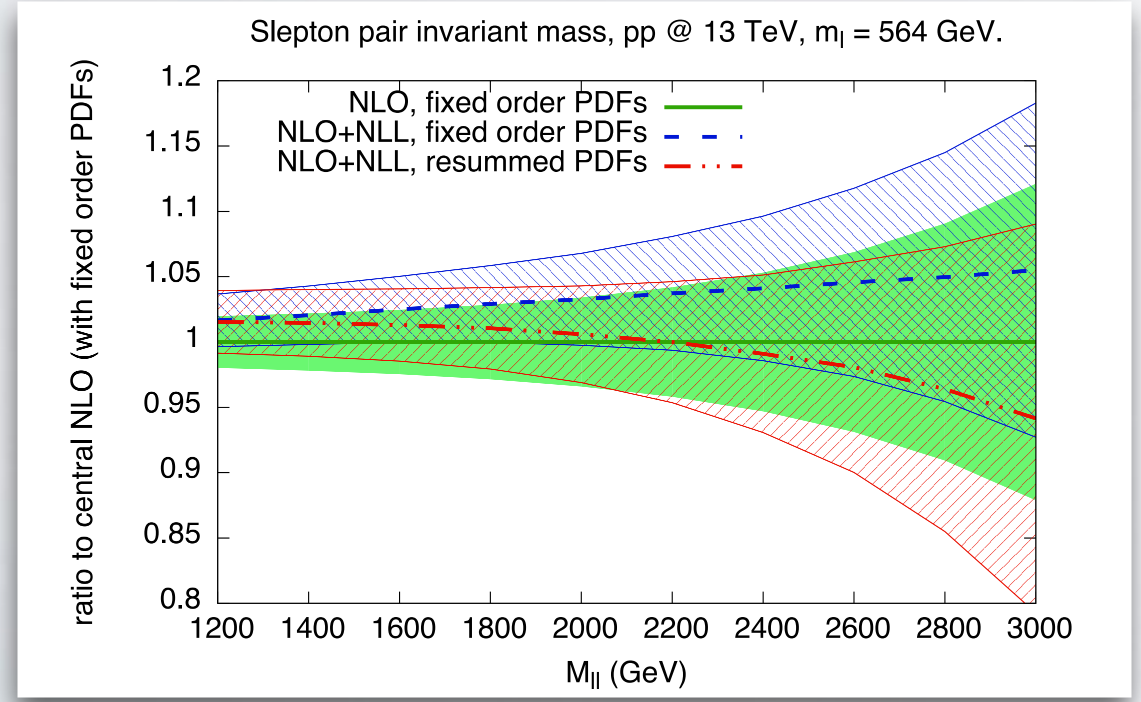
- as expected: visible effects at NLL+NLO are very much reduced at NNLO+NNLL
- χ^2 slightly worse because of DY fixed-target experiments

PHENOMENOLOGY

Bonvini *et al.* (2015), Beenakker *et al.* (2015)







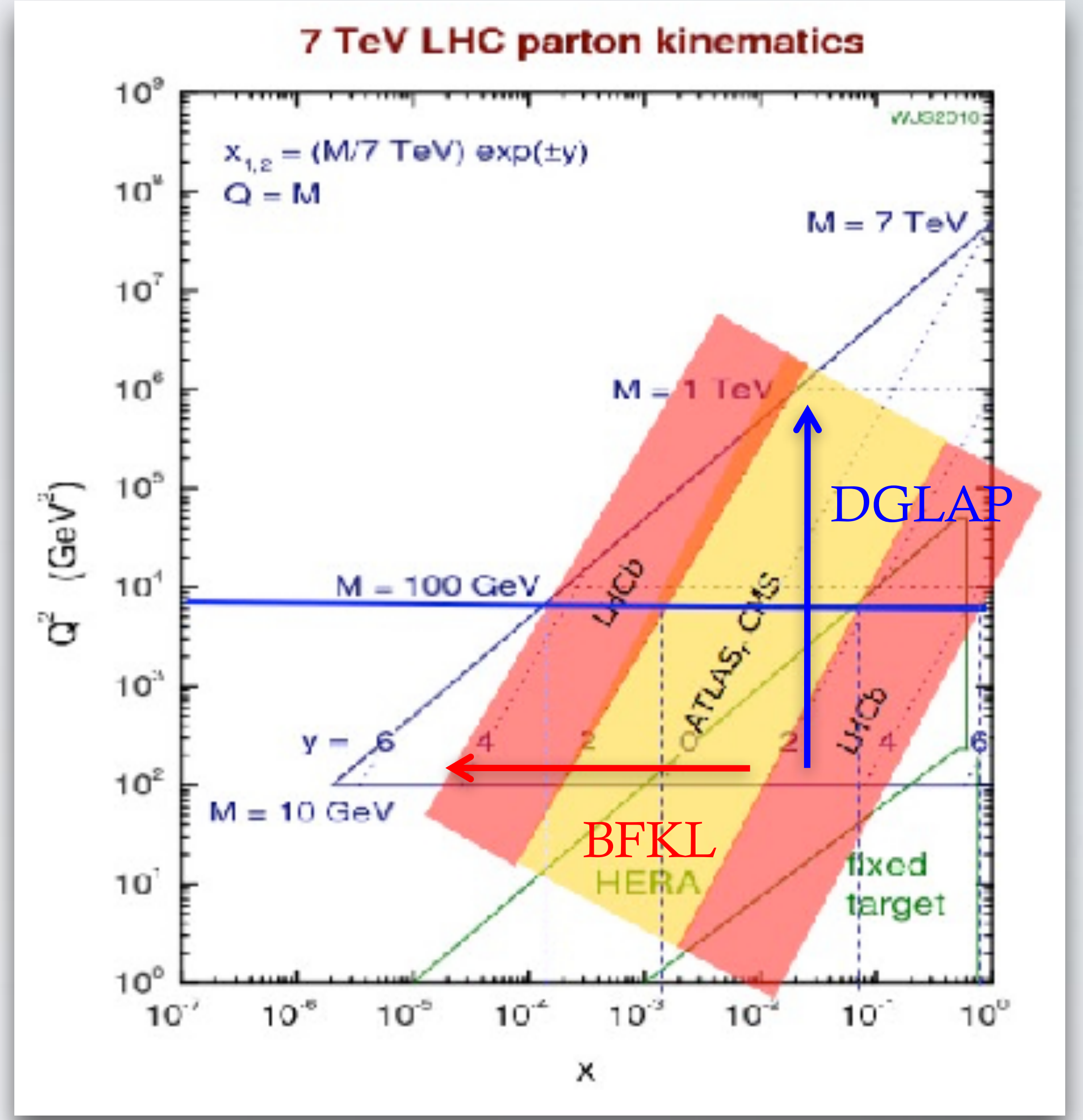
Take-home messages

- effects on SM Higgs negligible
- more pronounced for high-mass states, still within PDF errors
- resummed PDFs not (yet) competitive because of missing jet data

HIGH-ENERGY (SMALL-X) RESUMMATION

LHC KINEMATICS

- PDFs are largely unconstrained at low-x
- LHC does probe this region
- Is DGLAP enough to describe this region?
- Do we need BFKL?



DGLAP: Q^2 evolution for N moments of the parton density

$$\frac{d}{d\ln(Q^2/\mu^2)}G(N,Q^2) = \gamma(N,\alpha_s)G(N,Q^2)$$

BFKL: small-x evolution for M moments of the parton density

$$\frac{d}{d\ln(1/x)}G(x,M) = \chi(M,\alpha_s)G(x,M)$$

Mellin moments:

logs $\leftrightarrow$ poles

$$\ln^k \frac{Q^2}{\mu^2} \leftrightarrow \frac{1}{M^{k+1}}$$

$$\ln^k \frac{1}{x} \leftrightarrow \frac{1}{N^{k+1}}$$

	observable coefficient functions $C(\alpha_s(\mu))$	evolution splitting functions $P(\alpha_s(\mu))$
large-x	(N)NNLL	—
small-x	LLx	NLLx

BASICS OF HIGH-ENERGY RESUMMATION

- Evolution:
  - one constructs a resummed anomalous dimension from the BFKL kernel at next-to-leading log
  - however naive procedure leads to results not supported by HERA data *(too strong and too soon)*

Altarelli, Ball, Forte; Ciafaloni, Colferai, Salam, Stasto; White, Thorne.

- Coefficient functions:
  - the resummation here is known at leading log level
  - first developed for heavy quarks and DIS

Catani, Ciafaloni, Hautmann (1991); Catani, Hautmann (1994)

  - (subleading) running coupling terms are important Ball (2008)

- Recent work in SCET and work in progress for DIS applications

Rothstein and Stewart (2016) Pathak *at al.* in progress

SMALL-X PHENOMENOLOGY

- Problem studied by different groups in late ‘90s /early ‘00s
- The main interest was DIS at low x

  for a comparative review see HERA-LHC Proc. arXiv:0903.3861

- Key ingredients:
  - stable solution of the running coupling BFKL equation
  - match to standard DGLAP at large N (x)
  - important subleading effects
  - resummed coefficient functions (DIS, DY, HQ…)

Little phenomenology because a comprehensive code was missing

Small-x resummation from HELL

Marco Bonvini,[1a], Simone Marzani[2b], and Tiziano Peraro[3c]

Bonvini, SM, Peraro (2016)

HIGH ENERGY LARGE LOGARITHMS

https://www.ge.infn.it/~bonvini/hell/

Splitting function resummation

Ingredients (ABF):

- duality with BFKL evolution
- symmetry of the BFKL kernel
- momentum conservation
- resummation of (subleading, but fundamental) running coupling effects

- essentially follows ABF with stability improvements

Coefficient function resummation

High-energy (k_T) factorization:

$$\sigma \propto \int \frac{dz}{z} \int d^2\boldsymbol{k}\, \hat{\sigma}_g\left(\frac{x}{z}, \frac{Q^2}{\boldsymbol{k}^2}, \alpha_s(Q^2)\right) \mathcal{F}_g(z, \boldsymbol{k}) \qquad \begin{cases} \mathcal{F}_g(x, \boldsymbol{k}) : \text{unintegrated PDF} \\ \hat{\sigma}_g\left(z, \frac{Q^2}{\boldsymbol{k}^2}, \alpha_s\right) : \text{off-shell xs} \end{cases}$$

Defining

$$\mathcal{F}_g(N, \boldsymbol{k}) = U\left(N, \frac{\boldsymbol{k}^2}{\mu^2}\right) f_g(N, \mu^2)$$

we get [MB,Marzani,Peraro 1607.02153]

$$C_g(N, \alpha_s) = \int d^2\boldsymbol{k}\, \hat{\sigma}_g\left(N, \frac{Q^2}{\boldsymbol{k}^2}, \alpha_s\right) U\left(N, \frac{\boldsymbol{k}^2}{\mu^2}\right)$$

At LLx accuracy, U has a simple form, in terms of small-x resummed anom dim γ

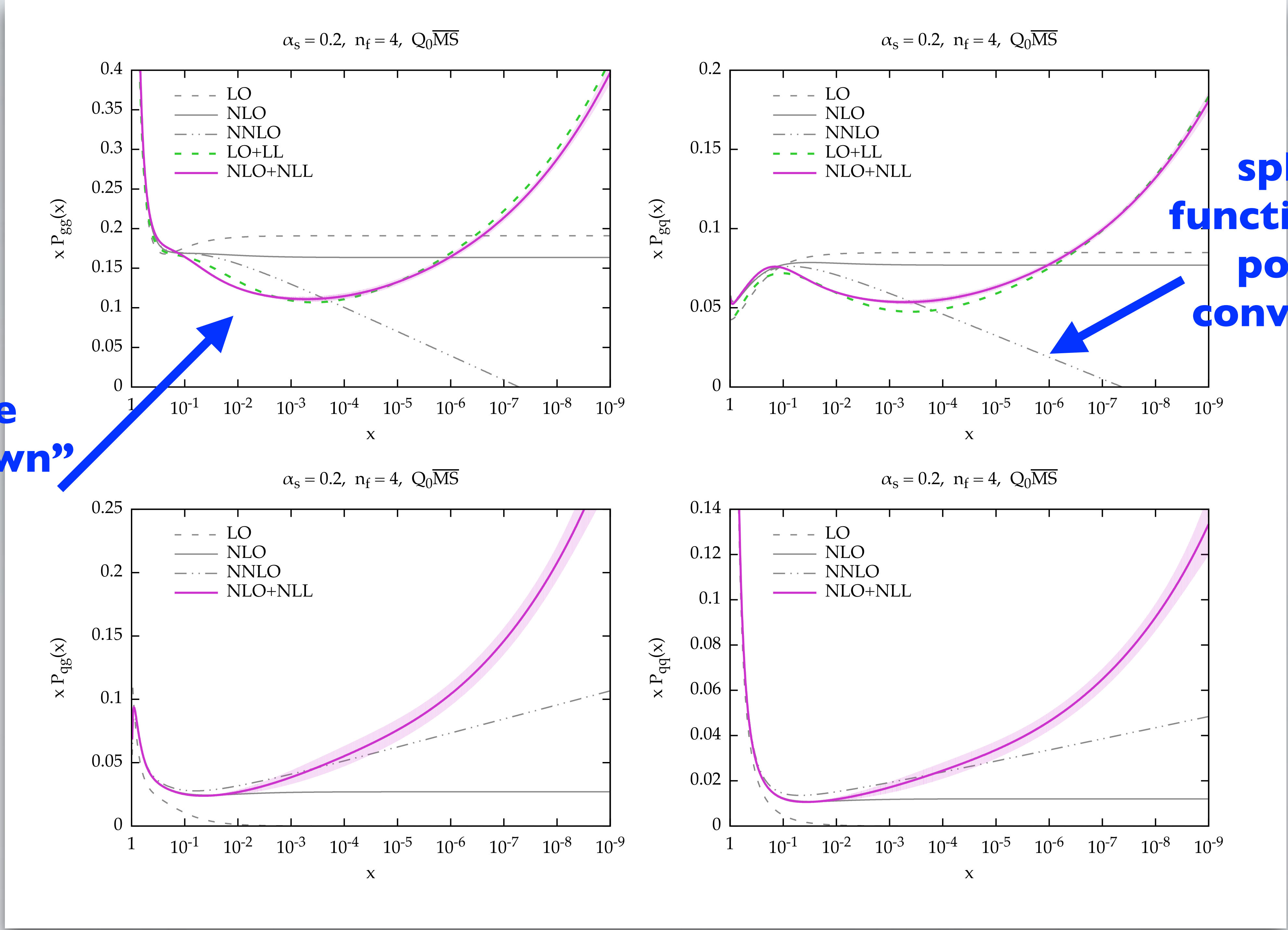
$$U\left(N, \frac{\boldsymbol{k}^2}{\mu^2}\right) \approx \boldsymbol{k}^2 \frac{d}{d\boldsymbol{k}^2} \exp \int_{\mu^2}^{\boldsymbol{k}^2} \frac{d\nu^2}{\nu^2} \gamma(N, \alpha_s(\nu^2))$$

- resummation in momentum space rather than using moments

(as Duff put it:
"The world is upside down")

- processes are easy to add

RESUMMED SPLITTING FUNCTIONS

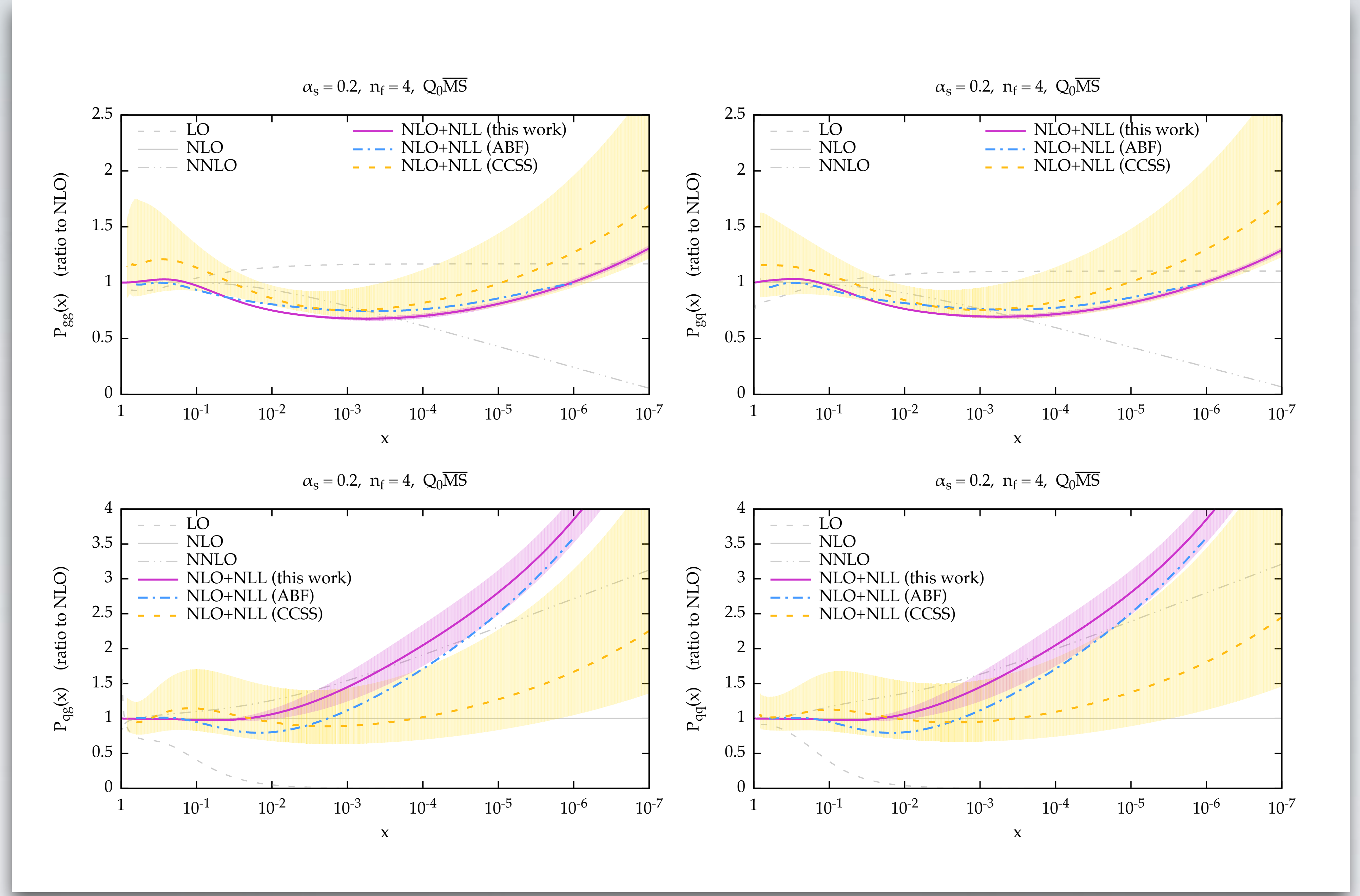


note the "well-known" dip

splitting functions have poor FO convergence

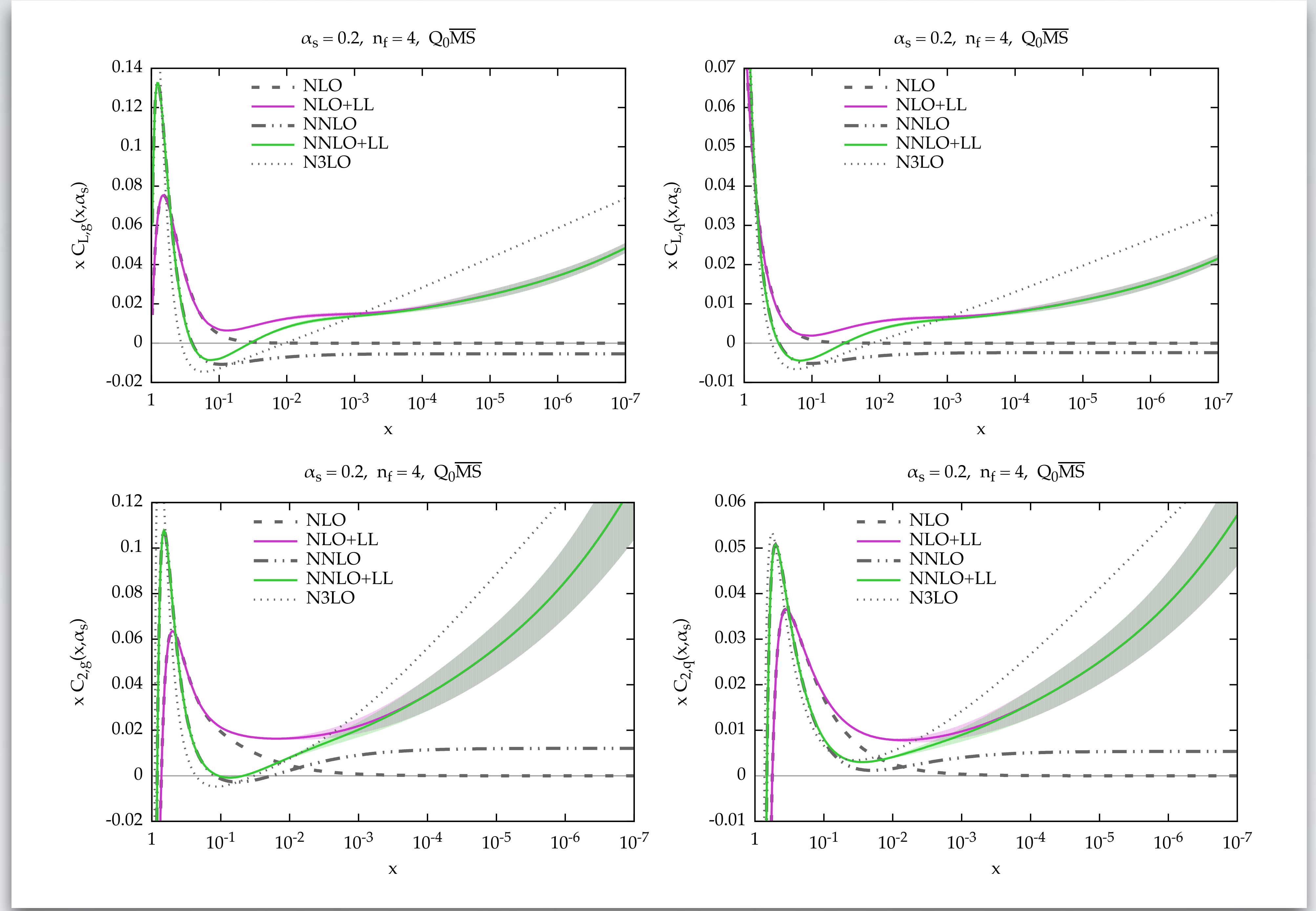
quark splitting functions have larger uncertainty: they start at NLL

COMPARISON TO PREVIOUS APPROACHES



quark splitting functions have larger uncertainty: they start at NLL

RESUMMED DIS COEFFICIENT FUNCTIONS

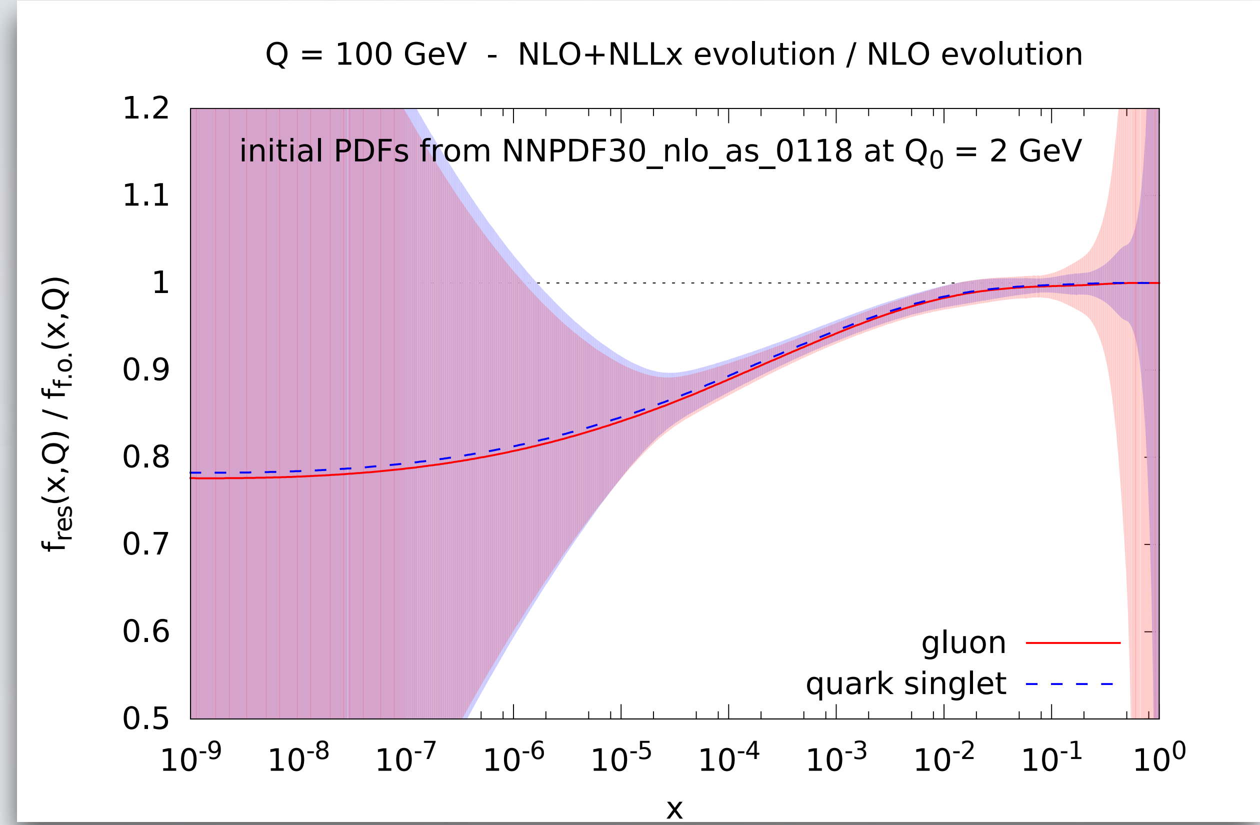


we are now implementing massive DIS and DY

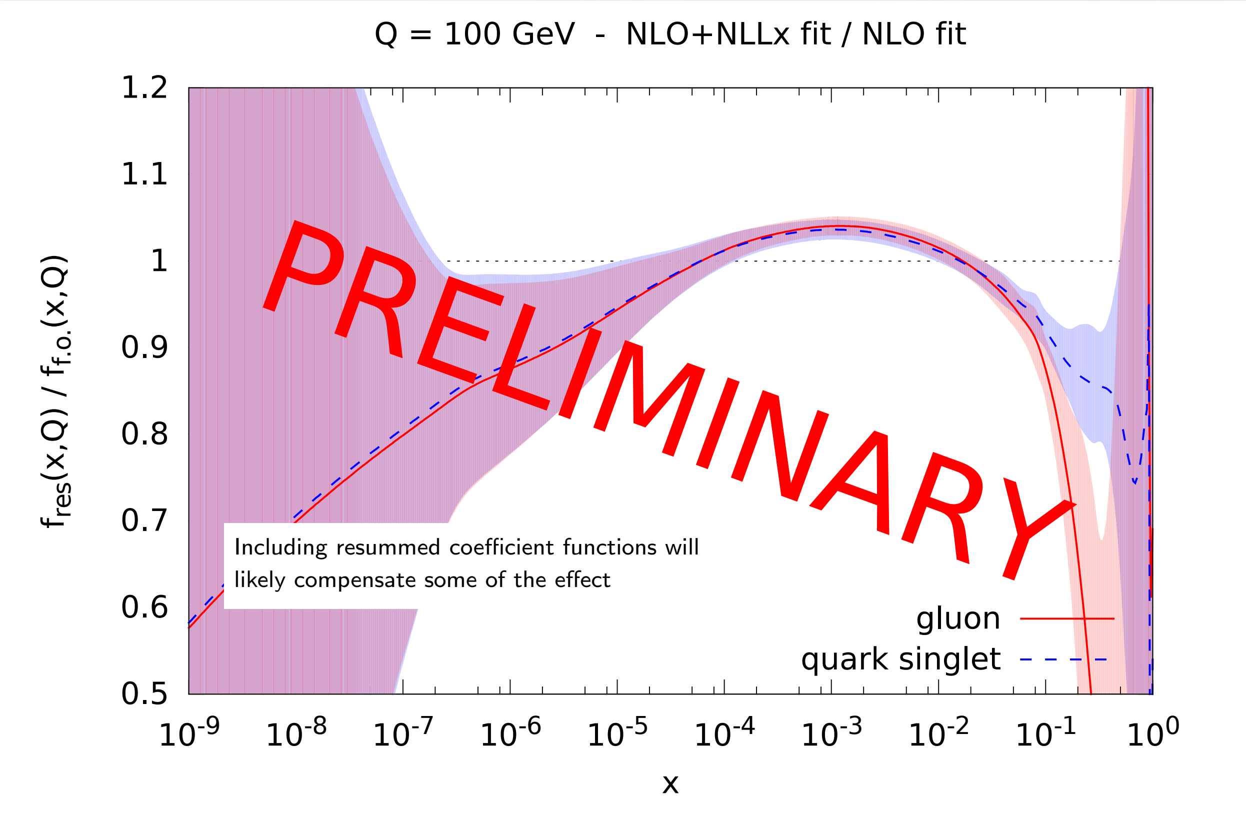
Thorne, White (2006); Ball, SM (2008)

TOWARDS A RESUMMED FIT

PDFs with resummed evolution



re-fitted PDFs
with resummed evolution



**work in progress
with NNPDF**

APFEL A PDF Evolution Library

A WORD ON Q_T RESUMMATION

TRANSVERSE MOMENTUM RESUMMATION

- One of the most studied distribution both in Higgs and DY
- High-accuracy NNLO+NLO calculations exist both in dQCD and SCET
- Codes available such DYQT, DYRES, Resbos, CuTe, *etc.*

e.g. Collins, Soper, Sterman; Catani *et al.;* Becher, Neubert; Neill, Rothstein Vaidya;

- resummation often performed in b-space

$$\delta^{(2)}\left(\sum_{i=1}^{n} \underline{k}_{Ti} + \underline{Q}_T\right) = \frac{1}{(2\pi)^2}\int d^2\underline{b}\, e^{i\underline{b}\cdot\underline{Q}_T}\prod_{i=1}^{n} e^{i\underline{b}\cdot\underline{k}_{Ti}}$$

resummed exponent

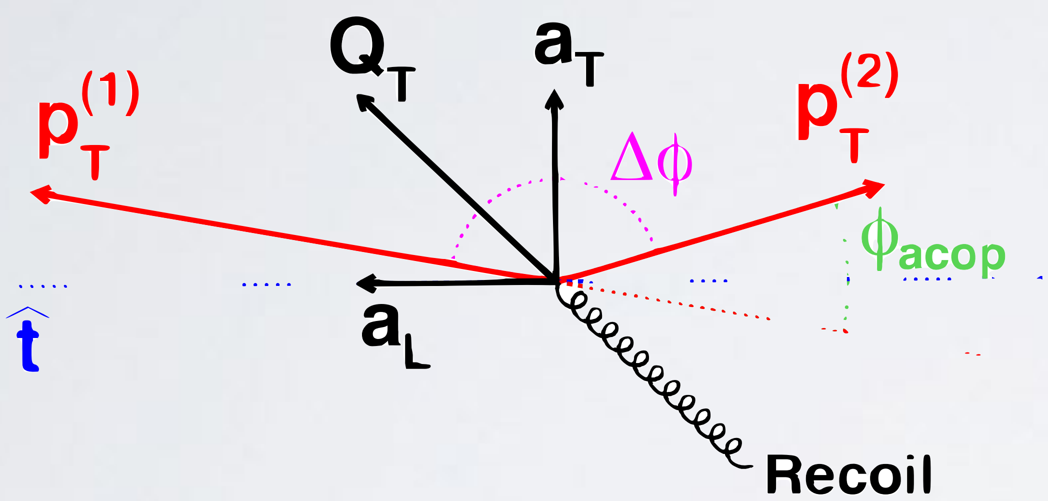
$$\frac{d\sigma}{dQ_T^2} \simeq \int_0^\infty db\, b\, J_0(bQ_T)e^{-R(b)}\Sigma(x_1, x_2, \cos\theta^*, bM)$$

non-log terms and PDFs

- alternative momentum-space method now exists at NNLL Monni, Re, Torielli

RELATED VARIABLES

- Variables introduced by the DØ collaboration for studying the transverse momentum of the Z boson
- Experimental viewpoint: one wants to measure angles rather than momenta



$$\phi^* = \tan\left(\phi_{\text{acop}}/2\right)\sin\theta^*$$

$$\underline{a}_T = \frac{\underline{Q}_T \times (\underline{p_T}^{(1)} - \underline{p_T}^{(2)})}{|\underline{p_T}^{(1)} - \underline{p_T}^{(2)}|}$$

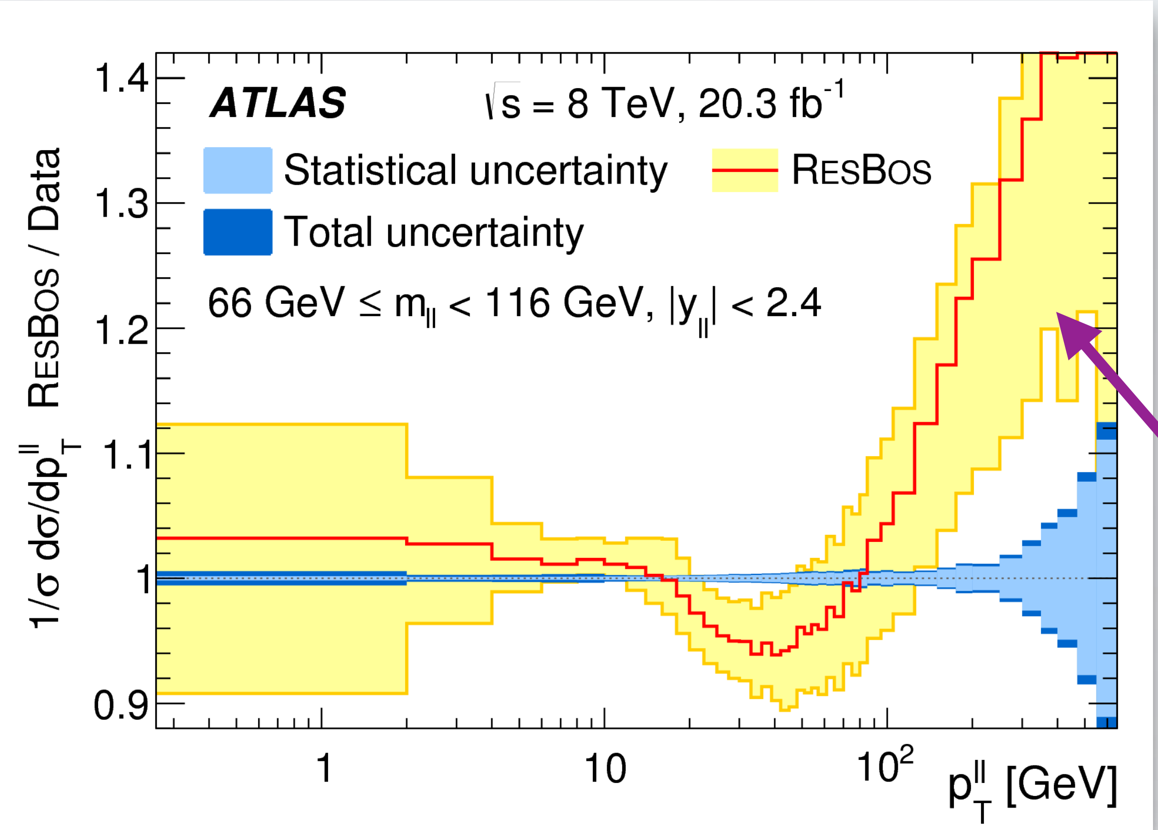
Vesterinen and Wyatt et al. (2008/10)

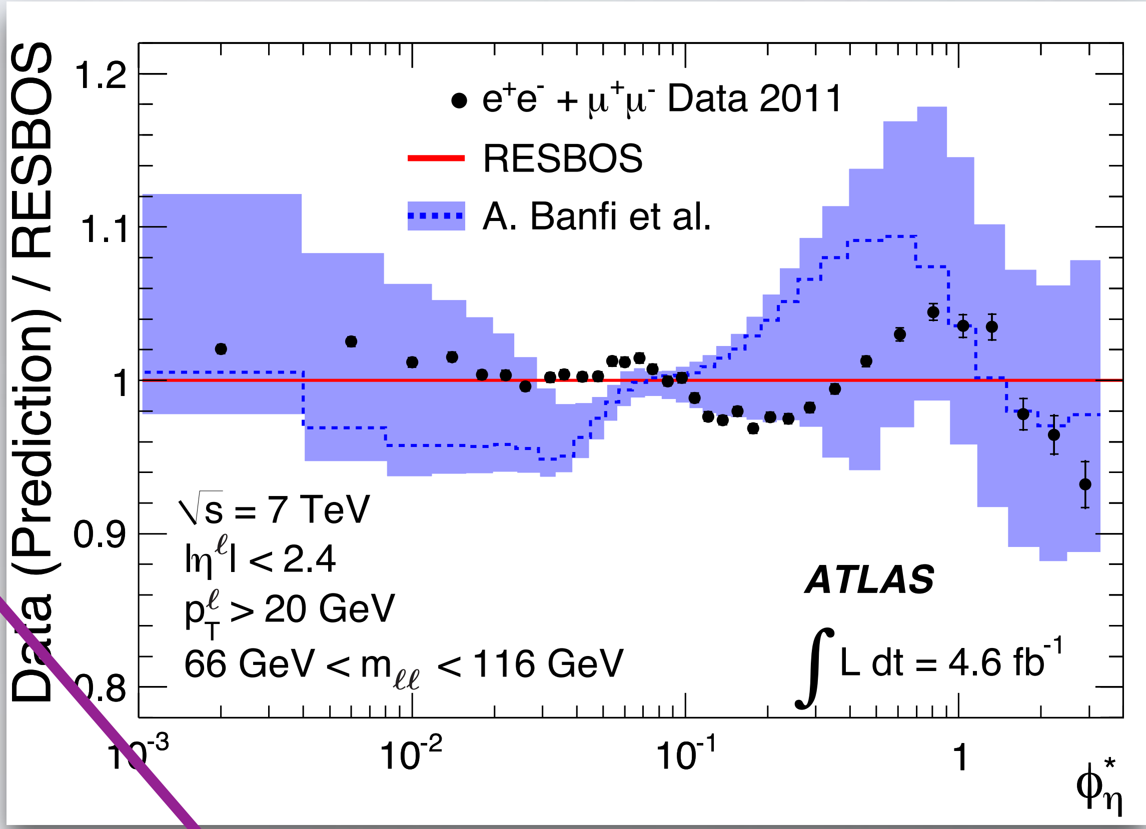
resummation closely related to Q_T

$$\frac{d\sigma}{d\phi^*} = \frac{\pi\alpha^2}{sN_c}\int_0^\infty d(bM)\cos(bM\phi^*)e^{-R(b)} \times\Sigma(x_1, x_2, \cos\theta^*, bM)$$

Banfi, Dasgupta, SM, Tomlinson (2010)
Guzzi, Nadolsky, Wang (2014)

COMPARISON TO DATA: NEED FOR BETTER PRECISION





"wrong" scale??

- experimental uncertainty below 1% !
- NLO+NNLL not adeguate (uncertainty 10%)

discussed here: (N)NLO+ N^{3}LL
(Huaxing's talk)

Boughezal *et al.* (2015), Gehrmann-De Ridder *et al.* (2016), Li, Zhu (2016)

Include these data in PDF fits ?

(Maria's talk)

CONCLUSIONS

- Better determinations of PDFs require both data and theory
- Resummation offers a complementary direction
- Large-x resummed fit already performed with restrict data set
- Small-x resummed DIS fit is work in progress

- Towards global resummed fits:
  - some work to be done for threshold (e.g. jets)
  - more work to be done for small-x
  - inclusion of low Q_T data requires resummation

THANK YOU !