Simplifications with Helicity at Leading and Next-to-Leading Power

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Outline

- Leading Power:
 - Organizing SCET Calculations with Helicity

• Next-to-Leading Power:

- SCET at Subleading Power
- Symmetry Constraints from Helicity Conservation
- Subleading Power Calculations

[Moult, Stewart, Tackmann, Waalewijn] 1508.02397

Leading Power

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[Bauer, Fleming, Pirjol, Stewart]

 SCET describes soft and collinear radiation in the presence of a hard scattering.



• Allows for a factorized description: Hard, Jet, Beam, Soft functions

 $\frac{d\sigma}{d\mathcal{M}_{1}\cdots} = \sum_{\{\kappa\}} \operatorname{tr} H_{\kappa} \mathcal{II} \mathcal{I}_{\mathcal{K}_{i}} \otimes \cdots \otimes \mathcal{I}_{\mathcal{K}_{j}} \mathcal{S}_{\kappa_{s}} \otimes f_{p/i} f_{p/j} \otimes f_{k \to H} \otimes \cdots \otimes f_{l \to H} \otimes F$

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• Hard scattering is described by operators in EFT



 At leading power, operator contains one quark or gluon jet field for each well separated jet direction.

$$\chi_{n,\omega}(x) = \left[\delta(\omega - \overline{\mathcal{P}}_n) W_n^{\dagger}(x) \xi_n(x)\right]$$
$$\mathcal{B}_{n,\omega\perp}^{\mu}(x) = \frac{1}{g} \left[\delta(\omega + \overline{\mathcal{P}}_n) W_n^{\dagger}(x) i D_{n\perp}^{\mu} W_n(x)\right]$$



• Many applications involve higher multiplicity:



Becomes tedious to make bases in terms of standard Lorentz/ Dirac structures:

$$\mathcal{O}^{abc\bar{\alpha}\beta} = \mathcal{B}^{\mu a}_{n_1 \perp} \mathcal{B}^{\nu b}_{n_2 \perp} \mathcal{B}^{\sigma c}_{n_3 \perp} \bar{\chi}^{\bar{\alpha}}_{n_4} \Gamma_{\mu\nu\sigma} \chi^{\beta}_{n_5}$$

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Helicity Amplitudes

- · Well known simplicity in on-shell helicity amplitudes
 - Compact expressions.
 - Manifest symmetry properties.
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- e.g. 4g scattering:

$$\mathcal{A}(1^{-}2^{-}3^{+}4^{+}) = g^{2} \sum_{\sigma \in S_{n}/\mathbb{Z}_{n}} \operatorname{Tr}(T^{a_{\sigma(1)}} \dots T^{a_{\sigma(4)}}) \\ \mathcal{A}(\sigma(1^{-}), \dots, \sigma(4^{+})) = \mathcal{A}(f^{+}) + \mathcal$$

$$A(1^{-}, 2^{-}, 3^{+}, 4^{+}) = \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle}$$

• Can we use similar tricks at the operator level?

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Helicity Operators in SCET

- SCET formulated as expansion about lightlike directions n_i : $\mathcal{B}_{n_i\perp}, \ \chi_{n_i,\omega}$
- Define natural directions to define helicities w.r.t.

• Can define jet fields of definite helicity:

Gluons: $\mathcal{B}_{i\pm}^{a} = -\varepsilon_{\mp\mu}(n_i, \bar{n}_i) \mathcal{B}_{n_i, \omega_i \perp_i}^{a\mu}$

Quark Bilinear :
$$J_{ij+}^{\bar{\alpha}\beta} = \frac{\sqrt{2} \, \varepsilon_{-}^{\mu}(n_i, n_j)}{\sqrt{\omega_i \, \omega_j}} \, \frac{\bar{\chi}_{i+}^{\bar{\alpha}} \, \gamma_{\mu} \chi_{j+}^{\beta}}{\langle n_i n_j \rangle} \,,$$

 $J_{ij-}^{\bar{\alpha}\beta} = - \frac{\sqrt{2} \, \varepsilon_{+}^{\mu}(n_i, n_j)}{\sqrt{\omega_i \, \omega_j}} \, \frac{\bar{\chi}_{i-}^{\bar{\alpha}} \, \gamma_{\mu} \chi_{j-}^{\beta}}{[n_i n_j]}$

An Example

- Example: pp > 3 jets, $q\bar{q}ggg$ channel.
- With standard operators, minimal basis hard to find, hard to match to:

$$\mathcal{O}^{abc\bar{\alpha}\beta} = \mathcal{B}^{\mu a}_{n_1 \perp} \mathcal{B}^{\nu b}_{n_2 \perp} \mathcal{B}^{\sigma c}_{n_3 \perp} \bar{\chi}^{\bar{\alpha}}_{n_4} \Gamma_{\mu\nu\sigma} \chi^{\beta}_{n_5}$$

• Helicity operators trivialize constructing a basis:

$$\begin{split} \mathcal{O}_{+++(\pm)}^{abc\bar{\alpha}\beta} &= \frac{1}{3!} \mathcal{B}_{1+}^{a} \mathcal{B}_{2+}^{a} \mathcal{B}_{3+}^{a} J_{45\pm}^{\bar{\alpha}\beta} \ , \ \mathcal{O}_{++-(\pm)}^{abc\bar{\alpha}\beta} &= \frac{1}{2} \mathcal{B}_{1+}^{a} \mathcal{B}_{2+}^{a} \mathcal{B}_{3-}^{a} J_{45\pm}^{\bar{\alpha}\beta} \\ \mathcal{O}_{--+(\pm)}^{abc\bar{\alpha}\beta} &= \frac{1}{2} \mathcal{B}_{1-}^{a} \mathcal{B}_{2-}^{a} \mathcal{B}_{3+}^{a} J_{45\pm}^{\bar{\alpha}\beta} \ , \ \mathcal{O}_{---(\pm)}^{abc\bar{\alpha}\beta} &= \frac{1}{3!} \mathcal{B}_{1-}^{a} \mathcal{B}_{2-}^{a} \mathcal{B}_{3-}^{a} J_{45\pm}^{\bar{\alpha}\beta} \end{split}$$

• Wilson coefficients, $C_{+\cdots-}^{a_1\cdots\bar{\alpha}\beta}$ found by matching.



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Matching, RG and Symmetries

- Basis has many nice features:
 - Wilson coefficients in EFT are given by finite part of color stripped helicity amplitudes

 \implies Trivial to interface with on-shell helicity amplitudes.

$$C_{+\cdots-}^{a_{1}\cdots\bar{\alpha}\beta}=-\mathrm{i}\mathcal{A}_{\mathrm{fin}+\cdots-}$$

Leading power SCET interactions preserve helicity of each sector
 RG evolution diagonal in helicity space (and no evanescent operators)



• Many symmetries manifest: *C*/*P*, angular momentum conservation, crossing symmetry.

Applications



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[Kolodrubetz, Moult, Stewart] 1601.02607 [Feige, Kolodrubetz, Moult, Stewart] Forthcoming [Moult, Rothen, Stewart, Tackmann, Zhu] Forthcoming

Next-to-Leading Power

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Introduction

 A large class of observables τ (p_T, threshold, event shapes, etc.) exhibit singularities in perturbation theory:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \sum_{m=0}^{2n-1} c_{nm}^{(0)} \left(\frac{\log^m \tau}{\tau}\right)_+ \\ + \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \sum_{m=0}^{2n-1} c_{nm}^{(2)} \log^m \tau \\ + \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \sum_{m=0}^{2n-1} c_{nm}^{(4)} \tau \log^m \tau \\ + \cdots$$

- Expansion about singular (soft/collinear) limits
 - Simplifies structure/ calculation
 - Often gives phenomenologically large contributions
 - Reveals universal structures in gauge theories.

Leading Power

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• Observables can be organized in an expansion in τ .

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = \frac{\mathrm{d}\sigma^{(0)}}{\mathrm{d}\tau} + \frac{\mathrm{d}\sigma^{(2)}}{\mathrm{d}\tau} + \frac{\mathrm{d}\sigma^{(4)}}{\mathrm{d}\tau} + \cdots$$

Leading power well understood for a wide variety of observables.



Subleading Power

- Subleading powers much less well understood.
- Are there factorization theorems at each power?

$$\frac{\mathrm{d}\sigma^{(n)}}{\mathrm{d}\tau} = \sum_{j} H_{j}^{(n_{Hj})} \otimes J_{j}^{(n_{Jj})} \otimes S_{j}^{(n_{Sj})}$$

- What is the degree of universality?
- Start by looking at Next-to-Leading Power (NLP):

$$\frac{\mathrm{d}\sigma^{(2)}}{\mathrm{d}\tau} = \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \sum_{m=0}^{2n-1} c_{nm}^{(2)} \log^m \tau$$

Soft Collinear Effective Theory

- SCET has proven to be a powerful framework for studying factorization.
- Allows a systematic expansion about soft p_s ~ (λ², λ², λ²) and collinear p_c ~ (1, λ, λ²) limits in a power counting parameter λ ~ √τ.
- In Collider context has been applied primarily at leading power.
- Fields/Lagrangians have a definite power counting in λ .

Operator	$\mathcal{B}^{\mu}_{n_i\perp}$	χ_{n_i}	$\mathcal{P}^{\mu}_{\perp}$	<i>q_{us}</i>	D^{μ}_{us}
Power Counting	$\dot{\lambda}$	λ	λ	λ^3	λ^2

$$\mathcal{L}_{\mathsf{SCET}} = \mathcal{L}_{\mathrm{hard}} + \mathcal{L}_{\mathrm{dyn}} = \sum_{i \ge 0} \mathcal{L}_{\mathrm{hard}}^{(i)} + \sum_{i \ge 0} \mathcal{L}^{(i)}$$

Leading Power SCET

- Leading Power SCET:
 - Leading Power Hard Scattering Operators: $\bar{\chi}_n \Gamma \chi_{\bar{n}}$
 - Leading power Feynman rules (eikonal/ collinear)
 - · Measurement function/ kinematics expanded to leading power



Subleading Power SCET

- Subleading Power in SCET:
 - Subleading Lagrangian Insertions



• Corrects dynamics of propagating soft/collinear particles.

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Subleading Power SCET

- Subleading Power in SCET:
 - Subleading Lagrangian Insertions
 - Subleading Hard Scattering Operators



• Local Corrections to scattering vertex. Multiple fields per collinear sector.

Subleading Power SCET

- Subleading Power in SCET:
 - Subleading Lagrangian Insertions
 - Subleading Hard Scattering Operators
 - Subleading Measurement function



• Corrects definition of measurement: $\tau = \tau^{(0)} + \tau^{(1)} + \tau^{(2)}$

Subleading Lagrangian

- Subleading Lagrangians are universal, and known.
- Correct the dynamics of soft and collinear particles. e.g.
 - Correction to eikonal emission:

• Emission of soft quarks:

$$\mathcal{L}_{\chi_n q_{us}}^{(1)} = \bar{\chi}_n \frac{1}{\bar{\mathcal{P}}} g \mathcal{B}_{n\perp} q_{us} + \text{h.c.}$$



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Subleading Hard Scattering Operators

- Hard Scattering Operators describe process dependence.
- Obtained by matching calculation.
- Need a complete basis of operators consistent with symmetries.
- Consider $e^+e^-
 ightarrow$ dijets:



Subleading Hard Scattering Operators

- Want basis to all orders in α_s . Finite by power counting.
- Multiple collinear fields per sector.



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Organize Using Helicities

- Operators, and interferences easily organized in terms of helicities.
- In SCET, all fields expanded about a common axis.
- Interesting constraints at subleading powers.
- Multiple collinear fields per sector:



• Wilson coefficient vanishes by basic QM.

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• Spinor helicity encodes spin projections (Wigner-D functions).

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- Spinor helicity encodes spin projections (Wigner-D functions).
- Also strong constraints for $gg \rightarrow H$

Relevant Hard Scattering Operators

For leading log (singularity), αⁿ_s log²ⁿ⁻¹(τ), two relevant hard scattering operators:



• $q\bar{q}$ in same sector has no LP analog.

Matching

- Wilson coefficients, $C(\omega_1, \omega_2)$ can be obtained by matching.
- $C(\omega_1, \omega_2)$ depends on large momentum fraction, i.e. z, 1-z.



Fixed Order Thrust at NLP

- Begin by studying structure in fixed order pert theory.
- Most interested in the leading log: $\alpha_s \log(\tau)$, $\alpha_s^2 \log^3(\tau)$, ...
- Simple playground is Thrust in e^+e^-



Exact NLO result known.

NLO Thrust at NLP

• Calculate one-loop graphs with NLP operators/ Lagrangians.

Collinear Gluon

Soft Gluon



NLO Thrust at NLP

• Calculate one-loop graphs with NLP operators/ Lagrangians.

Collinear Quarks Soft Quark $\frac{1}{\sigma_0} \frac{d\sigma_1^{(2)}}{d\tau} = -4C_F \left[\frac{1}{\epsilon} + \log\left(\frac{\mu^2}{Q^2 \tau}\right) \right] + 4C_F \left[\frac{1}{\epsilon} + \log\left(\frac{\mu^2}{Q^2 \tau^2}\right) \right]$ $= -4C_F \log(\tau)$

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NLO Thrust at NLP

• Sum four graphs, reproduces the NLP piece of well known NLO thrust result:



$$\frac{1}{\sigma_0} \frac{d\sigma_1}{d\tau} = 4C_F \log(\tau)$$

• Result gives directly (no expansions) the NLP contribution.

NNLO Thrust at NLP

- Proceed to NNLO.
- Dress NLP emission with leading power emission (collinear, soft, hard).



NNLO Thrust at NLP

- Interesting color structures for leading log (divergence).
- At LP, leading logarithmic divergence ~ C_F^n.
- More interesting structure at NLP.



Applications

- Goal: Understand all orders structure of NLP logs. Derive RG, etc.
- Fixed order is first step in understanding this.
- Already at fixed order NLP logs have interesting applications.

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Application to N-jettiness Subtractions

- NNLO calculations require cancellation of real/virtual poles.
- Use a physical resolution variable to slice phase space.
- Recently a general method allowing for jets in final state, based on N-jettiness (see also Xiaohui Liu's talk)

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[Boughezal, Focke, Petriello, Liu]
[Gaunt, Stahlhofen, Tackmann, Walsh]
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$$\sigma(X) = \int_{0}^{\infty} d\mathcal{T}_{N} \frac{d\sigma(X)}{d\mathcal{T}_{N}} = \int_{0}^{\mathcal{T}_{N}^{\text{cut}}} d\mathcal{T}_{N} \frac{d\sigma(X)}{d\mathcal{T}_{N}} + \int_{\mathcal{T}_{N}^{\text{cut}}}^{\infty} d\mathcal{T}_{N} \frac{d\sigma(X)}{d\mathcal{T}_{N}}$$

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N-Jettiness

• *N*-jettiness: Inclusive event shape to identify N - 2 jets.



$$\tau_N = \frac{2}{Q^2} \sum_k \min \left\{ q_a \cdot p_k, q_b \cdot p_k, q_1 \cdot p_k, \cdots, q_N \cdot p_k \right\}$$

- $\tau_N \ll 1 \implies N-2$ isolated jets.
- All orders factorization theorem:

$$\frac{d\sigma}{d\tau_N} = HB_a \otimes B_b \otimes \frac{S}{S} \otimes J_1 \otimes \cdots \otimes J_{N-1} + \mathcal{O}(\tau_N)$$

N-jettiness Subtractions

$$\sigma(X) = \int_{0}^{} d\mathcal{T}_{N} \frac{d\sigma(X)}{d\mathcal{T}_{N}} = \int_{0}^{\mathcal{T}_{N}^{\text{cut}}} d\mathcal{T}_{N} \frac{d\sigma(X)}{d\mathcal{T}_{N}} + \int_{\mathcal{T}_{N}^{\text{cut}}}^{} d\mathcal{T}_{N} \frac{d\sigma(X)}{d\mathcal{T}_{N}}$$

$$\int_{0}^{\mathcal{T}_{N}^{\text{cut}}} d\mathcal{T}_{N} \frac{d\sigma(X)}{d\mathcal{T}_{N}}$$

Compute using factorization in soft/collinear limits:

$$\frac{d\sigma}{d\tau_N} = HB_a \otimes B_b \otimes S \otimes J_1 \otimes \cdots \otimes J_{N-1}$$
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$$\int\limits_{\mathcal{T}_N^{\rm cut}} d\mathcal{T}_N \frac{d\sigma(X)}{d\mathcal{T}_N}$$

Additional jet resolved. Use NLO subtractions.

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New NNLO Results with *N*-jettiness

[Boughezal, Focke, Giele, Petriello, Liu]

• Impressive new results with jets in the final state: W/Z/H+jet at NNLO



- Implemented in MCFM for color singlet production at NNLO.
- Conceptually simple, extendable to higher orders.

Power Corrections

- Current subtractions use leading power result in singular region.
- Power corrections are dropped \implies small values of $\mathcal{T}_N^{\text{cut}}$ necessary.



 Use of a physical resolution variable analytically tractable.
 [Gaunt, Stahlhofen, Tackmann, Walsh], see also Xiaohui's Talk [Bourhezal, Petriello, Liu, et al.]

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NLO Beam Thrust at NLP

- Start with simplest case: 0-jettiness (beam thrust)
- All hard work in setup done for thrust: operators, Lagrangian, identical.
- Simply cross results, and take into account pdfs, measurement...

 $q\bar{q}$ channel

qg channel



NLO Beam Thrust at NLP



NNLO 0-Jettiness at NLP

- NNLO result for both $q\bar{q}$ and qg also obtained by crossing, taking into account pdf, and measurement.
- Find good agreement with NNLO fixed order (MCFM):



Conclusions

 Helicity can be used to organize operators in SCET at both LP and NLP.

LP: Facilitates matching to fixed order calculations.
 ⇒ Applications to processes with more legs.



- NLP: Simplifies organization of operator bases.
 - \implies Fixed order power corrections for NNLO subtractions.

Thanks!

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