The Analytic Structure of Non-global Logs

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Motivation

"Non-global logarithms are just a nuisance to obtaining reliable predictions for experimentalists and their favored observables."
–Strawman

This is true, and could not be more wrong.

- Manifestation of the color entanglement of QCD final states.
- Remarkable duality to saturation physics from conformal symmetry.
- Jets in QCD are an emergent phenomona, **requiring** an all-orders resummation of the perturbation theory.

Outline

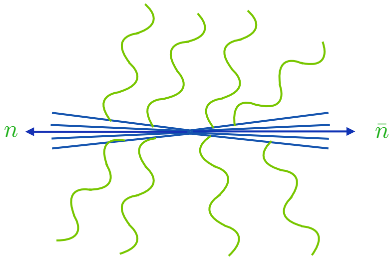
- What is an NGL?
- Banfi-Marchesini-Smye Equation.
- Expansion in resummed jets for BMS equation.
- The Buffer Region and breakdown of fixed order expansion.
- Resurrection of the fixed order expansion using conformal mappings.
- Conclusions.

Global Logarithms

Global logarithms: constraining all of phase space with a single measurement.

- Paradigmatic example: $e^+e^- \to hadrons$, measure thrust or N-jettiness.

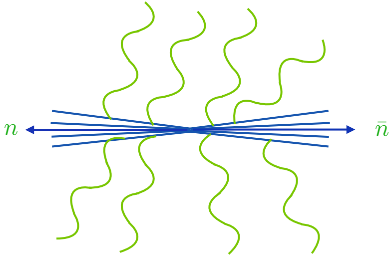
$$\tau \ll 1$$



Global Logarithms

- Momentum flow aligns with jet axis.
- One vetos the production of additional jets anywhere in phase space.

$$\tau \ll 1$$

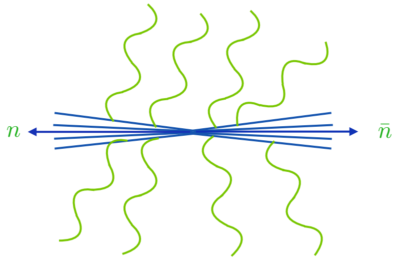


Global Logarithms

Factorize cross-section into distinct objects:

$$\frac{d\sigma}{d\tau} = H\Big(Q,\mu\Big) J_n\Big(\tau,\mu\Big) \otimes J_{\bar{n}}\Big(\tau,\mu\Big) \otimes S_{n\bar{n}}\Big(\tau,\mu\Big) + \ldots$$

$$\tau \ll 1$$



- How? Soft-Collinear Effective Field Theory: final state degrees of freedom precisely known.

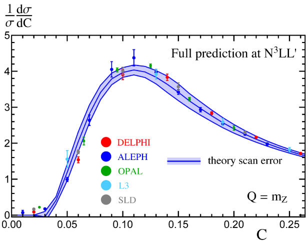
Global Logarithms

- Nothing happens (no on-shell final states emitted) between $q\bar{q}$ production at Q and thrust value $Q\tau$.

$\alpha_s \ln\tau$ resummed thru RG evolution:

$$\mu \frac{d}{d\mu} F(\mu) = \gamma(\mu) \otimes F(\mu)$$

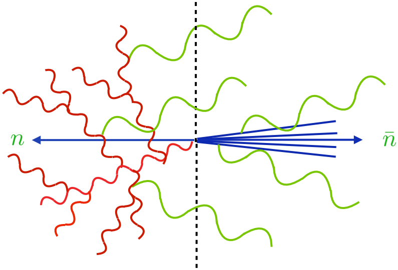
γ and F computable to high orders in fixed order perturbation theory.



Non-Global Logarithms

However: measure the invariant mass of each jet independently:

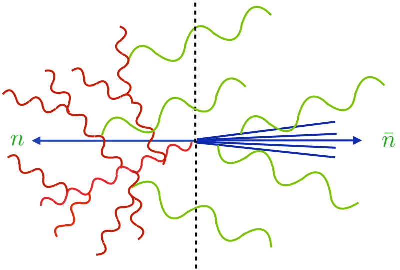
$$m_H \gg m_L$$



Between the scale m_H and m_L we can have active jet production. [Dasgupta, Salam]

- Distribution in m_L entangled with the "on-shell" branching history below m_H.
- NGL's sensitive to an *infinite* number of distinct degrees of freedom!(?)

$$m_H \gg m_L$$



- Simulate arbitrarily complicated secondary branching.
- Linear evolution equations no longer sufficient [BMS, Wiegert]

BMS equation at large N_c

Soft function expressable in terms of color dipole functions:

Non-global Log:

$$L = \frac{C_A}{\pi} \int_{m_L}^{m_H} \frac{d\mu}{\mu} \alpha_s(\mu)$$

$$S_{ab}(m_H, m_L; \mu) \sim V_{ab}(\mu, m_H, m_L) g_{ab}(L)$$

- S_{ab}: soft function appearing in dijet factorization with eikonal lines $a = (1, \hat{n}_a)$ and $b = (1, \hat{n}_b)$.
- g_{ab}: color dipole function with **global** or **Sudakov** evolution factored out.

BMS equation at large N_c

Evolution of color dipole functions:

Non-global Log:

$$L = \frac{C_A}{\pi}\int_{m_L}^{m_H}\frac{d\mu}{\mu}\alpha_s(\mu)$$

Evolution of Color Dipole Soft Function:

$$\partial_L g_{ab} = \int_J \frac{d\Omega_j}{4\pi} W_{ab}(j)\Big(U_{abj}(L) g_{aj} g_{jb} - g_{ab}\Big)$$

Within the active jet region J:

- color dipole g_{ab} with eikonal lines $a = (1, \hat{n}_a)$, $b = (1, \hat{n}_b)$.
- Decays to two new dipoles g_{aj}, g_{jb} with new eikonal line $j = (1, \hat{n}_j)$.
- Full color evolution $\rightarrow$ reduced density matrix [Wiegert, Caron-Huot, Nagy-Soper, Hatta et al.].
- EFT interpretation and small R[Becher et. al.]

BMS equation at large N_c

Evolution of Color Dipoles:

$$\partial_L g_{ab} = \int_J \frac{d\Omega_j}{4\pi} W_{ab}(j) \left(U_{abj}(L) g_{aj} g_{jb} - g_{ab} \right)$$

$$W_{ab}(j) = \frac{a \cdot b}{a \cdot j \; j \cdot b}$$

$$U_{abj}(L) = \exp\left(L \int_{S^2/J} \frac{d\Omega_q}{4\pi} W_{aj}(q) + W_{jb}(q) - W_{ab}(q) \right)$$

- Evolution equation can be derived from factorization theorems for **soft jet production**. [Larkoski, DN, Moult; DN]
- U_{abj} resummation dictated by RG structure for these factorization theorems.

Soft Jet Production

To make a soft jet:

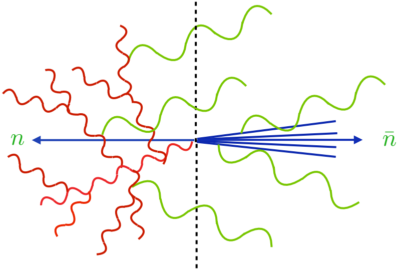
$$\frac{d\sigma}{dz\, d\theta de_{res} dm_L} = H(Q) H_{ab}\Big(z, n_{sj}\Big) J_a(e_{res}) \otimes J_{n_{sj}}(e_{res}) \otimes S_{abn_{sj}}(e_{res}; m_L) \otimes S_{n_{sj}\bar{n}_{sj}}\Big(e_{res}; R\Big) \otimes J_b(m_L)$$

- Fact. Th. *determines* real part of BMS kernel.
- KLN/zero-bins gives virtual.
- RG structure $\rightarrow U_{abj}$ resummation.
- Pheno. applications to jet substructure. [Larkoski, DN, Moult]

Infinite Degrees of Freedom and Existential Despair

- In principle, do we need to keep an infinite number of emissions?
- Factorization requires infinite number of soft operators [Becher et. al.].
- Dual to Balitsky's theory for forward scattering.
- Additional emissions *are not* α_s suppressed! (logarithmically enhanced)

$$m_H \gg m_L$$



Infinite Degrees of Freedom and Existential Despair

One does not need infinite number of emissions.

Order NGL cross section as an expansion in soft jets:

$$\frac{d\sigma}{dm_H dm_L} = d\sigma_{0-sj} + \int_1 d\sigma_{1-sj} + \int_2 d\sigma_{2-sj} + \ldots$$

Physical intuition:

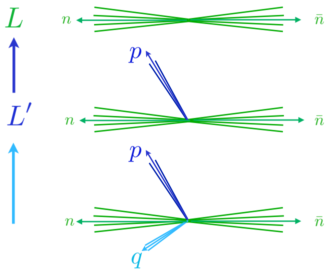
Add enough soft jets$\rightarrow$

no hierarchy between the D.O.F. [Larkoski,Moult,DN]

- Truncate the sum!
- Rapidity gaps, NGLs, and Glaubers: [Forshaw et. al.]. Talk to Simone.

Partons Versus Jets

$$\frac{d\sigma}{dm_H dm_L} = d\sigma_{0-sj} + \int_1 d\sigma_{1-sj} + \int_2 d\sigma_{2-sj} + \ldots$$



- Each term has a well-defined “subjet” factorization theorem.
- Nontrivial resummation of the virtual corrections associated with the additional jets.
- Defines an expansion in “jet” states, not partons.

The Dressed Gluon Expansion

Order NGL cross section as an expansion in soft jets:

$$\frac{d\sigma}{dm_H dm_L} = d\sigma_{0-sj} + \int_1 d\sigma_{1-sj} + \int_2 d\sigma_{2-sj} + \ldots$$

Formally, iterative solution to the BMS equation:

$$\partial_L g_{ab} = \int_J \frac{d\Omega_j}{4\pi} W_{ab}(j) \left(U_{abj}(L) g_{aj} g_{jb} - g_{ab} \right)$$

$$g_{ab} = 1 + \Phi_{ab}^{(1)} + \Phi_{ab}^{(2)} + \ldots$$

This is a uniformly convergent expansion $\forall L$.

The Dressed Gluon Expansion Converges $\forall L$

Formally, iterative solution to the BMS equation:

$$\partial_L g_{ab} = \int_J \frac{d\Omega_j}{4\pi} W_{ab}(j) \left(U_{abj}(L) g_{aj} g_{jb} - g_{ab} \right)$$

$$g_{ab} = 1 + \Phi_{ab}^{(1)} + \Phi_{ab}^{(2)} + \ldots$$

- Proof: logic follows Picard iteration proofs in Diff Eq. textbooks. [Larkoski,Moult,DN]
- Technical difficulties: Collinear sing. and *BFKL* physics(!) [DN]
- Formulate metric on function space that folds in collinear singularities.

But Why Dressed Gluon Expansion?

Why not use fixed order expansion?

$$g_{ab} = 1 + a_2 L^2 + a_3 L^3 + \ldots$$

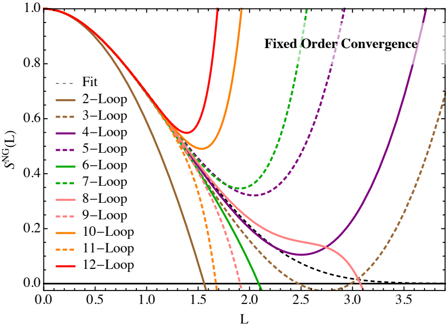
- Also truncates D.o.F. using free states.
- Beautiful illustration of modern Loop technology. [Schwartz, Zhu; Caron-Huot]

Finite Radius of Convergence

Fixed Order Expansion limited.

$$g_{ab} = 1 + a_2 L^2 + a_3 L^3 + \ldots$$

Back-to-Back hemisphere jets, compare to Dasgupta, Salam MC:



12 loop result: [Caron-Huot]

But Why Finite Radius of Convergence?

This is in contrast to Global Sudakov Logs:

$$\mu \frac{d}{d\mu} F(\mu) = \gamma(\mu) F(\mu)$$

$$F(\mu) = F(\mu_0) \exp\Big(\int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \gamma(\mu') \Big)$$

- The *exp* has an infinite radius of convergence.
- Fixed order series reproduces taylor series.

Resummation is expedient and practical,
but *formally* not necessary.

But Why Finite Radius of Convergence?

Radius of convergence → singularities in complex L-plane.

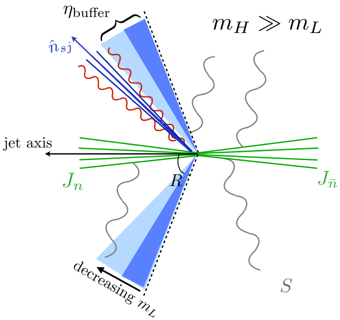
- Dressed Gluon Expansion converges $\forall$ L.
- *Necessarily* reproduces singularities of NGL distribution.

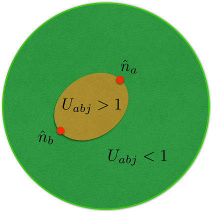
One Dressed Gluon:

$$g_{n\bar{n}} = 1 - \frac{1}{2}\Big(\gamma_E + \ln\Gamma(1+L)\Big) + \ldots$$

Singularities at $L = -1, -2, -3, \ldots$
Why such singularities?

The Buffer Region and Phenomonolgy of Soft jets





- Boundary Soft RG implies:
  $$U_{abj} \propto \left(1 - \frac{\tan^2 \frac{\theta_{sj}}{2}}{\tan^2 \frac{R}{2}}\right)^L$$
- Cross-section for production of a jet at the boundary vanishes!
- "Buffer Region" noticed in Monte Carlo by [Dasgupta,Salam].
- Boundary of buffer $\leftrightarrow$ saturation region.

One-Dressed Gluon

$$U_{abj} \propto \left(1 - \frac{\tan^2 \frac{\theta_{sj}}{2}}{\tan^2 \frac{R}{2}}\right)^L$$

$$\begin{aligned} \partial_L \Phi_{ab}^{(1)} &= \int_J \frac{d\Omega_j}{4\pi} W_{ab}(j) \Big(U_{abj} - 1\Big) \\ &\sim c \int^R d\theta \left(1 - \frac{\theta^2}{R^2}\right)^L + O\Big((1+L)^0\Big) \\ &\sim \frac{c}{1+L} + \ldots \end{aligned}$$

- $L < 0$ buffer region inverts!
- $L = -1 \rightarrow$ x-sec for edge-of-jet prod. unbounded!
- Generic (n): $\lim_{L \to -1} \Phi_{ab}^{(n)}(L) \sim \ln^{2^n}(1+L) + \ldots$

Beyond Leading Logs

At subleading orders in NGLs, F.O.P.T experiences stronger buffer regions effects.

$$U_{abj} \propto \left(1 - \frac{\tan^2 \frac{\theta_{sj}}{2}}{\tan^2 \frac{R}{2}}\right)^{L(\theta)} \leftarrow \text{depends on distance to edge of jet!}$$

$$L(\theta) \sim \frac{C_A}{\pi} \int_{m_L \Delta\theta}^{m_H \Delta\theta} \frac{d\mu'}{\mu'} \alpha_s(\mu')$$

$$\Delta\theta = 1 - \frac{\tan^2 \frac{\theta_{sj}}{2}}{\tan^2 \frac{R}{2}}$$

- Resummation of collinear splittings at jet boundary.
- N^kLL NGL F.O.P.T. behaves as $k!\alpha_s^k \ln\frac{m_H}{m_L}$
- Large Logs in phase space Integrate to large Constants.

NGLs and Resummation

For non-global logs, resummation is **necessary** since the resummed cross-section for soft jets reveals emergent behavior not captured at any order in perturbation theory.

- Jet states vanish at the boundary.
- Partonic states do not.
- Calculations with jet states reproduce singularities of the distribution.
- Partonic states do not.

Fixed Order Perturbation Theory is Dead!

Long live Fixed Order Perturbation Theory!
To revive F.O.P.T, we can simply use knowledge of singularites revealed by resummation.

- Construct conformal map where singularities are pushed to boundary.
- $g_{ab}(L) \to g_{ab}(u)$.
- $g_{ab}(u) = 1 + b_1 u + b_2 u^2 + \ldots$.

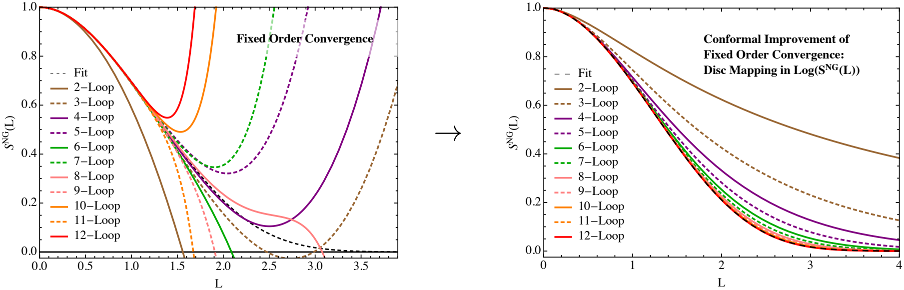
Example u's:

$$u(L) = \begin{cases} \ln(1+L), & \text{log mapping} \\ \frac{\sqrt{1+L}-1}{\sqrt{1+L}+1}, & \text{disc mapping} \end{cases}$$

Determine b's by matching taylor series at $L = 0$.

Fixed Order Perturbation Theory is Dead!

Long live Fixed Order Perturbation Theory!



Nota Bene: Conformal improvement done on $\ln g_{ab}$ not g_{ab}.
Large L behavior plays role here.

Conclusions

- Jets $\neq$ partons. They are emergent in a precisely definable sense.
- BMS equation was derived from considering strongly ordered fixed order P.T., but implies more than the simple sum of the series.
- To reproduce **all** of ***perturbative*** QCD phenomonology, resummation/Monte Carlo matched to fixed-order is *formally* necessary.
- Effective truncations exist using finite number of operators, beyond α_s or power suppression.